Chapter 4

Empirical Study

4.1 Bond Pricing

This section takes the bond-pricing formula shown in equation (4.1) to examine the proposed method. In equation (4.1), P_t is the bond price at time t; r_t is the market rate of interest at time t; F is the face value, which generally equals 100; T_0 is the term to maturity at time $t = 0$; C is the coupon payment, which equals $F\ r_c$; $\lfloor \bullet \rfloor$ rounds real numeric to the nearest integer towards minus infinity. Equation (4.1) is a nonlinear closed-form equation, which owns clear and definite characteristics for it.

$$P_t \equiv \sum_{k=\lfloor t+1 \rfloor}^{\lfloor T_0 \rfloor} \frac{C}{(1+r_t)^{k-t}} + \frac{F}{(1+r_t)^{T_0-t}} \tag{4.1}$$

As stated in (Sharpe and Alexander, 1990), Bond-pricing theorems characterize how bond prices move in response to changes in their yields-to-maturity: Periodic payment of fixed dollar amount and a lump sum payment at a stated date. The periodic payments are known as coupons, and the lump sum payment is known as the bond's principal (or par value or face value). A bond's coupon rate is calculated by dividing the dollar amount of the coupon payments a bondholder would receive over the course of a year by the principal. Lastly, the amount of time which is left until the last promised payment is made is known as the bond's term-to-maturity, whereas the discount rate which makes the present value of the all cash flows equal to the market price of the bond is known as the bond's yield-to-maturity (or, simply, yield).

Note that if a bond's market price equals to its par value, then its yield-to-maturity will equal to its coupon rate. However, if the market price is less than the par value (a situation where the bond is said to be selling at a discount), then the bond yield-to-maturity will be greater than the coupon rate. Conversely, if the bond price is greater than the par value (a situation where the bond is said to be selling a premium), then the yield-to-maturity will be less than the coupon rate.

Five theorems dealing with the bond-pricing have been derived (cf. Malkiel, 1962). Assume that there is one coupon payment per year (that is, coupon payments are made every 12 months). The theorems are as follows:

1. If a bond's market price increases, then its yield must decrease; conversely, if a bond's market price decreases, then its yield must increase. That is,

$$\frac{\partial P_t}{\partial r_t} < 0 \tag{4.2}$$

2. If a bond's yield does not change over its life, then the size of its discount or premium will decrease as its life gets shorter. That is,

$$\frac{\partial^2 P_t}{\partial T_t \partial r_t} < 0 \tag{4.3}$$

   where $T_t \equiv T_0 - t$ is the term to maturity at time t.

3. If a bond's yield does not change over its life, then the size of its discount or premium will decrease at an increasing rate as its life get shorter. That is,

$$\begin{cases} \dfrac{\partial^3 P_t}{\partial T_t^{\,2}} < 0 & \text{if } r_c > r_t \\ \dfrac{\partial^3 P_t}{\partial T_t^{\,2}} > 0 & \text{if } r_c < r_t \end{cases} \tag{4.4}$$

4. A decrease in a bond's yield will raise the bond's price by an amount which is greater in size than the corresponding fall in the bond's price which would occur if there is an equal-sized increase in the bond's yield. That is,

$$\frac{\partial^2 P_t}{\partial r_t^{\,2}} > 0 \tag{4.5}$$

5. The amount change in a bond's price due to a change in its yield will be higher if its coupon rate is higher. (Note: This theorem does not apply to bonds with a life of one year or to bonds that have no maturity date, known as consols, or perpetuities.)

That is,

$$\frac{\partial^2 P_t}{\partial r_c \partial r_t} < 0 \tag{4.6}$$

4.2 Data Collection and Method Application

We assume that there are 80 trading days and that coupon payments are made yearly, and derive r_t from a normal random number generator of $N(2\%, (0.1\%)^2)$,. Then we select 6 kinds of short-term bonds as shown in Table 4.1, and use the equation (4.1) for the bond pricing to generate the data with t = 1/80, 2/80, …, 80/80.

Table 4.1: The 6 selected short-term bonds

Term to maturity (T)	Contractual interest rate (r_c)
2	0.0%
4	1.5%
2	3.0%
4	0.0%
2	1.5%
4	3.0%

So there are 480 training samples with input variables T_t, r_c and r_t, and the desired output P_t. These variables are normalized via equation (4.7) (cf. Smith, 1993) to generate T_t', r_c', r_t' and P_t', stated in equations (4.8) to (4.11). In equation (4.7), the normalized variable *Tar* is a transformation of the raw value of the original variable *Val*, where *Vmax* and *Vmin* are the maximum and minimum values of the original variable respectively, and *Tmax* and *Tmin* are the desired maximum and minimum normalized values respectively.

$$Tar = Tmin + \left(\frac{Val - Vmin}{Vmax - Vmin}(Tmax - Tmin)\right) \tag{4.7}$$

$$P_t' = -0.9 + \left(\frac{P_t - 92}{106 - 92}(0.9 - (-0.9))\right) \tag{4.8}$$

$$T_t' = -1 + \left(\frac{T_t - 1}{4 - 1}(1 - (-1))\right) \tag{4.9}$$

$$r_c' = -1+(\frac{r_c - 0.00}{0.03 - 0.00}(1-(-1))) \tag{4.10}$$

$$r_t' = -1+(\frac{r_t - 0.016}{0.023 - 0.016}(1-(-1))) \tag{4.11}$$

We use the normalized variables to train 100 ANNs, each of which has 4 hidden nodes. The one with the minimum sum of square error is shown in Figure 4.1, where ${}_3q' =$ -0.055, ${}_2q_1' = -0.817$, ${}_2q_2' = -0.630$, ${}_2q_3' = -0.827$, ${}_2q_4' = 0.450$, ${}_3\mathbf{w}''^t = (-0.711, -0.256, 0.595, -0.112)$, ${}_2\mathbf{w}'^t_1 = (0.589, -0.545, 0.056)$, ${}_2\mathbf{w}'^t_2 = (0.218, -0.611, -0.129)$, ${}_2\mathbf{w}'^t_3 = (0.613, 0.680, -0.219)$, and ${}_2\mathbf{w}'^t_4 = (0.041, 0.757, 0.353)$. This ANN can be transformed back to following equations (4.12) to (4.16) with original variables:

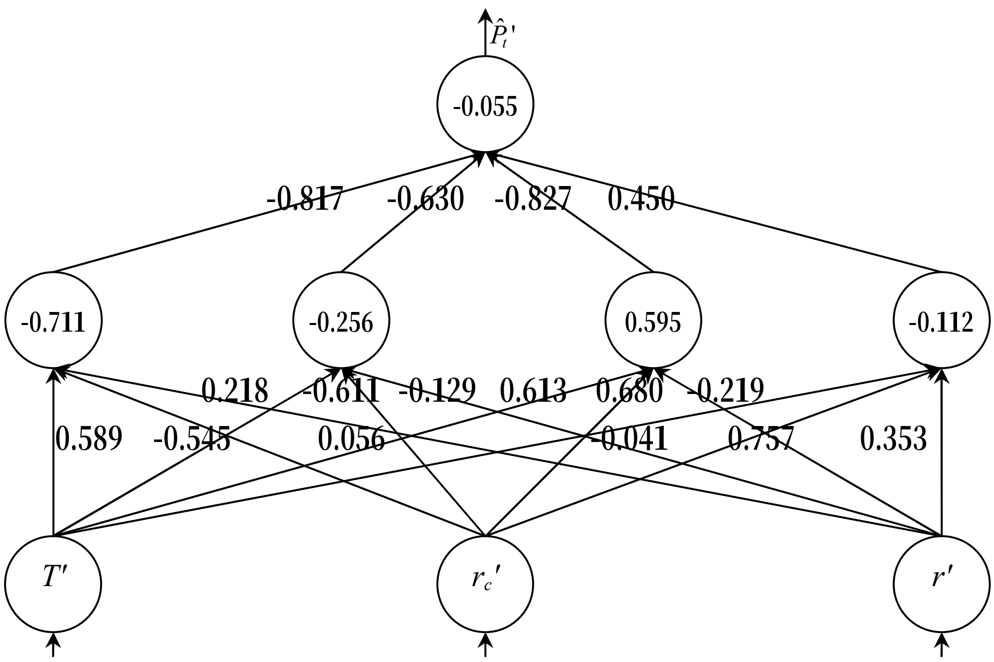

Figure 4.1: The structure and links of the ANN with 4 hidden nodes and the normalized bond pricing variables.

$h_1 = \tanh(-0.817 + 0.589T_t' - 0.545r_c' + 0.056r_t')$

$$= \tanh(-1.565 + 0.393T_t - 36.344r_c + 15.955r_t) \tag{4.12}$$

$h_2 = \tanh(-0.630 + 0.218T_t' - 0.611r_c' - 0.219r_t')$

$$= \tanh(0.335 + 0.145T_t - 40.733r_c - 36.784r_t) \tag{4.13}$$

$h_3 = \tanh(-0.827 + 0.613T_t' + 0.068r_c' - 0.219r_t')$

$$= \tanh(-1.310 + 0.409T_t + 45.318r_c - 62.477r_t) \quad (4.14)$$

$$h_4 = \tanh(0.450 + 0.041T_t' + 0.757r_c' + 0.353r_t')$$

$$= \tanh(-2.341 + 0.027T_t + 50.463r_c + 100.840r_t) \quad (4.15)$$

$$y_t = \hat{P}_t = 98.571 - 5.531h_1 - 1.995h_2 + 4.625h_3 - 0.871h_4 \quad (4.16)$$

Thus

$$t_1 = 0.393T_t - 36.344r_c + 15.955r_t \quad (4.17)$$

$$t_2 = 0.145T_t - 40.733r_c - 36.784r_t \quad (4.18)$$

$$t_3 = 0.409T_t + 45.318r_c - 62.477r_t \quad (4.19)$$

$$t_4 = 0.027T_t + 50.463r_c + 100.840r_t \quad (4.20)$$

$$\mathrm{g}_1(t_1)=\begin{cases} g_{11}(t_1) = 1.000 & \mathit{if}\ t_1 \geq 3.561 \\ g_{12}(t_1) = -2.183 + 1.788t_1 - 0.251t_1^2 & \mathit{if}\ 1.565 \leq t_1 \leq 3.561 \\ g_{13}(t_1) = -0.953 + 0.216t_1 + 0.251t_1^2 & \mathit{if}\ -0.431 \leq t_1 \leq 1.565 \\ g_{14}(t_1) = -1.000 & \mathit{if}\ t_1 \leq -0.431 \end{cases} \quad (4.21)$$

$$\mathrm{g}_2(t_2)=\begin{cases} g_{21}(t_2) = 1.000 & \mathit{if}\ t_2 \geq 1.661 \\ g_{22}(t_2) = 0.307 + 0.834t_2 - 0.251t_2^2 & \mathit{if}\ -0.335 \leq t_2 \leq 1.661 \\ g_{23}(t_2) = 0.363 + 1.170t_2 + 0.251t_2^2 & \mathit{if}\ -2.331 \leq t_2 \leq -0.335 \\ g_{24}(t_2) = -1.000 & \mathit{if}\ t_2 \leq -2.331 \end{cases} \quad (4.22)$$

$$\mathrm{g}_3(t_3)=\begin{cases} g_{31}(t_3) = 1.000 & \mathit{if}\ t_3 \geq 3.306 \\ g_{32}(t_3) = -1.744 + 1.660t_3 - 0.251t_3^2 & \mathit{if}\ 1.310 \leq t_3 \leq 3.306 \\ g_{33}(t_3) = -0.882 + 0.344t_3 + 0.251t_3^2 & \mathit{if}\ -0.686 \leq t_3 \leq 1.310 \\ g_{34}(t_3) = -1.000 & \mathit{if}\ t_3 \leq -0.686 \end{cases} \quad (4.23)$$

$$\mathrm{g}_4(t_4)=\begin{cases} g_{41}(t_4) = 1.000 & \mathit{if}\ t_4 \geq 4.337 \\ g_{42}(t_4) = -3.721 + 2.177t_4 - 0.251t_4^2 & \mathit{if}\ 2.341 \leq t_4 \leq 4.337 \\ g_{43}(t_4) = -0.970 - 0.173t_4 + 0.251t_4^2 & \mathit{if}\ 0.345 \leq t_4 \leq 2.341 \\ g_{44}(t_4) = -1.000 & \mathit{if}\ t_4 \leq 0.345 \end{cases} \quad (4.24)$$

$$y_t' = 98.571 - 5.531\mathrm{g}_1(t_1) - 1.995\mathrm{g}_2(t_2) + 4.625\mathrm{g}_3(t_3) - 0.871\mathrm{g}_4(t_4) \quad (4.25)$$

The extra constraints applied to the input variables are as follows:

$$(1 \leq T_t \leq 4) \text{ AND } (0 \leq r_c \leq 0.030) \text{ AND } (0.016 \leq r_t \leq 0.023) \quad (4.26)$$

For instance, the condition, $(-0.431 \leq t_1 \leq 1.565)$ AND $(-2.331 \leq t_2 \leq -0.335)$ AND $(-0.686 \leq t_3 \leq 1.310)$ AND $(0.345 \leq t_4 \leq 2.341)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, is expressed as the following form:

$$\mathbf{A}_{[3,3,3,3]}\,\mathbf{x} \geq \mathbf{b}_{[3,3,3,3]} \tag{4.27}$$

$$\mathbf{x} = \begin{bmatrix} T_t \\ r_c \\ r_t \end{bmatrix},\ \mathbf{A}_{[3,3,3,3]} = \begin{bmatrix} 0.393 & -36.344 & 15.955 \\ -0.393 & -36.344 & -15.955 \\ 0.145 & -40.733 & -36.784 \\ -0.145 & 40.733 & 36.784 \\ 0.409 & 45.318 & -62.477 \\ -0.409 & -45.318 & 62.477 \\ 0.027 & 50.463 & 100.840 \\ -0.027 & -50.463 & -100.840 \\ 1.000 & 0.000 & 0.000 \\ -1.000 & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & -1.000 & 0.000 \\ 0.000 & 0.000 & 1.000 \\ 0.000 & 0.000 & -1.000 \end{bmatrix},\ \mathbf{b}_{[3,3,3,3]} = \begin{bmatrix} -0.431 \\ -1.565 \\ -2.331 \\ 0.335 \\ -0.686 \\ -1.310 \\ 0.345 \\ -2.341 \\ 1.000 \\ -4.000 \\ 0.000 \\ -0.030 \\ 0.016 \\ -0.023 \end{bmatrix}$$

Minimize: *constant*

$$\text{Subject to: } \mathbf{A}_{[3,3,3,3]}\,\mathbf{x} \geq \mathbf{b}_{[3,3,3,3]} \tag{4.28}$$

There are 256 potential rules associated with ANN. For each rule, we can check if the region formed from the condition of the rule is empty via Simplex method. For instance, the equation (4.27) has a solution if and only if the LP problem (4.28) has an optimal solution. If equation (4.27) has a solution, then the corresponding rule exists. Otherwise, the rule fails to exist. With such analysis, we obtain the following 11 rules.

Rule 1:

If ($1.565 \leq t_1 \leq 2.561$) and ($-0.335 \leq t_2 \leq 1.661$) AND ($-0.686 \leq t_3 \leq 1.310$) and ($2.341 \leq t_4 \leq 4.337$) AND ($1 \leq T_t \leq 4$) AND ($0 \leq r_c \leq 0.030$) AND ($0.016 \leq r_t \leq 0.023$), then $y_t' = 109.193 - 2.061T_t + 426.212r_c - 389.823r_t - 1.955T_t r_c - 46.049T_t r_t - 4459.398r_c r_t + 0.419T_t^2 + 5605.512r_c^2 + 7785.223r_t^2$ (4.29)

Rule 2:

If ($1.565 \leq t_1 \leq 2.561$) AND ($-0.335 \leq t_2 \leq 1.661$) AND ($-0.686 \leq t_3 \leq 1.310$) AND

$(0.345 \leq t_4 \leq 2.341)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, then $y_t' = 106.798 - 2.033T_t + 477.854r_c - 286.627r_t - 3.154T_t r_c - 48.446T_t r_t + 8908.414r_c r_t + 0.418T_t^2 + 4492.314r_c^2 + 3339.985r_t^2$ (4.30)

Rule 3:

If $(1.565 \leq t_1 \leq 2.561)$ AND $(-2.331 \leq t_2 \leq -0.335)$ AND $(-0.686 \leq t_3 \leq 1.310)$ AND $(2.341 \leq t_4 \leq 4.337)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, then $y_t' = 109.081 - 2.110T_t + 439.863r_c - 377.496r_t + 9.905T_t r_c - 35.338T_t r_t - 7460.240r_c r_t + 0.398T_t^2 + 3944.0091r_c^2 + 6430.268r_t^2$ (4.31)

Rule 4:

If $(1.565 \leq t_1 \leq 2.561)$ AND $(-2.331 \leq t_2 \leq -0.335)$ AND $(-0.686 \leq t_3 \leq 1.310)$ AND $(0.345 \leq t_4 \leq 2.341)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, then $y_t' = 106.686 - 2.082T_t + 491.505\ r_c - 274.300\ r_t + 8.706T_t r_c - 37.735\ T_t r_t - 11909.255\ r_c r_t + 0.397\ T_t^2 + 2830.810\ r_c^2 + 1985.030r_t^2$ (4.32)

Rule 5:

If $(-0.341 \leq t_1 \leq 1.565)$ AND $(-0.335 \leq t_2 \leq 1.661)$ AND $(-0.686 \leq t_3 \leq 1.310)$ AND $(0.345 \leq t_4 \leq 2.341)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, then $y_t' = 99.997 - 0.327T_t + 319.917r_c - 217.292r_t + 76.108T_t r_c - 83.242T_t r_t - 5688.151r_c r_t - 0.010T_t^2 + 824.6264r_c^2 + 2633.130r_t^2$ (4.33)

Rule 6:

If $(-0.341 \leq t_1 \leq 1.565)$ AND $(-2.331 \leq t_2 \leq -0.335)$ AND $(1.310 \leq t_3 \leq 3.306)$ AND $(2.341 \leq t_4 \leq 4.337)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, then $y_t' = 98.294 + 0.840T_t + 419.799r_c - 498.237r_t + 3.154T_t r_c + 48.446T_t r_t + 8908.414r_c r_t - 0.418T_t^2 - 4492.314r_c^2 - 3339.985r_t^2$ (4.34)

Rule 7:

If $(-0.341 \leq t_1 \leq 1.565)$ AND $(-2.331 \leq t_2 \leq -0.335)$ AND $(-0.686 \leq t_3 \leq 1.310)$ AND $(2.341 \leq t_4 \leq 4.337)$ AND $(1 \leq T_t \leq 4)$ AND $(0 \leq r_c \leq 0.030)$ AND $(0.016 \leq r_t \leq 0.023)$, then $y_t' = 102.280 - 0.403T_t + 281.926r_c - 308.161r_t + 89.168T_t r_c - 70.135T_t r_t - 4239.977r_c r_t - 0.031T_t^2 + 276.322r_c^2 + 5723.413r_t^2$ (4.35)

Rule 8:

If ($-0.341 \leq t_1 \leq 1.565$) AND ($-2.331 \leq t_2 \leq -0.335$) AND ($-0.686 \leq t_3 \leq 1.310$) AND ($0.345 \leq t_4 \leq 2.341$) AND ($1 \leq T_t \leq 4$) AND ($0 \leq r_c \leq 0.030$) AND ($0.016 \leq r_t \leq 0.023$), then $y_t' = 99.885 - 0.376T_t + 333.567r_c - 204.965r_t + 87.968T_t r_c - 72.532T_t r_t - 8688.992r_c r_t - 0.031T_t^2 - 836.877r_c^2 + 1278.175r_t^2$ (4.36)

Rule 9:

If ($-0.341 \leq t_1 \leq 1.565$) AND ($-2.331 \leq t_2 \leq -0.335$) AND ($t_3 \leq -0.686$) AND ($2.341 \leq t_4 \leq 4.337$) AND ($1 \leq T_t \leq 4$) AND ($0 \leq r_c \leq 0.030$) AND ($0.016 \leq r_t \leq 0.023$), then $y_t' = 101.734 - 1.676T_t + 140.840r_c - 113.656r_t + 46.161T_t r_c - 10.845T_t r_t + 2334.218r_c r_t - 0.225T_t^2 - 2107.996r_c^2 + 1191.714r_t^2$ (4.37)

Rule 10:

If ($-0.341 \leq t_1 \leq 1.565$) AND ($-2.331 \leq t_2 \leq -0.335$) AND ($t_3 \leq -0.686$) AND ($0.345 \leq t_4 \leq 2.341$) AND ($1 \leq T_t \leq 4$) AND ($0 \leq r_c \leq 0.030$) AND ($0.016 \leq r_t \leq 0.023$), then $y_t' = 99.339 - 1.648T_t + 192.482r_c - 10.460r_t + 44.962T_t r_c - 13.241T_t r_t - 2114.797r_c r_t - 0.225T_t^2 - 3221.195r_c^2 - 3253.524r_t^2$ (4.38)

Rule 11:

If ($t_1 \leq -0.431$) AND ($-2.331 \leq t_2 \leq -0.335$) AND ($-0.686 \leq t_3 \leq 1.310$) AND ($2.341 \leq t_4 \leq 4.337$) AND ($1 \leq T_t \leq 4$) AND ($0 \leq r_c \leq 0.030$) AND ($0.016 \leq r_t \leq 0.023$), then $y_t' = 102.538 + 0.920T_t + 159.473r_c - 254.404_t + 49.537T_t r_c - 52.737T_t r_t - 5850.108r_c r_t + 0.184T_t^2 + 2110.166r_c^2 + 6076.841r_t^2$ (4.39)

Table 4.2 displays the rule number and corresponding coefficients of variables in each multivariate polynomial associated with each existing rule.

Table 4.2: The rule number and corresponding coefficients in each multivariate polynomial associated with each existing rule.

Rule No.	i				coefficients									
	i_1	i_2	i_3	i_4	Constant	T_t	r_c	r_t	$T_t r_c$	$T_t r_t$	$r_c r_t$	T_t^2	r_c^2	r_t^2
R1	2	2	3	2	109.193	-2.061	426.212	-389.823	-1.955	-46.049	-4459.398	0.419	5605.512	7785.223
R2	2	2	3	3	106.798	-2.033	477.854	-286.627	-3.154	-48.446	-8908.414	0.418	4492.314	3339.985
R3	2	3	3	2	109.081	-2.110	439.863	-377.496	9.905	-35.338	-7460.240	0.398	3944.009	6430.268
R4	2	3	3	3	106.686	-2.082	491.505	-274.300	8.706	-37.735	-11909.255	0.397	2830.810	1985.030
R5	3	2	3	3	99.997	-0.327	319.917	-217.292	76.108	-83.242	-5688.151	-0.010	824.626	2633.130
R6	3	3	2	2	98.294	0.840	419.799	-498.237	3.154	48.446	8908.414	-0.418	-4492.314	-3339.985
R7	3	3	3	2	102.280	-0.403	281.926	-308.161	89.168	-70.135	-4239.977	-0.031	276.322	5723.413
R8	3	3	3	3	99.885	-0.376	333.567	-204.965	87.968	-72.532	-8688.992	-0.031	-836.877	1278.175
R9	3	3	4	2	101.734	-1.676	140.840	-113.656	46.161	-10.845	2334.218	-0.225	-2107.996	1191.714
R10	3	3	4	3	99.339	-1.648	192.482	-10.460	44.962	-13.241	-2114.797	-0.225	-3221.195	-3253.524
R11	4	3	3	2	102.538	0.920	159.473	-254.404	49.537	-52.737	-5850.108	0.184	2110.166	6076.841

The 11 rules will be examined to obtain knowledge. For example, for rule $\mathbf{A}_{[3,3,3,3]}\ \mathbf{x} \geq \mathbf{b}_{[3,3,3,3]}$

$$\frac{\partial y_t'}{\partial r_t} = -204.965 - 72.532T_t - 8688.992r_c + 2556.351r_t \tag{4.40}$$

Thus to examine the feature $\left.\frac{\partial y_t'}{\partial r_t}\right|_{\mathbf{x}\in\{\mathbf{x}|\mathbf{A}_{[3,3,3,3]}\mathbf{x}\geq\mathbf{b}_{[3,3,3,3]}\}} < 0$ is to check whether the maximal value of the LP problem (4.41) is less than zero.

$$\text{Maximize: } \frac{\partial y_t'}{\partial r_t} = -204.965 - 72.532T_t - 8688.992r_c + 2556.351r_t$$

$$\text{Subject to: } \mathbf{A}_{[3,3,3,3]}\ \mathbf{x} \geq \mathbf{b}_{[3,3,3,3]} \tag{4.41}$$

As for examining the feature $\begin{cases} \frac{\partial^2 y_t'}{\partial T_t^2} < 0 & if\ r_c > r_t \\ \frac{\partial^2 y_t'}{\partial T_t^2} > 0 & if\ r_c < r_t \end{cases}$, there is a different analysis. For example, for rule $\mathbf{A}_{[3,3,3,3]}\ \mathbf{x} \geq \mathbf{b}_{[3,3,3,3]}$

$$\frac{\partial^2 y_t'}{\partial T_t^2} = \text{-}0.062 \tag{4.42}$$

Thus $\frac{\partial^2 y_t'}{\partial T_t^2} < 0$ if $r_c > r_t$ is true if only if $\frac{\partial^2 y_t'}{\partial T_t^2} < 0$ and the LP problem (4.43) has a optimal solution, or $\frac{\partial^2 y_t'}{\partial T_t^2} > 0$ if $r_c < r_t$ is true if only if $\frac{\partial^2 y_t'}{\partial T_t^2} > 0$ and the LP problem (4.44) has a optimal solution.

$$\text{Minimize: } \textit{constant}$$
$$\text{Subject to: } \begin{bmatrix} \mathbf{A}_{[3,3,3,3]} \\ [0,1,-1] \end{bmatrix} \mathbf{x} \geq \begin{bmatrix} \mathbf{b}_{[3,3,3,3]} \\ 0 \end{bmatrix} \qquad (4.43)$$

$$\text{Minimize: } \textit{constant}$$
$$\text{Subject to: } \begin{bmatrix} \mathbf{A}_{[3,3,3,3]} \\ [0,-1,1] \end{bmatrix} \mathbf{x} \geq \begin{bmatrix} \mathbf{b}_{[3,3,3,3]} \\ 0 \end{bmatrix} \qquad (4.44)$$

4.3 Results and Analysis

We can adopt the sign test (Hogg and Tains, 1997b), a binomial test about median of nonparametric methods, to the rules derived from the layered feed-forward neural network to find out features about the bond-pricing. Take as illustration of the sign test of the relationship between y_t' and r_t. Let the significance level of the test a be 0.01. The null hypothesis H_0 is that there is not a relationship between y_t' and r_t, while the alternative hypothesis H_1 is that there is a negative relationship between y_t' and r_t (that is, $\frac{\partial y'_t}{\partial r_t} < 0$). If H_0 is true, the conditional probability Pr($\frac{\partial y'_t}{\partial r_t} < 0 \mid H_0$) equal 0.5. If H_1 is true, the conditional probability Pr($\frac{\partial y'_t}{\partial r_t} < 0 \mid H_1$) is great than 0.5. Let n^- be the count of the maximal value of $\frac{\partial y'_t}{\partial r_t}$ associated with 11 LP problems stated in (4.45) that are less than 0.

$$\text{Maximize: } \frac{\partial y_t'}{\partial r_t}$$
$$\text{Subject to: } \mathbf{A}_i \, \mathbf{x} \geq \mathbf{b}_i \qquad (4.45)$$

If H_0 is true, then n^- has a binomial distribution, $b(11, 0.5)$. We reject H_0 and accept H_1 if only if n^- is greater than n_0^-, where n_0^- is 10 when $a = 0.01$. For example, if $n^- = 11$, we reject the null hypothesis H_0, and say that $\frac{\partial y'_t}{\partial r_t} < 0$ is significant at $a = 0.01$.

Table 4.3: The count of the optimal value of $\frac{\partial y'_t}{\partial r_t}$ associated with 11 LP problems that are greater or less than 0.

Characteristic	$\frac{\partial y'_t}{\partial T_t}$	$\frac{\partial y'_t}{\partial r_c}$	$\frac{\partial y'_t}{\partial r_t}$	$\frac{\partial^2 y'_t}{\partial T_t \partial r_c}$	$\frac{\partial^2 y'_t}{\partial T_t \partial r_t}$	$\frac{\partial^2 y'_t}{\partial r_c \partial r_t}$	$\frac{\partial^2 y'_t}{\partial T_t^2}$	$\frac{\partial^2 y'_t}{\partial r_c^2}$	$\frac{\partial^2 y'_t}{\partial r_t^2}$
> 0	1	11***	0	9*	1	2	5	7	9*
< 0	5	0	11***	2	10**	9*	6	4	2
otherwise	5	0	0	0	0	0	0	0	0
*, **, and *** denote that the sign test statistic is significant at a = 0.050, 0.010, and 0.001, respectively.									

Table 4.3 lists the results of sign tests associated the interested features of bond-pricing. Table 4.3 shows that $\frac{\partial y_t'}{\partial r_c} > 0$, $\frac{\partial y_t'}{\partial r_t} < 0$, $\frac{\partial^2 y_t'}{\partial T_t \partial r_c} > 0$, $\frac{\partial^2 y_t'}{\partial T_t \partial r_t} < 0$, $\frac{\partial^2 y_t'}{\partial r_c \partial r_t} < 0$ and $\frac{\partial^2 y_t'}{\partial r_t^2} > 0$ are significant features at $a = 0.050$. However, we make a mistake (Type I error) (Hogg and Tains, 1997a) for $\frac{\partial^2 y'}{\partial T_t \partial r_c} > 0$ because $\frac{\partial^2 y'}{\partial T_t \partial r_c} > 0$ is not an unequivocal characteristic in the bond-pricing field. There are some relationships between T_t and r_c by intuition; for example, the price increases if T_t and r_c both increase and the price decreases if T_t and r_c both decrease. But we do not know exactly whether the price increases or decreases when one of T_t as well as r_c decreases and the other increases. Thus $\frac{\partial^2 y_t'}{\partial T_t \partial r_c} > 0$ is not an unequivocal characteristic in the bond-pricing field.

On the other hand, we can investigate whether each rule satisfies the features associated with the bond-pricing. The results of the examination of rules are shown in Table 4.4. Table 4.4 shows that these rules have a mean of 100.00%

((100.00+100.00)/2) in satisfaction of both features $\frac{\partial y_t'}{\partial r_c}$ > 0 and $\frac{\partial y_t'}{\partial r_t}$ < 0, and a mean of 72.24% ((90.91+81.82+66.67+40.00+81.82)/5) in satisfaction of other four features. Besides, each rule has an average satisfaction of 82.90 % on these six features. In sum, the result is consistent with the notion that these extracted rules are reasonable.

Table 4.4: The results of the examination of rules

Characteristic \ Rule	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	**Ratio(%)[1]**
$\frac{\partial y'}{\partial r_c}>0$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	**100.00**
$\frac{\partial y'}{\partial r_t}<0$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	**100.00**
$\frac{\partial^2 y'}{\partial T_t \partial r_t}<0$	Yes	Yes	Yes	Yes	Yes	**No**	Yes	Yes	Yes	Yes	Yes	**90.91**
$\frac{\partial^2 y'}{\partial r_c \partial r_t}<0$	Yes	Yes	Yes	Yes	Yes	**No**	Yes	Yes	**No**	Yes	Yes	**81.82**
$\frac{\partial^3 y'}{\partial T_t^2}<0, if\ r_c>r_t$	-[3]	-	-	-	-	Yes	Yes	-	-	-	**No**	**66.67**
$\frac{\partial^3 y'}{\partial T_t^2}>0, if\ r_c<r_t$	Yes	Yes	Yes	Yes	**No**	**No**	**No**	**No**	**No**	**No**	-	**40.00**
$\frac{\partial^2 y'}{\partial r_t^2}>0$	Yes	Yes	Yes	Yes	Yes	**No**	Yes	Yes	Yes	**No**	Yes	**81.82**
Ratio[2]	**100**	**100**	**100**	**100**	**83.33**	**42.86**	**85.72**	**83.33**	**66.67**	**66.67**	**83.33**	-

1. (The number of "Yes") / (The total number of "Yes" and "No") in one characteristic.
2. (The number of "Yes") / (The total number of "Yes" and "No") in one rule.
3. There is not a region that $r_c > r_t$ or $r_c < r_t$.