Chapter 1

Introduction

American options are options that can be exercised before the maturity of the contract. They cannot be valued by closed-form formula and require the use of numerical methods. The main difficulty in the pricing of American put options is to describe the critical stock price, determined in David and Johnson (2000). At each time before maturity, if the value of the stock price under the critical stock price, the American put option should be exercised. There are two main approaches for pricing American put options, one is numerical analysis approach, and the other is analytic approximation model. The main numerical analysis approaches for valuing derivative securities are the lattice approach, the finite difference approach and the Monte Carlo simulation.

Cox, Ross and Rubinstein (1979) first introduced the binomial method. It is a well-known and widely used model for valuing standard options. It is quite flexible and easy to implement. It disparts both the time and state spaces in order to approximate the option price. But the convergence of binomial method is not smooth. The convergence of the binomial method for pricing American options is proved in Amin and Khanna (1994). Various extensions to the original binomial method have been proposed, such as the multinomial methods of Boyle (1988), Kamrad and Ritchken (1991), the flexible binomial option pricing model of Tian (1999), etc. The motivation of these extension methods is either to improve the rate of convergence or to generalize the method for pricing more complex derivatives.

Schwartz (1977) and Brennan and Schwartz (1978) introduced the finite difference

method for the valuation of American options. There are two alternative ways to implement the finite difference method. The first is the explicit finite difference method; the second is the implicit finite difference method. Brennan and Schwartz (1978) show that the explicit finite difference method is equivalent to a trinomial lattice approach and, in the limit, the implicit finite difference method corresponds to a multinomial lattice approach. Hull and White (1990) modify the explicit finite difference method and ensure the convergence of the calculated values to the correct solution. Houstis and Kortesis (1998) developed the Front-Tracking Finite Difference Method. It is an efficient method for approximating the option value and free boundary function.

Boyle (1977) first suggested the use of the Monte Carlo simulation for pricing options. It is very effective in pricing derivatives with strong path dependency, multiple factors, or non-Markovian features. But, it cannot deal with the problem of early exercise. To overcome this difficulty, Longstaff and Schwartz (2001) developed the Least-Squares Approach (LSM) to approximate the value of American options by simulation. The key point is to estimate the conditional expected payoff by using least squares to decide whether to exercise the option or not. And this is a useful way to value American put options.

There are many analytic approximation algorithms for pricing the American put option. Geske and Johnson (1984) introduced Richardson extrapolation to accelerate the computation of American option price. Barone-Adesi and Whaley (1987) consider quasi-analytical solutions. The key point is that the American put option can be decomposed into an otherwise equivalent European option plus an early exercise premium. Kim (1990), Jacka (1991), Carr et al. (1992) proved this decomposition of American put option. The early exercise premium is a function of the American option optimal exercise frontier. Based upon the decomposition of the American

option, Huang et al. (1996) introduced a recursive option pricing method to value the early exercise premium. Ibanez (2003) suggested a new algorithm to deal with the early exercise problem. With Richardson Extrapolation, the new algorithm is more accurate.

The purpose of the thesis is to compare several American put option pricing methods on the basis of speed and accuracy over the sets of option parameters gathered from related literatures. The evaluative method is generalized from Broadie and Detemple (1996). The option parameters in Broadie and Detemple (1996) were generated randomly. But this is unsuited and may be unreasonable. In order to identify representative options for option pricing, 186 parameters of option contracts were selected from related literatures, including options on stock with dividend, non-dividend, in-the-money, at-money and out-of-money, short maturity and long maturity.

There exists a great deal of methods for pricing American put option in related literatures. From literatures, we survey 27 methods. But a complete comparison of these methods is lacking. We implement 14 methods, including lattice approaches, finite difference methods, Monte Carlo simulations and analytic approximations, and apply these methods to value the 186 option contracts above. Numerical results will illustrate a set of widely cited option contracts and summarize the advantages and disadvantages of each method in terms of speed and accuracy.

The remainder of this thesis is organized as follows. Chapter 2 reviews the existing option pricing methodologies. Chapter 3 presents several numerical results and the comparison of all observations. Chapter 4 summarizes the thesis.