7. The Illustration

7.1 Definition

There are three neural networks illustrated in the following. The structures of three neural networks are referring to the frameworks in Figure 1, and they all include one input layer, one hidden layer and one output node.

Take Network I in Figure 5 as an illustration. The Network I has one hidden node, two input nodes and one output node. Let *net* to be the net input of the hidden node, and the definition of *net* is defined in (13).

7.2 The Network I

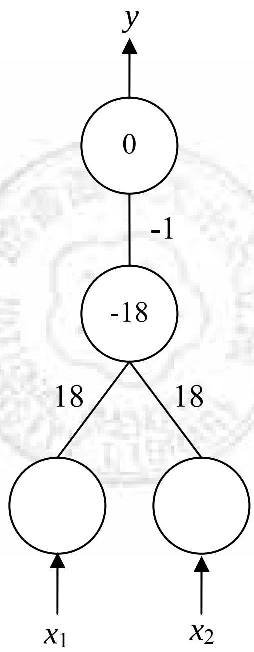

Figure 5: The Network I

$$net = 18x_1+18x_2-18 \qquad (13)$$

$$y = -\tanh(net) = -\tanh(18x_1+18x_2-18) \qquad (14)$$

7.2.1 The rule-extraction of the Network I

The area of round-off effect in the input space is $\{\mathbf{x}|\ 18x_1+18x_2-18 > \psi\} \cup \{\mathbf{x}|\ 18x_1+18x_2-18 < -\psi\}$. Let the definition of y is in (14) and the range of non-vague y where is $\{-1\} \cup [-tanh(\psi), tanh(\psi)] \cup \{1\}$. We can have the following rules:

1. $\mathbf{f}^{-1}(-1)$ equals $\{\mathbf{x}|\ x_1 + x_2 > 1+\frac{\psi}{18}\}$, which consists of an open polyhedron.

2. For any $y \in [-tanh(\psi), tanh(\psi)]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ 18x_1 + 18x_2 = tanh^{-1}(-y) + 18\}$, which consists of a line.

3. $\mathbf{f}^{-1}(1)$ equals $\{\mathbf{x}|\ x_1 + x_2 < 1-\frac{\psi}{18}\}$, which consists of an open polyhedron.

Let's take y = -0.5 as an example, the corresponding of observation can be depicted in Figure 6.

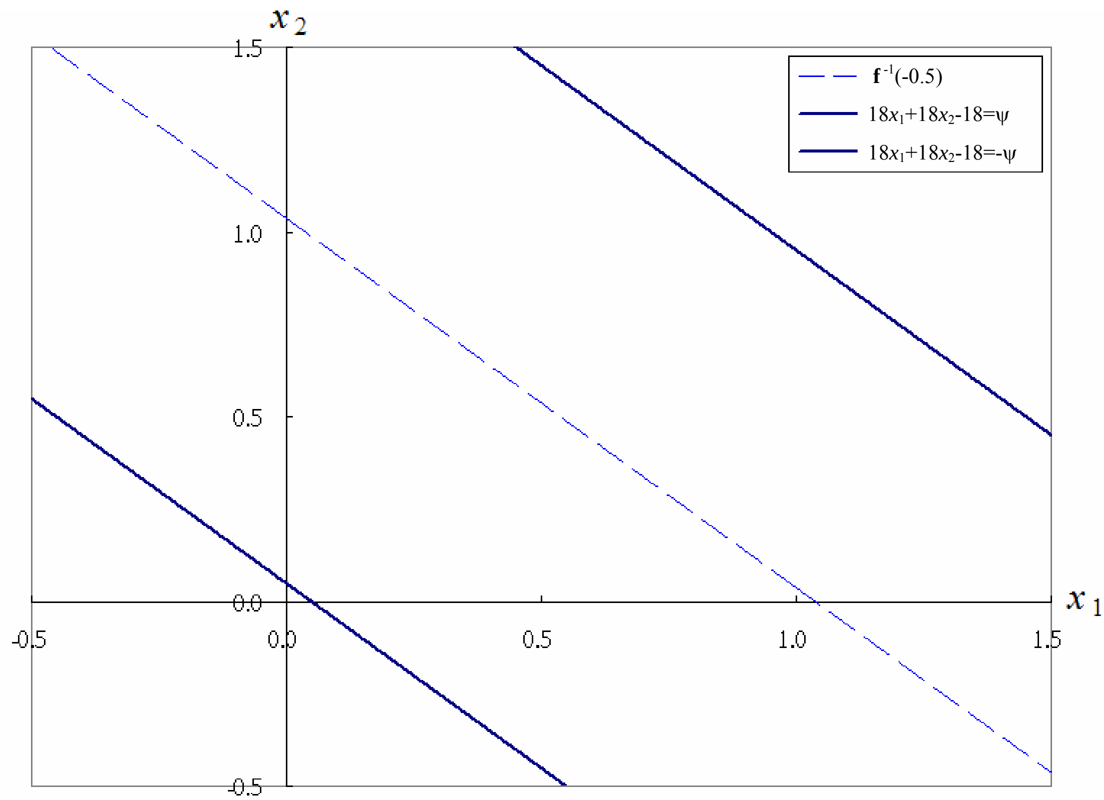

Figure 6: The observation of $f^{-1}(-0.5)$ in Network I.

7.2.2 The rule-extraction of the approximation Network I

Using g to approximate *tanh*, we can get equation (15).

$$\hat{y} = -g(net) \equiv \begin{cases} -1 & \text{if } net \geq \kappa \\ -\beta_1 net - \beta_2 net^2 & \text{if } 0 \leq net < \kappa \\ -\beta_1 net + \beta_2 net^2 & \text{if } -\kappa < net < 0 \\ 1 & \text{if } net \leq -\kappa \end{cases} \qquad (15)$$

For any $\hat{y}$, let Xa($\hat{y}$) be the set of x whose images under g are $\hat{y}$. Thus, we can have the following rules:

1. Xa(-1) equals $\{\mathbf{x} \mid x_1 + x_2 \geq 1 + \frac{\kappa}{18}\}$, which consists of an open polyhedron.
2. For any $\hat{y} \in (-1, 0]$, Xa($\hat{y}$) equals $\{\mathbf{x} \mid x_1 + x_2 = \frac{-\beta_1 + \sqrt{\beta_1^2 - 4\hat{y}\beta_2}}{36\beta_2} + 1\}$, which consists of a line.
3. For any $\hat{y} \in (0, 1)$, Xa($\hat{y}$) equals $\{\mathbf{x} \mid x_1 + x_2 = \frac{\beta_1 - \sqrt{\beta_1^2 + 4\hat{y}\beta_2}}{36\beta_2} + 1\}$, which consists of a line.
4. Xa(1) equals $\{\mathbf{x} \mid x_1 + x_2 \leq 1 - \frac{\kappa}{18}\}$, which consists of an open polyhedron.

Figure 7 illustrates the graph of Xa(-0.5).

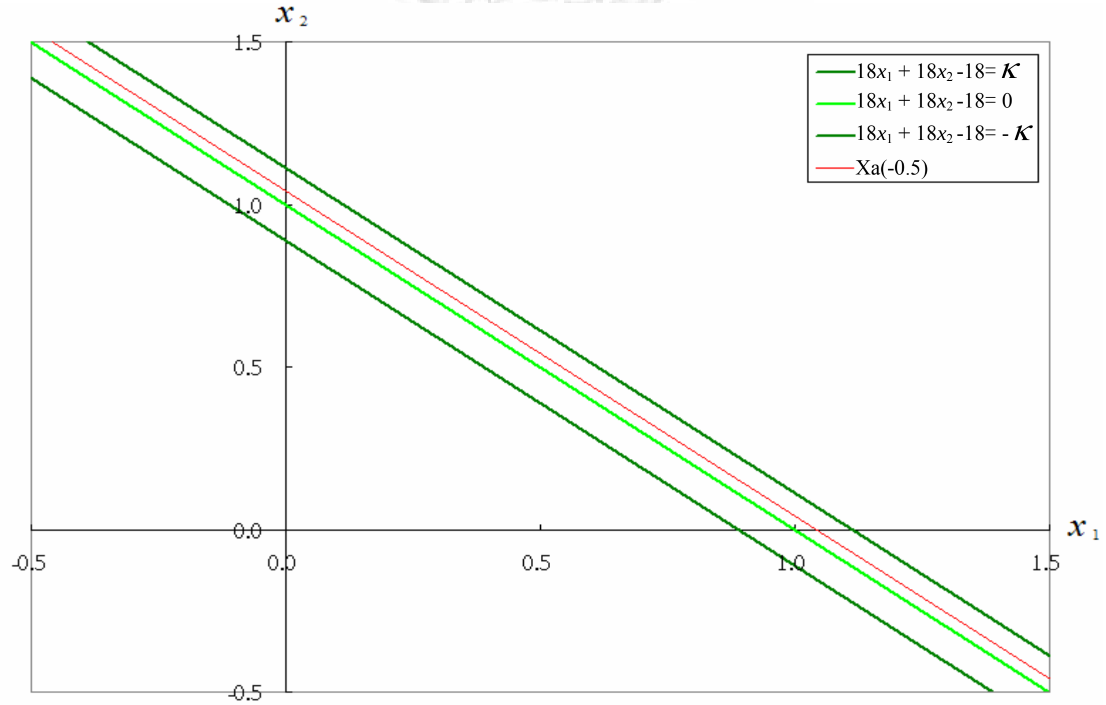

Figure 7: The observation of Xa(-0.5) in the approximation Network I

7.3 The Network II

The Network II in Figure 8 has two hidden nodes, two input nodes and one output node. Let net_1 and net_1 to be the net input of the hidden node, and the definitions of net_1 and net_2 are defined in (16).

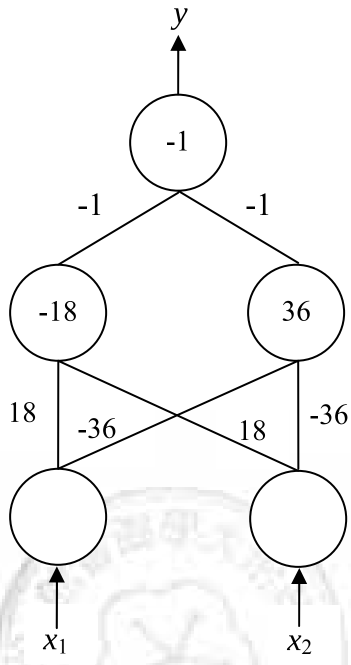

Figure 8: The Network II.

$$\begin{aligned} net_1 &= 18x_1 + 18x_2 - 18 \\ net_2 &= -36x_1 - 36x_2 + 36 \end{aligned} \quad (16)$$

$$y = -tanh\,(net_1) - tanh\,(net_2) - 1 \quad (17)$$

7.3.1 The rule-extraction of the Network II

The area of round-off effect for the net_1 is $\{\mathbf{x}|\ 18x_1+18x_2-18 > \psi\} \cup \{\mathbf{x}|\ 18x_1+18x_2-18 < -\psi\}$. The area of round-off effect for the net_2 is $\{\mathbf{x}|\ -36x_1-36x_2+36 > \psi\} \cup \{\mathbf{x}|\ -36x_1-36x_2+36 < -\psi\}$. Let the definition of y is in (17), and the range of non-vague y is $[\frac{\sqrt{\sqrt{5}-2}(\sqrt{5}-3)}{\sqrt{5}-1}-1, \frac{\sqrt{\sqrt{5}-2}(3-\sqrt{5})}{\sqrt{5}-1}-1]$[1]. We can have the following rules:

1. $\mathbf{f}^{-1}(\frac{\sqrt{\sqrt{5}-2}(\sqrt{5}-3)}{\sqrt{5}-1}-1)$ equals $\{x_1+x_2 = 1+\frac{tanh^{-1}(-\sqrt{\sqrt{5}-2})}{18}\}$, which consists of a line.
2. For any $y \in (\frac{\sqrt{\sqrt{5}-2}(\sqrt{5}-3)}{\sqrt{5}-1}-1, tanh(0.5\psi) - 2)$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1, a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of two parallel lines.
3. For any $y \in [tanh(0.5\psi)-2, tanh(0.5\psi)-tanh(\psi)-1]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(-y-2)}{18}+1\} \cup \{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1, a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of three parallel lines.
4. For any $y \in (tanh(0.5\psi)-tanh(\psi)-1, tanh(\psi)-2]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(-y-2)}{18}+1\} \cup \{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1, a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of three parallel lines.
5. For any $y \in (tanh(\psi)-2, -1)$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1, a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of two parallel lines.
6. $\mathbf{f}^{-1}(-1)$ equals $\{\mathbf{x}|\ x_1 + x_2 > 1+\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 < 1-\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 = 1\}$, which consists of two parallel open polyhedra and a line.
7. For any $y \in (-1, -tanh(\psi))$, $\mathbf{f}^{-1}(y)$ equals equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1, a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$ which consists of two parallel lines.
8. For any $y \in [-tanh(\psi), -tanh(0.5\psi)+tanh(\psi)-1)$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 =$

[1] For any $\mathbf{net} \in \{\mathbf{net}|\ -\psi \le net_1 \le \psi, -\psi \le net_2 \le \psi\}$: $a_1 = tanh(net_1)$, $-1 < a_1 < 1$, $a_2 = tanh(net_2)$, $-1 < a_2 < 1$, $y = -a_1-a_2-1$; $a_2 = tanh(-2tanh^{-1}(a_1)) = \frac{-2a_1}{1+a_1^2}$, $y = -a_1+\frac{2a_1}{1+a_1^2}-1$, $\frac{dy}{da_1} = a_1^4 + 4a_1^2 - 1 = 0$, $a_1 = \pm\sqrt{\sqrt{5}-2}$ and $y \in [\frac{\sqrt{\sqrt{5}-2}(\sqrt{5}-3)}{\sqrt{5}-1}-1, \frac{\sqrt{\sqrt{5}-2}(3-\sqrt{5})}{\sqrt{5}-1}-1]$.

$1+\dfrac{tanh^{-1}(-y)}{18}\}$ $\cup$ equals $\{\mathbf{x}|\ x_1 + x_2 = \dfrac{tanh^{-1}(a_1)}{18} + 1,\ -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1,\ a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of three parallel lines.

9. For any $y \in [-tanh(0.5\psi)+tanh(\psi)-1, -tanh(0.5\psi)]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = 1+\dfrac{tanh^{-1}(-y)}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 = \dfrac{tanh^{-1}(a_1)}{18} + 1,\ -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1,\ a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of three parallel lines

10. For any $y \in (-tanh(0.5\psi), \dfrac{\sqrt{\sqrt{5}-2}(\sqrt{5}-3)}{\sqrt{5}-1}-1)$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \dfrac{tanh^{-1}(a_1)}{18} + 1,\ -a_1 - tanh(-2tanh^{-1}(a_1)) = y + 1,\ a_1 \in [-tanh(0.5\psi), tanh(0.5\psi)]\}$, which consists of two parallel lines.

11. $\mathbf{f}^{-1}(\dfrac{\sqrt{\sqrt{5}-2}(3-\sqrt{5})}{\sqrt{5}-1}-1)$ equals $\{\mathbf{x}|\ x_1 + x_2 = 1+\dfrac{tanh^{-1}(\sqrt{\sqrt{5}-2})}{18}\}$, which consists of a line.

The corresponding of observation can be depicted in Figure 9.

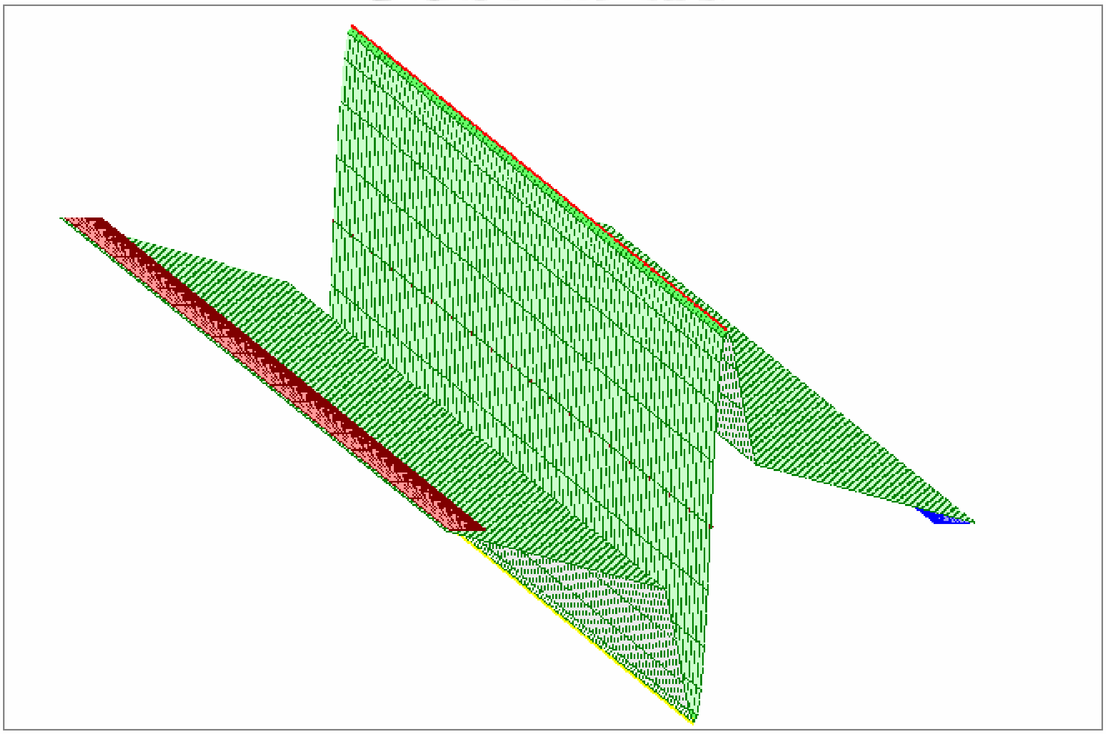

Figure 9: The observation of Network II.

7.3.2 The rule-extraction of the approximation Network II

Since $a_k = tanh(net_k)$ where k = 1 or 2, we can have equation (18) and (19), and the range of $\hat{y}$ is $[\frac{\beta_1^2}{6\beta_2} - \frac{\beta_1^2}{12\beta_2} - 1, \frac{-\beta_1^2}{6\beta_2} + \frac{\beta_1^2}{12\beta_2} - 1]$[2].

$$\hat{a}_k = g(net_k) \equiv \begin{cases} 1 & if\ net_k \geq \kappa \\ \beta_1 net_k + \beta_2 net_k^2 & if\ 0 \leq net_k < \kappa \\ \beta_1 net_k - \beta_2 net_k^2 & if\ -\kappa < net_k < 0 \\ -1 & if\ net_k \leq -\kappa \end{cases} \tag{18}$$

$$\hat{y} = -\hat{a}_1 - \hat{a}_2 - 1 \tag{19}$$

For any $\hat{y}$, let Xa($\hat{y}$) be the set of x whose images under g are $\hat{y}$. Thus, we can have the following rules:

1. Xa($\frac{\beta_1^2}{6\beta_2} - \frac{\beta_1^2}{12\beta_2} - 1$) equals $\{\mathbf{x} |\ x_1 + x_2 = \frac{\beta_1 - \sqrt{\beta_1^2 - 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of a line.
2. For any $\hat{y} \in (\frac{\beta_1^2}{6\beta_2} - \frac{\beta_1^2}{12\beta_2} - 1, 0.5\beta_1\kappa - 0.25\beta_2\kappa^2 - 2)$, Xa($\hat{y}$) equals $\{\mathbf{x} |\ x_1 + x_2 = \frac{\beta_1 \pm \sqrt{\beta_1^2 - 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of two parallel lines.
3. For any $\hat{y} \in [0.5\beta_1\kappa - 0.25\beta_2\kappa^2 - 2, -0.5\beta_1\kappa + 0.75\beta_2\kappa^2 - 1)$, Xa($\hat{y}$) equals $\{\mathbf{x} |\ x_1 + x_2 = \frac{\beta_1 - \sqrt{\beta_1^2 + 4(\hat{y}+2)\beta_2}}{36\beta_2} + 1\} \cup \{\mathbf{x} |\ x_1 + x_2 = \frac{\beta_1 \pm \sqrt{\beta_1^2 - 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of three parallel lines.
4. For any $\hat{y} \in [-0.5\beta_1\kappa + 0.75\beta_2\kappa^2 - 1, -1)$, Xa($\hat{y}$) equals $\{\mathbf{x} |\ x_1 + x_2 = \frac{\beta_1 - \sqrt{\beta_1^2 + 4(\hat{y}+2)\beta_2}}{36\beta_2} + 1\} \cup \{\mathbf{x} |\ x_1 + x_2 = \frac{\beta_1 \pm \sqrt{\beta_1^2 - 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which

[2] For any **net** $\in$ {**net**| $-\frac{\kappa}{2} < net_1 < 0, 0 \leq net_2 < \kappa$}, $\hat{a}_1 = \beta_1 net_1 - \beta_2 net_1^2$, $-\frac{\beta_1\kappa}{2} - \frac{\beta_2\kappa}{4} \leq \hat{a}_1 < 0$, $\hat{a}_2 = \beta_1 net_2 + \beta_2 net_2^2$, $0 \leq \hat{a}_2 < 1$, $\hat{y} = -\hat{a}_1 - \hat{a}_2 - 1$; $net_1 = -2net_2$, $\hat{y} = \beta_1 net_1 - 3\beta_2 net_1^2 - 1$, $\frac{d\hat{y}}{dnet_1} = \beta_1 - 6\beta_2 net_1 = 0$, $net_1 = \frac{\beta_1}{6\beta_2}$, $\hat{y} = \frac{\beta_1^2}{6\beta_2} - \frac{\beta_1^2}{12\beta_2} - 1$; For any **net** $\in$ {**net**| $0 \leq net_1 < \frac{\kappa}{2}, -\kappa \leq net_2 < 0$}, $\hat{a}_1 = \beta_1 net_1 + \beta_2 net_1^2$, $0 \leq \hat{a}_1 < g(\frac{\kappa}{2})$, $\hat{a}_2 = \beta_1 net_2 - \beta_2 net_2^2$, $-1 \leq \hat{a}_2 < 0$, $\hat{y} = -\hat{a}_1 - \hat{a}_2 - 1$; $net_1 = -2net_2$, $\hat{y} = \beta_1 net_1 + 3\beta_2 net_1^2 - 1$, $\frac{d\hat{y}}{dnet_1} = \beta_1 + 6\beta_2 net_1 = 0$, $net_1 = \frac{-\beta_1}{6\beta_2}$, $\hat{y} = \frac{-\beta_1^2}{6\beta_2} + \frac{\beta_1^2}{12\beta_2} - 1$ and $\hat{y} \in [\frac{\beta_1^2}{6\beta_2} - \frac{\beta_1^2}{12\beta_2} - 1, \frac{-\beta_1^2}{6\beta_2} + \frac{\beta_1^2}{12\beta_2} - 1]$.

consists of three parallel lines.

5. Xa(-1) equals $\{\mathbf{x}|\ 18x_1+18x_2-18 \geq \kappa\} \cup \{\mathbf{x}|\ 18x_1+18x_2-18 \leq -\kappa\} \cup \{\mathbf{x}|\ x_1 + x_2 = 1\}$, which consists of two open polyhedra and a line.
6. For any $\hat{y} \in (-1,\ 0.5\beta_1\kappa+0.75\beta_2\kappa^2-1]$, Xa($\hat{y}$) equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{-\beta_1 + \sqrt{\beta_1^2 - 4\hat{y}\beta_2}}{36\beta_2} + 1\} \cup \{\mathbf{x}|\ x_1 + x_2 = \frac{-\beta_1 \pm \sqrt{\beta_1^2 + 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of three parallel lines.
7. For any $\hat{y} \in (0.5\beta_1\kappa+0.75\beta_2\kappa^2-1,\ -0.5\beta_1\kappa-0.25\beta_2\kappa^2]$, Xa($y$) equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{-\beta_1 + \sqrt{\beta_1^2 - 4\hat{y}\beta_2}}{36\beta_2} + 1\} \cup \{\mathbf{x}|\ x_1 + x_2 = \frac{-\beta_1 \pm \sqrt{\beta_1^2 + 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of three parallel lines.
8. For any $\hat{y} \in (-0.5\beta_1\kappa-0.25\beta_2\kappa^2,\ \frac{-\beta_1^2}{6\beta_2} + \frac{\beta_1^2}{12\beta_2} - 1)$, Xa($\hat{y}$) equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{-\beta_1 \pm \sqrt{\beta_1^2 + 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of two parallel lines.
9. Xa($\frac{-\beta_1^2}{6\beta_2} + \frac{\beta_1^2}{12\beta_2} - 1$) equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{-\beta_1 + \sqrt{\beta_1^2 + 12(\hat{y}+1)\beta_2}}{108\beta_2} + 1\}$, which consists of a line.

The corresponding of observation can be depicted in Figure 10.

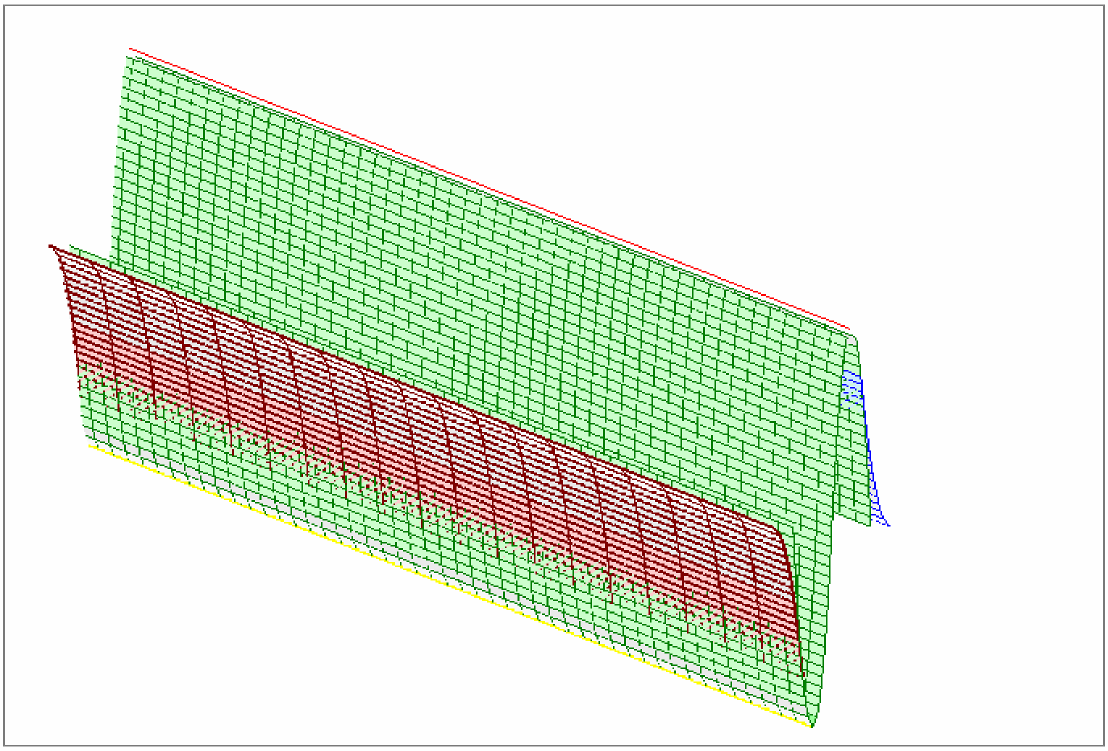

Figure 10: The observation of approximation Network II.

7.4 The Network III

Take Network III in Figure 4 as an illustration. The Network III has two hidden nodes, two input nodes and one output node. Let net_1 and net_2 to be the net input of the hidden node, and the definitions of net_1 and net_2 are defined in (10).

7.4.1 The rule-extraction of the Network III

The area of round-off effect for the net_1 is $\{\mathbf{x}|\ 18x_1+18x_2-18 > \psi\} \cup \{\mathbf{x}|\ 18x_1+18x_2-18 < -\psi\}$. The area of round-off effect for the net_2 is $\{\mathbf{x}|\ -18x_1+18x_2-18 > \psi\} \cup \{\mathbf{x}|\ -18x_1+18x_2-18 < -\psi\}$, and the range of non-vague y where is $\{-3\} \cup [-tanh(\psi)-2, tanh(\psi)] \cup \{1\}$. We can have the following rules:

1. $\mathbf{f}^{-1}(-3) = \{\mathbf{x}|\ x_1 + x_2 > 1+\frac{\psi}{18}, x_1 - x_2 < -1-\frac{\psi}{18}\}$ which is an open polyhedron in the input space.
2. For any $y \in [-tanh(\psi)-2, -2tanh(\psi)-1)$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = 1-\frac{tanh^{-1}(y+2)}{18}, x_1 - x_2 < -1-\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 - x_2 = -1+\frac{tanh^{-1}(y+2)}{18}, x_1 + x_2 > 1+\frac{\psi}{18}\}$, which includes two half-lines.
3. For any $y \in [-2tanh(\psi)-1, tanh(\psi)-2]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = 1-\frac{tanh^{-1}(y+2)}{18}, x_1 - x_2 < -1-\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 - x_2 = -1+\frac{tanh^{-1}(y+2)}{18}, x_1 + x_2 > 1+\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -x_1 + x_2 = \frac{tanh^{-1}(-a_1-y-1)}{18} + 1, a_1 \in [1-2tanh(\psi), tanh(\psi)]\}$, which includes two half-lines and a curve segment.
4. For any $y \in (tanh(\psi)-2, -1)$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1, -x_1 + x_2 = \frac{tanh^{-1}(-a_1-y-1)}{18} + 1, a_1 \in (-tanh(\psi), tanh(\psi))\}$, which includes a curve segment.
5. $\mathbf{f}^{-1}(-1) = \{\mathbf{x}|\ x_1 = \frac{tanh^{-1}(a_1)}{18}, x_2 = 1, a_1 \in [-tanh(\psi), tanh(\psi)]\} \cup \{\mathbf{x}|\ x_1 + x_2 > 1+\frac{\psi}{18}, x_1 - x_2 > -1+\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 < 1-\frac{\psi}{18}, x_1 - x_2 < -1-\frac{\psi}{18}\}$, which includes a line segment and two open polyhedra.

6. For any $y \in (-1, -tanh(\psi))$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1,\ -x_1 + x_2 = \frac{tanh^{-1}(-a_1 - y - 1)}{18} + 1,\ a_1 \in [-tanh(\psi), tanh(\psi)]\}$, which includes a curve segment.

7. For any $y \in [-tanh(\psi), 2tanh(\psi)-1]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 + x_2 = \frac{tanh^{-1}(a_1)}{18} + 1,\ -x_1 + x_2 = \frac{tanh^{-1}(-a_1 - y - 1)}{18} + 1,\ a_1 \in [-tanh(\psi), 2tanh(\psi)-1]\} \cup \{\mathbf{x}|\ x_1 - x_2 = -1+\frac{tanh^{-1}(y)}{18},\ x_1 + x_2 < 1-\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 = 1-\frac{tanh^{-1}(y)}{18},\ x_1 - x_2 > -1+\frac{\psi}{18}\}$, which includes a curve segment and two half-lines.

8. For any $y \in (2tanh(\psi)-1, tanh(\psi)]$, $\mathbf{f}^{-1}(y)$ equals $\{\mathbf{x}|\ x_1 - x_2 = -1+\frac{tanh^{-1}(y)}{18},\ x_1 + x_2 < 1-\frac{\psi}{18}\} \cup \{\mathbf{x}|\ x_1 + x_2 = 1-\frac{tanh^{-1}(y)}{18},\ x_1 - x_2 > -1+\frac{\psi}{18}\}$, which includes two half-lines.

9. $\mathbf{f}^{-1}(1) = \{\mathbf{x}|\ x_1 + x_2 < 1-\frac{\psi}{18},\ x_1 - x_2 > -1+\frac{\psi}{18}\}$, which is an open polyhedron in the input space.

Let's take y = -2.995, -2.5, -1.5, -1, -0.5 and 0.995 as examples, the corresponding of observation can be depicted in Figure 11.

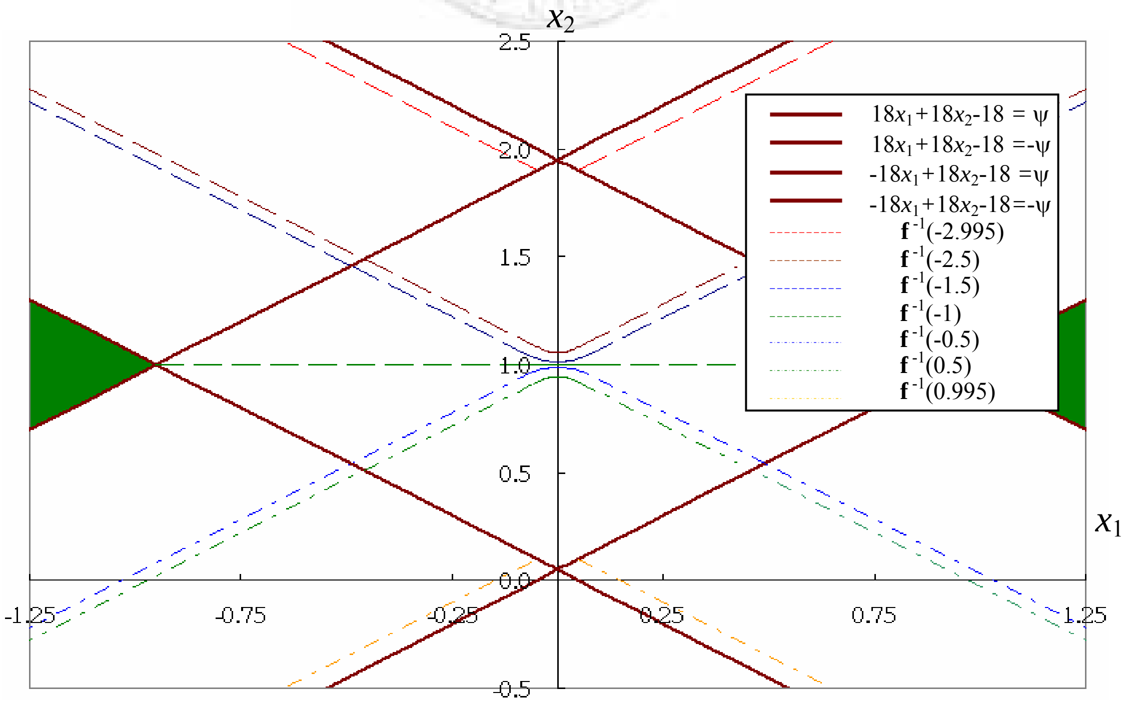

Figure 11: The observation of $f^{-1}(y)$ in Network III.

7.4.2 The rule-extraction of the approximation Network III

For any $\hat{y}$, let Xa($\hat{y}$) be the set of x whose images under g are $\hat{y}$. Thus, we can have the following rules:

1. Xa(-3) = {**x**| $18x_1+18x_2 \geq \kappa+18$, $-18x_1+18x_2 \geq \kappa+18$}, which is an open polyhedron in the input space.
2. For any $\hat{y} \in (-3, -2]$, Xa($\hat{y}$) equals {**x**| $18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 - 4(\hat{y}+2)\beta_2}}{2\beta_2} + 18$, $-18x_1+18x_2-18 \geq \kappa$ } $\cup$ {**x**| $-18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 - 4(\hat{y}+2)\beta_2}}{2\beta_2} + 18$, $18x_1+18x_2-18 \geq \kappa$} $\cup$ {**x**| $18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4\hat{a}_1\beta_2}}{2\beta_2} + 18$, $-18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2} + 18$, $0 \leq \hat{a}_1 < 1$}, which consists of two half-lines and a curve segment.
3. For any $\hat{y} \in (-2, -1)$, Xa($\hat{y}$) equals {**x**| $18x_1 + 18x_2 = \dfrac{\beta_1 - \sqrt{\beta_1^2 + 4(\hat{y}+2)\beta_2}}{2\beta_2} + 18$, $-18x_1+18x_2-18 \geq \kappa$} $\cup$ {**x**| $18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4\hat{a}_1\beta_2}}{2\beta_2} + 18$, $-18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2} + 18$, $0 \leq \hat{a}_1 < 1$} $\cup$ {**x**| $18x_1 + 18x_2 = \dfrac{\beta_1 - \sqrt{\beta_1^2 - 4\hat{a}_1\beta_2}}{2\beta_2} + 18$, $-18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2} + 18$, $-1 < \hat{a}_1 < 0$} $\cup$ {**x**| $-18x_1 + 18x_2 = \dfrac{\beta_1 - \sqrt{\beta_1^2 + 4(\hat{y}+2)\beta_2}}{2\beta_2} + 18$, $18x_1+18x_2-18 \geq \kappa$} $\cup$ {**x**| $18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4\hat{a}_1\beta_2}}{2\beta_2} + 18$, $-18x_1 + 18x_2 = \dfrac{\beta_1 - \sqrt{\beta_1^2 - 4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2} + 18$, $0 \leq \hat{a}_1 < 1$}, which consists of three curve segments and two half-lines.
4. For $\hat{y} = -1$, Xa(-1) equals {**x**| $18x_1+18x_2 \leq -\kappa+18$, $-18x_1+18x_2 \geq \kappa+18$} $\cup$ {**x**| $18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4\hat{a}_1\beta_2}}{2\beta_2} + 18$, $x_2 = 1$, $0 \leq \hat{a}_1 < 1$} $\cup$ {**x**| $18x_1 + 18x_2 = \dfrac{\beta_1 - \sqrt{\beta_1^2 - 4\hat{a}_1\beta_2}}{2\beta_2} + 18$, $x_2 = 1$, $-1 < \hat{a}_1 < 0$} $\cup$ {**x**| $18x_1+18x_2 \geq \kappa+18$, $-18x_1+18x_2 \leq -\kappa+18$}, which consists of two line segments and two open polyhedra.
5. For any $\hat{y} \in (-1, 0)$, Xa($\hat{y}$) equals {**x**| $18x_1 + 18x_2 = \dfrac{\beta_1 - \sqrt{\beta_1^2 + 4\hat{y}\beta_2}}{2\beta_2} + 18$, $-18x_1 + 18x_2 = \dfrac{-\beta_1 + \sqrt{\beta_1^2 + 4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2} + 18$, $-1 < \hat{a}_1 < 0$} $\cup$ {**x**| $-18x_1 + 18x_2$

$= \dfrac{-\beta_1+\sqrt{\beta_1^2-4\hat{y}\beta_2}}{2\beta_2} + 18$, $18x_1+18x_2-18 < -\kappa\}$ $\cup$ $\{\mathbf{x}|\ 18x_1 + 18x_2$ $= \dfrac{-\beta_1+\sqrt{\beta_1^2-4\hat{y}\beta_2}}{2\beta_2} + 18$, $-18x_1 + 18x_2 = \dfrac{\beta_1-\sqrt{\beta_1^2-4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2} + 18$, $0 \le \hat{a}_1 < 1\}$ $\cup$ $\{\mathbf{x}|\ 18x_1 + 18x_2 = \dfrac{\beta_1-\sqrt{\beta_1^2+4\hat{y}\beta_2}}{2\beta_2} + 18$, $net_2 = \dfrac{\beta_1-\sqrt{\beta_1^2-4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2}$, $-1 < \hat{a}_1 < 0\}$ $\cup$ $\{\mathbf{x}|\ 18x_1 + 18x_2 = \dfrac{-\beta_1+\sqrt{\beta_1^2-4\hat{y}\beta_2}}{2\beta_2} + 18$, $-18x_1+18x_2 \le -\kappa + 18\}$. Hence Xa($\hat{y}$) consists of three curve segments and two half-lines.

6. For any $\hat{y} \in (0, 1)$, Xa(y) equals $\{\mathbf{x}|\ 18x_1 + 18x_2 = \dfrac{\beta_1-\sqrt{\beta_1^2+4\hat{y}\beta_2}}{2\beta_2} + 18$, $net_2 = \dfrac{\beta_1-\sqrt{\beta_1^2-4(-\hat{y}-1-\hat{a}_1)\beta_2}}{2\beta_2}$, $-1 < \hat{a}_1 < 0\}$ $\cup$ $\{\mathbf{x}|\ -18x_1 + 18x_2 = \dfrac{\beta_1-\sqrt{\beta_1^2+4\hat{y}\beta_2}}{2\beta_2} + 18$, $18x_1+18x_2-18 \le -\kappa\}$ $\cup$ $\{\mathbf{x}|\ 18x_1 + 18x_2 = \dfrac{\beta_1-\sqrt{\beta_1^2+4\hat{y}\beta_2}}{2\beta_2} + 18$, $-18x_1+18x_2-18 \le -\kappa\}$), which consists of a curve segment and two half-lines.
7. Xa(1) equals $\{\mathbf{x}|\ 18x_1+18x_2 \le -\kappa+18,\ -18x_1+18x_2 \le -\kappa+18\}$, which is an open polyhedron in the input space.

Let's take $\hat{y}$ = -2.5, $\hat{y}$ = -1.5, $\hat{y}$ = -1, $\hat{y}$ = -0.5, $\hat{y}$ = 0.5 as examples in Figure 12.

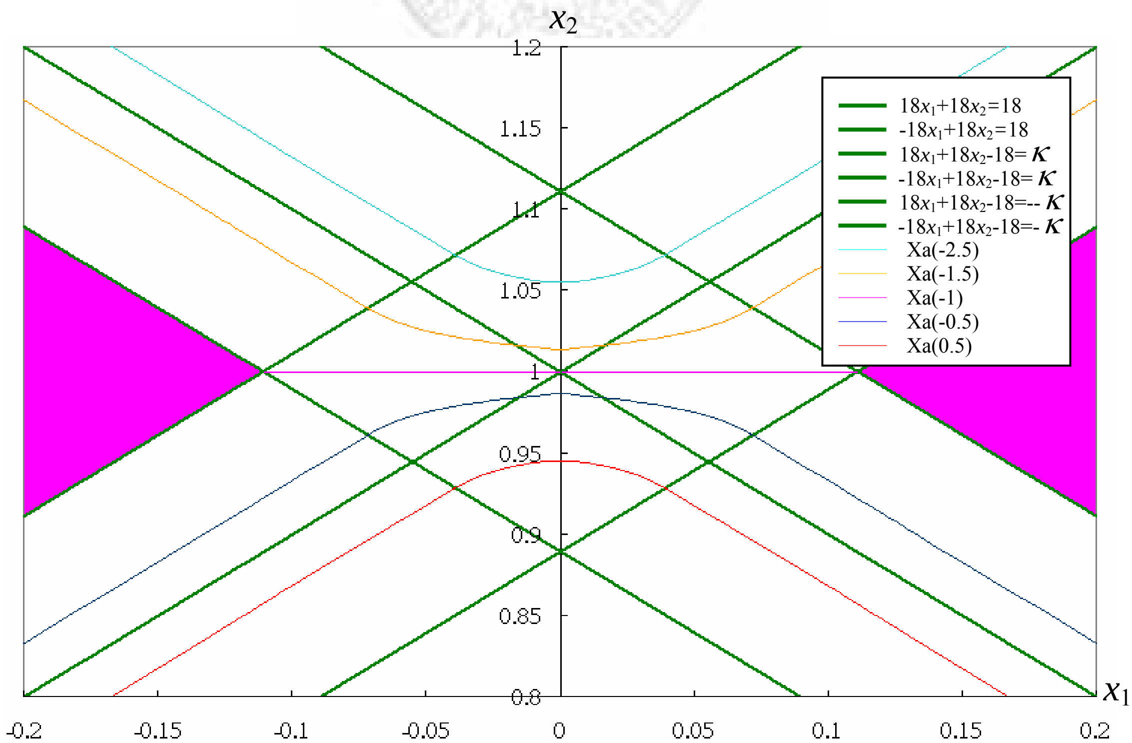

Figure 12: The observation of Xa($\hat{y}$) in Network III.