4. The Definition and Representation of Feed-forward Neural Networks

The layered feed-forward neural network includes one hidden layer, one input layer and one output node. In Figure 3, it is the framework of the feed-forward neural networks. The definition of neural network f is composed from the following equations (6) and (7), where $tanh(x) \equiv \frac{e^x - e^{-x}}{e^x + e^{-x}}$, m is the number of explanatory variables x_j's, p is the number of adopted hidden nodes, ${}_2\theta_{i0}$ is the bias value of the i^{th} hidden node a_i, ${}_2w_{ij}$ is the weight between the j^{th} explanatory variable x_j and the i^{th} hidden node a_i, ${}_3\theta_0$ is the bias value, ${}_3w_i$ is the weight between the i^{th} hidden node a_i and the output node, and y is the output value of the feed-forward neural networks which equals $f(\mathbf{x})$.

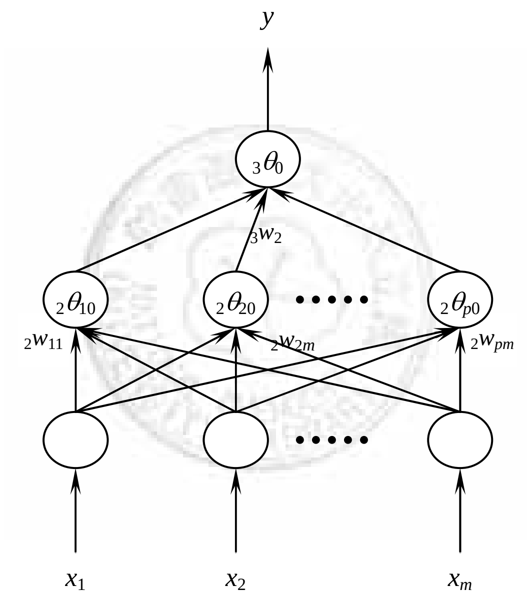

Figure 3: The framework of feed-forward neural network.

$$a_i(\mathbf{x}) \equiv tanh\ ({}_2\theta_{i0} + \sum_{j=1}^{m} {}_2w_{ij}\ x_j)\ , \tag{6}$$

$$f(\mathbf{x}) \equiv {}_3\theta_0 + \sum_{i=1}^{p} {}_3w_i\ a_i(\mathbf{x}) = {}_3\theta_0 + \sum_{i=1}^{p} {}_3w_i\ tanh\ ({}_2\theta_{i0} + \sum_{j=1}^{m} {}_2w_{ij}\ x_j). \tag{7}$$

Let net_i to be the net input of the i^{th} hidden node, and the definition of net_i is defined as follows (8):

$$net_i := {}_2\theta_{i0} + \sum_{j=1}^{m} {}_2w_{ij}\, x_j \tag{8}$$

Then, if $tanh(x)$ is given the b^{th} observation ${}_b\mathbf{x}$, the corresponding value of the i^{th} hidden node ${}_ba_i$ is $tanh(net_i)$, and the corresponding value of $f({}_b\mathbf{x})$ is ${}_3\theta_0 + \sum_{i=1}^{p} {}_3w_i\, {}_ba_i$.