

2. The model

In this section, we set up a model for the stock market involving three types of agents: the firm sector, the long-term shareholders, and the rational speculators. The firm sector offers the stock for sale in the stock market in order to raise money for further production plans, while the shareholders play a role as the stock demander paid with the dividends. The rational speculators in our model represent a broad class of agents who exploit the stock price changes to make profit. They sell the stock when expecting its price to fall and buy the stock when expecting a rising price. Therefore, the rational speculators can be viewed as not only stock suppliers but also stock demanders. After the description of those agents, we will describe the market-clearing condition and finish this section with a solution to this model.

2.1 The firm sector

Business activity depends on the availability of goods and services under the control of business management. When capital expenditure exceeds internal savings or retained income, firms often resort to finance their assets by issuing the stock. In other words, the firm sector tends to be the supplier of financial assets in the economy system.

In the following part, we will describe the relationship between the firm's production behavior and the stock market with the "Tobin's q" theory. According to Tobin (1969), the q ratio is obtained by dividing the market value of capital by its replacement cost. If the q ratio is greater than 1, firms will increase capital to make further production, and vice versa. In our model, we assume that the replacement cost of capital is fixed and issuing the stock is the only way for firms to raise money. In

this case, when the market value of capital gets higher, firms will issue more stocks because of their needs for further production.

Now let us consider the relationship between the market value of capital and the stock price. As we know, the stock price generally reflects the present value of the firm's expected stream of earnings yielded by the capital. From this point of view, we could say that a higher stock price implies a higher market value of capital and therefore cause the firm to issue more stocks. In other words, the higher the stock price is, the more quantity of the stock firms will issue. As a result, the supply function of the stock can be expressed as follows:

$$Q_t^s = \gamma p_t + u_t, \quad \gamma > 0, \quad (2.1)$$

where p is the log form of the stock price P , and Q^s is the quantity supplied of the stock. As mentioned above, we can assure that $\gamma > 0$, which represents a positive connection between the quantity supplied of the stock and its price. Moreover, u_t denotes a random shock with $E(u_t) = 0$, $E(u_t^2) = \sigma_u^2$. It is called the issuing shock which summarizes all factors that alter the firms' requirement for funds, such as unanticipated industrial policy.

2.2 Long-term shareholders

In our model, the long-term shareholders are the main sources of funds in the stock market. These agents' need for the stock depends on the dividends rather than the speculation because their expertise in this area is limited. In this part, we'll represent these agents with an interpretation of their demand for the stock.

When the long-term shareholders buy the stock, it implies that they have a right

to claim on the proportional share of the firm's outputs. Therefore, let π be the profit distributed to each stock and is defined as: $\pi = \alpha'Y$, where Y denotes the output of a unit of capital, and α' denotes the proportion of the output Y ; that is, the larger α' becomes, the more proportion of output the shareholders will have.

Suppose that the rate of return of the dividends is d , then:

$$\frac{\alpha'Y}{P} = d. \quad (2.2)$$

Transforming Eq.(2.2) by linear approximation and express it in log forms¹, we have:

$$d_0(y - p) = d, \quad (2.3)$$

where y denotes the log form of Y , and $d_0 \equiv \alpha' \frac{Y_0}{P_0}$. (Y_0 and P_0 are the initial value in equilibrium of Y and P respectively.)

Further, let's consider the cost for the long-term shareholders to buy the stock. Here we suppose that they borrow money under the fixed interest rate level R by signing a contract to the bank in order to lock their risk at a fixed level, then the rate of return on the holding shares in period t is $d_0(y_t - p_t) - R$. Hence, the long-term

¹ See M. L. Chu and T. W. Tsaur (1990).

From Eq.(2.1), we can obtain $Pd = \alpha'Y$, expanding it in a Taylor series for first order around the initial value, then:

$$P_0(d - d_0) + d_0(P - P_0) = \alpha'(Y - Y_0),$$

dividing both side by P_0 , thus

$$(d - d_0) + d_0 \frac{(P - P_0)}{P_0} = \alpha' \frac{(Y - Y_0)}{P_0},$$

as in equilibrium, $P_0 d_0 = \alpha' Y_0$, therefore $\alpha' = P_0 d_0 / Y_0$, substituting it into the above equation, we obtain:

$$(d - d_0) + d_0 \frac{(P - P_0)}{P_0} = d_0 \frac{(Y - Y_0)}{Y_0},$$

integrating the above equation and expressing it in log forms, then it approximates to:

$$d = d_0(y - p) + (a + d_0), \text{ where } a \text{ is a constant;}$$

let a be $-d_0$, then $d = d_0(y - p)$ and $d_0 = \alpha' Y_0 / P_0$.

shareholders' demand for the stock is assumed to be a linear function of the rate of return:

$$Q_t^d = A[d_0(y_t - p_t) - R] = \alpha(y_t - p_t) - AR, \quad (2.4)$$

$$\alpha = Ad_0 = A \frac{\alpha Y_0}{P_0} > 0,$$

where A expresses the sensitivity of the stock demand to the rate of stock return; that is, if the long-term shareholders become more sensitive to the rate of stock return, A will become higher. We also assume that the output of a unit of capital at time t is $y_t = y_f + \varepsilon_t$, where y_f is the normal level of the firm's output, and ε_t denotes the the shock which affects the firm's output level (such as business cycle or wars) and then affects the dividend that is distributed to each stock with $E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma_\varepsilon^2$. Therefore, we call it the dividend shock. Then, we have:

$$Q_t^d = \alpha(y_f + \varepsilon_t - p_t) - AR, \quad (2.5)$$

The above equation shows that the stock demand varies with the unexpected shock ε_t , the model parameter α , and the exogenous variable y_f, R .

2.3 Rational speculators

In addition to the stock suppliers and the long-term creditors in the market, we posit the existence of the rational speculators who are fully informed and attempt to profit from expected changes in the stock price. These speculators buy the stock in period t at price p_t and hope to sell it for a profit in period $t+1$ at price p_{t+1} . Moreover, suppose the rational speculators use margin account to buy the stock and

have to pay the cost r_t , therefore the rate of return of a unit of the stock can be defined as:

$$p_{t+1} - p_t - r_t, \quad (2.6)$$

where $p_{t+1} - p_t$ denotes the capital gains that the speculator earns in each unit of the stock. In addition, let r_t be $\bar{r} + \Delta_t$, which is combined by two parts: the margin rate \bar{r} and the margin-rate shock Δ_t which represents the factors that change the margin rate with $E(\Delta_t) = 0$, $Var(\Delta_t) = \sigma_\Delta^2$. To earn these profits the speculator establish a position of size h_t in the period t , which is measured in units of the stock. Then the rate of stock return for the rational speculator can be expressed as:

$$\pi_{t+1} = h_t(p_{t+1} - p_t - \bar{r} - \Delta_t). \quad (2.7)$$

In addition, since the rational speculators involving in the short-term trading might cause short-term fluctuation in the stock price, the government has to take some actions to stabilize the stock market. In this case, suppose that the government puts a tax rate τ on the capital gains of the rational speculators to see whether the taxation stabilizes or destabilizes the market. Then, the rate of stock return of the rational speculator after taxation is:

$$\pi_{t+1} = h_t[(1 - \tau)(p_{t+1} - p_t) - \bar{r} - \Delta_t]. \quad (2.8)$$

Further, to obtain the optimal holding h_t , we have the following welfare function² that the speculators want to maximize:

$$W_t = E(\pi_{t+1}) - (\theta/2)Var(\pi_{t+1}), \quad (2.9)$$

where $E(\pi_{t+1})$ denotes the expected level, and $Var(\pi_{t+1})$ denotes the variance, of a speculator's next-period stock return conditional on the information available at time

² This welfare function is adopted from Carlson and Osler (2000).

t . The term θ is defined as a speculator's level of absolute risk aversion. Now substituting Eq.(2.8) into Eq.(2.9), we obtain:

$$W_t = h_t[(1-\tau)(E_t p_{t+1} - p_t) - \bar{r} - \Delta_t] - (\theta/2)h_t^2(1-\tau)^2 \text{Var}(p_{t+1}), \quad (2.10)$$

where $\text{Var}(p_{t+1})$ is the variance of the stock price conditional on the information at time t . The information set for period t includes the structural parameters of the model, σ_u^2 , σ_ε^2 , and σ_Δ^2 .

Maximization of the above expression with respect to h_t yields the first-order condition:

$$h_t = b_t \left[\frac{(E_t p_{t+1} - p_t)}{1-\tau} - \frac{(\bar{r} + \Delta_t)}{(1-\tau)^2} \right], \quad (2.11)$$

where

$$b_t = \frac{1}{\theta \text{Var}(p_{t+1})}. \quad (2.12)$$

From Eq.(2.12), we know that b_t is composed of θ and $\text{Var}(p_{t+1})$. In our analysis, we assume that θ is fixed. This indicates that b_t varies with $\text{Var}(p_{t+1})$. Given the value of structural parameters and the constant assumption of σ_u^2 , σ_ε^2 , and σ_Δ^2 , we can obtain a fixed value of $\text{Var}(p_{t+1})$ and therefore a constant b_t . Hence, a change in the structural parameter will cause a change in $\text{Var}(p_{t+1})$, and then a change in b_t .

2.4 Market Solution

Suppose that there are N speculators in the stock market, so their desired holdings of the stock can be written as: $Nh_t = I_t$. Now combining the stock purchases and sales of the speculators to the stock supplies and demands, we derive the period t

market-clearing condition:³

$$I_t + Q_t^d = I_{t-1} + Q_t^s. \quad (2.13)$$

Substituting Eq.(2.1), (2.5), and (2.11) into the above equation, we obtain:

$$\begin{aligned} & -\alpha y_f + AR + (\alpha + \gamma)p_t + u_t - \alpha \varepsilon_t \\ & = Nb_t \left[\frac{E_t p_{t+1} - p_t}{1 - \tau} - \frac{(\bar{r} + \Delta_t)}{(1 - \tau)^2} \right] - Nb_{t-1} \left[\frac{E_{t-1} p_t - p_{t-1}}{1 - \tau} - \frac{\bar{r}}{(1 - \tau)^2} \right]. \end{aligned} \quad (2.14)$$

To simplify our analysis, let N to be 1 and therefore we have Nb_t to be b_t . As we mentioned earlier, since σ_u^2 , σ_ε^2 , and σ_Δ^2 are all constant, b_t will be the same for any period of t if structural parameters don't change and hold constant. As a result, we can assume b_t to be a constant, b , and then Eq.(2.14) can be re-expressed as:

$$\left[\frac{-1}{(1 - \tau)} - \frac{\omega}{b} \right] p_t + \frac{1}{1 - \tau} [E_t p_{t+1} - E_{t-1} p_t + p_{t-1}] = -X_t, \quad (2.15)$$

where $X_t = -\frac{c}{b} - \frac{1}{b} u_t + \frac{\alpha}{b} \varepsilon_t - \frac{\Delta_t}{(1 - \tau)^2}$, $\alpha + \gamma = \omega$, and $c = -\alpha y_f + AR$.

To obtain the expression for p_t as a function of p_{t-1} and the unexpected shocks, we use the lag operator and substitute $E_{t-1} p_t = \lambda p_{t-1}$ into the above difference Eq.(2.15), then we have⁴:

$$p_t = \frac{(1 - \tau)^{-1}(1 - \lambda)}{(1 - \tau)^{-1}(1 - \lambda) + \frac{\omega}{b}} p_{t-1} + \frac{1}{(1 - \tau)^{-1}(1 - \lambda) + \frac{\omega}{b}} X_t. \quad (2.16)$$

Rational consistency therefore requires that:

$$\lambda = \frac{(1 - \tau)^{-1}(1 - \lambda)}{(1 - \tau)^{-1}(1 - \lambda) + \frac{\omega}{b}}, \quad (2.17)$$

³ This market-clearing condition is adopted from Muth (1961).

⁴ See appendix A1.

since $E_{t-1}p_t = \frac{(1-\tau)^{-1}(1-\lambda)}{(1-\tau)^{-1}(1-\lambda) + \frac{\omega}{b}} p_{t-1}$.

Eq.(2.17) is denoted RE as a reminder that it is a necessary condition for the expectation process to be rational. Rearrangement of RE equation allows it to be expressed as:

$$\frac{(1-\lambda)^2}{(1-\tau)} = \frac{\lambda\omega}{b}. \quad (2.18)$$

Taking the smaller root of the above characteristic equation, we obtain a positive fraction λ . Then, we combine Eq.(2.16) and Eq.(2.17) to obtain our main expression for the stock price's dynamics⁵:

$$p_t = \lambda p_{t-1} + (1-\lambda)\bar{p} - \frac{1-\lambda}{\omega} u_t + \frac{\alpha(1-\lambda)}{\omega} \varepsilon_t - \frac{\lambda}{(1-\lambda)(1-\tau)} \Delta_t, \quad (2.19)$$

where \bar{p} is the long-run equilibrium value of the stock price and $\bar{p} = \frac{-c}{\omega}$.

⁵ See appendix A2.