Appendix

Appendix A1: The equation of the stock price.

The solution begins with Eq. (2.15) from the text, which we repeat here for convenience as Eq.(A.1):

$$[\frac{-1}{(1-\tau)}-\frac{\omega}{b}]p_t+\frac{1}{1-\tau}[E_t p_{t+1}-E_{t-1}p_t+p_{t-1}]=-X_t\,, \tag{A.1}$$

where $X_t=-\frac{c}{b}-\frac{1}{b}u_t+\frac{\alpha}{b}\varepsilon_t-\frac{\Delta_t}{(1-\tau)^2}$, $\alpha+\gamma=\omega$, and $c=-\alpha y_f+AR$.

Take expectations of Eq. (A.1) as of time $t-1$, we obtain:

$$[\frac{-2}{(1-\tau)}-\frac{\omega}{b}]E_{t-1}p_t+\frac{1}{1-\tau}(E_{t-1}p_{t+1}+E_{t-1}p_{t-1})=-E_{t-1}X_t\,. \tag{A.2}$$

This expression is a difference equation in the expectation terms. Use of the lag operator L allows the substitute:

$$E_{t-1}p_{t-1}=LE_{t-1}p_t\,,\qquad E_{t-1}p_{t+1}=L^{-1}E_{t-1}P_t\,. \tag{A.3}$$

We therefore obtain:

$$\left\{L^2-[2+\frac{\omega(1-\tau)}{b}]L+1\right\}E_{t-1}P_t=0 \tag{A.4}$$

Therefore, either we require that $E_{t-1}P_t=0$ or we require that the quadratic expression in L be identically equal to zero. As a result, a nontrivial solution requires that:

$$L^2-[2+\frac{\omega(1-\tau)}{b}]L+1=0\,. \tag{A.5}$$

Denote the roots of this quadratic equation by λ_i, $i=1,2$, then:

$$\lambda_1\lambda_2 = 1 \quad \text{and} \quad \lambda_1 + \lambda_2 = 2 + \frac{\omega(1-\tau)}{b} > 2\,. \tag{A.6}$$

From Eq. (A.6), we can assure that one root is greater than 1 and the other root is less than1. Furthermore, the general solution to Eq. (A.1) is given by:[1]

$$E_{t-1}P_{t+j} = c\lambda_1^j + d\lambda_2^j\,, \tag{A.7}$$

with c and d determined by an initial-value condition. In the special case of $E_{t-1}P_{t-1} = P_{t-1}$, we can therefore substitute $j = -1$ into Eq. (A.7) and obtain:

$$P_{t-1} = c\lambda_1^{-1} + d\lambda_2^{-1}\,. \tag{A.8}$$

The constant (c or d) corresponding to the greater root is assumed to be equal to zero to ensure a convergent system. Now define $\lambda = \lambda_1 < \lambda_2$, where λ_1 is selected as the smaller of the two roots[2]. In this case, d is supposed to be zero so that Eq. (A.8) is express as:

$$E_{t-1}P_{t-1} = P_{t-1} = c\lambda^{-1}\,. \tag{A.9}$$

We can re-express it as: $c = \lambda P_{t-1}$. Therefore, the rational price expectation process is given by:

$$E_{t-1}P_{t+j} = \lambda^{j+1}P_{t-1}\,, \tag{A.10}$$

and so

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1 See Sargent (1987, Ch.9).

2 In calculation, we adopt the "backward" method to obtain $0 < \lambda_1 < 1$ and $\lambda_2 > 1$, and then select $\lambda_1 = \lambda$ to ensure a convergent system. However, if the "forward" method is adopted, we may obtain another two roots: $0 < \dfrac{1}{\lambda_2} < 1$ and $\dfrac{1}{\lambda_1} > 1$, and then choose $\dfrac{1}{\lambda_2} = \lambda$ to ensure a convergent system. In our following analysis, we only use the property of $0 < \lambda < 1$, rather than the value of λ. Therefore, either backward solution or forward solution will lead to the same conclusions.

$$E_{t-1}P_t = \lambda P_{t-1} \quad \text{and} \quad E_t P_{t+1} = \lambda P_t . \tag{A.11}$$

Substitute Eq. (A.11) into Eq. (A.1), we obtain:

$$p_t = \frac{(1-\tau)^{-1}(1-\lambda)}{(1-\tau)^{-1}(1-\lambda)+\frac{\omega}{b}} p_{t-1} + \frac{1}{(1-\tau)^{-1}(1-\lambda)+\frac{\omega}{b}} X_t . \tag{A.12}$$

Appendix A2: The dynamic equation of the stock price.

To obtain the stock price's dynamic equation, we first have to compute the long-run equilibrium $\overline{p}$. In the long run, all unexpected shocks will not exist anymore and the stock price in each period will be equal to the long-run equilibrium stock price. Judging from the above, we therefore set $u_t = \varepsilon_t = \Delta_t = 0$, and replace p_t and p_{t-1} by $\overline{p}$ in Eq.(A.1) to obtain:

$$\overline{p} = \frac{-1}{\omega}(-\alpha y_f + AR) = \frac{-c}{\omega}, \tag{A.13}$$

where $c = -\alpha y_f + AR$.

In addition, we can rewrite the RE equation as follows:

$$\frac{(1-\lambda)^2}{1-\tau} = \frac{\lambda\omega}{b} . \tag{A.14}$$

Substitute Eq.(A.13) and Eq.(A.14) into Eq.(A.1), we obtain:

$$p_t = \lambda p_{t-1} + \frac{\lambda(1-\tau)}{1-\lambda}\left[\frac{-c}{b} - \frac{1}{b}u_t + \frac{\alpha}{b}\varepsilon_t - \frac{\Delta_t}{(1-\tau)^2}\right]$$

$$= \lambda p_{t-1} + (1-\lambda)\overline{q} - \frac{(1-\lambda)}{\omega}u_t + \frac{\alpha(1-\lambda)}{\omega}\varepsilon_t - \frac{\lambda}{(1-\lambda)(1-\tau)}\Delta_t . \tag{A.15}$$

Appendix A3: Derivation of $\frac{d\lambda}{d\tau}$.

From Eq.(3.3) and Eq.(3.4), we have:

$$b(1-\tau)^{-1}(1-\lambda)^2 = \lambda\omega \quad , \qquad \text{(RE)} \qquad \text{(A.16)}$$

$$b(1-\tau)^{-1}(1-\lambda)^2 = \frac{(1-\tau)^{-1}(1-\lambda)^2}{\theta Var(p_{t+1})} = f \,. \qquad \text{(SB)} \qquad \text{(A.17)}$$

Differentiating the above two expressions with respect to b, λ, τ, and then expressing them in matrix notation, we obtain:

$$\begin{bmatrix} (1-\tau)^{-1}(1-\lambda)^2 & -2b(1-\tau)^{-1}(1-\lambda)-\omega \\ (1-\tau)^{-1}(1-\lambda)^2 & -2b(1-\tau)^{-1}(1-\lambda)-f_\lambda \end{bmatrix} \cdot \begin{bmatrix} db \\ d\lambda \end{bmatrix} = \begin{bmatrix} -b(1-\tau)^{-2}(1-\lambda)^2 d\tau \\ \{-b(1-\tau)^{-2}(1-\lambda)^2 + f_\tau\}d\tau \end{bmatrix},$$

(A.18)

where f denotes the SB equation:

$$f = \frac{(1-\lambda)^2}{\theta(1-\tau)\{\frac{(1-\lambda)^2}{\omega^2}\cdot Var(u) + \frac{\alpha^2(1-\lambda)^2}{\omega^2}\cdot Var(\varepsilon) + \frac{\lambda^2}{(1-\lambda)^2(1-\tau)^2}\cdot Var(\Delta)},$$

(A.19)

f_λ denotes the partial derivative of f with respect to λ:

$$f_\lambda = \frac{-1}{\theta(1-\tau)[\frac{1}{\omega^2}\cdot Var(u) + \frac{\alpha^2}{\omega^2}\cdot Var(\varepsilon) + \frac{\lambda^2}{(1-\lambda)^4(1-\tau)^2}\cdot Var(\Delta)]^2} \cdot$$

$$\frac{2\lambda(1+\lambda)}{(1-\lambda)^5(1-\tau)^2}\cdot Var(\Delta)\,, \qquad \text{(A.20)}$$

and f_τ denotes the partial derivative of f with respect to τ:

$$f_\tau = \frac{-1}{\theta[\frac{(1-\tau)}{\omega^2}Var(u) + \frac{\alpha^2(1-\tau)}{\omega^2}Var(\varepsilon) + \frac{\lambda^2}{(1-\lambda)^4(1-\tau)^2}Var(\Delta)]^{-2}} \cdot$$
$$[\frac{-1}{\omega^2}Var(u) - \frac{\alpha^2}{\omega^2}Var(\varepsilon) + \frac{\lambda^2}{(1-\lambda)^4(1-\tau)^2}Var(\Delta)]. \tag{A.21}$$

Thus, application of Cramer's rule yields derivation of:

$$\frac{d\lambda}{d\tau} = \frac{f_\tau}{\omega - f_\lambda}. \tag{A.22}$$

Appendix A4: Determination of $\frac{d\lambda}{d\tau}$ ***under different types of shocks.***

(1) The issuing shock

Substituting $Var(\varepsilon) = Var(\Delta) = 0$ into f_λ and f_τ, we have:

$$f_\lambda = 0, \text{ and } f_\tau = \frac{1}{\theta(1-\tau)^2 \cdot \frac{1}{\omega^2} \cdot Var(u)} > 0. \tag{A.23}$$

Therefore,

$$\frac{d\lambda}{d\tau} = \frac{f_\tau}{\omega - f_\lambda} = \frac{1}{\theta(1-\tau)^2 \cdot \frac{1}{\omega} \cdot Var(u)} > 0. \tag{A.24}$$

(2) The dividend shock

Substituting $Var(u) = Var(\Delta) = 0$ into f_λ and f_τ, we have:

$$f_\lambda = 0, \text{ and } f_\tau = \frac{1}{\theta(1-\tau)^2 \cdot \frac{\alpha^2}{\omega^2} \cdot Var(\varepsilon)} > 0. \tag{A.25}$$

Therefore,

$$\frac{d\lambda}{d\tau} = \frac{f_\tau}{\omega - f_\lambda} = \frac{1}{\theta(1-\tau)^2 \cdot \frac{\alpha^2}{\omega} \cdot Var(\varepsilon)} > 0\,. \qquad \text{(A.26)}$$

(3) The margin-rate shock

Substituting $Var(u) = Var(\varepsilon) = 0$ into f_λ and f_τ, we have:

$$f_\lambda = \frac{-2(1-\tau)(1-\lambda)^3(1+\lambda)}{\theta\lambda^3 Var(\Delta)} < 0, \text{ and } f_\tau = \frac{-(1-\lambda)^4}{\theta\lambda^2 Var(\Delta)} < 0\,. \qquad \text{(A.27)}$$

Therefore,

$$\frac{d\lambda}{d\tau} = \frac{f_\tau}{\omega - f_\lambda} = \frac{-(1-\lambda)^4}{\theta\omega\lambda^2 Var(\Delta) + \frac{2(1-\tau)(1-\lambda)^3(1+\lambda)}{\lambda}} < 0\,. \qquad \text{(A.28)}$$

Appendix A5: Determination of $\frac{dh_t}{d\tau}$ ***under different types of shocks.***

(1) The issuing shock

From Eq.(3.7) and Eq.(3.8), we have:

$$h_\tau = \frac{(1-\lambda)}{\theta Var(p_{t+1})(1-\tau)} \cdot (\overline{p} - p_t) = \frac{(1-\lambda)^2 u_t}{\theta\omega(1-\tau)Var(p_{t+1})}\,. \qquad \text{(A.29)}$$

Thus, differentiating h_τ with respect to τ, we obtain:

$$\frac{dh_t}{d\tau} = \frac{u_t}{\theta\omega} \cdot \frac{d}{d\tau}\left[\frac{1}{Var(p_{t+1})} \cdot \frac{(1-\lambda)^2}{1-\tau}\right]$$

$$= \frac{u_t}{\omega\theta} \cdot \left\{\frac{-1}{Var^2(p_{t+1})} \cdot \frac{dVar(p_{t+1})}{d\tau} \cdot \frac{(1-\lambda)^2}{1-\tau} + \frac{1}{Var(p_{t+1})} \cdot \left[-2\frac{1-\lambda}{1-\tau} \cdot \frac{d\lambda}{d\tau} + \frac{(1-\lambda)^2}{(1-\tau)^2}\right]\right\}$$

$$= \frac{u_t}{\omega\theta \cdot \frac{(1-\lambda)^2 Var(u)}{\omega^2}} \cdot \left\{\frac{-1}{\frac{(1-\lambda)^2}{\omega^2} \cdot Var(u)} \cdot \frac{-2(1-\lambda)}{\omega^2} \cdot \frac{d\lambda}{d\tau} \cdot Var(u) \cdot \frac{(1-\lambda)^2}{1-\tau}\right.$$

$$-\frac{2(1-\lambda)}{1-\tau}\cdot\frac{d\lambda}{d\tau}+\frac{(1-\lambda)^2}{(1-\tau)^2}\}$$

$$= \frac{\omega\cdot u}{(1-\tau)^2 Var(u)} > 0\,. \qquad \text{(A.30)}$$

(2) The dividend shock

From Eq.(3.11) and Eq.(3.12), we have:

$$h_t = \frac{(1-\lambda)}{\theta Var(p_{t+1})(1-\tau)}\cdot(\overline{p}-p_t) = \frac{-\alpha(1-\lambda)^2\varepsilon_t}{\theta\omega(1-\tau)Var(p_{t+1})}\,. \qquad \text{(A.31)}$$

Thus, differentiating h_τ with respect to τ, we obtain:

$$\frac{dh_t}{d\tau} = \frac{-\alpha\varepsilon_t}{\omega\theta}\cdot\frac{d}{d\tau}[\frac{1}{Var(p_{t+1})}\cdot\frac{(1-\lambda)^2}{1-\tau}]$$

$$= \frac{-\alpha\varepsilon_t}{\omega\theta}\cdot\{\frac{-1}{Var^2(p_{t+1})}\cdot\frac{dVar(p_{t+1})}{d\tau}\cdot\frac{(1-\lambda)^2}{1-\tau}+\frac{1}{Var(p_{t+1})}\cdot[-2\frac{1-\lambda}{1-\tau}\cdot\frac{d\lambda}{d\tau}+\frac{(1-\lambda)^2}{(1-\tau)^2}]\}$$

$$= \frac{-\alpha\varepsilon_t}{\omega\theta\cdot\frac{\alpha^2(1-\lambda)^2 Var(\varepsilon)}{\omega^2}}\cdot\{\frac{-1}{\frac{\alpha^2(1-\lambda)^2}{\omega^2}\cdot Var(\varepsilon)}\cdot\frac{-2\alpha^2(1-\lambda)}{\omega^2}\cdot\frac{d\lambda}{d\tau}\cdot Var(\varepsilon)\cdot\frac{(1-\lambda)^2}{1-\tau}$$

$$-\frac{2(1-\lambda)}{1-\tau}\cdot\frac{d\lambda}{d\tau}+\frac{(1-\lambda)^2}{(1-\tau)^2}\}$$

$$= \frac{-\omega\varepsilon}{\alpha(1-\tau)^2 Var(\varepsilon)} < 0\,. \qquad \text{(A.32)}$$

(3) The margin-rate shock

From Eq.(3.16) and Eq.(3.21) , we have:

$$h_t = \frac{-(1-\lambda)^2}{\theta Var(p_{t+1})\lambda(1-\tau)}\cdot(\overline{p}-p_t) = \frac{(\lambda-1)\Delta_t}{\theta(1-\tau)^2 Var(p_{t+1})}\,. \qquad \text{(A.33)}$$

Similarly, let us differentiating the above expression with respect to τ, we obtain:

$$\frac{dh_t}{d\tau} = \frac{\Delta_t}{\theta}\cdot\frac{d}{d\tau}[\frac{1}{\mathrm{var}(p_{t+1})}\cdot\frac{\lambda-1}{(1-\tau)^2}]$$

$$= \frac{\Delta_t}{\theta} \cdot \{ \frac{-1}{Var^2(p_{t+1})} \cdot \frac{dVar(p_{t+1})}{d\tau} \cdot \frac{\lambda - 1}{(1-\tau)^2} + \frac{1}{Var(p_{t+1})} [\frac{1}{(1-\tau)^2} \cdot \frac{d\lambda}{d\tau} + \frac{2(\lambda - 1)}{(1-\tau)^3}]\}$$

$$= \frac{\Delta_t}{\theta \cdot \frac{\lambda^2 Var(\Delta)}{(1-\lambda)^2(1-\tau)^2}} \cdot \{ \frac{-1}{\frac{\lambda^2}{(1-\lambda)^2(1-\tau)^2} \cdot Var(\Delta)} \cdot [\frac{2\lambda^2}{(1-\lambda)^2(1-\tau)^3} + \frac{2\lambda}{(1-\lambda)^3(1-\tau)^2} \cdot \frac{d\lambda}{d\tau}] Var(\Delta) \cdot$$
$$\frac{\lambda - 1}{(1-\tau)^2} + \frac{1}{(1-\tau)^2} \cdot \frac{d\lambda}{d\tau} + \frac{2(\lambda - 1)}{(1-\tau)^3}\}$$

$$= \frac{\Delta_t}{\theta Var(p_{t+1})} \cdot \frac{\lambda + 2}{\lambda(1-\tau)^2} \cdot \frac{d\lambda}{d\tau} < 0 , \qquad \text{(A.34)}$$

since $\frac{d\lambda}{d\tau} < 0$ in this case.