4.0 BELIEF UPDATING

4.1 CHANGE OF IDEOLOGY

After having the one-shot equilibrium, we are now able to see more. So far the choice media firms face is a one shot choice. If we allow citizens to update their ideology according to the news they receive, the degree of media bias may get worse. In this way, we can go further to see what will happen with possibility of belief updating.

Before turning to a mathematical description, a few remarks should be made. First, imagine someone who has chosen firm X, as time goes by, his or her ideology will more and more close to X, let alone the speed for the moment. Second, for those who have more extreme ideologies, it is more difficult for them to be influenced.[1] The following analysis is based on these two ideas.

[1] Here we regard positions farther from the center as more extreme ideologies. Related discussions are shown in Section 5.

Let us make some change to the basic model. Assume that firm X locates on x_t and firm Y locates on y_t in period t (both are best response). Like section 3.2, because we focus on the polarization case, $x_t < y_t$. For citizens' choices, once the choices have been made they cannot be changed until next period. After reading firm X's report, some of X's readers will take one step near to x_t, and the rest stay the same. Similarly, some readers of Y will move one step near to y_t, and the others do not change. Knowing the updating way of readers, in period $t+1$, each media firm can move one step toward the right or the left or stay.

In order to simplify the computation, let us replace the original continuous distribution with a discrete distribution. That is, suppose that the distribution of citizens, θ, is divided into n segments. Moreover, suppose that there exists a location function, $l(r)$, which defines

$$l(r) = s \text{ if } \frac{s-1}{n} \leq r < \frac{s}{n},\ s \in \{1, 2, ..., n\},\ n \in N,$$

where $0 \leq r \leq 1$ representing the ideological position of citizens in the continuous case. According to this definition, let the best stand of firm X in period t be k^*, and the best stand of firm Y in period t be $k^{*\prime}$, that is, $l(x_t) = k^*$, $l(y_t) = k^{*\prime}$. Because we assume that $x_t < y_t$ and $\theta(\cdot)$ is symmetric, we may have that $k^* < k^{*\prime}$ and $k^{*\prime} = n - k^*$.

Next, let us introduce the way of ideology updating.[2] In period t, individual i's location is i_t. First, see the individual i on the left of X, e.g., more liberal, than firm X's stand. If $l(i_t) = k^* - m$, $1 \leq m \leq k^* - 1$, then

$$l(i_{t+1}) = \begin{cases} k^* - m + 1 & \text{w.p. } p_m \\ \\ k^* - m & \text{w.p. } 1 - p_m \end{cases},$$

where $p_m > p_{m'}$ if $m < m'$. For the individual i on the right of X, if $l(i_t) = k^* + m$, $1 \leq m \leq \frac{k^* + k^{*\prime}}{2} - k^*$,[3] then

$$l(i_{t+1}) = \begin{cases} k^* + m - 1 & \text{w.p. } q_m \\ \\ k^* + m & \text{w.p. } 1 - q_m \end{cases},$$

where $q_m > q_{m'}$ if $m < m'$. After reading X, for those who are more liberal, p_m is the probability of moving to the right for a unit. For those who are more conservative, q_m is the probability of moving to the left for a unit. The condition $p_m > p_{m'}$ and $q_m > q_{m'}$ if $m < m'$ means that the more close to X, the greater the probability of being influenced is. Furthermore, suppose that for readers lying in the opposite two sides at the same distance,

[2]For the reasons analogous to Ellison and Fudenberg (1993), we adopt a "rule-of-thumb" learning to replace the Bayesian learning.

[3]For the sake of simpicity, assume that if the distance between a consumer and firm Y is smaller than the distance from firm X, then this consumer will not buy news from firm X. Qualitatively, this assumption will not affect our result.

the influence power of firm X's report is weaker for those who with more extreme ideology. Hence, we may have

$$p_m < q_m.$$

Finally, assume that the greater the distance between any consumer and firm Y, the more sensitive to the movement of X. That is,

$$\begin{aligned} & \left| f\left(\left| h-k \right|, \left| h-k' \right| \right) - f\left(\left| h-k+1 \right|, \left| h-k' \right| \right) \right| \\ \geq \; & \left| f\left(\left| h'-k \right|, \left| h'-k' \right| \right) - f\left(\left| h'-k+1 \right|, \left| h'-k' \right| \right) \right| \end{aligned}$$

where $|h-k'| \geq |h'-k'|$. It accords with our intuition since if someone locates farther from Y, the influence power of firm Y is smaller to him or her, thus the influence power of firm X is greater.[4]

4.2 DYNAMIC ANALYSIS

Now we are interested in what media firms would behave while facing the updated distribution of ideology. We are going to begin with computing the new pmf, θ_{t+1}, in every location.

[4] Of course, this statement is not suitable for citizens $i > k^{*\prime}$. It has no problem to say that because we have excluded citizens not smaller than $\frac{k^{*}+k^{*\prime}}{2}$.

Then we have to solve the best responses by comparing the market shares of the media firms. Finally, to compare the media firms' stand before and after.

Still the same, by symmetric argument, we focus on firm X only. In period $t+1$,

$$\theta_{t+1}(h) = \begin{cases} \begin{array}{c} \theta_t(h)\left[1 - f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) \cdot p_{k^*-h}\right] \\ +\theta_t(h-1) \cdot f\left(\left|(h-1)-k^*\right|, \left|(h-1)-k^{*\prime}\right|\right) \cdot p_{k^*-(h-1)} \end{array} & h < k^* \\ \begin{array}{c} \theta_t(h)\left[1 - f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) \cdot q_{h-k^*}\right] \\ +\theta_t(h+1) \cdot f\left(\left|(h+1)-k^*\right|, \left|(h+1)-k^{*\prime}\right|\right) \cdot q_{(h+1)-k^*} \end{array} & h \geq k^* \end{cases},$$

At location h, on one hand, the term $\theta_t(h)\left[1 - f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) \cdot p_{k^*-h}\right]$ and the term $\theta_t(h)\left[1 - f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) \cdot q_{h-k^*}\right]$ represent original citizens minus readers switching after reading firm X's report. In other words, they are citizens did not buy any news in period t, plus citizens did not leave after reading the story firm X told. On the other hand, $\theta_t(h-1) \cdot f\left(\left|(h-1)-k^*\right|, \left|(h-1)-k^{*\prime}\right|\right) \cdot p_{k^*-(h-1)}$ and $\theta_t(h+1) \cdot f\left(\left|(h+1)-k^*\right|, \left|(h+1)-k^{*\prime}\right|\right) \cdot q_{(h+1)-k^*}$ represent citizens moving over from the next location.[5]

After knowing the pmf in every location, media firms can decide their ideological standing according to the market shares they will get. In period t, firm X stands on k^*. Its market

[5] In fact, at the position of both ends concerned, namely, $l = 1$ and $l = \frac{k+k'}{2}$, noboby will move over. However, qualitatively, it has only a minor effect to take them into account. Hence, for the sake of simplicity, it is not undue to omit its effect.

share is the summation of all pmf times their purchasing probability, that is,

$$\pi_t(x_t = k^*) = \sum_{h=1}^{\frac{k^*+k^{*\prime}}{2}} \theta_t(h) \cdot f(|h - k^*|, |h - k^{*\prime}|).$$

It is important to keep in mind that k^* is the best stand of firm X in period t. Thus we know that

$$\pi_t(x_t = k^*) \geq \pi_t(x_t = k^* - 1),$$

and

$$\pi_t(x_t = k^*) \geq \pi_t(x_t = k^* + 1).$$

To investigate how media firms would respond, all we have to do is to compare $\pi_{t+1}(x_{t+1} = k^*)$, $\pi_{t+1}(x_{t+1} = k^* - 1)$, and $\pi_{t+1}(x_{t+1} = k^* + 1)$. If $\pi_{t+1}(x_{t+1} = k^* - 1)$ is the greatest one, then we can come to a conclusion that media inclines to polarize more.

Proposition 3 Let $(k^*, n - k^*)$ be the equilibrium choices of both X and Y in Proposition 1 (without belief update). Consider the case that citizens update their beliefs. There exists $\overline{p}, \underline{q} \in (0,1)$ such that if $p_m \leq \overline{p}, q_{m'} \geq \underline{q}$ $(1 \leq m, m' \leq n)$, then the equilibrium choices $(k^{**}, n - k^{**})$ of X and Y has the property that $k^{**} < k^*$.

Proof. Suppose at some period t that $x_t = k^*$, $y_t = k^{*\prime} = n - k^*$.

$$
\begin{aligned}
&\pi_{t+1}\left(x_{t+1} = k^*\right) - \pi_{t+1}\left(x_{t+1} = k^* - 1\right) \\
= &\sum_{h=1}^{\frac{k^*+k^{*\prime}}{2}} \theta_{t+1}(h) f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - \sum_{h=1}^{\frac{k^*-1+k^{*\prime}}{2}} \theta_{t+1}(h) f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right) \\
= &\sum_{h=1}^{\frac{k^*+k^{*\prime}}{2}} \theta_{t+1}(h) \cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right] \\
= &\sum_{h=1}^{k^*-1} \theta_{t+1}(h) \cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right] \\
&+ \sum_{h=k^*}^{\frac{k^*+k^{*\prime}}{2}} \theta_{t+1}(h) \cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right] \\
= &\sum_{h=1}^{k^*-1} \begin{bmatrix} \theta_t(h) - \theta_t(h) f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) p_{k^*-h} \\ +\theta_t(h-1) f\left(\left|(h-1)-k^*\right|, \left|(h-1)-k^{*\prime}\right|\right) p_{k^*-(h-1)} \end{bmatrix} \\
&\cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right] \\
&+ \sum_{h=k^*}^{\frac{k^*+k^{*\prime}}{2}} \begin{bmatrix} \theta_t(h) - \theta_t(h) f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) q_{k^*-h} \\ +\theta_t(h+1) f\left(\left|(h+1)-k^*\right|, \left|(h+1)-k^{*\prime}\right|\right) q_{(h+1)-k^*} \end{bmatrix} \\
&\cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right] \\
= &\sum_{h=1}^{k^*-1} \begin{bmatrix} \theta_t(h-1) f\left(\left|(h-1)-k^*\right|, \left|(h-1)-k^{*\prime}\right|\right) p_{k^*-(h-1)} \\ +\theta_t(h)\left[1 - f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) p_{k^*-h}\right] \end{bmatrix} \\
&\cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right] \\
&+ \sum_{h=k^*}^{\frac{k^*+k^{*\prime}}{2}} \begin{bmatrix} \theta_t(h+1) f\left(\left|(h+1)-k^*\right|, \left|(h+1)-k^{*\prime}\right|\right) q_{(h+1)-k^*} \\ +\theta_t(h)\left[1 - f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) q_{k^*-h}\right] \end{bmatrix} \\
&\cdot \left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right].
\end{aligned}
$$

[6]Let

$$
\begin{aligned}
A(h) &= \left.\left\{\begin{array}{c} \theta_t(h-1) f\left(\left|(h-1)-k^*\right|, \left|(h-1)-k^{*\prime}\right|\right) p_{k^*-(h-1)} \\ \\ +\theta_t(h)\left[1-f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) p_{k^*-h}\right] \end{array}\right\}\right|_{h=1,2,\ldots,k^*-1}, \\
B(h) &= \left.\left\{\begin{array}{c} \theta_t(h+1) f\left(\left|(h+1)-k^*\right|, \left|(h+1)-k^{*\prime}\right|\right) q_{(h+1)-k^*} \\ \\ +\theta_t(h)\left[1-f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) q_{k^*-h}\right] \end{array}\right\}\right|_{h=k^*,k^*+1,\ldots,\frac{k^*+k^{*\prime}}{2}}, \\
C(h) &= \left.\left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right]\right|_{h=1,2,\ldots,k^*-1}, \\
D(h) &= \left.\left[f\left(\left|h-k^*\right|, \left|h-k^{*\prime}\right|\right) - f\left(\left|h-(k^*-1)\right|, \left|h-k^{*\prime}\right|\right)\right]\right|_{h=k^*,k^*+1,\ldots,\frac{k^*+k^{*\prime}}{2}}.
\end{aligned}
$$

Among them, $A(h) > 0$ and $B(h) > 0$. $C(h) < 0$ because for citizens left to x_t, the purchasing probability is greater when X moves to left. On the contrary, $D(h) > 0$ because for citizens right to x_t, X's left move increase the distance between them, thus reduce the purchasing probability. Note that $|C(k^* - m)| > |D(k^* - m)|$ because we have assumed that the influence power of firm X is greater when a citizen is farther from Y. Therefore our question has been simplified into comparing A and B. If $A(k^* - m)$ is greater than

[6]In fact, the number of items should be revised when combining $\pi_{t+1}(x_{t+1} = k)$ and $\pi_{t+1}(x_{t+1} = k - 1)$ into the same summation. But there is only an insignificant effect to ignore the error here as long as n is large enough.

$B(k^*+m)$, combined with $|C(k^*-m)|>|D(k^*+m)|$, then the sign of $\pi_{t+1}(x_{t+1}=k^*)-\pi_{t+1}(x_{t+1}=k^*-1)$ can have been affirmed.[7]

Remember the assumption $p_m<q_m$. Think about the most extreme example first. If $p\to 0$ and $q\to 1$, then A becomes $\theta_t(h)$, B becomes $\theta_t(h+1)f(|(h+1)-k^*|,|(h+1)-k^{*\prime}|)+\theta_t(h)[1-f(|h-k^*|,|h-k^{*\prime}|)]$. Hence the difference of market shares becomes

$$
\begin{aligned}
&\pi_{t+1}(x_{t+1}=k^*)-\pi_{t+1}(x_{t+1}=k^*-1)\\
=&\sum_{h=1}^{k^*-1}\theta_t(h)\cdot[f(|h-k^*|,|h-k^{*\prime}|)-f(|h-(k^*-1)|,|h-k^{*\prime}|)]\\
&+\sum_{h=k^*}^{\frac{k^*+k^{*\prime}}{2}}\left[\begin{array}{c}\theta_t(h+1)f(|(h+1)-k^*|,|(h+1)-k^{*\prime}|)\\ \\ +\theta_t(h)[1-f(|h-k^*|,|h-k^{*\prime}|)]\end{array}\right]\\
&\cdot[f(|h-k^*|,|h-k^{*\prime}|)-f(|h-(k^*-1)|,|h-k^{*\prime}|)].
\end{aligned}
$$

Let us look at B. Since $\theta_t(h+1)$ will be less than or equal to $\theta_t(h)$ eventually, and $f(|(h+1)-k^*|,|(h+1)-k^{*\prime}|)<f(|h-k^*|,|h-k^{*\prime}|)$ when $h\geq k^*$, we have $B<A$ when $p\to 0$ and $q\to 1$. As a result, we can definitely conclude that there exists an upper bound of p and a lower bound of q, namely $\overline{p}$ and $\underline{q}$, and $\overline{p},\underline{q}\in(0,1)$ such that if $p_m\leq\overline{p}$ and $q_{m'}\geq\underline{q}$ $(1\leq m,m'\leq n)$, then $B<A$, thus $\pi_{t+1}(x_{t+1}=k^*)-\pi_{t+1}(x_{t+1}=k^*-1)<0$. By the same

[7] We should notice that this statement holds only when the numbers of items are not widely different, otherwise the inequality would be no longer tenable.

computations we can also know that $\pi_{t+1}(x_{t+1} = k^* + 1) - \pi_{t+1}(x_{t+1} = k^* - 1) < 0$. We omit this part as it is similar to the above arguments. Thus we have a conclusion which means that the choice of firm X in period $t+1$, k^{**}, will lie on a position left of k^*. Q.E.D. $\square$