數學附錄

A. 本國廠商成本函數

本國廠商i的成本函數求法為在固定生產成本c_i^h為限制式下，極大化本國廠商i產出的一階條件，將求得的生產要素函數代回限制式c_i^h，求導過程如下：

$$\max\ x_i(k_i^h, k_i^m) = (k_i^h)^{1-s}(k_i^m)^s$$
$$s.t.\quad c_i^h = r^h k_i^h + exr^m k_i^m$$

其中r^h為國內生產要素的單位成本，r^m為國外進口要素的單位成本，ex為匯率（以本國貨幣表示的每單位外國貨幣價格）。

設 $\mathrm{L} = (k^h)^{1-s}(k^m)^s + \lambda(c_i^h - r^h k_i^h - exr^m k_i^m)$，一階條件為：

$$\frac{\partial \mathrm{L}}{\partial k_i^h} = (1-s)(\frac{k_i^m}{k_i^h})^s - \lambda r^h = 0 \rightarrow (1-s)(\frac{k_i^m}{k_i^h})^s = \lambda r^h \quad \text{(A1)}$$

$$\frac{\partial \mathrm{L}}{\partial k_i^m} = s(\frac{k_i^m}{k_i^h})^{s-1} - \lambda exr^m = 0 \rightarrow s(\frac{k_i^m}{k_i^h})^{s-1} = \lambda exr^m \quad \text{(A2)}$$

$$\frac{\partial \mathrm{L}}{\partial \lambda} = c_i^h - r^h k_i^h - exr^m k_i^m = 0 \rightarrow c_i^h = r^h k_i^h + exr^m k_i^m \quad \text{(A3)}$$

$$\frac{\text{(A1)}}{\text{(A2)}} = \frac{1-s}{s}(\frac{k_i^m}{k_i^h}) = \frac{r^h}{exr^m} \qquad \therefore k_i^m = \frac{r^h}{exr^m}\frac{s}{1-s}k_i^h \quad \text{(A4)}$$

將(A4)式代回$x_i(k_i^h, k_i^m) = (k_i^h)^{1-s}(k_i^m)^s$中，可得：

$$x_i = (\frac{r^h}{exr^m})^s(\frac{s}{1-s})^s k_i^h \qquad \therefore k_i^h = (\frac{exr^m}{r^h})^s(\frac{1-s}{s})^s x_i \quad \text{(A5)}$$

將(A5)式代回(A4)式，可得：

$$\therefore k_i^m = (\frac{exr^m}{r^h})^{s-1}(\frac{1-s}{s})^{s-1} x_i \quad \text{(A6)}$$

將(A5)式與(A6)式代入$c_i^h = r^h k_i^h + er^m k_i^m$後，可得：

$$c_i(r^h, r^m, e, x_i) = A(r^h)^{1-s}(exr^m)^s x_i \quad , \quad A = \frac{(1-s)^{s-1}}{s^s}$$

假設本國廠商i生產的產品可分為內銷與外銷兩部分，所以$x_i = x_i^h + x_i^e$，x_i^h為內銷的產量，x_i^e為外銷的產量。因此，上式又可寫成：

$$c_i(r^h, r^m, e; x_i^h, x_i^f) = A(r^h)^{1-s}(er^m)^s(x_i^h + x_i^e) \quad , \quad A = \frac{(1-s)^{s-1}}{s^s}$$

B. $x_i^h, x_i^e, x_j^f, x_j^m$ 均衡值的數學推導

本文(5a)與(5b)兩式分別為本國廠商 i 與外國廠商 j 的利潤函數：

$$\pi_i^d = P^d(X^d)x_i^h + exP^w(X^w)x_i^e - c_i^h(r^h, r^m, ex, x_i^h, x_i^e) \quad , \quad i = 1, 2, \cdots, n \tag{5a}$$

$$\pi_j^w = P^d(X^d)x_j^m + exP^w(X^w)x_j^f - exc_j^w(x_j^f, x_j^m) \quad , \quad j = 1, 2, \cdots, n^* \tag{5b}$$

由本文(4a)與(4b)式可得：

$$P^d(X^d) = a - bX^d \quad , \quad a 、b > 0 \tag{4a}$$

$$P^w(X^w) = a^w - b^w X^w \quad , \quad a^w 、b^w > 0 \tag{4b}$$

由於 $X^d = \sum_{i=1}^{n} x_i^h + \sum_{j=1}^{n^*} x_j^m$ 、 $X^w = \sum_{i=1}^{n} x_i^e + \sum_{j=1}^{n^*} x_j^f$ ，所以前二式又可寫成：

$$P^d(X^d) = a - b(\sum_{i=1}^{n} x_i^h + \sum_{j=1}^{n^*} x_j^m) \tag{B1}$$

$$P^w(X^w) = a^w - b^w(\sum_{i=1}^{n} x_i^e + \sum_{j=1}^{n^*} x_j^f) \tag{B2}$$

將(B1)式與(B2)式代入(5a)式與(5b)式，並把 $c_i^h(r^h, r^m, ex; x_i^h, x_i^e)$ 與 $c_j^w(x_j^f, x_j^m)$ 分別以 $A(r^h)^{1-s}(er^m)^s(x_i^h + x_i^e)$ 與 $c^w(x_j^f + x_j^m)$ 替代後可得：

$$\begin{aligned}\pi_i^d = & \left[a - b(\sum_{i=1}^{n} x_i^h + \sum_{j=1}^{n^*} x_j^m) \right] x_i^h + ex\left[a^w - b^w(\sum_{i=1}^{n} x_i^e + \sum_{j=1}^{n^*} x_j^f) \right] x_i^e \\ & - A(r^h)^{1-s}(exr^m)^s(x_i^h + x_i^e) \quad , \quad i = 1, 2, \cdots, n\end{aligned} \tag{B3}$$

$$\pi_j^w = \left[a - b(\sum_{i=1}^{n} x_i^h + \sum_{j=1}^{n^*} x_j^m) \right] x_j^m + ex\left[a^w - b^w(\sum_{i=1}^{n} x_i^e + \sum_{j=1}^{n^*} x_j^f) \right] x_j^f$$
$$- exc^w(x_j^f + x_j^m) \quad , \quad j = 1, 2, \cdots, n^* \tag{B4}$$

對(B3)式 x_i^h 與 x_i^e 、(B4)式 x_j^f 與 x_j^m 作偏微分，並令其一階導函數為零，可得：

$$\frac{\partial \pi_i^d}{\partial x_i^h} = a - b(x_1^h + x_2^h + \cdots + 2x_i^h + \cdots + x_n^h) - b\sum_{j=1}^{n^*} x_j^m - A(r^h)^{1-s}(exr^m)^s = 0$$

$$\therefore b(x_1^h + x_2^h + \cdots + 2x_i^h + \cdots + x_n^h) = a - b\sum_{j=1}^{n^*} x_j^m - A(r^h)^{1-s}(exr^m)^s \tag{B5}$$

$$\frac{\partial \pi_i^d}{\partial x_i^e} = exa^w - exb^w(x_1^e + x_2^e + \cdots + 2x_i^e + \cdots + x_n^e) - exb^w\sum_{j=1}^{n^*} x_j^f - A(r^h)^{1-s}(exr^m)^s = 0$$

$$\therefore exb^w(x_1^e + x_2^e + \cdots + 2x_i^e + \cdots x_n^e) = exa^w - exb^w\sum_{j=1}^{n^*} x_j^f - A(r^h)^{1-s}(exr^m)^s \tag{B6}$$

$$\frac{\partial \pi_j^w}{\partial x_j^f} = exa^w - exb^w\sum_{i=1}^{n} x_i^e - exb^w(x_1^f + x_2^f + \cdots + 2x_j^f + \cdots x_{n^*}^f) - exc^w = 0$$

$$\therefore exb^w(x_1^f + x_2^f + \cdots + 2x_j^f + \cdots x_{n^*}^f) = exa^w - exb^w\sum_{i=1}^{n} x_i^e - exc^w \tag{B7}$$

$$\frac{\partial \pi_j^w}{\partial x_j^m} = a - b\sum_{i=1}^{n} x_i^h - b(x_1^m + x_2^m + \cdots + 2x_j^m + \cdots x_{n^*}^m) - exc^w = 0$$

$$\therefore b(x_1^m + x_2^m + \cdots + 2x_j^m + \cdots x_{n^*}^m) = a - b\sum_{i=1}^{n} x_i^h - exc^w \tag{B8}$$

由(B5)式可推得：

$$b(2x_1^h + x_2^h + \cdots + x_n^h) = b(x_1^h + 2x_2^h + \cdots + x_n^h) = \cdots = b(x_1^h + x_2^h + \cdots + 2x_n^h)$$
$$= a - b\sum_{j=1}^{n^*} x_j^m - A(r^h)^{1-s}(exr^m)^s$$

$\therefore x_1^h = x_2^h = \cdots = x_i^h = \cdots = x_n^h$　同理可推得：

$x_1^e = x_2^e = \cdots = x_i^e = \cdots = x_n^e$ ， $x_1^f = x_2^f = \cdots = x_j^f = \cdots = x_{n^*}^f$ ，
$x_1^m = x_2^m = \cdots = x_j^m = \cdots = x_{n^*}^m$

所以(B5)式、(B6)式、(B7)式、(B8)式可改寫為：

$$b(n+1)x_i^h = a - bn^* x_j^m - A(r^h)^{1-s}(exr^m)^s \quad \text{(B9)}$$

$$exb^w(n+1)x_i^e = exa^w - exb^w n^* x_j^f - A(r^h)^{1-s}(exr^m)^s \quad \text{(B10)}$$

$$exb^w(n^*+1)x_j^f = exa^w - exb^w n x_i^e - exc^w \quad \text{(B11)}$$

$$b(n^*+1)x_j^m = a - bnx_i^h - exc^w \quad \text{(B12)}$$

由(B9)式與(B12)式可推得：

$$x_i^h = x_j^m + \frac{exc^w - A(r^h)^{1-s}(exr^m)^s}{b} \quad \text{(B13)}$$

將(B13)式代入(B12)式，可得：

$$x_j^{m*} = \frac{a - exc^w(1+n) + A(r^h)^{1-s}(exr^m)^s n}{bN} \quad , \quad N = 1 + n + n^*$$

將上式代入(B13)式，可得：

$$x_i^{h*} = \frac{a - A(r^h)^{1-s}(exr^m)^s(1+n^*) + exc^w n^*}{bN}$$

由(B10)式與(B11)式可推得：

$$x_j^f = x_i^e + \frac{A(r^h)^{1-s}(exr^m)^s - exc^w}{exb^w} \quad \text{(B14)}$$

將(B14)代入(B11)式，可得：

$$x_i^{e*} = \frac{exa^w - A(r^h)^{1-s}(exr^m)^s(1+n^*) + exc^w n^*}{exb^w N}$$

將上式代入(B14)式，可得：

$$x_j^{f*} = \frac{exa^w - exc^w(1+n) + A(r^h)^{1-s}(exr^m)^s n}{exb^w N}$$