5 Examples

Example 5.1 Let $\mathbb{T} = [0,1] \cup \mathbb{Z}$. We consider the following PBVP on $\mathbb{T}$

$$\begin{aligned} &x^{\Delta} + (t+1)x^{\sigma}(t) = \frac{x}{10e^t}, \quad t \in J := [0,4]_{\mathbb{T}}, t \neq \frac{1}{2}, \\ &x\left(\frac{1}{2}+\right) - x\left(\frac{1}{2}-\right) = \frac{1}{4}\sin\left(x\left(\frac{1}{2}-\right)\right), \\ &x(0) = x(\sigma(4)). \end{aligned}$$

Let

$$p(t) = t+1, \;\; f(t,x) = \frac{x}{10e^t}, \text{ and } I(x) = \frac{1}{4}\sin x.$$

It is easy to see that

$$|f(t,u) - f(t,v)| \leq \frac{1}{10}|u-v|, \text{ for all } t \in [0,\sigma(4)]_{\mathbb{T}} \text{ and } u,v \in \mathbb{R},$$

and

$$|I(u) - I(v)| \leq \frac{1}{4}|u-v| \text{ for all } u,v \in \mathbb{R}.$$

Also, by a simple computation, we get $A = 360e^{\frac{3}{2}}/(360e^{\frac{3}{2}} - 1)$ and hence

$$A\left[\sigma(4)\frac{1}{10} + \frac{1}{4}\right] < 1.$$

Hence by Theorem 4.4 the PBVP has at least one solution.

Example 5.2 Let $\mathbb{T} = [0, \frac{1}{2}] \cup 2^{\mathbb{N}_0}$. We consider the following PBVP on $\mathbb{T}$

$$\begin{aligned} &x^{\Delta} + p(t)x^{\sigma}(t) = f(t,x), \quad t \in J := [0,4]_{\mathbb{T}}, t \neq \frac{1}{4}, \\ &x(\frac{1}{4}+) - x(\frac{1}{4}-) = I(x(\frac{1}{4}-)), \\ &x(0) = x(\sigma(4)), \end{aligned}$$

where

$$p(t)=\begin{cases} t, & t\in[0,\frac{1}{2}], \\ 1, & t\in 2^{\mathbb{N}_0}, \end{cases}, \quad f(t,x)=\frac{2\sin t}{x^2+1}, \text{ and } I(x)=\frac{1}{24}x.$$

It is easy to see that

$$|f(t,x)|\le 2 \text{ for all } t\in[0,\sigma(4)]_{\mathbb{T}} \text{ and } x\in\mathbb{R},$$

and

$$|I(x)|\le\frac{1}{24}|x| \text{ for all } x\in\mathbb{R}.$$

By a simple computation, we get $A=75e^{\frac{1}{8}}/(75e^{\frac{1}{8}}-2)$ and so $A/24<1$. Then by Theorem 4.6, the PBVP has at least one solution.

Example 5.3 Let $\mathbb{T}=\mathbb{N}_0^2\cup[6,8]$. We consider the following PBVP on $\mathbb{T}$

$$\begin{aligned} &x^{\Delta}+p(t)x(\sigma(t))=f(t,x), \quad t\in J:=[0,8]_{\mathbb{T}}, t\neq 7, \\ &x(7+)-x(7-)=I(x(7-)), \\ &x(0)=x(\sigma(8)), \end{aligned}$$

where

$$p(t)=\begin{cases} 1, & t\in\{0,1,4\}, \\ t, & t\in[6,8], \end{cases}, \quad f(t,x)=\frac{x}{16t+1}, \text{ and } I(x)=\sin x.$$

It is easy to see that

$$|f(t,x)|\le\frac{1}{48}|x| \text{ for all } t\in[0,\sigma(8)]_{\mathbb{T}} \text{ and } x\in\mathbb{R},$$

and

$$|I(x)|\le 1 \text{ for all } x\in\mathbb{R}.$$

Also, by a simple computation, we get $A = \dfrac{24e^{14}}{24e^{14}-1}$ and so

$$\frac{1}{48}A\sigma(8) < 1.$$

Then by Theorem 4.5, the PBVP has at least one solution.

Example 5.4 Let $\mathbb{T}$ be a time scale and let $0, T \in \mathbb{T}$. We consider the following PBVP on $\mathbb{T}$

$$x^{\Delta} + x^{\sigma} = e^{\frac{1}{x}} \sin t, \quad t \in J := [0,T]_{\mathbb{T}}, \ t \neq t_k, \ k = 1, \dots, m,$$

$$x(t_k+) - x(t_k-) = x(t_k-)^{\frac{1}{2}}, \quad k = 1, \dots, m,$$

$$x(0) = x(\sigma(T)),$$

where $t_k \in (0,T)_{\mathbb{T}}$, $k = 1, \dots, m$, are right-dense for all $k = 1, \dots, m$. Let $f(t,x) = e^{\frac{1}{x}} \sin t$ and $I_k(x) = x^{\frac{1}{2}}$. Then it is easy to see that

$$\lim_{|x|\to\infty} \frac{f(t,x)}{x} = 0 \ \text{ and } \lim_{|x|\to\infty} \frac{I_k(x)}{x} = 0.$$

Hence it follows from Theorem 4.8 that the PBVP has least one solution .