1 Introduction

In recent years, the theory of impulsive equations has become an important field due to its strong potential for practical applications (see, for example, the excellent book [2] for impulsive differential equations and [9], [10] for impulsive dynamic equations). Motivated by [4] and [8], in this paper, we study the existence of solutions of periodic boundary value problems of first-order impulsive dynamic equations.

Let $\mathbb{T}$ be a time scale, i.e., a nonempty closed subset of $\mathbb{R}$, and let $0, T \in \mathbb{T}$. Throughout this paper, $[0,T]_{\mathbb{T}}$ represents an interval on $\mathbb{T}$, i.e., $[0,T]_{\mathbb{T}} = [0,T] \cap \mathbb{T}$. Other types of intervals on $\mathbb{T}$ can be represented by a similar way. This paper is concerned with the existence of solutions to the following periodic boundary value problems (PBVPs for short) for first-order impulsive dynamic equations

$$x^{\Delta} + p(t)x^{\sigma} = f(t,x), \quad t \in J := [0,T]_{\mathbb{T}}, \ t \neq t_k, \ k = 1, \dots, m, \tag{1}$$

$$x(t_k+) - x(t_k-) = I_k(x(t_k-)), \quad k = 1, \dots, m, \tag{2}$$

$$x(0) = x(\sigma(T)), \tag{3}$$

where $f \in C([0,\sigma(T)]_{\mathbb{T}} \times \mathbb{R}, \mathbb{R})$, $I_k \in C(\mathbb{R},\mathbb{R})$, $p : [0,\sigma(T)]_{\mathbb{T}} \to [0,\infty)$ is rd-continuous and regressive with $p \not\equiv 0$, $t_k \in (0,T)_{\mathbb{T}}$, $0< t_1 < \cdots < t_m < T$ on $\mathbb{T}$ and t_k are right-dense for all $k = 1, \dots, m$.

For convenience, we shall refer to (1)-(2)-(3) as (NP).

Let

$$PC = \{x : [0,\sigma(T)]_{\mathbb{T}} \to \mathbb{R} | x_k \in C(J_k) \text{ and both } x(t_k+) \text{ and } x(t_k-)$$
$$\text{exist such that } x(t_k-) = x(t_k), \ \forall k = 1, \dots, m\}.$$

Here x_k is the restriction of x to J_k, $\forall k = 0, \dots, m$, where $J_0 = [0,t_1]_{\mathbb{T}}$,

$J_k = (t_k, t_{k+1}]_{\mathbb{T}},\ \forall k = 1, \ldots, m-1$, and $J_m = (t_m, \sigma(T)]_{\mathbb{T}}$.

We introduce the Banach space $X = \{x : x \in PC,\ x(0) = x(\sigma(T))\}$ with the norm $\|x\|_X = \sup_{[0,\sigma(T)]_{\mathbb{T}}} |x(t)|$.

Definition 1.1 *A function x is said to be a solution of (NP) if and only if $x \in PC \bigcap C^1(J \backslash \{t_1, t_2, ..., t_m\},\ \mathbb{R})$ and satisfies (1)-(2)-(3).*