2 Preliminary

For completeness, we state some fundamental definitions and results concerning time scales that we will use in the sequel. More details can be found in [11], [12], and [13]. In this section, we assume that $\mathbb{T}$ is an arbitrary time scale.

Definition 2.1 *The mappings $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ defined by*

$$\sigma(t) := inf\{s \in \mathbb{T} | s > t\}, \qquad \rho(t) := sup\{s \in \mathbb{T} | s < t\}$$

are called the forward and backward jump operators respectively. In this definition, we put $inf \varnothing = sup\mathbb{T}$ and $sup\varnothing = inf\mathbb{T}$. Graininess function $\mu : \mathbb{T} \to [0, \infty)$ is defined by $\mu(t) := \sigma(t) - t$.

Definition 2.2 *A point t in $\mathbb{T}$ is said to be right-dense if $t < sup\mathbb{T}$ and $\sigma(t) = t$, and left-dense if $t > inf\mathbb{T}$ and $\rho(t) = t$.*

Definition 2.3 *A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive provided that $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$.*
The set of all regressive and rd-continuous functions will be denoted by $\mathcal{R}$.

Definition 2.4 *We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by*

$$\mathcal{R}^+ = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0 \ for \ all \ t \in \mathbb{T}\}.$$

Definition 2.5 *A function on $\mathbb{T}$ is called rd-continuous if it is continuous at all right-dense points in $\mathbb{T}$ and its left-sided limits exist at all left-dense points in $\mathbb{T}$. The set of all rd-continuous functions on $\mathbb{T}$ is denoted by $C_{rd}(\mathbb{T})$.*

Definition 2.6 *If $p \in \mathcal{R}$, then the delta exponential function is given by*

$$e_p(t, s) = exp(\int_s^t g(\tau)\Delta\tau),$$

where

$$g(\tau) = \begin{cases} p(\tau), & if \ \ \mu(\tau) = 0, \\ \dfrac{1}{\mu(\tau)} Log(1 + p(\tau)\mu(\tau)), & if \ \ \mu(\tau) \neq 0. \end{cases}$$

Lemma 2.7 *If* $p \in \mathcal{R}$, *then*

(1) $e_p(t,t) \equiv 1$;

(2) $e_p(t,s) = \dfrac{1}{e_p(s,t)}$;

(3) $e_p(t,u)e_p(u,s) = e_p(t,s)$;

(4) $e_p^{\Delta}(t,t_0) = p(t)e_p(t,t_0)$, *for* $t \in \mathbb{T}^k$ *and* $t_0 \in \mathbb{T}$.

Lemma 2.8 *If* $p \in \mathcal{R}^+$ *and* $t_0 \in \mathbb{T}$, *then* $e_p(t,t_0) > 0$ *for all* $t \in \mathbb{T}$.

Lemma 2.9 *If f is continuous, then f is rd-continuous.*

We shall apply the well-known Banach's fixed point theorem and Schaefer's theorem to estabish the existence and uniqueness theorem for (NP). For readers' convenience, we provide these two theorems here.

Lemma 2.10 *(Banach's fixed point theorem [14]) A contraction* f *of a complete metric space* S *has a unique fixed point* $p \in S$.

Lemma 2.11 *(Schaefer's theorem [14]) Let* S *ba a normed linear space, and let operator* $F : S \to S$ *be compact. If the set*

$$H(F) = \{x \in S : x = \mu F(x), \ \ for \ \ some \ \ \mu \in (0,1)\}$$

is bounded, then F *has a fixed point.*