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Intermediated Contracts, Efficiency Gains and Market Mechanisms

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Abstract

This paper tries to illustrate by examples that intermediated contractual arrangements make efficiency gains under price mechanisms. Three examples of pure exchange environments, one in partial equilibrium and two in general equilibrium, are provided. When asymmetric information causes adverse selection problems, intermediated contractual arrangements with state-contingent payments and randomized information-processing make efficiency gains. Moreover, in some of these examples, intermediated contractual arrangements can be decentralized and coordinate with competitive market prices of goods. A policy implication is that the presence of imperfect information does not necessarily gives reasons for the government's intervention in the market. The market itself can handle information problems. When an appropriate information technology presents in some environments with adverse selection problems, market mechanisms together with intermediated contractual arrangements perform well just as they do in the perfect information world. However, market mechanisms sometimes does not work well enough to eliminate all adverse selection problems. In numerical example 3, we show a case in which contractual arrangements through decentralized intermediaries alone can solve the problem; however, such arrangements cannot coordinate with market prices of goods.

Keyword: intermediated contract, state-contingent contract, market mechanisms, adverse selection, asymmetric information
JEL Classification: D2, D8

This paper tries to illustrate by examples that intermediated contractual arrangements make efficiency gains under price mechanisms. Three examples of pure exchange environments, one in the context of partial equilibrium and two in the context of general equilibrium, are provided. When asymmetric information presents, intermediated contractual arrangements together with randomized information-processing make efficiency gains. Moreover, in some of these examples, intermediated contractual arrangements can be decentralized through market mechanisms. A policy implication is that imperfect information does not necessarily gives reasons for government's intervention in the market. The market itself can handle informational problems. When an appropriate information technology presents in an environment with ex ante asymmetric information, market mechanisms together with intermediated contractual arrangements perform well just like they do in a perfect information world.

To our knowledge, no existing literature has explored this aspect of intermediation. Factors such as redundant monitoring costs due to asymmetric information, reduction of search costs and trading externalities have been used to explain the existence and functions of intermediation. For example, Diamond (1984) uses incentive problems due to asymmetric information to discuss financial intermediation and shows that intermediation is an efficient way to avoid redundant monitoring costs. Rubinstein and Wolinsky (1987) discuss how middlemen can reduce search costs for both sides of transactions. Bhattacharya and Hagerty (1987), extending Diamond (1982)'s search model, discuss how trading externalities give rise to specialization of agents into producers and dealers in a general equilibrium framework. Assuming that the defected products from a producer badly influence consumers' perception about other products middlemen sell and that it is costless to replace defected products, Biglaiser and Friedman(1994) show that by dropping defected products from the shelves of his store, middlemen act as guarantors of quality and effectively solve the moral hazard problem.

Biglaiser(1993)'s discussion is close to our analysis in some aspects. He shows conditions under which intermediation arrangement is welfare-improving in an environment similar to ours. However, our paper differs to his paper in, at least, two aspects. First, state-contingency is crucial for intermediation arrangement being efficient while his paper does not touch this issue at all. This is because in his environment sellers know the true quality of their goods while in our environment sellers know only the probability of their goods being of high quality. Second, in this paper middlemen uncover the true quality of goods and thus avoid misallocation of resources, while in his paper middlemen reduces search costs by shrinking waiting time through which sellers signal qualities of their goods. One feature of our paper, perhaps the most important one, is that we illustrate how and to what extent price mechanisms and intermediated contractual arrangements interact to resolve adverse selection. Biglaiser(1993)'s discussion abstracts from this important feature completely.

Numerical Example 1

In this numerical example we provide a partial equilibrium framework in which the problem of adverse selection arises because of private information . A decentralized intermediated contractual arrangement is provided to ameliorate the problem. Such decentralized intermediation endogenously emerges.

Consider the market of a good with two different quality levels, high(h) and low(ℓ). The good is supplied in two types, the type-$\hat{h}$ good and the type $\hat{\ell}$ good. The type $\hat{h}$ good is of high quality with probability 0.99 and type $\hat{\ell}$ good is of high quality with probability 0.01. There are a large number of consumers and suppliers in the sense that each suppliers is a price-taker in the market. Consumers are risk neural and the high-quality good is worth one dollar to consumers. The low-quality good is worthless to consumers; that is the value of one unit of the low-quality good is zero dollar. Suppose that the suppliers are to maximize the value of sale and get nothing from consuming the good.

No Private Information

When the type of goods is public information, consumers can distinguish type $\hat{h}$ from type $\hat{\ell}$. Each consumers would not pay more than \$ 0.99 per unit of type $\hat{h}$ good and \$ 0.01 per unit of type $\hat{\ell}$ good.

Private Information

When the type of goods is the supplier's private information, consumers are unable to tell type $\hat{h}$

good from type $\hat{\ell}$ good. They would pay the same price for both types of good. The price depends upon the expected utility they would obtain from consuming one unit of the good from the market. Thus the price would reflect the amounts of goods of both types available in the market. Denote the amount of type $\hat{h}$ and $\hat{\ell}$ by $Q(\hat{h})$ and $Q(\hat{\ell})$ respectively. The price a consumer would pay is $\frac{Q(\hat{h})}{Q(\hat{\ell})+Q(\hat{h})} \times 0.99 + \frac{Q(\hat{\ell})}{Q(\hat{\ell})+Q(\hat{h})} \times 0.01$. Suppose both type $\hat{h}$ and $\hat{\ell}$ have equal amount available, the price is \$0.5.

Inspection Technology and No Intermediation

Suppose that there is an inspection technology which requires \$0.05 to inspect one unit of good and reveal the quality level of the inspected unit as public information. Will the inspection technology be used? Who will use it?

Each of inspected units would be revealed as of high-quality or low-quality and the types of all uninspected units remain indistinguishable to consumers. Thus, for consumers, there are three different types of goods, high-quality(h), low-quality(ℓ) and unknown quality(u), available in the market. Due to constant marginal utilities, the price of the high-quality good is \$1.00 and the price of the low-quality good is zero. Thus the type $\hat{\ell}$ supplier would have no incentive to inspect its good because the unit cost of inspection (\$0.05) exceeds the expected revenue from one inspected unit \$0.01 ($= 0.01 \times \$1 + 0.99 \times \$0$). The price of type u good would depend on the expected utility it yields. Suppose there are equal amounts of type $\hat{h}$ and $\hat{\ell}$ goods. The proportion of type $\hat{h}$-good *not* being inspected is denoted by $\theta(\hat{h})$. Then the price of type u good is $\frac{0.99\times\theta(\hat{h})+0.01}{\theta(\hat{h})+1}$. Given the market price, the type $\hat{h}$-firm will use the inspection technology only if the profit from inspection technology ($=$ 0.99 - 0.05) exceeds the profit from not inspecting his good: $0.95 > \frac{0.99\times\theta(\hat{h})+0.01}{\theta(\hat{h})+1}$ (or $\theta(\hat{h}) < 23.5$). Since $\theta(\hat{h})$ is less than one, the type $\hat{h}$ firm always pays to inspect his goods. Thus the market equilibrium would be that all type $\hat{h}$ firms use the inspection technology but no type $\hat{\ell}$ firm uses the inspection technology. The equilibrium prices, denoted by $q(s), s \in \{h, \ell, u\}$, are $q(h) = 1$, $q(\ell) = 0$ and $q(u) = 0.01$. The type $\hat{h}$ suppliers receive net revenue of \$0.94 per unit of good and the type $\hat{\ell}$ suppliers receive revenue of \$0.01 per unit of good.

Intermediated Contractual Arrangement

and Randomized Inspection

In the above discussion, the cost of distinguishing both type $\hat{h}$ and type $\hat{\ell}$ goods is to inspect all type $\hat{h}$ goods. In the following discussion, we are to show that there is an institutional arrangement which can cut down this cost by invoking randomized inspection.

Consider a profit-driven intermediary which proposes a state-contingent contract $(\xi, \tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0))$ to buy both types of indistinguishable goods. Whenever a supplier offers a good with unknown quality, the probability that the intermediary inspects the unknown quality good is ξ. When inspection takes place the intermediary will charge \$0.05 per unit inspected, and pay $\tilde{q}(h)$ if the outcome is high quality, pay $\tilde{q}(\ell)$ if the outcome is low quality. If inspection does not take place, the intermediary will pay $\tilde{q}(0)$. The intermediary will sell the uninspected goods and inspected goods with high-quality in the market and throw away inspected goods with low-quality. The good has four distinctive types available in the market, high quality(h), low quality(ℓ), unknown quality from intermediaries$(\bar{u})$ and unknown quality from original suppliers$(\hat{u})$.

Given the market prices of four different distinctive types, $q(h)$, $q(\ell)$, $q(\bar{u})$ and $q(\hat{u})$, intermediaries design state-contingent contracts to maximize profits such that type $\hat{h}$ suppliers have incentive to come to him and type $\hat{\ell}$ suppliers are discouraged away. Such incentive compatibility can be formalized as

$$\xi\,(0.99 \times \tilde{q}(h) + 0.01 \times \tilde{q}(\ell) - 0.05) + (1-\xi)\tilde{q}(0) \geq q(\hat{u}), \qquad \text{and}$$

$$\xi\,(0.01 \times \tilde{q}(h) + 0.99 \times \tilde{q}(\ell) - 0.05) + (1-\xi)\tilde{q}(0) \leq q(\hat{u}).$$

Notice that incentive compatibility rules out the possibility of type $\bar{u}$ good being of type $\hat{\ell}$. Thus the price of type $\bar{u}$ good consumers would pay is $q(\bar{u}) = 0.99$. Assume that inspection technology is accessible to every agents in the market. Then free entry drives intermediaries' profits to zero. We are to show that in equilibrium all type $\hat{h}$ suppliers sell their goods to intermediaries and the minimum value of probability of inspection(ξ) for that incentive compatibility holds in the following steps:

(1) Since incentive compatibility holds for type $\hat{\ell}$ suppliers, and $q(h) = 1$, $q(\ell) = 0$, $q(\bar{u}) = 0.99$, the zero profit condition makes the expected amount received by the type $\hat{h}$ supplier is $\xi \times 0.99 + (1-\xi) \times 0.99$. Thus the expected revenue net of inspection cost, $R(\hat{h})$, is

$$R(\hat{h}) = \xi \times (0.99 - 0.05) + (1 - \xi) \times 0.99.$$

(2) All type $\hat{\ell}$ goods are sold directly, without going through intermediaries. Denote the proportion of type $\hat{h}$ goods do *not* go through intermediaries by $\theta(\hat{h})$. Then the price of good $\hat{u}$ is $q(\hat{u}) = \frac{0.99\times\theta(\hat{h})+0.01}{\theta(\hat{h})+1}$.

(3) Incentive compatibility for type $\hat{h}$ suppliers holds if and only if $R(\hat{h}) \leq q(\hat{u})$, or $\xi \leq (-q(\hat{u}) + 0.99)/0.05$. Notice that $\xi \in [0,1]$ and $q(\hat{u})$ is increasing in $\theta(\hat{h})$. The incentive compatibility condition for type $\hat{h}$ holds at inequality for any value of $\theta(\hat{h}) \in [0,1]$. Thus all type $\hat{h}$ goods are sold to intermediaries and $q(\hat{u}) = 0.01$.

(4) From the zero profit condition

$$\xi(1 \times 0.99 + 0 \times 0.01) + (1 - \xi) \times 0.99 = \xi(\tilde{q}(h) \times 0.99 + \tilde{q}(\ell) \times 0.01) + (1 - \xi) \times \tilde{q}(0),$$

we know that $\tilde{q}(h) = 1$, $\tilde{q}(\ell) = 0$ and $\tilde{q}(0) = 0.99$.

(5) In order that incentive compatibility for the type $\hat{\ell}$ suppliers holds, ξ must satisfy

$$\xi\,(0.01 \times 1 + 0.99 \times 0 - 0.05) + (1 - \xi)0.99 \leq 0.01, \qquad \text{or} \qquad \xi \geq 0.98/1.03.$$

The lower value ξ takes, the more profitable type $\hat{h}$'s trade with intermediaries. Competition in intermediation business would drive ξ as low as possible. Thus ξ converges to $0.98/1.03 (\approx 0.95)$.

The state-contingent contract invokes randomized inspection, reduces the cost of allocation of resources, and pays the inspected good its market value. Incentive compatibility makes uninspected goods from intermediaries have more value than those directly from suppliers. Without intermediation inspection is conducted on all units of type $\hat{h}$ goods, while with intermediation only 95 percent of type $\hat{h}$ goods are inspected in our parametric environment. Moreover, such a state-contingent contract works through competition in the market.

Numerical Example 2: Intermediated Contract and Efficiency Gains

A pure exchange economy is provided in which some traders have private information about their own endowments. There exists an inspection technology which is costly and can alleviate incentive problems caused by private information. The inspection activity generates valuable information to allocate goods into right agents and makes efficiency gains. As in Example 1, we are to show in this example that endogenous emergence of intermediated contractual arrangements ameliorates incentive problems.

Consider a pure exchange economy in which there are one good with different quality levels and two types of agents, type C and type U, differing in preferences and endowments. The good has three quality levels, high(h), constant(c) and low(ℓ). Type C agents distinguish the quality levels of the good and their preferences are represented by the utility function,

$$V_C = 2 \cdot x(C, h) + x(C, c) + 0 \cdot x(C, \ell),$$

where $x(C, s)$ is the quantity of the good with s-quality consumed by the type C agent, $s = h, c$, or ℓ. The high-quality good yields a marginal utility level of two and the constant-quality good yields a marginal utility level of one. The low-quality good is worthless to type C agents. Type U agents do not distinguish the quality levels of the good. Their preferences are also linear and can be represented by

$$V_U = x(U, h) + x(U, c) + x(U, \ell).$$

Each type C agent is endowed with one unit of constant-quality good, while each type U agent is endowed with one unit of uncertain-quality good. Each type U agent receives a signal, either $\hat{h}$ or $\hat{\ell}$, about the quality of his endowment. The signal $\hat{h}$ indicates that the endowment is of high-quality with probability 0.99 and low-quality with probability 0.01. The signal $\hat{\ell}$ indicates that the endowment is high quality with probability 0.01 and low quality with probability 0.99. The set of type C agents is of measure one and the set of type U agents is also of measure one. Suppose that there are equal numbers of type U agents with signal $\hat{h}$ and with signal $\hat{\ell}$. Those type U agents with signal $\hat{h}$ are called type $\hat{h}$ agents and those with signal $\hat{\ell}$ are called type $\hat{\ell}$ agents.

Signals are public information

When signals about type U agents' endowments are public, trades between type C agents and type $\hat{\ell}$ agents won't happen at all. Thus type $\hat{\ell}$ agents remain autarky. There are two different goods relevant to C agents' consumption decision. One is from type $\hat{h}$ agents and the other one is from type C agents. Each unit of type $\hat{h}$ endowments yields 1.98 of expected marginal utility level for type C agents. Thus type C agents would compete to exchange their own endowments for type

$\hat{h}$ agents' endowments. Since type C agents outnumber type $\hat{h}$ agents, competition would drive up the exchange rate to the level at which no further increase is acceptable to type C agents; i.e. the exchange rate would be 1.98 units of the quality-c good for one unit of type $\hat{h}$ endowments.

All gains from exchange are appropriated by type $\hat{h}$ agents. Notice that inspection technology does not give any benefit to both types of agents because it is "too" costly and signals are public.

Signals are private information

Type C agents cannot distinguish both type $\hat{h}$ and type $\hat{\ell}$ endowments. When no inspection technology exists, whether or not trades between type C agents and type U agents taking place depends on the ratio of the population of type $\hat{h}$ agents to that of type $\hat{\ell}$ agents. When there are too few type $\hat{h}$ agents, say the above ratio is only one half, then type C agents would rather stay in autarky than trade with type U agents. In our example both type $\hat{h}$ agents and type $\hat{\ell}$ have equal population, the expected utility a type C agent obtains from each unit of good u is one, so they are indifferent between trade and no-trade.

Inspection without Intermediation

Suppose there exists an inspection technology which uses 0.05 unit of the constant quality good to uncover the true quality of one inspected good. The inspection technology enables type $\hat{h}$ agents to separate themselves from type $\hat{\ell}$ agents and to obtain some welfare gains. Consider the following trade contract:

(1) one type C agent and one type U agent get together and use 0.05 unit of constant-quality good to inspect the type U agent's endowment;

(2) if the outcome is high quality, the type U agent owns 0.95 unit of constant-quality good and t units of the inspected unit (good h). The type C agent obtains the remaining inspected unit.

(3) if the outcome is low quality, the type C agent owns the rest of constant-quality good (0.95 unit) and destroy the inspected unit such that the type U agent obtains nothing.[1]

Would there exists a t such that the contract is attractive to type $\hat{h}$ agents but is not interesting

[1] Given that the type-$\hat{\ell}$ endowment is low quality with probability 0.99, it is necessary to destroy some part of the low-quality unit in order to make this contract not attractive to type $\hat{\ell}$ agents. Appendix 1 illustrates the computation process of t.

to type $\hat{\ell}$ agents? Notice that each type U agent has three alternatives: (1) goes for the contract, (2) goes autarky, or (3) goes to the market and directly sells his endowments without inspection. It is sufficient to show that there exists a t such that the contract offers the highest payoff for each type $\hat{h}$ agent among these three alternatives and the contract offers type $\hat{\ell}$ agents a payoff which is less than what autarky offers.

Notice that the type C agent outnumbers the type $\hat{h}$ agent; thus type $\hat{h}$ agents will appropriate fully gains from bilateral exchange between both types of agents. Thus we can consider $t \approx 0.5$ such that the type $\hat{h}$ agent reaches 1.435 as his utility level from the bilateral contract with inspection while the type C obtains one as its utility level. The contract with $t \approx 0.5$ offers the type $\hat{\ell}$ agents the expected utility level of 0.0145, and would not be attractive enough to the type $\hat{\ell}$ agent. Would the type $\hat{h}$ agent go for direct sale of his endowment? It would depend upon what he can get from direct sale.

Let $\theta(\hat{h})$ and $\theta(\hat{\ell})$ denote respectively the proportion of type $\hat{h}$ and $\hat{\ell}$ agents who do sell their endowments directly. Then the maximum amount of the constant quality good the type C agent would pay to exchange for one unit of the uninspected good is

$$q(\hat{u}) = \frac{\theta(\hat{h}) \cdot 1.98 + \theta(\hat{\ell}) \cdot 0.02}{\theta(\hat{h}) + \theta(\hat{\ell})},$$

where $\hat{u}$ denotes the uninspected uncertain quality good and $q(\hat{u})$ is equal to the type C agent's expected utility from the uninspected uncertain quality good. It is straightforward to show that $q(\hat{u})$ is non-decreasing in $\theta(\hat{h})$ and is non-increasing in $\theta(\hat{\ell})$. The type $\hat{h}$ agent goes for $q(\hat{u})$ if and only if $q(\hat{u}) > 1.435$. However if $q(\hat{u}) > 1.435$, all type $\hat{\ell}$ agents go for direct sale and $q(\hat{u})$ is no longer greater than 1.435 and no type $\hat{h}$ agent goes for direct sale($\theta(\hat{h}) = 0$). Consequently, no type $\hat{\ell}$ has incentive to go for direct sale, because that $q(\hat{u})$ would be less than one when no type $\hat{h}$ agent directly sells his endowment. In sum, a contract described above with $t = 0.5$ attracts all type $\hat{h}$ agents to trade with type C agents and type $\hat{\ell}$ will remain autarky.

The existence of inspection technology makes all good $\hat{h}$'s be inspected and, thus, become distinguishable from good $\hat{\ell}$'s. The gains from inspection are appropriated fully by the type $\hat{h}$ agent due to the fact that the type C agent outnumbers the type $\hat{h}$ agents.

Intermediated Contractual Arrangement and

Randomized Inspection

There is still another way, randomizing inspecting the type $\hat{h}$ good, to separate type $\hat{h}$ endowments from type $\hat{\ell}$ endowments and to yield even more gains. Consider an intermediary which offers to the type U agent the following state-contingent contract:

(1) inspect the type U agent's endowment with probability ξ,

(2) when inspection takes place,

- if the inspection outcome is high quality, the intermediary would give the type U agent slightly less than ($1.\overline{95}$ units of any good), and call this state-contingent payment $\tilde{q}(h)$;
- if the outcome is low quality, the type U agent gets nothing back; that is the state-contingent payment is zero when the inspection outcome is low quality, *i.e.*, $\tilde{q}(\ell) = 0$;

(3) if inspection does not take place, the type U agent gets 1.98 units of good; that is the state-contingent payment is 1.98 when no inspection occurs, *i.e.*, $\tilde{q}(0) = 1.98$.

For each type C agent who presents his endowment to the intermediary, the intermediary gives him 0.495 ξ unit of the high-quality good, $0.5 \times (1 - \xi)$ unit of the uninspected type $\hat{h}$ good and slightly greater than 0.01 unit of good c such that each type C agent obtains a utility level of slightly greater than one.

It is sufficient to show that there exists a probability ξ to make the above mentioned state-contingent payment, $(\tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0)) = (1.\overline{95}, 0, 1.98)$, feasible and incentive compatible. First we show it is always feasible to find an ξ such that the contract is incentive compatible for the type $\hat{h}$ agent. The contract is incentive compatible for the type $\hat{h}$ agent if and only if his expected payoff from the contract is greater than the opportunity cost (max $\{ q(\hat{u}), 1\}$), where $q(\hat{u}) = \frac{\theta(\hat{h}) \cdot 1.98 + \theta(\hat{\ell}) \cdot 0.02}{\theta(\hat{h}) + \theta(\hat{\ell})}$. As previously analyzed, $q(\hat{u})$ would not be greater than one; otherwise all type $\hat{\ell}$ agents sell their endowments without inspection and this would enforce $q(\hat{u})$ to be less than one. If $q(\hat{u})$ is less than one, no type $\hat{h}$ agent goes for direct sale of his endowment and $\theta(\hat{h}) = 0$. Thus the probability of inspection, ξ, has to be chosen such that

$$\xi \times 0.99 \times 1.\overline{95} + (1 - \xi) \times 1.98 \geq 1,$$

This condition always holds due to the fact that $\xi \in [0, 1]$. Second, by the incentive compatibility condition for the type ℓ agent:

$$\xi \times 0.01 \times 1.\overline{95} + (1 - \xi) \times 1.98 \leq 1,$$

we know that when ξ is greater than $0.499 \approx 0.5$, the contract is not interesting to the type $\hat{\ell}$ agent.

Notice that type C agents cannot tell type $\hat{h}$ agents from type $\hat{\ell}$ agents. So he would not offer a price of $1.98 - \varepsilon$ units of constant-quality good per unit of type $\hat{h}$ good to attract type $\hat{h}$ agents away from the above contractual arrangement. The type C has no incentive to deviate from this contract and directly trades with the type $\hat{h}$ agent because that he is unable to tell good $\hat{h}$ from good $\hat{\ell}$.

This intermediated contractual arrangement yields for type $\hat{h}$ agents a higher level of expected utility (1.96) than the levels obtained without intermediation (1.435). Moreover, this intermediated contract arrangement can be decentralized. Suppose that the inspection technology is accessible to anyone in the economy and every agent in the economy is small in the sense that no one can affect the prices in good markets. Now using the inspection technology and offering the aforementioned state-contingent contract except that the payment when no inspection occurs is less than 1.98. Then the agent who run such intermediation could have some goods left for himself. This benefit of running intermediation business would attract more and more agents coming into the business by offering more attractive contract to type $\hat{h}$ agents. The new entrance into the intermediation business won't stop until the benefit is driven to zero.

Intermediated Contracts and Goods Markets

The intermediated contract just discussed can also work with the market of goods. Consider a market economy in which the type C agent sells his endowment in the market and uses the income to buy the good he wants to consume. The intermediary buys the constant-quality good from the commodity market at its market price ($q(c)$) for inspection and offers the state contingent contract (ξ, $\tilde{q}(h)$, $\tilde{q}(\ell)$, $\tilde{q}(0)$) in which the state-contingent payment is expressed in the quantity of the good of low quality. For example $\tilde{q}(\hat{h}) = 1.\overline{95}$ means the payment made to the $\hat{h}$ agent is equivalent to the market value of $1.\overline{95}$ units of the low-quality good. The intermediary sells the inspected and the uninspected goods in the commodity market at the market prices. Then there are four types of commodities traded in the market: the high-quality, the low-quality and the uninspected goods from the intermediary and the constant-quality good from the C agent. Let $q(h)$, $q(c)$, $q(\ell)$, $q(\hat{h})$ and $q(u)$ denote respectively the market prices of the high-quality, constant-quality, low-quality, uninspected good from intermediary and uninspected good from the type ℓ agent. We show that $(q(h), q(\hat{h}), q(c), q(\ell), q(u) = (2, 1.98, 1, 1, 1)$ supports the aforementioned state-contingent contract $(\xi, \tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0)) = (0.5, 1.\overline{95}, 0, 1.98)$ as a market equilibrium with a number of price-taking intermediaries in which all types of agents are market prices-takers and intermediaries also take the state-contingent contracts as given. As previously shown, given the state-contingent contract, the best choice for the $\hat{h}$ agent is to trade with the intermediary. The type ℓ agent is indifferent between remaining autarky and selling his endowment directly in the market. The type C agent obtains the same level of marginal utility from each unit of incomes spending on the high-quality, constant-quality and uninspected good is identical and is greater than that on the low-quality and the uninspected good from the ℓ agent. Thus the consumption allocation in the previous analysis is consistent with the market prices, $(q(h), q(\hat{h}), q(c), q(\ell), q(u)$ $= (2, 1.98, 1, 1, 1)$. The intermediated state-contingent is compatible with goods market. That is market mechanisms together with state-contingent contracts can resolve the adverse selection problem arisen from asymmetric information in this numerical example. Notice that in such an environment, intermediation emerges endogenously.

Nonetheless market mechanisms are not necessary a panacea for every environment with adverse selection. The next economic environment provides an example in which market mechanisms do not work and the intermediated state-contingent fails to coordinate with market prices of goods.

Numerical Example 3: The Failure of Market Mechanisms

Consider a pure exchange economy in which there are one good with different quality levels and two types of agents, type C and type U, differing in their preferences and endowments. The good has two quality levels, high(h) and low(ℓ). The type C agent distinguishes the quality levels of the good and his preferences are represented by the utility function,

$$V_C = 2 \cdot x(C,h) + x(C,\ell),$$

where $x(C,s)$ is the quantity of the good with s-quality consumed by type C agents, $s = h,\ \ell$. The high-quality good yields a marginal utility level of two and the low-quality good yields a marginal utility level of one. Type U agents do not distinguish the quality levels of the good. Their preferences are also linear and can be represented by

$$V_U = x(U,h) + x(U,\ell).$$

Each type C agent is endowed with one unit of constant, low quality good (good ℓ), while each type U agent is endowed with one unit of the good with uncertain-quality (good u). Each type U agent receives as private information a signal, either $\hat{h}$ or $\hat{\ell}$, about the quality of his endowment. The signal $\hat{h}$ indicates that the endowment is high-quality with probability 0.9 and low-quality with probability 0.1. The set of type C agents is of measure 0.1 and the set of type U agents is of measure 0.9. Suppose that there are equal numbers of type U agents with signal $\hat{h}$ and with signal $\hat{\ell}$. The type U the agent with signal $\hat{h}$ is called the type $\hat{h}$ agent and the type U agent with signal $\hat{\ell}$ is called the $\hat{\ell}$ agent.

No private information is revealed

When no signal is available about the quality of good u, the price a type C agent would pay for a unit of good u is not greater than 1.5 units of the good ℓ. As long as the type C agent pays more than one unit of good ℓ for one unit of good u, trade between type C agents of type U agent occurs. In fact, type C agents are outnumbered by type U agents, and the price of good u would be driven down to one.

Bilateral Exchange and Gains from Inspection

Suppose that there exists a technology of inspection which costs 0.05 unit of the good to uncover the quality level of the inspected unit. The inspection technology enables type $\hat{h}$ agents to separate themselves from type $\hat{\ell}$ agents and to obtain some welfare gains. Consider the following trade contract $(\tilde{q}(h),\ \tilde{q}(\ell))$:

(1) one type C agent and one type U agent get together and use 0.05 unit of constant-quality good to inspect the type U agent's endowment;

(2) if the outcome is high quality, the type U agent gets $\tilde{q}(h)$ units of any good while the type C agent gets 1.95 - $\tilde{q}(h)$ units of goods. Assign the high quality (good h) to the type C agent first and, if there is some left, is assigned to the type U agent;

(3) if the outcome is low quality, the type U agent gets $\tilde{q}(\ell)$ units of any good while the type C agent gets 1.95 - $\tilde{q}(\ell)$ units of goods. The uninspected good (good u) is assigned to the type C agent first and then, if some left, is assigned to the type U agent.

The state-contingent contract ($\tilde{q}(h)$, $\tilde{q}(\ell)$) is incentive compatible if and only if

$$0.9 \cdot \tilde{q}(h) + 0.1 \cdot \tilde{q}(\ell) \geq 1, \qquad \text{and}$$

$$0.1 \cdot \tilde{q}(h) + 0.9 \cdot \tilde{q}(\ell) < 1.$$

It is clear that the incentive compatible contract exists. Due to the fact that type C agents are outnumbered by type h agents, the contract will be such that the type $\hat{h}$ agent obtains one as her expected utility. As a result, the contract $(\tilde{q}(h), \tilde{q}(\ell))$ = (1/0.9, 0) is the incentive compatible contract with which the arrangement of bilateral exchange between the type $\hat{h}$ agent and the type C agent would end up. The type C agent obtains 1.705 (=0.9×(1.95 - 1/0.9)×2+0.1×1.95×1) as her expected utility. The contractual, bilateral exchange makes efficiency gains by inspecting measure ten percents of type $\hat{h}$ agents' endowments, all of those present to type C agents.

Intermediated Contractual Arrangement
and Randomized Inspection

As in the previous example, randomized inspection together with intermediated contractual arrangement gives more efficiency gains than inspecting all good u with signal h which are traded. Let (ξ, $\tilde{q}(h)$, $\tilde{q}(\ell)$, $\tilde{q}(0)$) denote a contract with a probability of ξ to inspect good u. When the inspection outcome is h, gives $\tilde{q}(h)$ units of goods to the type U agent; when the inspection outcome is ℓ, gives $\tilde{q}(\ell)$ units of goods to the type u agent. When no inspection occurs, gives $\tilde{q}(0)$ units to the type u agent. The type U agent can at most get one as utility when directly selling his endowment to the type C agent because that both subtypes of type U agents outnumber the type C agent. Thus we can write the incentive compatibility conditions for a contract $(\xi, \tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0))$ as

$$\xi \cdot (0.9 \cdot \tilde{q}(h) + 0.1 \cdot \tilde{q}(\ell)) + (1 - \xi) \cdot \tilde{q}(0) \geq 1, \qquad \text{and}$$

$$\xi \cdot (0.1 \cdot \tilde{q}(h) + 0.9 \cdot \tilde{q}(\ell)) + (1 - \xi) \cdot \tilde{q}(0) < 1.$$

Consider an economic agent (either a type U agent or a type $\hat{h}$ agent) who on the one hand signs contracts with many type C agents in which this agent exchanges with the type C agent 0.451335 unit of the high-quality good and 0.597567 unit of the uninspected $\hat{h}$ good for the type C agent's endowment, and on the other hand signs the state-contingent contract $(\xi, \tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0))$ = (0.09733, 1.10870, 0, 1.00023) with many type U agents. First note that the proposed contract is incentive compatible, because from the contract a type $\hat{h}$ agent would reach slightly greater than one of expected utility, while a type $\hat{\ell}$ agent would obtain expected utility of 0.91367. As a result, only type $\hat{h}$ agent would go for the contract.[2]

Notice that this state-contingent contractual arrangement with randomized inspection is Pareto superior to the bilateral exchange with inspection due to the fact that the randomization of inspection enables the society to make differences between type $\hat{h}$ agents and type $\hat{\ell}$ agents at lower costs. The foregoing state contingent contractual arrangement inspects only measure of 0.048665 (= 0.5×0.09733) of $\hat{h}$ goods, versus measure of 0.1 in the bilateral exchange, and the type C agent obtains an even higher expected utility, 1.89757 versus 1.705.

Failure of Market Mechanisms

In this economic environment, the intermediated contractual arrangement with randomized inspection fails to coordinate with competitive markets of commodities, although it is Pareto improving to bilateral exchange.

Given the state-contingent contract $(\xi, \tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0))$ = (0.09733, 1.10870, 0, 1.00023) and the contract between the intermediary and the type C agent, the consumption allocation is such that the type $\hat{h}$ agent consumes both of the uninspected $\hat{h}$ good and the low-quality good and the

[2] Appendix II provides details for the calculation of this example.

type C agent consumes both of the high-quality and the uninspected goods. The prices of goods consistent with the type C agent's consumption pattern would be $q(h) = 2/1.9$, $q(\hat{h}) = 1$, and $q(\ell) = 1$. However such a set of prices is inconsistent with competitive equilibrium. Given such a set of goods prices and the foregoing state-contingent payment, on the one hand, the type C agent would not use up his incomes (his income is 1 while his spending is about 0.99872). This means both high-quality and intermediately uninspected $\hat{h}$ goods markets are in shortage, the price of these two goods would be adjusted up, and then the goods prices would be inconsistent with the type $\hat{h}$ agent's consumption pattern. On the other hand, the intermediary would make positive profits (≈ 0.097), implying that there will be more agents entering into the intermediation business by increasing state-contingent payments and the price of good h and $\hat{h}$(due to the shortage of both goods). Such a competition process will end up with nonexistence of competitive equilibrium. Consider a market economy in which all goods are traded in the market. Appendix III discusses how to compute the equilibrium and show the nonexistence of market equilibrium in this numerical example.

Summary and Concluding Remarks

This paper tries to illustrate by examples that intermediated contractual arrangements make efficiency gains under price mechanisms. Three examples of pure exchange environments, one in partial equilibrium and two in general equilibrium, are provided. When asymmetric information causes adverse selection problems, intermediated contractual arrangements with state-contingent payments and randomized information-processing make efficiency gains. Moreover, in some of these examples, intermediated contractual arrangements can be decentralized and coordinate with competitive market prices of goods. A policy implication is that the presence of imperfect information does not necessarily gives reasons for the government's intervention in the market. The market itself can handle information problems. When an appropriate information technology presents in some environments with adverse selection problems, market mechanisms together with intermediated contractual arrangements perform well just like they do in a perfect information world. However, market mechanisms sometimes does not work well enough to eliminate all adverse selection problems. In numerical example 3, we show a case in which contractual arrangements through decentralized intermediaries alone can solve the problem; however, such arrangements cannot coordinate with market prices of goods.

Appendix I

This appendix shows how we compute allocation of goods in Numerical 2.

- ***Inspection Without Intermediation***

Given this state-contingent contract in the paper, the payoff each agent obtains is:

$$\hat{h}:\ 0.99 \cdot (0.95 + t) + 0.99 \cdot 0$$

$$\hat{C}:\ 0.99 \cdot (1 - t) \cdot 2 + 0.01 \cdot 0.95$$

$$\hat{\ell}:\ 0.01 \cdot (0.95 + t) + 0.99 \cdot 0$$

Since the C agents outnumbers the $\hat{h}$ agents, gains from bilateral exchange are fully appropriated by the $\hat{h}$ agent. Consequently, t satisfies:

$$0.99 \cdot (1 - t) \cdot 2 + 0.01 \cdot 0.95 = 1 \qquad \text{or} \qquad t = \frac{(1.98 + 0.0095 - 1)}{1.98} = 0.499\overline{74} \approx 0.5.$$

- ***Randomized Inspection***

The computation proceeds in the following steps:

i Given a state contingent contract $(\xi,\ \tilde{q}(h),\ \tilde{q}(\ell),\ \tilde{q}(0))$, we figure out the amounts of different goods supplied by the the intermediary.

ii Distribute goods to the type C agent such that each type C agent's utility is slightly greater than one. As a result, every type C agent going to the centralized intermediary would obtain utility slightly higher than he would obtain if remains autarky. Moreover the distribution of goods to the type C agent is proceeded in the order of the high-quality, the uninspected $\hat{h}$ and C.

iii Figure out the intermediated state-contingent payment scheme by using feasibility condition and incentive compatibility for the type ℓ agent.

The results of the first two steps are summarized as follows:

Goods	The Amount Available	The Amount Assigned to each of C	The Amount Left for $\hat{h}$
h	$0.5 \times 0.99 \times \xi$	$0.495 \times \xi$	0
$\hat{h}$	$0.5 \times (1 - \xi)$	$0.5 \times (1 - \xi)$	0
c	$1 - 0.5 \times 0.05 \times \xi$	0.01	$0.99 - 0.025 \times \xi$
ℓ	$0.5 \times 0.01 \times \xi$	0	$0.005 \times \xi$

The type C agent's utility from the transaction with the intermediary is $0.5 \times 0.99 \times \xi \times$

$+0.5 \times (1-\xi) \times 1.98 + 0.01 = 1$. When giving each type C agent slightly more than 0.01 units of the constant-quality good, the type C agent will attain a utility level slightly higher than one and, thus, the type C agent is happy to make transaction with the intermediary.

After allocating goods to the type C agents, the remained amounts of goods have to be assigned to type $\hat{h}$ agents. Since no distinction between different quality levels is made by the $\hat{h}$ agent, we only need to count the usage and availability of goods. Given the state-contingent payment scheme (ξ, $\tilde{q}(h)$, $\tilde{q}(\ell)$, $\tilde{q}(0)$). Feasibility requires

$$0.5 \times [0.99 \times \xi \times \tilde{q}(h) + (1-\xi) \times \tilde{q}(0)] = (0.99 - 0.025 \times \xi) + 0.005 \times \xi \qquad \text{and}$$

$$\Longleftrightarrow \quad [0.495 \times \tilde{q}(h) - 0.5 \times \tilde{q}(0)] \times \xi + 0.5 \times \tilde{q}(0) = -0.02 \times \xi + 0.99.$$

Thus $\tilde{q}(0) = 1.98$ and $\tilde{q}(h) = 1.\overline{95}$. Now we check incentive compatibility. Now we check incentive compatibility. Notice since $\xi \in [0,1]$, the state-contingent payment $(\tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0)) = 1.\overline{95}, 0, 1.98)$ always satisfies incentive compatibility of the type $\hat{h}$ agent:

$$\xi \times 0.99 \times \tilde{q}(h) + (1-\xi) \times \tilde{q}(0) \geq \max\{q(u), 1\} (= \ 1)$$

We use the type $\hat{\ell}$ agent's incentive compatibility to compute the probability of inspection:

$$\xi \times 0.01 \times \tilde{q}(h) + (1-\xi) \times \tilde{q}(0) \leq \max\{q(u), 1\} (= \ 1)$$

$$\Longleftrightarrow \quad (0.01 \times 1.\overline{95} - 1.98) \times \xi \leq -0.98$$

$$\Longleftrightarrow \quad \xi \geq 0.4998724$$

As a result the state contingent contract (ξ, $\tilde{q}(h)$, $\tilde{q}(\ell)$, $\tilde{q}(0)$) = (0.5, $1.\overline{95}$, 0, 1.98) is feasible and incentive compatible.

Appendix II: Calculation of Numerical Example 3

Consider the intermediary offers an incentive compatible state contingent contract, $(\xi,\ \tilde{q}(h),\ \tilde{q}(\ell),\ \tilde{q}(0))$ to the type U agent and offers the type C agent a contract of exchanging the high-quality good and the uninspected $\hat{h}$ good for the low-quality good such that the type C agent would attain one as as his utility level. Due incentive compatibility, all type $\hat{h}$ agent take the contract from the intermediary and all type $\hat{\ell}$ choose not to go through intermediation. The goods available from intermediation and the allocation of these goods such that the type C agent attains one as his utility level are given in the following table:

Goods	The amount available	The amount left for $\hat{\ell}$	The amount assigned to each of C
h	$0.5 \times 0.9 \times \xi$	0	$0.5 \times 0.9 \times \xi$
$\hat{h}$	$0.5 \times (1-\xi)$	0	$\mu_c \left[\frac{0.5\times 0.9\times\xi}{0.1} + \frac{1-2\times\frac{0.5\times0.9\times\xi}{0.1}}{1.9}\right]$
ℓ	$0.1 + 0.05\,\xi - 0.025\,\xi$	$0.1 + 0.05\,\xi - 0.025\,\xi$	0

The calculation of the incentive compatible contract $(\xi,\ \tilde{q}(h),\ \tilde{q}(\ell),\ \tilde{q}(0))$ is shown below. First to penalize the bad inspection outcome, we set $\tilde{q}(\ell) = 0$ and use feasibility condition to compute the state contingent payments $\tilde{q}(h)$ and $\tilde{q}(0)$. Then we use the type $\hat{\ell}$'s incentive compatibility condition to compute the lowest possible value of ξ.

Feasibility

$$0.5\ (\tilde{q}(0)\ (1-\xi) + 0.9\,\tilde{q}(h)\,\xi)$$

$$= \overbrace{0.45\,\xi}^{h} + \underbrace{(0.5 - 0.5\,\xi)}_{\hat{h}} + \overbrace{(0.1 + 0.05\,\xi - 0.025\,\xi)}^{\ell} - \underbrace{(0.0526316 - 0.0236842\,\xi)}_{\text{dist. to C}}$$

$$\Longrightarrow 0.5\,\tilde{q}(0) + (0.45\,\tilde{q}(h) - 0.5\,\tilde{q}(0))\ \xi = 0.547368 - 0.0013158\,\xi$$

$$\Longrightarrow 0.5\,\tilde{q}(0) = 0.547368 \qquad \Longrightarrow \qquad \underline{\tilde{q}(0) = 1.094736}$$

$$(0.45\,\tilde{q}(h) - 0.5\,\tilde{q}(0))\ \xi = -0.0013158\,\xi \ \Longrightarrow\ -0.547368 + 0.45\,\tilde{q}(h) = -0.0013158$$

$$\Longrightarrow \qquad \underline{\tilde{q}(h) = 1.2134493}$$

Incentive compatibility of $\hat{\ell}$

$$\xi\,\pi(h|\hat{\ell})\,\tilde{q}(h) + (1-\xi)\,\tilde{q}(0) < 1$$

$$\Longrightarrow 0.1 \times 1.21345\,\xi + (1-\xi) \times 1.094736 < 1$$

$$\Longrightarrow 1.09474 - 0.973391\xi < 1 \Longrightarrow \qquad \xi > 0.0973253 \qquad \text{We set } \underline{\xi = 0.09733}.$$

The state-contingent contract from this calculation is

ξ	$\tilde{q}(h)$	$\tilde{q}(\ell)$	$\tilde{q}(0)$
0.009733	1.2134493	0	1.094736

Under this contract the type $\hat{h}$ agent's expected utility is 1.09448 and the type $\hat{\ell}$ agent's expected utility is 0.999995.

Assume the inspection technology is accessible to every agent and the entrance into the intermediation business is free. Thus the competition in the market makes zero profits for intermediation business. Due to the fact that the type $\hat{h}$ agent exceeds the type C agent in number, competition would make the $\hat{h}$ agent's expected utility is just about one, the autarky level, and the type C agent appropriates all benefits from inspection. As a result, the state contingent contract becomes

ξ	$\tilde{q}(h)^*$	$\tilde{q}(\ell)^*$	$\tilde{q}(0)^*$
0.009733	1.10869951	0	1.0000234015

* Dividing the previous payments by 1.0944798474 we get this new set of payments.

Under this contract the type $\hat{\ell}$ agent's expected utility is 0.91368, and incentive compatibility holds. Corresponding to this new set of state-contingent payments, the intermediary offers the type C agent a contract in which each type C agent get 0.437966 units of the high-quality good and 0.537702 units of the uninspected $\hat{h}$ good. The type C agent's utility under this contract is 1.89757. The allocation of goods is summarized in the following table:

Goods	The Amount Available	The Amount left for $\hat{h}$	The Amount Assigned to Each of C
h	0.043797	0	0.043797
$\hat{h}$	0.451334	0.397567	0.053770
ℓ	0.102433	0.102433	0

• ***Market Prices of Goods and Intermediated Contracts***

The following is to show the intermeidated contractual arrangement mentioned above cannot be supported as a competitive equilibrium in which all transactions of goods take place in the market of goods. The prices of goods consistent with the consumption pattern are:

$q(h)$	$q(\hat{h})$	$q(\ell)$
2/1.9	1	1

The calculation of the type C agent's spending and the intermeidary's profits are as the following tables:

The intermediary's profit:

Sale Revenue:	0.5998719398149570
State-contingent Payment:	0.5000000000000000
Inspection Cost:	0.0024331435158123
Profits:	0.0974387962991450

The type C agent's spendings and incomes:

Payments:	0.9987193981495720
Incomes:	1.0000000000000000

Appendix III

This appendix addresses the discussion on the non-existence of general equilibrium in Numerical Example 3. The environment is characterized by three types of agents, each type endowed with one unit of the good and are distinguished by their endowments and preferences . The type C agent is endowed with one unit of the good with certain, low (denoted by ℓ) quality. The type $\hat{h}$ agent is endowed with one unit of the good with uncertain quality and private information $\pi(h|\hat{h})$. The type ℓ agent is endowment with one unit of the good with uncertain quality and private information $\pi(h|\hat{\ell})$. Private information $\pi(h|\hat{h})$ is the probability of the type $\hat{h}$ agent's endowment being of high quality and $\pi(h|\hat{\ell})$ is the probability of the type ℓ agent's endowment being of high quality. Assume that $\pi(h|\hat{h}) > \pi(h|\hat{\ell})$. The type C agent obtains different utility levels from different qualities of the good; more specifically, There the a contiuum of agents of which the measures of type C, $\hat{h}$, and ℓ agent are μ_c, $\pi(\hat{h})$ and $\pi(\ell)$ respectively.

$$U_C = u_h \cdot x(C,h) + u_\ell \cdot x(C,\ell)$$

The type $\hat{h}$ agent and the type ℓ agent do not make differences between high quality and low quality:

$$U_{\hat{h}} = x(\hat{h},h) + x(\hat{h},\ell), \qquad \text{and}$$

$$U_{\hat{\ell}} = x(\hat{\ell},h) + x(\hat{\ell},\ell).$$

where $x(i,j)$ is agent i's consumption of good j. There exists an inspection technology which requires m units of the good with low quality to uncover the true quality of one unit uncertain-quality good.

- **Decentralized Intermediation with Randomized Inspection**

If randomized inspection takes place and not all goods with uncertain quality are inspected, there are four types of goods are traded in the market: high quality (h), low quality (ℓ), intermediated uninspected good ($\hat{h}$) (due to incentive compatibility, the goods go through intermediation must be the good with probability $\pi(h|\hat{h})$) and non-intermediated uninspected good ($\hat{u}$).

Let $(\xi, \tilde{q}(h), \tilde{q}(\ell), \tilde{q}(0))$ be a state-contingent payment scheme with randomized-inspection and $(q(h)$, $q(\hat{h})$, $q(\ell)$, $q(u))$ be the market prices of goods of different qualities.

An equilibrium involves the consumption of every agent, the decision of each agent on how to trade his endowment, the state contingent contract and market prices. The assumption of atomic agents in the environment makes the agent is a price-taker in the market. In equilibrium the state contingent contract satisfies incentive constraints:

$$(\text{IC}_{\hat{h}}) \qquad \xi \cdot \left[\pi(h|\hat{h})\tilde{q}(\hat{h}) + \pi(\ell|\hat{h})\tilde{q}(\ell)\right] + (1-\xi)\cdot\tilde{q}(0) \geq \max\{q(u), 1\}$$

$$(\text{IC}_{\hat{\ell}}) \qquad \xi \cdot \left[\pi(h|\hat{\ell})\tilde{q}(h) + \pi(\ell|\hat{\ell})\tilde{q}(\ell)\right] + (1-\xi)\cdot\tilde{q}(0) < \max\{q(u), 1\};$$

and zero-profit condition for intermediation:

$$\begin{aligned} \xi \cdot \left[\pi(h|\hat{h})\tilde{q}(h) + \pi(\ell|\hat{h})\tilde{q}(\ell)\right] + (1-\xi)\cdot\tilde{q}(0) \\ = \xi \cdot \left[\pi(h|\hat{h})q(h) + \pi(\ell|\hat{h})q(\ell) - m\right] + (1-\xi)\cdot q(\hat{h}) \end{aligned}$$

- **Solving Equilibirum**

1. First check zero-profit condition to find out the relation between the prices of goods, ($q(h)$, $q(\ell)$, $q(\hat{h})$, $q(u)$), and the state-contingent payment, (ξ, $\tilde{q}(h)$, $\tilde{q}(\ell)$, $\tilde{q}(\hat{h})$). Given a ξ, the comparison of both sides of the zero-profit condition gives us:

   $\tilde{q}(0) = q(\hat{h})$, and

   $\pi(h|\hat{h})\tilde{q}(\hat{h}) + \pi(\ell|\hat{h})\tilde{q}(\ell) = \pi(h|\hat{h})q(h) + \pi(\ell|\hat{h})q(\ell) - m$

Penalizing agents who presenting the good with a bad inspection outcome by setting $\tilde{q}(\ell) = 0$, we obtain that

$$\tilde{q}(h) = q(h) + \frac{\pi(\ell|\hat{h})}{\pi(h|\hat{h})}q(\ell) - \frac{m}{\pi(h|\hat{h})}, \qquad \tilde{q}(u) = q(\hat{h}).$$

2. Computing the equilibrium prices of goods h, ℓ, $\hat{h}$, and u, and check incentive constraints.

The first part can be done by using market clearings, consumption decision rules and budget constraints. The second part is done by plugging the outcomes in the first part into incentive constraints to see if they are satisfied. There are four possible outcomes for equilibrium; nonetheless, after checking incentive constraints, only one case survives.

Since in equilibrium incentive constraints hold, the type ℓ agent would be in autarky. It is enough to classify equilibrium outcomes by only taking into account type C and type $\hat{h}$ agent's consumption.

(1) The type C agent consumes good h only, while type $\hat{h}$ agent consumes good h, $\hat{h}$, and ℓ.

Then the prices are:

$q(h) = q(\hat{h}) = q(\ell) = 1$ (by the consumption decision rules of type $\hat{h}$ agent)

The consumption allocation for type C and type $\hat{h}$ agents:

$x(C, h) = 1$, $x(C, \hat{h}) = x(C, \ell) = 0$ (by the type C agent's budget constraint and consumption decision rule.)

The state contingent payments are $\tilde{q}(h) = (1 - m)/\pi(h|\hat{h})$, $\tilde{q}(\ell) = 0$, and $\tilde{q}(0) = 1$.

The type $\hat{h}$ agent's expected utility is less than one, the incentive constraint does not holds:

$$\begin{aligned} U_{\hat{h}} &= \xi \cdot \left[\pi(h|\hat{h}) \cdot \tilde{q}(h) + \pi(\ell|\hat{h}) \cdot \tilde{q}(\ell)\right] + (1 - \xi) \cdot \tilde{q}(0) \\ &= \xi \cdot \frac{1 - m}{\pi(h|\hat{h})} + (1 - \xi) \\ &= 1 - \xi \cdot m < 1. \end{aligned}$$

The incentive compatibility for the type $\hat{h}$ agent fails.

Thus, it cannot be an equilibrium outcome that both type C and type $\hat{h}$ agents consume good $\hat{h}$

(2) The type C agent consumes good h only, while type $\hat{h}$ agent consumes good $\hat{h}$, and ℓ. Then: the prices are: $q(h) = \left(\frac{\xi\pi(\hat{h})\pi(h|\hat{h})}{\mu_c}\right)^{-1}$, (from market-clearing) and $q(\hat{h}) = q(\ell) = 1$.

The state-contingent payments are $\tilde{q}(h) = \mu_c/(\xi \cdot \pi(\hat{h}) \cdot \pi(h|\hat{h})) + \pi(\ell|\hat{h})/\pi(h|\hat{h}) - m/\pi(h|\hat{h})$, $\tilde{q}(\ell) = 0$ and $\tilde{q}(0) = 1$. The type C agent's utilities is

$$U_c = \frac{\pi(\hat{h})}{\mu_c} \cdot \xi \cdot \pi(h|\hat{h}) \cdot u_h$$

$$\begin{aligned} U_{\hat{h}} &= \xi \cdot \pi(h|\hat{h}) \cdot \left(\frac{\mu_c}{\xi \cdot \pi(\hat{h}) \cdot \pi(h|\hat{h})} + \frac{\pi(\ell|\hat{h})}{\pi(h|\hat{h})} - \frac{m}{\pi(h|\hat{h})}\right) + (1 - \xi) \\ &= \frac{\mu_c}{\pi(\hat{h})} + \xi \cdot \pi(\ell|\hat{h}) - m + (1 - \xi) \end{aligned}$$

Note that (1) when $IC_{\hat{h}}$ holds and (2)consumers in the market know if an uninspected goods if from the intermeidary, those from the type ℓ agents would be recognized and thus would worth no more than $E(U|\hat{\ell})$ which is less than 1. So $\max\{q(u), 1\} = 1$. It is clear that $IC_{\hat{h}}$ holds if and only if $\xi \leq \frac{\frac{\mu_c}{\pi(\hat{h})} - q(u) + 1}{1 - \left(\pi(\ell|\hat{h}) - m\right)}$ From the type ℓ agent's incentive compatibility

$$\xi \cdot \pi(h|\hat{\ell}) \cdot \left(\frac{\mu_c}{\xi \cdot \pi(\hat{h}) \cdot \pi(h|\hat{h})} + \frac{\pi(\ell|\hat{h})}{\pi(h|\hat{h})} - \frac{m}{\pi(h|\hat{h})}\right) + (1 - \xi) \leq 1,$$

one can see that $\xi \geq \dfrac{\frac{\mu_c}{\pi(\hat{h})}\frac{\pi(h|\hat{\ell})}{\pi(h|\hat{h})} - q(u) + 1}{1 - \frac{\pi(h|\hat{\ell})}{\pi(h|\hat{h})}(\pi(\ell|\hat{h}) - m)}$; if $1 - \dfrac{\pi(h|\hat{\ell})}{\pi(h|\hat{h})}(\pi(\ell|\hat{h}) - m) > 0$

In sum,

$$\underline{\xi} \leq \xi \leq \overline{\xi} \text{ where } \underline{\xi} = \frac{\frac{\mu_c}{\pi(\hat{h})}\frac{\pi(h|\hat{\ell})}{\pi(h|\hat{h})}}{1 - \frac{\pi(h|\hat{\ell})}{\pi(h|\hat{h})}(\pi(\ell|\hat{h}) - m)} \text{ and } \overline{\xi} = \frac{\frac{\mu_c}{\pi(\hat{h})}}{1 - \left(\pi(\ell|\hat{h}) - m\right)}.$$

Note that $\pi(h|\hat{h}) > \pi(h|\hat{\ell})$ and thus $\underline{\xi} < \overline{\xi}$. Competition in intermediate business would drive ξ to $\underline{\xi}$. What remained to check is that if the above prices, and the consumption pattern are consistent with utility rationality of all types of agents.

(3) The type C agent consume good h, $\hat{h}$ and the type $\hat{h}$ agent consumes good $\hat{h}$, and ℓ. Then the price are: $q(h) = \dfrac{u_h}{E(u|\hat{h})}$ and $q(\hat{h}) = q(\ell) = 1$.

The expected utility levels and consumption allocations are:

$U_c = E(U|\hat{h})$

$x(C,h) = \xi \cdot \frac{\pi(\hat{h})}{\mu_c} \cdot \pi(h|\hat{h})$; $x(C,\hat{h}) = 1 - \xi \cdot \frac{\pi(\hat{h})}{\mu_c} \cdot \pi(h|\hat{h}) \cdot \frac{u_h}{E(U|\hat{h})}$

$U_{\hat{h}} = 1 - (\pi(h|\hat{h}) + m) \cdot \xi + \dfrac{\pi(h|\hat{h}) \cdot u_h}{E(U|\hat{h})} \cdot \xi$ (from IC-state-contingent payment scheme)

$$= 1 - \left[\pi(h|\hat{h}) + m - \frac{\pi(h|\hat{h}) \cdot u_h}{E(U|\hat{h})}\right] \cdot \xi$$

$$= 1 + \left[\frac{\pi(h|\hat{h}) \cdot u_h}{E(U|\hat{h})} - \pi(h|\hat{h}) - m\right] \cdot \xi$$

$x(\hat{h},\ell) = \xi \cdot \pi(\ell|\hat{h}) + \frac{\mu_c - m \cdot \xi \cdot \pi(\hat{h})}{\pi(\hat{h})}$ $x(\hat{h},\hat{h}) = 1 - \xi - \frac{\mu_c}{\pi(\hat{h})} + \xi \cdot \frac{\pi(h|\hat{h}) \cdot u_h}{E(U|\hat{h})}$

This is the average consumption of the type $\hat{h}$ agent. In equilibrium (if it exists) not every type $\hat{h}$ agent consumes the same amount of goods. The consumption depends on state-contingent incomes.

What remained to check is that if incentive compatibility holds and if prices and the conusmption pattern are consistent with utility-rationality. For example, from the type $\hat{h}$ agent's incentive compatibility($IC_{\hat{h}}$), we know that when $U_{\hat{h}} = 1 + \left[\dfrac{\pi(h|\hat{h}) \cdot u_h}{E(U|\hat{h})} - \pi(h|\hat{h}) - m\right] \cdot \xi < 1$, ($IC_{\hat{h}}$) fails.

So, it is clear when $\left[\dfrac{\pi(h|\hat{h}) \cdot u_h}{E(U|\hat{h})} - \pi(h|\hat{h}) - m\right] < 0$, ($IC_{\hat{h}}$) fails and equilibrim outcomes cannot have the consumption pattern in which the type C agent consumes goods h and $\hat{h}$ and the type $\hat{h}$ agent consumes goods $\hat{h}$ and ℓ.

(4) The agent C consume good h and $\hat{h}$ and the agent $\hat{h}$ consume good ℓ.

This cannot be an equilibrium. When incentive comaptibility holds, the uninspected good $\hat{h}$ can be told from ℓ; thus the uninspected good ℓ would have a price of $E(U|\hat{\ell}) < 1$ if it were purchased by type C agents. Then the type ℓ agent has no incentive to trade and stays in autarky. So when type $\hat{h}$ consumes only good ℓ, the total amount of good ℓ available would not be enough to make $IC_{\hat{h}}$ sustain.

From the above discussion, the equilibrium outcome has two possible consumption patterns: (1) the type C agent consumes good h only, while the type $\hat{h}$ consumes good $\hat{h}$ and ℓ; (2) the type C agent consumes good h and $\hat{h}$, while the type $\hat{h}$ agent consumes good $\hat{h}$ and ℓ.

In the last part of this appendix we will follow the above discussion to check the equilibrium outcome of Numerical Example 3. The conclusion is that there is no competitive equilibrium in this numerical example. The computational result is reported in the following table.

Numerical Example 3:

Parameter Values

$\pi(\hat{h})$	$\pi(\ell)$	$\mu(c)$	m	
0.5	0.5	0.1	0.05	
$\pi(h\|\hat{h})$	$\pi(h\|\ell)$	u_h	u_ℓ	$E(U\|\hat{h})$
0.9	0.1	2	1	1.9

Two possible equilibrium outcomes:

(1) The type C agent consumes good h.
The type $\hat{h}$ agent consumes good $\hat{h}$ and ℓ.

The Prices of Goods			
$q(h)$	$q(\hat{h})$	$q(\ell)$	$q(u)$
9.944444	1	1	1
The State-Contingent Contract			
ξ	$\tilde{q}(h)$	$\tilde{q}(\ell)$	$\tilde{q}(0)$
0.022346	10	0	1

Utility-Rationality does not hold:

$$\frac{u_h}{q(h)} \approx 0.20112 \leq \frac{E(u|\hat{h})}{q(\hat{h})} = 1.9$$

(2) The type C agent consumes good h and $\hat{h}$.
The type $\hat{h}$ agent consumes good $\hat{h}$ and ℓ.

The Prices of Goods			
$q(h)$	$q(\hat{h})$	$q(\ell)$	$q(u)$
1.05632	1	1	1
The State-Contingent Contract			
ξ	$\tilde{q}(h)$	$\tilde{q}(\ell)$	$\tilde{q}(0)$
	1.108187	0	1

The incentive compatibility of type $\hat{h}$ agent's does not hold:

$$\left[\frac{\pi(h|\hat{h})\cdot u_h}{E(U|\hat{h})} - \pi(h|\hat{h}) - m\right] \approx -0.0026316 < 0.$$

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