Interest-rate rules and transitional dynamics in an endogenously growing open economy

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Abstract

This paper sets up an endogenous growth model of an open economy in which the monetary authority implements a gradualist interest-rate rule with targets for both inflation and economic growth. We show that, under a passive rule, a monetary equilibrium exists and is unique. Moreover, the equilibrium is locally determinate. By contrast, an active rule implies either two equilibria, one high-growth and one low-growth, or none. In the case of two equilibria, the high-growth equilibrium is locally determinate, while the low-growth equilibrium is a source. Besides, the stabilization and growth effects of alternative target policies are also explored in this study. Moreover, in departing from the existing literature, we turn to focus on the analysis of transition with a particular emphasis on the case of imperfect credibility.
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1. Introduction

The interest of economists in interest-rate rules has been sparked by early studies by Sargent and Wallace (1975) and McCallum (1981) and revived by Taylor (1993) in the light of

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description for U.S. monetary policy.[1] Taylor (1993) shows that since 1987 U.S. monetary policy can be well described in terms of a simple rule whereby the central bank sets the short-term nominal interest rate as an increasing linear function of a measure of inflation and the output gap with coefficients of 1.5 and 0.5, respectively. According to various coefficients of inflation, interest-rate rules can be divided into two distinctive styles. If a feedback rule has an inflation coefficient greater than unity (which satisfies the Taylor principle), it is dubbed an *active* interest-rate rule (or Taylor's rule). By contrast, a *passive* rule involves an inflation coefficient of less than unity. Taylor (1999) claims that the active feedback rule is widely judged to have been unusually successful in the United States, suggesting that the rule is worth adopting as a principle of behavior. In fact, in many industrialized countries their monetary policies can be also classified into various styles of interest-rate rules (see Clarida et al., 2000).

Ever since Taylor's (1993) seminal work, the related issues have been extensively analyzed in a wide variety of models that include interest-rate rules aimed at inflation targets and/or output targets.[2] Nevertheless, this is still a contentious issue that deserves more careful investigation. There exist two facts which are neglected by the theoretical literature.

First, traditionally, interest-rate rules followed by central banks have been regarded as devices for macroeconomic stabilization in the *short run*. However, in practice, central banks do not focus solely on welfare gains from stabilization in economic environments with uncertainty (the short run goal), but they also pursue *long run* development goals, such as economic growth.[3] For example, since 1997 the Bank of England's *Inflation Report* has reported projections for four-quarter inflation and real GDP growth for an eight-quarter forecast horizon. Since 1992 Sweden's Riksbank has published forecasts for Q4/Q4 real GDP growth and December/December CPI inflation for the current year and the two following "out" years (see Kuttner, 2004). Similar projections were also made by the central bankers of New Zealand, Australia and Canada though some of these projections may be not available publicly.

Second, there is an evident fact that in an open-economy environment, real-world central banks have the option to trade foreign assets at a world interest rate. The presence of foreign assets, traded freely by home agents (including the central bank), and the prevalence of a (fixed) world interest rate, will redefine the mechanics that underlie interest-rate targeting by central banks. Once this extra device in the hands of central bankers — the foreign assets — creates an essential mechanism for managing real money balances, the capital account and the foreign interest rates will become very important forces behind the impact of interest-rate targeting on the macro-economy. However, in spite of its importance, with few exceptions (Taylor, 2001; McCallum and Nelson, 2001; Erceg, 2002), economists have paid relatively little attention to the discussion regarding the performance of interest-rate rules in an open economy.

Based on these two facts, this paper makes a theoretical attempt to analyze the macroeconomic and growth implications by viewing interest-rate targeting as a *policy regime* that can

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[1] Sargent and Wallace (1975) claim that interest-rate rules are undesirable, because they lead to price level indeterminacy in models with forward-looking agents. However, McCallum (1981) argues that determinacy is possible if the interest rate feeds back to endogenous state variables such as the price level.

[2] See Benhabib and Farmer (1999), McCallum (2003), and Woodford (2003) for a literature review.

[3] To our knowledge, in some recent empirical studies, such as Erceg and Levin (2003), Smithin (2002), and Kuttner (2004), the growth rate target is embedded in the interest-rate rules.

also influence sustainable long-run growth. In our analysis it is crucial that the private sector reacts to the interest-rate rule with long-run growth targets in mind.[4] Specifically, we extend the interest-rate rule that is *gradualist*, in the sense that it feeds back to both the *inflation and income-growth targets* in an *open* economy with endogenously-determined growth rates. Moreover, in this open-economy setup, the roles of the capital account and foreign interest rates become particularly important.

The core questions we ask are: (i) what happens to the macro-economy in the long run under the gradualist passive and active interest-rate rules with respect to inflation targeting in an open economy? (ii) what are the stability properties and the transition dynamics in each case? and (iii) what is the role of policy credibility in answering the above two questions? i.e. what is the impact of the non-credibility of policy (along the lines of Lahiri, 2000) with special emphasis on the impact of the duration of credibility for the resulting paths and steady state(s)? These issues are obviously unexplored in the existing literature in which interest-rate rules are thought of as devices for macroeconomic stabilization in *the short run*. Besides, more recent studies focus on whether appropriately designed policy *institutions* reduce the probability of asset pricing bubbles and other self-fulfilling prophecies that can be generated in models with multiple equilibria (see, for example, Benhabib and Farmer, 1999; McCallum, 2003; Woodford, 2003; Benhabib et al., 2003). Our paper will depart from this issue and focus on the analysis of transition.

In order to come up with these answers, our model comprises some novel characteristics.[5] First of all, we specify that the central bank's operating procedure is implementing a *gradualist* interest-rate rule, i.e. it gradually adjusts the nominal interest rate toward the targeted level, rather than adjusting the interest rate by an immediate once-and-for-all jump. As is commonly believed by economists, this is the way several central bankers have conducted their monetary policy. Such a belief is also supported by evidence that the official interest rates in major countries have been adjusted by small amounts with infrequent reversals (see, for example, Goodhart, 1996; Sack, 1998; Woodford, 1999; Martin, 1999). Second, we adopt the "*Ak*" production function as the source of endogenous growth in a *deterministic* environment.[6] Such a specification is the simplest resolution to abstracting from externalities, which can blur the role of the policy analysis for growth.[7] Yet, technically, the AK technology can lead to a problem in an open economy, namely, that this technology with fixed physical capital returns tends to lead to such consumption/investment decisions of households that imply instant transition to a steady state (infinite speed of convergence). To avoid this problem, we follow the resolution of Barro and Sala-i-Martin (1995, ch. 3), and include adjustment costs for investment, which becomes an essential ingredient for our analysis.

Third, money is introduced into our model by employing a transactions' cost technology a-la Sims. Specifically, a higher velocity of money comes together with an extra cost in terms of the consumable good. As a result, money is no longer a simple "veil" of the economy, leading our model to transcend the (neo)classical dichotomy between real and monetary variables. Compared to other

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[4] From a practical standpoint, Bernanke et al. (1999) describe inflation targeting as a "framework" rather than as a rule. In a similar vein, Gavin (2003) also characterizes inflation targeting as a "management objective," rather than as a well-defined policy rule. Along this standpoint, in this study we view interest-rate rules as a policy regime.

[5] Some characteristics of our framework are similar to those of Shaw et al. (2005).

[6] Benhabib et al. (2001a,b, 2003) also adopt such a modeling strategy, abstracting uncertainty from their models.

[7] For an example of the closed economy, the reader can refer to Zhang (1996).

popular approaches to breaking the classical dichotomy, namely, the money-in-the-utility-function and cash-in-advance approaches, the transaction costs approach seems to be more plausible because its assumption is less restrictive.[8] In addition, Feenstra (1986) has shown that the transaction costs and the money-in-the-utility-function approaches can be functionally equivalent. Wang and Yip (1992) have also proved that these three approaches can yield the same qualitative results. According to their finding, our main results can be easily applied to the other two approaches.

By characterizing the balanced growth equilibrium, several interesting findings emerge from our analysis which are summarized as follows. First of all, a well-known result obtained in the literature is that an equilibrium is locally unique if the central bank follows an active rule, and locally indeterminate if the passive rule is implemented (see, for example, Clarida et al., 2000). This result is challenged by more recent studies. Generally speaking, the indeterminacy in dynamic general equilibrium models associated with the interest-rate feedback rule hinges crucially on the specific economic environment in which money enters preferences and technology (Benhabib et al., 2001a,b), the monetary-fiscal regime (Benhabib et al., 2001a,b), the investment and its adjustment costs of investment (Dupor, 2001, 2002; Carlstrom and Fuerst, 2005), the endogenous labor supply (Meng and Yip, 2004), and the exact definition of inflation measurement (Bernanke and Woodford, 1997; Benhabib et al., 2001b, 2003; Carlstrom and Fuerst, 2000, 2005), etc. Apart from the issues raised in this literature, this paper shows that, in an *open-economy endogenous growth* model, whether interest-rate rules are active or passive plays a decisive role in affecting the overall economy's dynamic stability properties. If the central bank implements a passive rule, then the monetary equilibrium exists and is unique. Moreover, this unique passive monetary equilibrium is locally determinate. However, if the central bank implements an active rule, then the open economy either has two equilibria, one high-growth and one low-growth, or none. In the case of two equilibria, the high-growth equilibrium is locally determinate, while the low-growth equilibrium is a source.

By focusing on the steady-state effects, we find that there can be an appropriately chosen interest-rate rule that feeds back to both inflation and economic growth, which can lead to a double gain: it can lead to both more price stability and to higher economic growth. To be more specific, we find that, in the long run, an anticipated rise (rather than decline) in the inflation target unexpectedly leads to this double gain. This result holds true for unanticipated policies as well.

For the transitional dynamics, we show that in response to an anticipated permanent decrease in the inflation target, the growth rate of the open economy will decline on impact and then continuously fall toward its new stationary level. However, the nominal interest rate and the inflation rate will exhibit a mis-adjustment: they will fall during the period between the policy's announcement and its implementation; once the lower inflation target is realized, the nominal interest and inflation rates will start to increase toward their new and higher stationary levels.

Finally, as argued by Gavin (2003), inflation targeting has been successful because the policy-maker decides on objectives and announces them. In line with Calvo (1986) and Drazen and Helpman (1988), the framework with imperfect credibility of the monetary authority is analyzed. We point out that the economy can enjoy higher growth during the entire transition if

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[8] The money-in-the-utility-function approach assumes that money is a commodity that yields utility directly. This assumption does not seem to be very reasonable, as money appears as a primitive need of real-world people and not as a mean of transactions. The cash-in-advance approach also restricts the monetary base to being only cash. This element of the cash-in-advance formulation is also somewhat restrictive.

the commitment in regard to the growth target policy is imperfectly credible. This provides a theoretical explanation as to why the government often overstates its intended growth target and implements a non-credible policy. In particular, we show that in response to different credibility horizons, the economy will experience quite distinct transitional dynamics.

2. The model

Consider a small open economy consisting of a representative household, a government, and a central bank. The domestic economy produces and consumes a single traded good, the foreign price of which is given in the world market. In the absence of any impediments to trade, the law of one price is assumed to hold. By denoting P as the domestic price level, P^* as the world price level, and E as the nominal exchange rate, the law of one price can be described in percentage change terms as follows:

$$\pi = \pi^* + \varepsilon, \tag{1}$$

where $\pi\left(\equiv \dot{P}/P\right)$ is the rate of inflation of the good in terms of the domestic currency, $\pi^*\left(\equiv \dot{P}^*/P^*\right)$ is the rate of inflation of the good in terms of the foreign currency, and $\varepsilon\left(\equiv \dot{E}/E\right)$ is the rate of depreciation of the domestic currency.

2.1. The household

The representative household with an infinite planning horizon has access to perfect world capital markets, being able to lend internationally. Let us denote M and B^* as the stocks of the nominal money balances and net nominal foreign bonds, respectively. The net nominal foreign bond earns a fixed rate of return, namely, the world nominal interest rate, R^*, in the world bond market. When the world capital market is perfect, the following asset arbitrage condition must hold:

$$R^* - \pi^* = R - \pi, \tag{2}$$

where R is defined as the domestic nominal interest rate. Eq. (2) is essentially the interest-rate parity. The household chooses consumption c, investment i, physical capital k, real money balances $m(\equiv M/P)$, and real net foreign bonds $b^*(\equiv EB^*/P)$ so as to maximize the discounted sum of future instantaneous utilities:

$$\int_0^\infty \frac{c^{1-\theta} - 1}{1 - \theta} \mathrm{e}^{-\rho t}\mathrm{d}t, \quad \theta > 0, \ \rho > 0,$$

where ρ represents the constant rate of time preference and θ is the inverse of the intertemporal elasticity of substitution which measures the curvature of the utility function. For simplicity, the individual's labor supply is assumed to be fixed inelastically.

The domestic output y is produced by means of a simple Ak production technology, i.e. $y = Ak$, where k denotes the domestic capital stock (it can also be treated as a composite of physical and human capital, as suggested by Lucas, 1988) and $A > 0$ is the total factor productivity. The process by which the firm sells its product to the consumers involves transactions' costs. Firms, as argued by Dornbusch and Frenkel (1973), hold money balances in order to facilitate transactions. Following Sims (1989) and Zhang (1996), we summarize

the transactions' cost technology in terms of the rate of loss in real output using the following form:

$$\Phi = \Phi(m, y) \quad \text{with } \Phi_m < 0 \text{ and } \Phi_y > 0.$$

To sustain a macroeconomic equilibrium with ongoing growth, we further assume that Φ is homogeneous of degree 0 in m and y; more specifically, $\Phi(m, y) = \phi(m/y)$ with $\phi' < 0$, $\phi'' \geq 0$, $\lim_{m/y \to 0} \phi(m/y) = 1$, and $\lim_{m/y \to \infty} \phi(m/y) = \overline{\phi} \in (0, 1)$.

Furthermore, in line with Hayashi (1982), Abel and Blanchard (1983), and Turnovsky (1996), the accumulation of physical capital involves adjustment costs Ψ (or installation costs) with a quadratic convex function, i.e.:

$$\Psi(i, k) = \frac{h}{2} \frac{i^2}{k},$$

where $h(>0)$ is a constant parameter of adjustment costs. The existence of adjustment costs for investment can help us avoid some of the counterfactual results from the open-economy version of the Ramsey model (see Barro and Sala-i-Martin, 1995, ch. 3). Turnovsky (2000) points out that the linear homogeneity of the adjustment cost function is necessary to give rise to non-degenerate dynamics and to be sustained if the stationary equilibrium exhibits ongoing growth.[9] Given that the capital stock does not depreciate, the following physical capital accumulation constraint should be satisfied:

$$\dot{k} = i. \tag{3}$$

At each instant of time, the representative household is bound by a flow constraint linking wealth accumulation to any difference between its gross income and its expenditure. Thus, the household's flow budget constraint is given by:

$$\dot{m} + \dot{b}^* = y - c - i\left(1 + \frac{h}{2}\frac{i}{k}\right) - \phi\left(\frac{m}{y}\right)y + (R^* + \varepsilon - \pi)b^* - \pi m + \tau, \tag{4}$$

where the τ are real transfers from the government. Eq. (4) states that, if the sum of output, interest income on net foreign bonds, and transfer income from the government exceeds that of expenditure on consumption, investment, transactions' costs, and inflation tax, then the representative agent will increase his holdings of either real money balances or real net foreign bonds.

If we denote the real financial asset as $a \equiv m + b^*$, then the current-value Hamiltonian for the representative agent's optimization problem is given by:

$$H = \frac{c^{1-\theta} - 1}{1 - \theta} + \lambda\left[(R^* - \pi^*)a + Ak - c - i\left(1 + \frac{h}{2}\frac{i}{k}\right) - \phi\left(\frac{m}{Ak}\right)Ak - Rm + \tau\right] + q'i,$$

where λ is the shadow value (marginal utility) of wealth in the form of the real financial asset and q' is the shadow value of the capital stock. Let the shadow value of wealth be the

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[9] Adjustment costs that depend upon investment relative to the capital stock can be justified by learning-by-doing in the installation process. As addressed by Feichtinger et al. (2001, p. 255), "if the capital stock is large, a lot of machines have been installed in the past so that this firm has a lot of experience, implying that it is more efficient in installing new machines."

numéraire. Accordingly, $q \equiv q'/\lambda$ is defined as the market value of capital in terms of the (unitary) price of the real financial asset.

The optimum conditions necessary for this optimization problem are:

$$c^{-\theta} = \lambda, \tag{5}$$

$$\frac{\dot{k}}{k} = \frac{i}{k} = \frac{(q'/\lambda) - 1}{h} \equiv \frac{q - 1}{h}, \tag{6}$$

$$-\phi'\left(\frac{m}{y}\right) = R, \tag{7}$$

$$\frac{\dot{q}}{q} + \frac{1}{q}\left\{ \left[1 - \phi\left(\frac{m}{y}\right) + \phi'\left(\frac{m}{y}\right)\frac{m}{y} \right] A + \frac{(q-1)^2}{2h} \right\} = R^* - \pi^*, \tag{8}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - (R^* - \pi^*), \tag{9}$$

together with Eqs. (3) and (4), and the transversality conditions of a and k are:

$$\lim_{t \to \infty} \lambda a e^{-\rho t} = 0, \tag{10a}$$

$$\lim_{t \to \infty} q' k e^{-\rho t} = \lim_{t \to \infty} \lambda q k e^{-\rho t} = 0. \tag{10b}$$

Eq. (5) indicates that the household equates the marginal utility of consumption with the marginal utility of the real financial asset. Eq. (6) represents the "Tobin q" theory of investment. Eq. (7) indicates that the marginal benefit of holding real money balances equals the domestic nominal interest rate. Eq. (8) is an arbitrage condition, which indicates that the real rate of return on marginal investment should equal the real rate of return on the real financial asset. The former is the sum of the expected capital gains, $\dot{q}/q$, the value of the marginal product of capital, and the value of marginal savings on installation costs.

2.2. The government and the central bank

There are no commercial banks in the economy and thus the central bank lends only to the government. Under a regime of flexible exchange rates, foreign reserves are constant over time. Following Agénor and Montiel (1996, pp. 323–326), we normalize the constant level of reserves to zero. Accordingly, given that D is the nominal stock of domestic credit, changes in the real money balances are equal to changes in the real credit stock, i.e.:

$$\dot{m}/m = \dot{d}/d = \mu - \pi, \tag{11}$$

where $d(\equiv D/P)$ is the real domestic credit and $\mu(\equiv \dot{D}/D)$ is the growth rate of the nominal credit stock. In addition, let us assume that the government forgoes the issuance of domestic bonds to finance its deficit, and distributes seigniorage to the representative household as a transfer payment in a lump-sum manner. Thus, the flow budget constraint of the government can be written as:

$$\tau = \mu m, \tag{12}$$

Putting Eqs. (4), (11) and (12) together, the open economy's consolidated budget constraint can be obtained as follows:

$$\dot{b}^* = \left[1 - \phi\left(\frac{m}{y}\right)\right]y - c - i\left(1 + \frac{h}{2}\frac{i}{k}\right) + (R^* - \pi^*)b^*. \tag{13}$$

Eq. (13) indicates that the economy's accumulation of net foreign bonds is equal to the current account balance, which in turn equals the balance of trade plus the interest payment on net foreign bonds.

The existing studies dealing with Taylor's rule specify that the central bank has an implicit "intermediate target" $\overline{R}$ and follows an interest-rate feedback rule.[10] The "traditional" Taylor's rule specifies that the central bank adjusts the nominal interest rate to inflation and *output* gaps. By contrast, this paper specifies that the nominal interest rate reacts to inflation and *growth* gaps. This specification reflects the fact that the central bank is pursuing both inflation and growth stabilization goals. Theoretically, such a modification is necessary in models with unceasing growth. It is also in accordance with Dupor's (2001, p. 104) generalized interest-rate rule. Practically, many developed countries have been adjusting interest rates in an attempt to accommodate the economic growth rate. The degree of participation in this policy varies from country to country, with the US Federal Reserve being the most aggressive, Canada and the U.K. moving slightly more cautiously, and the European Union taking a significantly more modest stance on adjusting rates. A recent example is that where on April 22, 2004 Greenspan said that he expected "solid economic growth" ahead and noted that the Fed's key interest rate would have to rise "at some point". In line with Smithin (2002), we therefore specify the following monetary policy rule:

$$\overline{R} = \alpha_0 + \alpha_\pi(\pi - \overline{\pi}) + \alpha_\gamma(\gamma - \overline{\gamma}), \tag{14}$$

where α_0 is the constant intercept term and $\overline{\pi}$ and $\overline{\gamma}$ represent the targets for the inflation rate and economic growth rate set by the monetary authorities, respectively. The coefficients $\alpha_\pi > 0$ and $\alpha_\gamma > 0$ measure the extent of the central bank's response to the inflation gap and the growth gap, respectively. Following Leeper (1991), Meng (2002), Benhabib et al. (2001b), and Dupor (2001), we refer to monetary policy as passive if $\alpha_\pi < 1$ and as active if $\alpha_\pi > 1$.

In line with the way in which central bankers in many countries practically conduct their monetary policy (see Woodford, 1999), the interest-rate rule is implemented gradually. Under such a gradualist monetary rule, the central bank implements a partial adjustment in the nominal interest rate toward the targeted level $\overline{R}$, rather than adjusting the interest rate by an immediate once-and-for-all jump. This causes the nominal interest rate to move in a particular direction over sustained periods of time. Following Goodhart (1996), Sack (1998), and Woodford (1999), we capture this gradual behavior by specifying that the nominal interest rate follows a law of motion as in the following form:

$$\dot{R} = -\delta(R - \overline{R}), \tag{15}$$

where the coefficient $\delta(> 0)$ measures the degree of inertia in the central bank's response. Eq. (15) indicates that the change in the nominal interest rate only partially offsets a deviation from the targeted rate, with the lagged change in the interest rate exerting a significant effect on the dynamics. At each instant of time, the central bank adjusts the credit supply to whatever level is needed for the actual interest rate R to prevail.

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[10] The intermediate target will be associated with an "operating target" level for the federal funds rate.

Given the international financial assets arbitrage condition in Eq. (2) and the nominal interest-rate rule in (14), (15) can be rewritten as:

$$\dot{R} = -\delta(R - \alpha_0) + \delta\alpha_\pi[(R - \overline{\pi}) - (R^* - \pi^*)] + \delta\alpha_\gamma(\gamma - \overline{\gamma}). \qquad (16)$$

Eq. (16) highlights the importance of the international real interest spread in determining the behavior of the nominal interest rate in an open economy.

3. Balanced growth path and stability properties

Differentiating Eq. (5) with respect to time and plugging the resulting equation into (9), the optimal change in consumption is derived as:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(R^* - \pi^* - \rho) \equiv \gamma_c. \qquad (17)$$

This is a standard Keynes–Ramsey rule, which indicates that consumption rises (falls) as the real rate of return on the real financial asset $R^* - \pi^*$ exceeds (falls short of) the rate of time preference ρ.

Moreover, from the optimal condition for holdings of real money balances (7), the real balances-output ratio can be derived as:

$$m/y \equiv v(R); \quad v' \equiv \partial v/\partial R = -1/\phi'' < 0.$$

Substituting the above equation into (8), the evolution of the market value of capital q can be rewritten as:

$$\frac{\dot{q}}{q} = R^* - \pi^* - \frac{1}{q}\left\{[1 - \phi(v(R)) + \phi'(v(R))v(R)]A + \frac{(q-1)^2}{2h}\right\}. \qquad (8a)$$

In order to satisfy the household's intertemporal budget constraint, given that q exponentially approaches $\tilde{q}$ along the transitional adjustment path, the transversality condition (10b) holds if and only if[11]:

$$\frac{\tilde{q} - 1}{h} < R^* - \pi^*, \qquad (18)$$

where $\tilde{q}$ is the stationary value of q, which is determined below. Eq. (18) shows that in the long run the rate of capital growth is less than the real rate of return on the real financial asset.

We now turn to the evolution of the output growth rate. Given the Ak production function and by combining the definition $\gamma \equiv \dot{y}/y$ with Eq. (6), we have:

$$\gamma \equiv \dot{y}/y = \dot{k}/k = (q-1)/h. \qquad (19)$$

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[11] Solving Eqs. (6) and (9) yields $k(t) = k_0 \exp\left[\int_0^t (q-1)/h\, \mathrm{d}\xi\right]$ and $\lambda(t) = \lambda_0 \exp[\rho t - (R^* - \pi^*)t]$, where λ_0 is the endogenously-determined initial marginal utility of wealth and k_0 is a given initial stock of domestic capital. Equipped with $k(t)$ and $\lambda(t)$, (10b) can be rewritten as:

$$\lim_{t \to \infty} \lambda q k e^{-\rho t} = \lambda_0 k_0 \tilde{q} \lim_{t \to \infty} \exp\left\{\int_0^t [(q-1)/h] - (R^* - \pi^*)\mathrm{d}\xi\right\} = 0.$$

This implies that the relationship reported in Eq. (18) is true.

Differentiating Eq. (19) with respect to time and combining the resulting equation with (8a), the change in the output growth rate is derived as:

$$\dot{\gamma} = \frac{\dot{q}}{h} = (R^* - \pi^*)\left(\gamma + \frac{1}{h}\right) - [1 - \phi(v(R)) + \phi'(v(R))v(R)]\frac{A}{h} - \frac{\gamma^2}{2}. \tag{20}$$

Eqs. (16) and (20) construct a 2×2 (γ, R) dynamic system. Recursively, the evolution of q, $\gamma_{b^*}(\equiv \dot{b}^*/b^*)$ and $\gamma_m(\equiv \dot{m}/m)$ can be determined.[12] It should be noted that the transversality condition (10a) is distinct from (10b). Combining Eq. (17) with $\theta\dot{c}/c = -\dot{\lambda}/\lambda$ gives the no-Ponzi-game condition $\lim_{t \to \infty} b^* \exp[-(R^* - \pi^*)t] = 0$. Thus, following Turnovsky (1996, 1997), the transversality condition (10a) must be satisfied and it will hold if and only if[13]:

$$c_0 = (R^* - \pi^* - \gamma_c)\left\{ b_0^* + \frac{k_0\left[A' - \left((\tilde{q}_0^2 - 1)/2h\right)\right]}{R^* - \pi^* - \tilde{\gamma}_0} \right\}, \tag{21a}$$

$$R^* - \pi^* > \gamma_c, \tag{21b}$$

$$R^* - \pi^* > \tilde{\gamma} \equiv (\tilde{q} - 1)/h, \tag{21c}$$

where A' is defined as $A(1 - \phi(v_0))$ with v_0 denoting the initial real money balances-output ratio, $\tilde{\gamma}$ is the stationary value of the output growth rate, and $\tilde{\gamma}_0$ is the initial equilibrium value of γ. Eq. (21a) determines the feasible initial level of real consumption. Eq. (21b) imposes an upper bound on the consumption growth rate. Eq. (21c) is equivalent to Eq. (18).

We are ready to analyze the equilibrium characteristic of the dynamic system to which we now turn. At the steady-growth equilibrium, the economy is characterized by $\dot{R} = \dot{\gamma} = 0$ and R and γ are at their stationary values, namely, $\tilde{R}$ and $\tilde{\gamma}$, respectively. It then follows from Eqs. (7) and (19) that $\tilde{\gamma}_m = \tilde{\gamma}_k = \tilde{\gamma}_y = \tilde{\gamma}(= \tilde{q} - 1/h)$ should hold, meaning that, along the balanced growth path, real money balances, physical capital, and real output grow at the same rate $\tilde{\gamma}$.[14]

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[12] Differentiating Eq. (7) with respect to time and combining the resulting equation with (16) and (19), the optimal change in m is expressed as:

$$\dot{m}/m = \gamma + \left\{ \delta(R - \alpha_0) - \delta\alpha_\pi[R - (R^* - \pi^*) - \overline{\pi}] - \delta\alpha_\gamma(\gamma - \overline{\gamma}) \right\}/\phi_0''v_0,$$

where ϕ_0'' and v_0 are the endogenously-determined initial values of ϕ'' and v, respectively. In addition, from Eqs. (2) and (11) the growth rate of the nominal credit stock is derived as:

$$\mu = \gamma + R - (R^* - \pi^*) + \left\{ \delta(R - \alpha_0) - \delta\alpha_\pi[R - (R^* - \pi^*) - \overline{\pi}] - \delta\alpha_\gamma(\gamma - \overline{\gamma}) \right\}/\phi_0''v_0.$$

[13] From the assumption of Ak production technology and $\dot{R} = 0$ in the steady state, we learn that y, k, and m grow at the same rate. By substituting $c(t)$ in (17) and i and $k(t)$ in (14) into (13), we have: $\dot{b}^*(t) = (R^* - \pi^*)b^* + A'k_0e^{\gamma t} - c_0e^{\gamma_c t} - \gamma(1 + h\gamma/2)k_0e^{\gamma t}$. Given an initial stock b_0^*, we solve this differential equation for $b^*(t) = e^{(R^* - \pi^*)t}\{b_0^* + \int_0^t \{[A' - ((q^2 - 1)/2h)]k_0e^{\gamma\xi} - c_0e^{\gamma_c\xi}\}e^{-(R^* - \pi^*)\xi}d\xi\}$. By substituting the above expression of $b^*(t)$ and $\lambda(t)$ in (9) into (10a), (21) thus can be derived. See Turnovsky (1996) for details.

[14] From (17), the stationary growth rate of real consumption is $\tilde{\gamma}_c = (R^* - \pi^* - \rho)/\theta$. In addition, analogous to Turnovsky (1996), the rate of growth of net real foreign bonds γ_{b^*} converges asymptotically to $\max(\tilde{\gamma}_y, \tilde{\gamma}_c)$.

It follows from Eqs. (16) and (20) that the steady-state values $\tilde{R}$ and $\tilde{\gamma}$ satisfy the following stationary relationships:

$$-\delta(\tilde{R} - \alpha_0) + \delta\alpha_\pi[\tilde{R} - (R^* - \pi^*) - \overline{\pi}] + \delta\alpha_\gamma(\tilde{\gamma} - \overline{\gamma}) = 0, \tag{16a}$$

$$(R^* - \pi^*)\left(\tilde{\gamma} + \frac{1}{h}\right) - \left[1 - \phi(v(\tilde{R})) + \phi'(v(\tilde{R}))v(\tilde{R})\right]\frac{A}{h} - \frac{\tilde{\gamma}^2}{2} = 0. \tag{20a}$$

As depicted in Fig. 1, the loci $\dot{R} = 0$ and $\dot{\gamma} = 0$ trace all combinations of $\tilde{\gamma}$ and R that satisfy Eqs. (16a) and (20a), respectively. When the central bank implements a passive monetary rule ($\alpha_\pi < 1$), $\dot{R} = 0$ is upward sloping in the (γ, R) space, while it is downward sloping when the monetary rule is active ($\alpha_\pi > 1$). The locus $\dot{\gamma} = 0$ is a convex curve with a negative slope since the transversality condition must hold.[15] In referring to panel A of Fig. 1, if the central bank implements a passive interest-rate rule, then there exists only one intersection, namely, Q_0, of loci $\dot{R} = 0$ and $\dot{\gamma} = 0$. However, if the central bank implements an active interest-rate rule, there are either two intersections, namely Q_0 and Q_0', or no intersection, between loci $\dot{R} = 0$ and $\dot{\gamma} = 0$, as displayed in panel B of Fig. 1.[16] These results lead us to establish the following proposition:

Proposition 1. *If the central bank implements a passive interest-rate rule ($\alpha_\pi < 1$), then the monetary equilibrium of the open economy exists and is unique. However, if the central bank implements an active interest-rate rule ($\alpha_\pi > 1$), the open economy either has two equilibria or does not have any equilibrium.*

Next, based on Eqs. (16) and (20), we identify the local stability properties of these equilibria presented above. First of all, we linearize the dynamic system (γ, R) around the steady state and derive:

$$\begin{bmatrix} \dot{R} \\ \dot{\gamma} \end{bmatrix} = \tilde{J}\begin{bmatrix} R - \tilde{R} \\ \gamma - \tilde{\gamma} \end{bmatrix} - \begin{bmatrix} \delta\alpha_\pi \\ 0 \end{bmatrix}(\overline{\pi} - \overline{\pi}_0) - \begin{bmatrix} \delta\alpha_\gamma \\ 0 \end{bmatrix}(\overline{\gamma} - \overline{\gamma}_0), \tag{22}$$

where

$$\tilde{J} = \begin{bmatrix} -\delta(1 - \alpha_\pi) & \delta\alpha_\gamma \\ \tilde{v}A/h & \tilde{\eta} \end{bmatrix},$$

and $\tilde{\eta} \equiv R^* - \pi^* - \tilde{\gamma} > 0$ is required by (21c). Let s_1 and s_2 be the two characteristic roots of the Jacobian matrix $\tilde{J}$ in (22). It follows from (22) that the trace and the determinant of $\tilde{J}$, respectively, are:

$$\mathrm{Tr}(\tilde{J}) = s_1 + s_2 = \tilde{\eta} - \delta(1 - \alpha_\pi) \gtreqless 0, \tag{23a}$$

$$\mathrm{Det}(\tilde{J}) = s_1 s_2 = -\delta\left[(1 - \alpha_\pi)\tilde{\eta} + \alpha_\gamma\tilde{v}A/h\right] \gtreqless 0. \tag{23b}$$

It is easy to learn from (23) that, when the central bank implements a passive rule ($\alpha_\pi < 1$), the Jacobian matrix J has two real roots with opposite signs since $\mathrm{Det}(\tilde{J}) < 0$. Moreover, because the dynamic system reported in (22) has one jump variable γ and one gradual adjustment variable R,

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[15] From (16a), we have $\partial R/\partial\gamma|_{\dot{R}=0} = \alpha_\gamma/(1 - \alpha_\pi) \gtreqless 0$ as $\alpha_\pi \lesseqgtr 1$ and $\partial^2 R/\partial\gamma^2|_{\dot{R}=0} = 0$. From (20a), we have $\partial R/\partial\gamma|_{\dot{\gamma}=0} = (\tilde{\gamma} - R^* + \pi^*)/(A\tilde{v}/h) < 0$ and $\partial^2 R/\partial\gamma^2|_{\dot{\gamma}=0} = h\{1 - (R^* - \pi^* - \tilde{\gamma})^2(\tilde{v}'h/A\tilde{v}^2)\}/A\tilde{v} > 0$.

[16] Of course, it is also possible that the locus $\dot{\gamma} = 0$ may be tangent to the locus $\dot{R} = 0$ if $\alpha_\pi > 1$. Under such a situation, the economy may be characterized by one monetary equilibrium.
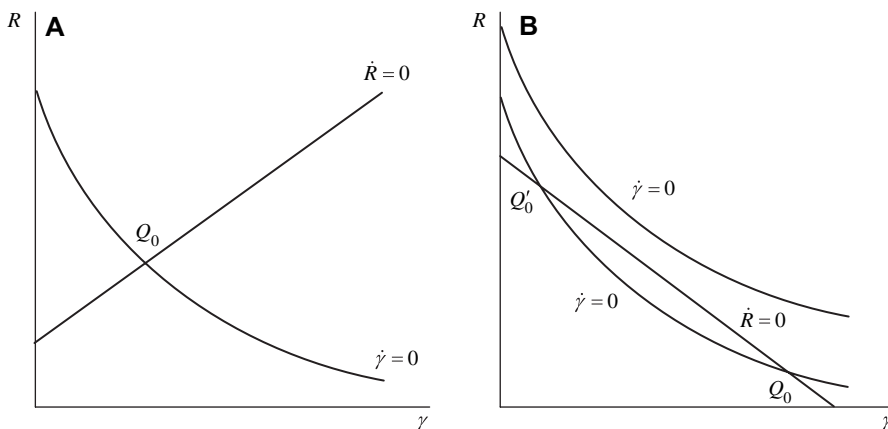
Fig. 1. Existence of BGP. (A) Passive rule and (B) active rule.

it thus exhibits saddle-point stability. As a result, the unique passive steady-state equilibrium Q_0 depicted in panel A of Fig. 1 is locally determinate and there exists a unique growth path converging to it.

By contrast, when the central bank implements an active rule ($\alpha_\pi > 1$), we have $\mathrm{Tr}(\tilde{J}) > 0$ and $\mathrm{Det}(\tilde{J}) \gtrless 0$ from (23). In this case, the two real roots of J either have opposite signs or are both positive. Furthermore, the sign of $\mathrm{Det}(\tilde{J})$ is either negative or positive, depending on whether locus $\dot{R} = 0$ is steeper or flatter than locus $\dot{\gamma} = 0$ at the steady-state equilibrium (if equilibria exist).[17] Accordingly, we can conclude that under an active rule the equilibrium with higher growth, such as Q_0 in panel B of Fig. 1, is a saddle and thus is locally determinate, whereas the equilibrium with lower growth, such as Q_0' in panel B of Fig. 1, is a source.[18] To sum up, we have:

Proposition 2. *Under a passive monetary rule, the unique monetary equilibrium of the open economy is locally determinate. Under an active monetary rule, the high-growth equilibrium is locally determinate, while the low-growth equilibrium is a source.*

4. Transitional dynamics and target policies

In this section we discuss the economic performance in the face of different target policies. To make the analysis meaningful, in what follows we only focus on the case where there are stable equilibria, i.e. point Q_0 in both panel A and panel B of Fig. 1.

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[17] Given $\partial R/\partial \gamma|_{\dot{R}=0} = \alpha_\gamma/(1-\alpha_\pi) < 0$ and $\partial R/\partial \gamma|_{\dot{\gamma}=0} = -h\tilde{\eta}/A\tilde{v} < 0$, (23b) can be rewritten as $\mathrm{Det}(\tilde{J}) = -\delta(1-\alpha_\pi)(\tilde{v}A/h)[\partial R/\partial \gamma|_{\dot{R}=0} - \partial R/\partial \gamma|_{\dot{\gamma}=0}] \gtrless 0$. Under an active monetary rule, the sign of $\mathrm{Det}(\tilde{J})$ is equal to that of $[\partial R/\partial \gamma|_{\dot{R}=0} - \partial R/\partial \gamma|_{\dot{\gamma}=0}]$.

[18] The existing literature on the stabilization properties of interest-rate rules points out that forward-looking rules give rise to aggregate instability while backward-looking rules contribute to protecting the economy from embarking on expectations-driven fluctuations. In our model, the assumption $\delta > 0$ implies that the interest-rate rule is, by nature, backward-looking. When we relax this assumption and focus on a forward-looking rule, the interest-rate-smoothing parameter δ will be negative (see Benhabib et al., 2003, p. 23). The sign of δ is crucial to the local equilibrium dynamic property. Specifically, when $\delta < 0$, it is found that: (i) under a passive rule, the monetary equilibrium remains unique but becomes a source; (ii) under an active rule, the low-growth equilibrium is locally determinate, while the high-growth equilibrium is either a saddle or a sink (in this case, local indeterminacy emerges), depending upon $|\delta| \lessgtr \tilde{\eta}/(\alpha_\pi - 1)$.

For expository convenience, we assume that $s_1 < 0 < s_2$. It follows from (22) that the general solutions for R and γ are, respectively:

$$R(t) = \tilde{R}(\overline{\pi}, \overline{\gamma}) + B_1 e^{s_1 t} + B_2 e^{s_2 t}, \tag{24a}$$

$$\gamma(t) = \tilde{\gamma}(\overline{\pi}, \overline{\gamma}) - \frac{\tilde{\eta} - s_1}{\tilde{v}A/h} B_1 e^{s_1 t} - \frac{\tilde{\eta} - s_2}{\tilde{v}A/h} B_2 e^{s_2 t}, \tag{24b}$$

where B_1 and B_2 are unknown parameters. Based on Eqs. (24a) and (24b), in Fig. 2 we show two cases: panel A presents the case of a passive monetary rule ($\dot{R} = 0$ is upward sloping), while panel B presents the case of an active monetary rule ($\dot{R} = 0$ is downward sloping). As indicated by the direction of the arrows in Fig. 2, the loci SS and UU represent the stable and unstable branches, respectively. Evidently, the convergent saddle path SS is downward sloping, while the divergent branch UU is upward sloping.[19]

Since the dynamic behavior of the economy *around the stable equilibrium* in association with both passive and active rules will experience very similar transitional dynamics, in this section we only focus on the case of the active rule and abstract from the case of the passive rule.[20] The detailed analysis concerning the case of the passive rule is available upon request from the authors.

4.1. Anticipated reduction in the inflation target

Assume that at the "present" time $t = 0$ the central bank pre-announces that its inflation target will decrease from $\overline{\pi}_0$ to $\overline{\pi}_1 (< \overline{\pi}_0)$ at a specific future date $t = T$. In Fig. 3, the initial equilibrium is established at Q_0, where the $\dot{\gamma} = 0$ locus intersects $\dot{R} = 0(\overline{\pi}_0)$. The corresponding initial rates of growth and nominal interest are $\tilde{\gamma}_0$ and $\tilde{R}_0$, respectively. Upon an anticipated permanent decrease in the inflation target, the $\dot{\gamma} = 0$ locus remains intact, while $\dot{R} = 0(\overline{\pi}_0)$ shifts upward to $\dot{R} = 0(\overline{\pi}_1)$.[21] As a result, the new steady-growth equilibrium is at point Q_1, with γ and R being $\tilde{\gamma}_1$ and $\tilde{R}_1$, respectively.

Before we proceed to study the dynamic adjustment, three points should be addressed. First, we denote 0^+ as the instance after the announcement is made by the central bank, as well as T^- and T^+ as the respective instants before and after the policy's implementation. Second, during the period between 0^+ and T^-, the inflation target has not yet been reduced and thus point Q_0 should be treated as the reference point to govern the dynamic adjustment. Third, as the inflation target decreases from $\overline{\pi}_0$ to $\overline{\pi}_1$ at the moment T^+, the economy should move to a point exactly on the stable branch SS at that instant of time to ensure that the system will converge.

Based on the above information, Fig. 3 reveals that, at the instant the policy announcement is made, the economy will jump from point Q_0 to Q_{0^+} on impact. Notice that since the central bank implements a gradual adjustment in the interest rate, the nominal interest rate will remain intact at the original level $\tilde{R}_0$. However, with perfect foresight the public can correctly anticipate that the central bank will eventually raise the nominal interest rate as a result of implementing a lower inflation target $\overline{\pi}_1$. Because this increase in the nominal interest rate will decrease the net marginal product of capital in the future,[22] the

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[19] Given $B_2 = 0$ and $B_1 = 0$ in (24), we have $\partial R/\partial \gamma|_{SS} = \delta \alpha_\gamma/[\delta(1 - \alpha_\pi) + s_1] = h(s_1 - \tilde{\eta})/\tilde{v}A$ and $\partial R/\partial \gamma|_{UU} = \delta \alpha_\gamma/[\delta(1 - \alpha_\pi) + s_2] = h(s_2 - \tilde{\eta})/\tilde{v}A$. Because $s_1 - \tilde{\eta} < 0$ and $s_2 - \tilde{\eta} > 0$, SS is downward sloping, while UU is upward sloping.

[20] Since the low-growth equilibrium under an active rule is a source, we will abstract it from our analysis.

[21] It is clear from Eqs. (16) and (20) that $\partial R/\partial \overline{\pi}|_{\dot{R}=0} = 1/(\alpha_\pi - 1) > 0$ and $\partial R/\partial \overline{\pi}|_{\dot{\gamma}=0} = 0$ due to $\alpha_\pi > 1$ under the active monetary rule.

[22] By differentiating the net marginal product of capital $[1 - \phi(v(\tilde{R})) + \phi'(v(\tilde{R}))v(\tilde{R})]$ with respect to the steady-state nominal interest rate $\tilde{R}$, the negative relationship is confirmed.
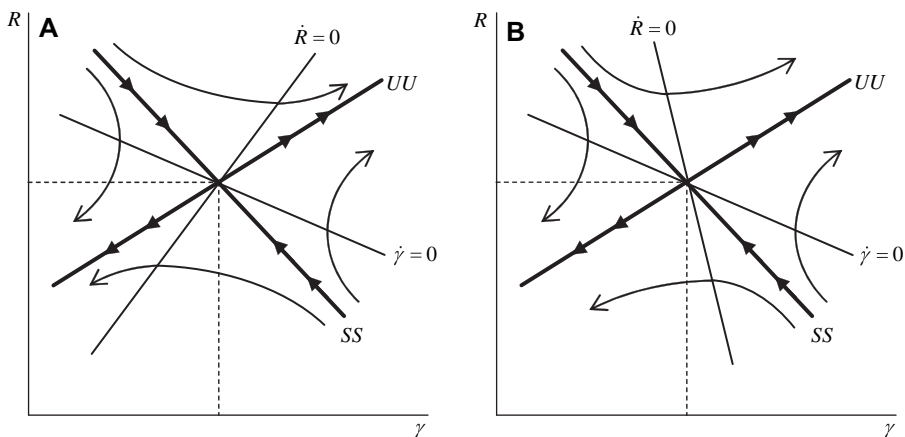
Fig. 2. Phase diagram. (A) Passive rule and (B) active rule.

agents with perfect foresight will instantly decrease their demand for capital, and in turn the market price q will also fall. As a result, Fig. 3 indicates that the rate of economic growth instantly decreases from $\tilde{\gamma}_0$ to γ_{0^+}.

The anticipation of a future reduction in the inflation target also leads to a continuous fall in the demand for capital during the period between 0^+ and T^-. The decumulation of capital will depress the economic growth rate until the date of the policy's implementation. However, on the other hand, the interest-rate feedback rule (14) indicates that the pre-announced effect will induce the central bank to temporarily lower the nominal interest rate. Thus, from 0^+ to T^- both R and γ fall monotonically, and the economy moves gradually from Q_{0^+} to Q_T, as shown by the arrows in Fig. 3.

Once the lower inflation target $\overline{\pi}_1$ is realized, there is a continuous rise in the nominal interest rate. The persistent rise in the nominal interest rate means a persistent increase in the opportunity cost of holding money, thereby leading to a continuous reduction in the demand
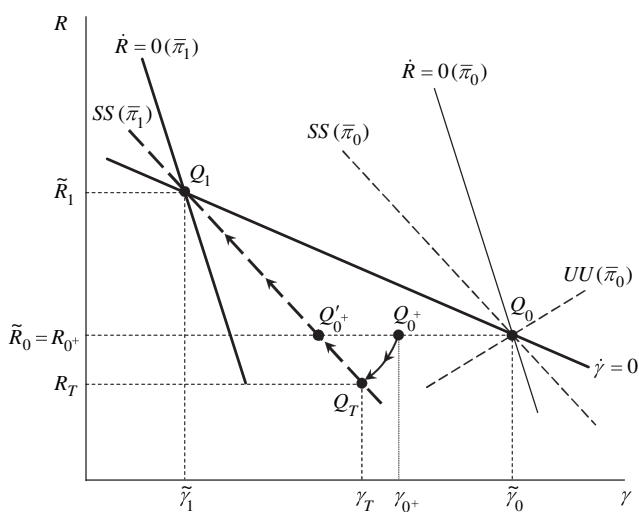

Fig. 3. Dynamics responses to an anticipated permanent decrease in the inflation target.

for money. Consequently, the transactions' cost rises. Since the net marginal product of capital decreases due to the higher transactions' cost, the rate of economic growth is further depressed. Thus, the economic growth rate keeps on falling and the nominal interest rate starts to rise as the economy moves along the $SS(\overline{\pi}_1)$ locus toward its new long-run equilibrium Q_1.

We summarize the above results as:

Proposition 3. *In response to an anticipated permanent decrease in the inflation target, the transitional dynamics of the monetary equilibrium exhibit the following properties:*

(i) *instantaneously, the growth rate of the open economy falls and the nominal interest rate remains unchanged;*

(ii) *in transition, the economic growth rate falls monotonically toward its new stationary value. The nominal interest rate exhibits a mis-adjustment: it falls during the period between the policy's announcement and its implementation and then starts to increase toward a higher new stationary value after the lower inflation target is realized;*

Note that Proposition 3 is also true if the central bank implements a passive monetary rule.

It is in our interest to further explore the transitional dynamics of the inflation rate π (which equals the depreciation rate ε given that the purchasing power parity (1) holds continuously). From the interest-rate parity $R^* - \pi^* = R - \pi$ in (2), we learn that the evolutionary process of the inflation rate π coincides with that of R at each instant in time. By combining this with the results in Proposition 3, we know that, upon an anticipated permanent reduction in the inflation target, the inflation rate will exhibit mis-adjustment in transition and will rise in the long run.[23,24] Consequently, the rates of inflation and economic growth exhibit a positive relationship during the period between the policy's announcement and its implementation. Once the lower inflation target is realized, the rates of inflation and economic growth begin to be negatively correlated. Such a negative correlation between π and γ also prevails in the long run.

The above results lead us to establish the following proposition:

Proposition 4. *In response to an anticipated permanent change in the inflation target, in the long run the relationship between the rates of inflation and economic growth is negative. However, in the transition the relationship is*

(i) *positive during the period between the policy's announcement and its implementation;*

(ii) *negative after the new inflation target is realized.*

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[23] From   (16a)   we   have   the   stationary   relations $\tilde{R}_0 = \alpha_0 + \alpha_\pi(\tilde{\pi}_0 - \overline{\pi}_0) + \alpha_\gamma(\tilde{\gamma}_0 - \overline{\gamma}_0)$ and $\tilde{R}_1 = \alpha_0 + \alpha_\pi(\tilde{\pi}_1 - \overline{\pi}_1) + \alpha_\gamma(\tilde{\gamma}_1 - \overline{\gamma}_0)$ corresponding to the target rates of inflation $\overline{\pi}_0$ and $\overline{\pi}_1$, respectively. Under the initial condition, $\tilde{\pi}_0 = \overline{\pi}_0$ and $\tilde{\gamma}_0 = \overline{\gamma}_0$, the following relationship must hold:

$$\tilde{\pi}_1 = \overline{\pi}_1 + (\tilde{R}_1 - \tilde{R}_0)\big/\alpha_\pi + (\tilde{\gamma}_0 - \tilde{\gamma}_1)\alpha_\gamma/\alpha_\pi.$$

Given $\tilde{R}_1 > \tilde{R}_0$ and $\tilde{\gamma}_1 < \tilde{\gamma}_0$ as stated in Proposition 3, the above relationship indicates that the new stationary rate of inflation $\tilde{\pi}_1$ exceeds the new target rate of inflation $\overline{\pi}_1$.

[24] In the case of an unanticipated decline in the inflation target, the economy will jump immediately from Q_0 to Q'_{0+} on the stable branch $SS(\overline{\pi}_1)$ in Fig. 3 at the moment of policy implementation. From then on, the economy will travel along $SS(\overline{\pi}_1)$ to regain the steady state Q_1. Clearly, the mis-adjustment of the inflation rate no longer appears under unanticipated policies.

A corollary to Proposition 4 is that, under a nominal interest-rate rule that feeds back to both inflation and economic growth, regardless of whether it is passive or active, there is a double gain in terms of price stability and economic growth expansion both in the transitional period and in the long run, if the central bank *immediately* (without pre-announcement) raises the inflation target. However, if the central bank chooses to implement a *pre-announced* inflation target policy, the economy will face a tradeoff between price stability and economic growth prior to the date of realization of the new inflation target.

At first glance it may be surprising that such a double gain is realized by raising (rather than lowering) the inflation target. The intuition is as follows. When the monetary authorities raise the inflation target $\overline{\pi}$, the interest-rate feedback rule (14) indicates that the nominal interest rate should be lowered. The lower rate of interest will decrease the opportunity cost of holding money balances, and thereby encourage the public's money holdings. This will in turn reduce transactions' costs and thus raise the net marginal product of capital. On the one hand, the increase in the net marginal product of capital stimulates domestic investment and hence economic growth; on the other hand, it attracts more international capital to inject into the domestic economy, which leads to an appreciation of domestic currency and further stimulates domestic economic growth. Owing to purchasing power parity, domestic currency appreciation should be accompanied by domestic disinflation. Therefore, an expansionary monetary policy (a lower nominal interest rate due to a higher inflation target) boosts economic growth; moreover, by inducing capital inflows, it lowers the inflation rate in the economy as a whole.

4.2. A change in the growth target with imperfect credibility

The approach in the Section 4.1 can be applied easily to analyze the announcement effect of a change in $\overline{\gamma}$. However, a more interesting issue is to explore the effect of a target policy regarding $\overline{\gamma}$ in the presence of the central bank's credibility problem, to which we now turn.

Indeed, the central bank's policy often involves some kind of credibility problem and it has also received a considerable amount of attention in the literature (see, for instance, Calvo, 1986; Drazen and Helpman, 1988; Calvo and Végh, 1993; Calvo and Drazen, 1998; Lahiri, 2000, 2001. There is a typical example in this context which is that the authorities in many countries often overstate the intended growth target and hence modify their announced policy in the future. This leads the authorities to a loss of credibility with respect to their disposition or ability to commit to their previously announced policies, and thereby gives rise to the imperfect credibility problem.

To shed light on this problem, three further assumptions are necessary: (i) at $t = 0$, the policy authority announces that it is raising the target rate of economic growth *permanently* from $\overline{\gamma}_0$ to a higher $\overline{\gamma}_1$; (ii) due to the lack of credibility of the authority, the public, however, expects that after a certain period of time the policy-maker will modify $\overline{\gamma}_1$ toward a lower level, say, $\overline{\gamma}_2$, which is considered here to be such that $\overline{\gamma}_1 > \overline{\gamma}_2 > \overline{\gamma}_0$[25]; and (iii) the public is uncertain about the precise timing of the policy modification, T_p, and has a subjective probability distribution regarding T_p with its distribution function being given by $F(T_p)$. To simplify the analysis without loss of generality, in line with Lahiri (2000), we assume that the distribution has a mass at T^*.

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[25] We assume for simplicity that the public accurately expects that the modified level of the growth target is $\overline{\gamma}_2$. If we relax this assumption, the precise dynamic trajectory will depend on the true value of the modified growth target. However, except for the discrete jump at the instant when the true modified growth target is revealed, the qualitative characteristics of the dynamic path will remain the same as in our analysis in this section.

We first consider the case in which the public accurately expects the true credibility horizon, that is, $T^* = T_p$. Under such a simplified situation, during the transition the economy will evolve exactly as follows:

$$\overline{\gamma} = \begin{cases} \overline{\gamma}_1; & 0 \leq t \leq T^{*-} \\ \overline{\gamma}_2; & t \geq T^{*+} \end{cases}. \tag{24}$$

Fig. 4 presents the economy's dynamic response to the policy (24), in which the initial equilibrium Q_0 is the same as that in the previous subsection. Upon the policy change, the $\dot{R} = 0(\overline{\gamma}_0)$ locus shifts downward to $\dot{R} = 0(\overline{\gamma}_1)$, leaving the $\dot{\gamma} = 0$ locus unchanged.[26] The intersection of loci $\dot{R} = 0(\overline{\gamma}_1)$ and $\dot{\gamma} = 0$ is at point Q_1, with γ and R being $\tilde{\gamma}_1$ and $\tilde{R}_1$, respectively. Point Q_1 is the reference point that governs the dynamic adjustment during the period of the temporary target rate of economic growth $\overline{\gamma}_1$. The $SS(\overline{\gamma}_1)$ path is its corresponding saddle path. Once the growth target is modified to $\overline{\gamma}_2$, the dynamic system will travel along the saddle path $SS(\overline{\gamma}_2)$ in order for the system to be convergent. The associated steady-state equilibrium is Q_2, with the stationary rates of growth and nominal interest being $\tilde{\gamma}_2$ and $\tilde{R}_2$, respectively.

As indicated in Fig. 4, given different values of T^*, the economy will have very different responses on impact (more specifically, it will jump to a point located between points A and B). If T^* is larger, namely T_L^*, then the economy will jump rightward to Q_{0+}^L, a point that is closer to point A.[27] By contrast, if T^* is smaller, namely T_S^*, then the economy will jump rightward to Q_{0+}^S, a point that is closer to point B. As a consequence, at instant 0^+ the nominal interest rate remains intact while the economic growth rate immediately leaps from $\tilde{\gamma}_0$ to γ_{0+}^L (or γ_{0+}^S). Intuitively, under the gradualist monetary rule, the nominal interest rate will remain intact at the original level $\tilde{R}_0(= R_{0+})$ at instant 0^+. However, with perfect foresight, the public correctly anticipates that, in response to the higher growth target $\overline{\gamma}_1$, the central bank will eventually reduce the nominal interest rate and that this will cause the net marginal product of capital to rise. Therefore, the agent will increase his demand for capital instantly. In turn, this will raise the market price of capital, q, and cause the economic growth rate to rise instantly from $\tilde{\gamma}_0$ to γ_{0+}.

It is interesting to note and worth emphasizing that the initial rise in the economic growth rate increases with the credibility horizon. It follows from the interest-rate feedback rule (14) that the initial boost in the economic growth rate induces the central bank to accelerate the reduction in the nominal interest rate. This tends to reduce the opportunity cost of holding money and thus encourages money holdings. As a result, the transactions' costs decline and thus capital accumulation and the economic growth rate are given a boost. Graphically, during the period between 0^+ and T^{*-}, there is a continuous rise in the economic growth rate from $\gamma_{0+}^L (\gamma_{0+}^S)$ to $\gamma_{T^*}^L (\gamma_{T^*}^S)$ and a continuous fall in the nominal interest rate from $\tilde{R}_0(= R_{0+})$ to $R_{T^*}^L (R_{T^*}^S)$. As indicated by the arrows in Fig. 4, the economy proceeds along the unstable path linking the points $Q_{0+}^L (Q_{0+}^S)$ and $Q_{T^*}^L (Q_{T^*}^S)$ to exactly reach the point $Q_{T^*}^L (Q_{T^*}^S)$ at $t = T^*$.

From T^{*+} onward, for $T_L^* (T_S^*)$, the economy will travel along the saddle path $SS(\overline{\gamma}_2)$ from point $Q_{T^*}^L (Q_{T^*}^S)$ toward the new steady-state equilibrium Q_2. The economic growth rate will correspondingly fall (rise) continuously toward $\tilde{\gamma}_2$ while the nominal interest rate will rise (fall) monotonically toward $\tilde{R}_2$. Evidently, for a different credibility horizon, the system will adjust exactly in the opposite direction. Intuitively, for $T_L^* (T_S^*)$, at the moment of time T^{*+}, the

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[26] From (16) and (20), we know that under an active rule $(\alpha_\pi > 1) \partial R / \partial \overline{\gamma}|_{\dot{R}=0} = \beta/(\alpha_\pi - 1) > 0$ and $\partial R / \partial \overline{\pi}|_{\dot{\gamma}=0} = 0$.

[27] We can find a critical credibility horizon T_c such that the economic growth rate exhibits neither overshooting nor undershooting; that is, in response to T_c, the economy will jump to point C on impact. We refer to all $T^* > T_c$ as a *long* credibility horizon and to all $T^* < T_c$ as a *short* credibility horizon.
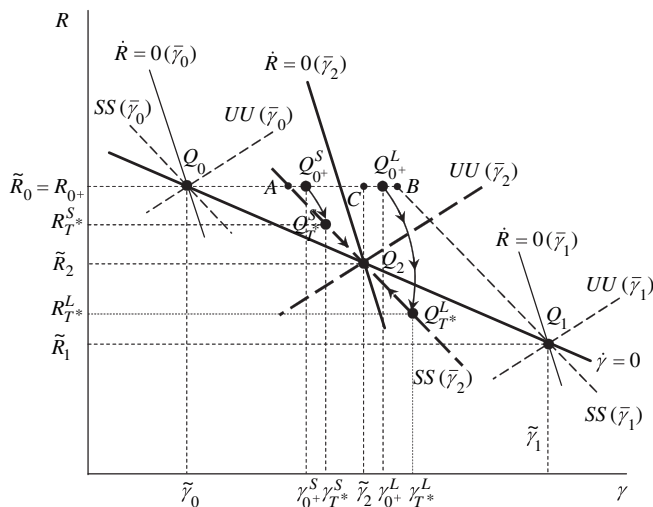
Fig. 4. Dynamic responses to a non-credible rise in the growth target.

economic growth rate will be higher (lower) than the new target rate $\overline{\gamma}_2$. According to (14), this will induce the central bank to raise (lower) the nominal interest rate. Such a rise (decline) in R discourages (encourages) money holdings and thus leads to an increase (decrease) in the transactions' cost. Consequently, investment is discouraged (encouraged) and the economic growth rate declines (rises). Due to the gradualism of the monetary rule, such a dynamic adjustment will persist until the new stationary $(\tilde{\gamma}_2, \tilde{R}_2)$ is achieved.

By comparing $(\tilde{\gamma}_2, \tilde{R}_2)$ with $(\tilde{\gamma}_0, \tilde{R}_0)$, the following proposition is established:

Proposition 5. *In response to a non-credible increase in the growth target, the transitional dynamics of the monetary equilibrium exhibits the following properties:*

(i) *instantaneously, the growth rate of the open economy rises while the nominal interest rate remains unchanged; moreover, over a long (short) credibility horizon, the economic growth rate overshoots (undershoots) its long-run response;*

(ii) *in transition:*

    (a) *during the entire transition, the rates of economic growth and nominal interest exhibit a negative relationship;*

    (b) *during the dates before the policy's modification, the economic growth rate exhibits a mis-adjustment over a long credibility horizon, while over a short credibility horizon, the economic growth rate exhibits a monotonic rise;*

    (c) *from T* onward, over a long (short) credibility horizon, the economic growth rate will continuously fall (rise) toward its higher new stationary value.*

Given that the evolutionary process of the inflation rate coincides with that of the nominal interest rate at each instant in time, from Proposition 5 we derive the following proposition:

Proposition 6. *Under a nominal interest-rate rule regime that feeds back to both inflation and economic growth, regardless of whether it is active or passive, in the event of an non-credible*

growth target policy change, the rates of inflation and economic growth are negatively corre-
lated in the long run.

Propositions 5 and 6 together point out that, under a nominal interest-rate rule that feeds back to both inflation and economic growth, an increase in the growth target can in the long run lead to both price stability and higher economic growth.

In terms of Fig. 4, we show that the monetary authority always has incentives to overstate its intended growth target. For this purpose, let us first consider a fully credible policy in which the policy authority announces at $t = 0$ an immediate permanent rise in the growth target from $\overline{\gamma}_0$ to $\overline{\gamma}_2$. At the moment the shock occurs, the economy initially jumps from Q_0 to point A in Fig. 4 and thereafter converges along the $SS(\overline{\gamma}_2)$ locus toward the new steady-state equilibrium Q_2. Although credible and non-credible policies lead to the same long-run outcome, they actually bring out distinct transitional dynamics. By comparing the adjustment paths corresponding to credible (the Q_0AQ_2 trajectory) and non-credible policies (the $Q_0Q_{0+}^L Q_{T*}^L Q_2$ trajectory or the $Q_0Q_{0+}^S Q_{T*}^S Q_2$ trajectory), it is easily found that the economy enjoys higher growth during the entire transition period when the growth target policy is non-credible. Moreover, the longer the credibility horizon, the more the economy will profit from the non-credibility of the policy. Thus, we have:

Proposition 7. *In the absence of reputational costs, there are always incentives for the policy authority to overstate its intended growth target and implement an incredible policy, since the economy can thereby enjoy higher growth during the entire transition.*

Finally, we would like to remind readers that the above discussions proceed under the assumption that the public accurately predicts the true credibility horizon. If the precise date of policy modification, T_p, differs from the public's expected date, T^*, then the transitional dynamics becomes more complicated. Let us consider the case where policy modification occurs prior to T^* (i.e. $T^* > T_p$). After the initial jump from Q_{0-}^L to Q_{0+}^L (from Q_{0-}^S to Q_{0+}^S), the economy starts to travel along the unstable branch passing through Q_{0+}^L and Q_{T*}^L (Q_{0+}^S and Q_{T*}^S), as shown in Fig. 4. However, before reaching $Q_{T*}^L(Q_{T*}^S)$ (i.e. the economy is located somewhere between Q_{0+}^L and Q_{T*}^L (Q_{0+}^S and Q_{T*}^S)), the policy modification occurs. At this moment, the economy jumps horizontally from the unstable branch to $SS(\overline{\gamma}_2)$. Accordingly, the economic growth rate falls instantly, leaving the nominal interest rate unchanged. From then on, the economy will travel along $SS(\overline{\gamma}_2)$ to asymptotically regain the steady state Q_2. As is evident from Fig. 4, as long as the expectation error, $T^* - T_p$, is not too large, from T_p onward, the economic growth rate will fall (rise) continuously toward $\tilde{\gamma}_2$, while the nominal interest rate will rise (fall) monotonically toward $\tilde{R}_2$. In the case where $T^* = T_L^*$ and $T_L^* - T_p$ is sufficiently large (in the sense that the nominal interest rate is still above $\tilde{R}_2$ at the moment of policy modification), then, from T_p onward, the economic growth rate will rise continuously and the nominal interest rate will fall continuously until the new stationary state $(\tilde{\gamma}_2, \tilde{R}_2)$ is reached.[28]

5. Conclusion

This paper sets up an endogenous growth model of an open economy in which the central bank implements a gradualist interest-rate rule with targets for inflation and economic growth.

We show that under a passive rule the monetary equilibrium exists and is unique; moreover, it is locally determinate. By contrast, an active rule implies either two equilibria, one high-growth

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[28] By contrast, if $T^* < T_p$, then at time T^* the public will modify its expectations to a longer credibility horizon. To save space, this paper does not discuss this situation. Detailed discussions are available from the authors upon request.

and one low-growth, or none. In the case of two equilibria, the high-growth equilibrium is locally determinate, while the low-growth equilibrium is a source. Our results clearly differ from those in previous studies (see, for example, Carlstrom and Fuerst (2000, 2005) and Fiore and Liu (2002)), and hence provide new insights for policy implications.

We have also shown that, under a nominal interest-rate rule that feeds back to both inflation and economic growth, regardless of whether it is passive or active, an immediate rise in the inflation target stabilizes the price level and promotes economic growth both in the transitional period and in the long run. Nevertheless, under a pre-announced inflation target policy, price stability and the expansion of economic growth cannot be achieved simultaneously during the period between the policy's announcement and its implementation. We also show that, under such an interest-rate rule, a higher growth target has a favorable effect on both inflation and economic growth in the long run. In addition, the policy authority always has incentives to overstate its intended growth target and hence will implement a non-credible policy. The credibility of the central bank's commitment is shown to play an important role in influencing the transitional dynamics of the macroeconomic variables.

Acknowledgements

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