

THE SYNCHRONIZATION OF OUTPUT FLUCTUATIONS BETWEEN TAIWAN AND THE UNITED STATES

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摘 要

本文在於探討於1961年至1987年這一段期間臺灣與美國總體產出波動的特性。希望能夠檢定(1)確定趨勢過程 (deterministic trend process) 或隨機趨勢過程 (stochastic trend process) 何者較適於代表臺灣與美國總體產出變數演化 (evolution) 的過程, (2)在臺灣與美國總體產出的波動中, 趨勢成分 (trend component) 或循環成分 (cycle component) 何者扮演比較重要的角色。

比較以 OLS 模型及 ARIMA 模型所導出的產出循環 (output cycles), 吾人發現以 OLS 模型導出的產出循環較以 ARIMA 模型導出的產出循環波動幅度 (magnitudes) 較大但頻率 (frequencies) 較低。依據理論的背景, 應以 ARIMA 模型而非 OLS 模型所導出的產出循環來代表臺灣與美國之產出波動的一致性。

Abstract

This paper investigates the nature of output fluctuations in both Taiwan and the U.S. for the period from 1961 to 1987. It attempts to test (1) whether the evolution of the output series is better represented by the deterministic trend process or by the stochastic trend process, and (2) which component, trend or cycle, plays a more important role in the output fluctuations.

Using the Dickey-Fuller test and the Phillips-Perron test to examine three different unit root models, the results show that we cannot reject the hypothesis that the evolutions of Taiwan's and U. S. real output follow a stochastic trend process. Both estimates of the structural model and the ARIMA model indicate that the trend component plays a more important role than the cycle component in both countries' output fluctuations. In addition, innovations in output have persistent effect on the output fluctuations of Taiwan and the U. S.

At the end, the output cycles derived by the OLS and ARIMA models are compared. We find that the output cycles derived by the OLS model have a larger magnitudes but lower frequencies than the

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output cycles derived by the ARIMA model. Referring to theoretical background, the output cycles derived by the ARIMA model rather than by the OLS model should be used to represent the synchronization of output fluctuations between Taiwan and the U. S.

1. Decomposition of Output Fluctuations

The international economic interdependence is usually reflected by the synchronization of economic fluctuations among countries. There are two major measurements of the synchronization of economic fluctuations. One of them is the so-called "diffusion index" [3,4,19] or "weighted diffusion index," [17] which measures the uniformity of the direction of change in a set of economic variables, and hence the extent of synchronization of economic fluctuations among countries.

Another measurement is referred to as the "growth cycle." It can be estimated either in terms of the growth rate of such economic variables as output or in terms of relative deviation from such a variable. The conventional measurement of economic fluctuations (or business cycles) assumes that the loci of the evolution of output follow a simple polynomial function of time. Under this assumption, if output is regressed on time, the fitted value of the output therefore can be viewed as its trend value, and the residual is viewed as the cycle component of the variable. This approach implies that the trend is deterministic and economic fluctuations are attributed entirely to the variation from the trend.

If the evolution of a variable follows a trend stationary process, the linear detrending approach can be used for the separation of its trend and cycle components. On the other hand, if the variable's evolution does not follow a trend stationary but a random walk process, the use of linear polynomial model to decompose a time series into the trend and the cycle components will produce pseudo-cyclical and purely artificial fluctuation patterns of the variables.¹

¹ If the linear least square is used to detrend a random walk time series, the first few lags' autocorrelations will be spurious positive, this implies that the residuals are not white noise, periodicity is hence meaningless and the genuine dynamics of economy is overlooked [7,22,23]. Nelson and Kang [22] conclude that

Conventional tests for trend are strongly biased toward finding a trend when none is present, and this effect is only partially mitigated by Cochrane-Orcutt correction for autocorrelation.

Beveridge and Nelson [2], and Nelson and Plosser [21] criticize the conventional approach of decomposing economic time series into the trend and the cycle components in measuring the business cycle. Nelson and Plosser interpret most U. S. economic time series as belonging to the random walk model,² hence treating these time series as the deterministic trend model and using the ordinary least square to detrend and decompose the time series into the trend and the cycle components are inappropriate.

The conventional decomposition approach assumes that the trend component is deterministic in nature and the cycle component is stochastic and stationary around a trend. This implies that the variation in macroeconomic time series is determined entirely by the variation of the cycle component instead of the trend component. On the other hand, the Nelson and Plosser approach assumes that the trend component is a non-stationary stochastic process and the cycle component is a stationary process. Under this approach, the variation in macroeconomic time series can be attributed to both components. The accumulation of the variance of the innovations in the trend component will increase without bound as time becomes larger. If the Nelson and Plosser approach was correct, the variation of macroeconomic time series would be mostly attributed to the trend component in the long run.

In macroeconomic analysis, monetary disturbances are assumed to have only a transitory impact on the economy. The conventional decomposition approach implies that economic fluctuations are mainly caused by monetary disturbances, not real disturbances. Economic theory assumes that real disturbances have long-run real effect on the economy, hence it associates with the trend component of macroeconomic time series. Therefore, the Nelson and Plosser approach attributes economic fluctuations in the long run mainly to the real disturbances not to the approach.³ The latter considers that long-run evolution of economic variable is business cycle [18].

The Nelson and Plosser approach also has different implications for the persistence of economic fluctuations from the conventional decomposition

² Nelson and Plosser use the U. S. historical annual data (with starting dates from 1860 to 1909 and ending in 1970) to analyze and conclude that the evidence is consistent with the random walk process. But, they recognize that none of the tests can have power against a trend stationary process alternative with root of autoregressions arbitrarily close to unity.

approach.³ The latter considers that long-run evolution of economic variable is deterministic and perfectly predictable, the innovations will not have a persistent effect on the variable. On the other hand, if the evolution of economic variable is random and not deterministic, then its deviation from any deterministic path will grow when time increases (because random walk is an accumulation of disturbances), the innovations will have persistent effect on the variable.⁴ Thus, before choice of the method of detrending is made, we should test whether the evolution of a variable being considered belongs to the deterministic trend or to the stochastic trend process.

2. Evolution Process of Output

If the seasonal swings and the variation in the irregular component do not depend on the level of the series, the output at time t , Y_t , can be properly expressed by the additive decomposition approach. That is

$$Y_t = T_t + C_t + S_t + E_t, t = 1, \dots, N, \quad (1)$$

where T_t , C_t , S_t , and E_t are trend, cyclical, seasonal, and irregular (or error) components respectively. If seasonal element is adjusted (or deseasoned) and irregular component is ignored, the output variable can be expressed as⁵

$$Y_t = T_t + C_t, t = 1, \dots, N. \quad (2)$$

We assume that the process generating the trend takes the following form

$$\begin{aligned} T_t &= T_{t-1} + \beta_{t-1} + \eta_t, \\ \beta_t &= \beta_{t-1} + \delta_t, \end{aligned} \quad (3)$$

³ Campbell and Mankiw [6] define the persistence as

a shock to an economy may be considered persistent if it lasts for more than one period... or continuing for a long time into the future.

⁴ Campbell and Mankiw [5] argue that the fluctuations in the U. S. output appear highly persistent. They also show that most industrial countries' (Canada, France, Germany, Italy, and Japan) output fluctuations are more persistent than the U. S. [6]. But, Cochrane [9] argues that little long-term persistence in the U. S. output fluctuations.

⁵ The following analysis mainly follows Harvey [15] and Harvey and Todd [16].

where η_t and δ_t are normally distributed independent white noise terms with zero means and variances σ_η^2 and σ_δ^2 respectively, and β is drift.

The cycle component C_t can be assumed to be generated by a stationary discrete linear stochastic process, which can be either an autoregressive process (AR), or a moving average process (MA), or an autoregressive moving average process (ARMA). The mathematical expression of C_t is

$$\phi(B)C_t = \theta(B)u_t, t = 1, \dots, N, \quad (4)$$

where u_t is a normally distributed and independent white noise term with zero mean and variance σ_u^2 , B is the backward-shift operator, $\phi(B)$ and $\theta(B)$ are polynomials in B that satisfy the conditions for stationary and invertibility. If $\theta(B) = 1$, C_t is an autoregressive process; if $\phi(B) = 1$, C_t is a moving average process; if $\theta(B) \neq 1$ and $\phi(B) \neq 1$, C_t is an autoregressive moving average process. Combining equations (2), (3), and (4), we obtain a structural model for output evolution

$$\begin{aligned} Y_t &= T_t + C_t, \\ T_t &= T_{t-1} + \beta_{t-1} + \eta_t, \\ \beta_t &= \beta_{t-1} + \delta_t, \\ \phi(B)C_t &= \theta(B)u_t. \end{aligned} \quad (5)$$

In equation (5), the observed series Y_t is decomposed into unobserved components of T_t and C_t . It is hence called the “unobserved components model” (a simplified model to represent the evolution process of time series Y_t). If $\sigma_\delta^2 = \sigma_\eta^2 = 0$, equation (5) reduces to a deterministic (linear) trend process. Furthermore, if C_t follows a stationary and invertible linear process, equation (5) becomes a linear trend-stationary model and can be expressed as

$$Y_t = a + bT + C_t, \quad (6)$$

where a and b are parameters, T is time trend, C_t is the cycle component representing the deviation from trend.

If equation (6) is the true evolution process of an output variable, then ordinary least square (OLS) can be used to detrend the time series Y_t . The fitted value of Y_t (i.e., $\hat{Y}_t = \hat{a} + \hat{b}T$) is regarded as the trend component, and the residual

is viewed as the cycle component.

Equation (5) can be rewritten as follow if $\sigma_\delta^2 = 0$, then $\beta_t = \beta_{t-1} = \beta$

$$\begin{aligned} Y_t &= T_t + C_t, \\ T_t &= T_{t-1} + \beta + \eta_t, \\ \phi(B)C_t &= \theta(B)u_t. \end{aligned} \tag{7}$$

Taking the first difference of Y_t , we get

$$\Delta Y_t = \beta + \eta_t + \Delta C_t, \tag{8}$$

where ΔY_t and ΔC_t are the first difference of both the output variable and its cycle component respectively.

Provided that $\sigma_\eta^2 > 0$, and η_t follows a stationary and invertible ARMA process, then equation (8) proves that equation (7) is stationary in first difference.

In essence, equation (6) is a deterministic trend model where the evolution of Y_t follows a linear trend-stationary process. Nelson and Plosser dub it the "trend-stationary model." The evolution of Y_t in the long-run is completely deterministic (i.e. ignoring the short-run fluctuations caused by the irregular or error component). The long-term expectations of Y_t depend only on its mean ($a + bT$), neither current nor past events will alter long-term expectations, thus uncertainty is bounded, even in the infinite future. Therefore, the deterministic trend model implies that the variation of Y_t comes from the cycle component only and its variance is finite.

Equation (7) is a stochastic trend model where the evolution of Y_t follows a difference-stationary process, and the stochastic comes from the stochastic of trend component. Nelson and Plosser call it as the "difference-stationary model."⁶ The stochastic trend model implies that the variation of Y_t comes from both the trend component and the cycle component. If the trend component follows a process assumed to be a random walk with drift, then the accumulation of

⁶ The deterministic trend model is of course difference-stationary, thus the difference-stationary model is not a good term to equation (7).

variation of the trend component (hence the series) will increase over time.⁷

3. Identification and Estimation of Output Evolution

The identification of a macroeconomic time series include the identification of the model (the deterministic trend model or the stochastic trend model) and the identification of the evolution of the series. That is, if the series is identified as the stochastic trend model, then what kind of stationary discrete linear stochastic process causes the generation of the series?

There are two main approaches to identify and estimate a macroeconomic time series. One is the single equation approach, while the other is the structural (or unobserved components) approach. Using the single equation approach, Dickey and Fuller [10,11] suggest a simple way to test the deterministic trend hypothesis against the stochastic trend hypothesis, that is to employ OLS to run the following equations

$$Y_t = \rho Y_{t-1} + \gamma(Y_{t-1} - Y_{t-2}) + \epsilon_t, \quad (9)$$

$$Y_t = \alpha + \rho Y_{t-1} + \gamma(Y_{t-1} - Y_{t-2}) + \epsilon_t, \quad (10)$$

$$Y_t = \alpha + \beta T + \rho Y_{t-1} + \gamma(Y_{t-1} - Y_{t-2}) + \epsilon_t, \quad (11)$$

where α is the deterministic (or drift) term, T is the time trend, ϵ_t is the error term. Equation (9) is a pure random walk model, equation (10) is a random walk with drift α model, and equation (11) is a random walk with drift α and linear time trend T model. Equations (9), (10), and (11) are a nested test for equations (6) and (8). The null hypothesis of equation (9) is $H_0: (\rho) = (1)$, that is the model is a pure random walk process; equation (10) is $H_0: (\alpha, \rho) = (0, 1)$, that is, the model is a pure random walk process; equation (11) is $H_0: (\alpha, \beta, \rho) = (\alpha, 0, 1)$, that is, the model is a random walk with drift process, or $H_0: (\alpha, \beta, \rho) = (0, 0, 1)$, that is, the model is a pure random walk process. All the null hypotheses of these

⁷ The deterministic trend model can be expressed as: $Y_t = f(T) + u_t$, where $f(T)$ is a function of time T , u_t is white noise with normal and independent distribution. The variance of Y_t equals the variance of u_t . The stochastic trend model can be expressed as (assume no cycle component): $Y_t = Y_{t-1} + \beta + \eta_t$, where β is the (fixed) mean of first difference of Y_t , η_t is white noise with normal and independent distribution. Y_t can be expressed as the sum of initial value Y_0 and time trend, that is, $Y_t = Y_0 + \beta T + \sum_{i=1}^T \eta_i$. Thus, the variance of Y_t is accumulated sum of the variance of η_i .

tests are that Y_t follows the stochastic trend (or random walk) process.

Because the traditional t -statistic test tends to reject the stochastic trend hypothesis when it is true, Dickey and Fuller developed a revised t -statistics (τ -statistics) [10] and likelihood ratio statistics [11] as alternative.⁸ Phillips and Perron also developed a nonparametric approach to test this hypothesis. Their test statistics are in fact the same as Dickey and Fuller's τ -statistics and likelihood ratio statistics [25,26,27]. By testing equations (9) to (11), many empirical studies conducted in the past show that the hypothesis that most of U. S. macroeconomic time series are characterized by the stochastic trend model cannot be rejected at some (5%) significance level.⁹ But, as Nelson and Plosser has pointed out, the acceptance of the stochastic trend hypothesis is not a disproof of the alternative deterministic trend hypothesis.

Another approach to identifying the evolution process of a macroeconomic time series is based on the sample autocorrelations of the first difference of the variable. Under the assumption that the trend component follows a random walk process, and the innovations of trend component and cycle component are independent, if the sample autocorrelations in the first differences of the variable are positive and significant at lag one only, this implies that the variable follows a stochastic trend process. (But this is a very narrowly defined approach.)¹⁰

The Box-Jenkins approach can be used to identify and estimate the difference-stationary time series if it is generated by the discrete linear stochastic process. For selection of an appropriate ARIMA model to represent the evolution process of a macroeconomic time series, the principle of parsimony will prove to be useful.

A general representation of a macroeconomic time series is a structural model. But, if a priori considerations are imposed on the structural model, the structural

⁸ Schwert [30] points out that the test for unit roots of the first difference macroeconomic time series data should adopt ARMA not just AR model (as equations (9), (10), and (11)), because the macroeconomic time series variables will not be a pure autoregressive processes. If the variables were a MA only model, the Dickey-Fuller test would lead to a false conclusion.

⁹ For example, with the same data set, both Nelson and Plosser by the Dickey-Fuller approach [10] and Perron [25] by the nonparametric approach find that the hypothesis that most U. S. macroeconomic time series follow a stochastic trend process cannot be rejected at the 5% significance level.

¹⁰ If the innovations of trend component and cycle component are uncorrelated, equation (8) becomes: $\Delta Y_t = \beta + \xi_t + \theta \xi_{t-1}$, where ξ_t is white noise with normal and independent distribution, and exact maximum likelihood estimation is very possible to produce $\theta = -1$. See Harvey [14], p. 170 and Sargan and Bhargava [28].

model will be reduced to a single equation of the ARIMA model. For example, Nelson and Plosser assume that the innovations of the trend component and the cycle component are uncorrelated, the structural model reduces to an ARIMA(0, 1, 1) model. If we assumed that the innovations of the trend component and the cycle component are perfectly correlated, the structural model reduces to an unconstrained ARIMA model [31].

To formulate the macroeconomic time series by the Box-Jenkins approach, the ARIMA model is built according to the principle of parsimony on the basis of the correlogram of the data. This approach assumes that the structure of the series through out the whole time period analysed is constant and attempts to catch the true mechanism generating the series. But it puts the prior considerations on the model and can not give direct interpretation to the components of which the series is composed. On the other hand, the structural model aims not to represent the underlying data generation process but to represent and interpret the series in terms of unobserved components directly.

The ARIMA models are ordinarily estimated by the maximum likelihood method. This method is restricted by the condition that the absolute value of MA roots must be less than unity, MA parameters therefore must satisfy the condition of invertibility. If the MA parameters are close to, or lie on, the boundary of unit root, the ARIMA models should be estimated by exact maximum likelihood method [14]. (However, one still cannot distinguish a unit root from a root near unity.)

The structural model can be written in the form of state space. Then, the Kalman filter can be used to construct the likelihood function, and the parameters in the structural model can be estimated by the maximum likelihood method. Following these procedures, equation (5) or (7) can be estimated. By the fitted value of the variances of the trend component and the cycle component, the evolution of the series can be classified as the deterministic trend model or the stochastic trend model.

4. Test of Sino-American Output Evolutions

In order to find out the nature of the output evolutions for Taiwan and the U. S. during the period from 1961 to 1987, the seasonally adjusted, logarithmic

quarterly data of both countries' real *GNP*, and Taiwan's real industrial production are tested.¹¹ The procedure of the test is as follows.

4.1 Unit Root Test

First the ordinary least square (OLS) is used to estimate equations (9), (10), and (11), i. e. three different unit root models are being tested.¹² The results of these works are listed in Table 1, Table 2, and Table 3. In those tables, the model (3) is a random walk with drift and linear time trend process, the model (2) is a random walk with drift process, and the model (1) is a pure random walk process. α is the coefficient of the drift term, β is the coefficient of the linear time trend term, ρ is the coefficient of the lagged one period dependent variable, γ is the coefficient of the lagged first difference, RSS is the residual sum of squares and the numbers in parentheses are standard errors.

The τ_i are Dickey and Fuller's revised t -statistics used to test whether ρ in the three different models mentioned above is significantly different from unity. $\hat{\tau}_\tau$, $\hat{\tau}_\mu$, and $\hat{\tau}$ are τ -statistics when the regression is a random walk with drift and linear time trend, a random walk with drift, or a pure random walk respectively. LR statistics are the likelihood ratio test (or F statistics). Φ_3 is the statistic used to test the null hypothesis $H_0: (\alpha, \beta, \rho) = (\alpha, 0, 1)$, that is, the model is a random walk with drift process. Φ_2 is the statistic used to test the null hypothesis $H_0: (\alpha, \beta, \rho) = (0, 0, 1)$, that is, the model is a pure random walk process. Φ_1 is the statistic used to test the null hypothesis $H_0: (\alpha, \rho) = (0, 1)$, that is, the model is a pure random walk process.

In order to prove the robustness of the Dickey-Fuller test, the models (1) and (2) are again tested by the Phillips and Perron's nonparametric approach which imposes a weaker conditions on the innovations of the system and allows for

¹¹ The U. S. data are from Citibase which had been seasonally adjusted at annual rates by the moving average method. Taiwan's data are from *Quarterly National Income Statistics in Taiwan Area, the Republic of China (1961-1984)* (Taipei: DGBAS, June 1986), and *Quarterly National Economic Trend Taiwan Area, the Republic of China: Quarterly National Income Estimates* (Taipei: DGBAS, February 1988) respectively. Taiwan's data are seasonally adjusted with *TSP* package at annual rates by the moving average method. We put emphasis on realizing the fluctuations of Taiwan's output, hence both Taiwan's real *GNP* and real industrial production are tested.

¹² These models are restrictive, more general models are tested after these tests.

weakly dependent and heterogeneously distributed time series. τ and τ_μ are the Phillips and Perron's nonparametric statistics to test $\rho=1$, the distributions of τ and τ_μ are the same as $\hat{\tau}$ and $\hat{\tau}_\mu$ respectively.¹³ (The distributions of the τ -statistics are in Fuller [13], p. 373; the distributions of the likelihood ratio statistics are in Dickey and Fuller [11], p. 1063.)

Table 1: Unit Root Test for U. S. Real *GNP*

Model	α	β	ρ	γ	RSS	τ -statistics	LR Statistics
Model (3)	.54 (.19)	.0005 (.0002)	.929 (.026)	.27 (.09)	.0089	$\hat{\tau}_\tau = -2.79^*$	$\Phi_3 = 3.32^*$ $\Phi_2 = 3.13^*$
Model (2)	.06 (.03)		.994 (.004)	.26 (.09)	.0095	$\hat{\tau}_\mu = -1.44^*$ $\tau_\mu = -1.63^*$	$\Phi_1 = 1.3^*$
Model (1)			1.0007 (.00015)	.29 (.09)	.0097	$\hat{\tau} = 4.7^\bullet$ $\tau = .04^*$	

*: Accept the null hypothesis at the 10% significance level.

•: Reject the null hypothesis at the 1% significance level.

In Table 1, all the Dickey-Fuller tests, except the pure random walk model, show that U. S. real *GNP* follows a random walk process. The nonparametric test shows that $\rho = 1$ in the models (1) and (2) can be accepted at the 10% significance level. Both approaches fail to reject the stochastic trend hypothesis for the evolution of U. S. real *GNP*.

Table 2: Unit Root Test for Taiwan's Real *GNP*

Model	α	β	ρ	γ	RSS	τ -statistics	LR Statistics
Model (3)	2.2 (.70)	.004 (.001)	.806 (.063)	-.49 (.10)	.117	$\hat{\tau}_\tau = -3.06^*$	$\Phi_3 = 4.36^*$ $\Phi_2 = 3.49^*$
Model (2)	.08 (.06)		.996 (.005)	-.14 (.10)	.127	$\hat{\tau}_\mu = -.80^*$ $\tau_\mu = -.70^*$	$\Phi_1 = .81^*$
Model (1)			1.002 (.0003)	-.142 (.10)	.129	$\hat{\tau} = 6.12^\bullet$ $\tau = .23^*$	

*: Accept the null hypothesis at the 10% significance level.

•: Reject the null hypothesis at the 1% significance level.

In Table 2, the Dickey-Fuller test and the nonparametric test support the stochastic trend hypothesis, and show that the evolution of Taiwan's real *GNP* can be appropriately represented by a random walk with drift model.

¹³ τ and τ_μ are calculated by the choice of the truncation lag parameter $l=1$.

Table 3: Unit Root Test for Taiwan's Real Industrial Production

Model	α	β	ρ	γ	RSS	τ -statistics	LR Statistics
Model (3)	.65 (.36)	.002 (.001)	.938 (.036)	.015 (.10)	.303	$\hat{\tau}_\tau = -1.7^*$	$\Phi_3 = 1.11^\bullet$ $\Phi_2 = 1.82^*$
Model (2)	.124 (.07)		.992 (.006)	-.015 (.10)	.310	$\hat{\tau}_\mu = -1.39^*$ $\tau_\mu = -1.2^*$	$\Phi_1 = 1.6^*$
Model (1)			1.002 (.0005)	-.002 (.10)	.319	$\hat{\tau} = 4.48^{\bullet\bullet}$ $\tau = .25^*$	

*: Accept the null hypothesis at the 10% significance level.

•: Reject the null hypothesis at the 5% significance level.

••: Reject the null hypothesis at the 1% significance level.

As shown in Table 3, both the Dickey-Fuller and the nonparametric tests support the hypothesis that the evolution of Taiwan's real industrial production follows a random walk process.

From our empirical study, we may conclude that the evolution of output in both Taiwan and the U. S. can be well described by the stochastic trend process.¹⁴ (Hence, the fluctuations of Taiwan's and U. S. real output variables should have the nature attributed to this model.) It suggests that the trend component plays a more important role than the cycle component in the fluctuations of real output in those two countries. In the following, the structural model and ARIMA model are used to test this nature.

4.2 Structural Model Test

A test of the structural model shown in equation (5) is performed by the use of the Clark approach [8]. In this approach, the Kalman algorithm is used to estimate the parameters of $\phi(B)$ and the standard deviations of u_t , δ_t , and η_t .¹⁵ The first step of the application of the Kalman filter algorithm is to construct the joint distribution of parameters and dependent variables at time t given the

¹⁴ Ignoring the seasonal autocorrelations, only lag one sample autocorrelation is significant for Taiwan's first-difference logarithmic real *GNP*. The sample autocorrelations of the first-difference of real output variables also provides an evidence to support the hypothesis that the evolutions of Taiwan's and U. S. real output follow the stochastic trend process.

¹⁵ Clark's model assumes that the cycle component is generated by AR(2) process not general ARMA process, that is $\theta(B) = 1$ in equation (5).

informations available at time $t - 1$. Under the assumptions of normality and independence, the joint distribution of parameters and dependent variables can be specified by the mean and covariance matrix.

Next, the distribution of parameters at time t given information available at time t is constructed. This can be gotten by the conditional distribution from the joint distribution of parameters and dependent variables. The joint distribution of parameters and dependent variables and the conditional distribution of parameters together form a Kalman filter algorithm. With the help of the Kalman filter algorithm, it is easy to build the log likelihood function of the observations. The maximum likelihood method can then be used to estimate the conditional means and variances of the stochastic parameters for any given values of the fixed parameters in the state space vector.

For the sample $t = 1, \dots, T$, the starting values for the Kalman filter can be constructed from the first n observations, the likelihood function for $n + 1, \dots, T$ is then constructed by the one-step-ahead prediction error decomposition. In this way, the means and variances of the dependent variables and the stochastic parameters in any time period can be computed recursively, given information available in the previous period and the fixed parameters of the model.

The choice of starting values for the Kalman filter does not have much influence on the final results [24]. In fact there is an easy way to choose the starting value for the Kalman filter by initiating the Kalman filter at $t = 0$ with a diagonal covariance matrix in which the diagonal elements are large but finite in number [16]. Clark's model adopts this assumption.

The estimates of equation (5) for U. S. real *GNP*, Taiwan's real *GNP* and real industrial production from 1961:1 to 1987:4 are listed in Table 4.¹⁶ In this table, ϕ_1 and ϕ_2 are AR(2) parameters of the cycle component, σ_δ , σ_η , and σ_u are the standard deviations of innovations of the drift, linear time trend component, and cycle component respectively, $2\ln$ likelihood is a double of log likelihood function.

Table 4 shows that for U. S. real *GNP*, the standard deviation of innovations in the trend component (.039) is greater than the standard deviation of innovations in the cycle component (.013), this means that even for a single quarter (as well as in the long run) the trend component plays a more important role than the

¹⁶ We are grateful to Peter K. Clark for use his programs.

Table 4: Estimates of Structural Model

Variable	ϕ_1	ϕ_2	σ_δ	σ_η	σ_u	2ln likelihood
The U. S. Real GNP	1.03	-.038	.000077	.039	.013	569.7
Taiwan's Real GNP	1.47	-.767	.00032	.406	.000022	87.7
Taiwan's Real Industrial Production	1.14	-.191	.0013	.2838	.018	164.3

cycle component in the fluctuations of U. S. real *GNP* for the period from 1961 to 1987. This result supports Nelson and Plosser's point of view that the U. S. output fluctuations are mainly caused by the trend rather than the cycle component.¹⁷

In the case of Taiwan, our estimation shows that the standard deviation of innovations in the trend component of real *GNP* (.406) is far larger than the corresponding standard deviation of innovations in the cycle component (.000022). This indicates that the trend component plays a much more important role than the cycle component in the fluctuations of Taiwan's real *GNP*.¹⁸

Because industrial production such as mining, manufacturing, utilities, and constructions are more cyclical than other productions in the economy, the standard deviation of innovations in the cycle component theoretically should be larger in the real industrial production than in the real *GNP*. This expectation is confirmed by Taiwan's experience. The standard deviation of innovations in the cycle component for real industrial production (.018) is much larger than the value for real *GNP* (.000022).¹⁹ But, the trend component also plays a more important role than the cycle component in the fluctuations of Taiwan's real industrial production, although the difference of standard deviation of innovations in the trend com-

¹⁷ This outcome is different from Clark's estimate for U. S. real *GNP* in the period from 1947:1 to 1985:4. His estimate shows that the standard deviation of innovations in the cycle component is greater than the standard deviation of innovations in the trend component, this implies that the cycle component plays a more important role than the trend component in the fluctuations of U. S. real *GNP*, this confirms conventional point of view that output fluctuations are caused mainly by the cycle component not the trend component.

¹⁸ We are surprised by the small value of standard deviation of innovations in the cycle component in Taiwan's real *GNP*. One possibility is that the data are smoothed excessively, but we do not have any information to draw such an inference.

¹⁹ Clark also finds that the standard deviation of innovations in the cycle component is larger in the industrial production than in the real *GNP* for U. S. data.

ponent and the cycle component in Taiwan's real industrial production is not as large as in Taiwan's real *GNP*.

It is notable that the estimated cycle components of U. S. real *GNP* and Taiwan's real industrial production in Table 4 are very close to a random walk. Therefore, they do not deserve to be called "cyclical." In addition, the stationary condition requires the roots of the characteristic equation of AR(2) to lie outside the unit circle. We calculated that the roots of ϕ_1 and ϕ_2 in Table 4 for U. S. real *GNP* are 26.2 and 1.01; for Taiwan's real *GNP*, both are .89, for Taiwan's real industrial production are 4.92 and 1.06 respectively. The existence of roots on the boundary of unit circle or less than unity indicates that the specification of this structural model is not good. Thus the results of this model is just tentative and referential. Further evidence is needed to confirm the relative importance of the trend component and the cycle component on Taiwan's and U. S. output fluctuations.

4.3 ARMA Model Test

The above estimated structural model belongs to a restricted ARMA model. In the following, a more general, unrestricted ARMA model is estimated to identify the nature of output fluctuations in both Taiwan and the U. S.²⁰ The model makes use of the exact maximum likelihood method which the log likelihood function of the model is built by the Kalman filter algorithm as a sum of conditional log likelihood. We estimated twelve ARMA (p, q) models (where $p = 0$ to 3, $q = 0$ to 3) for the first-difference logarithmic of U. S. real *GNP*, Taiwan's real *GNP* and real industrial production, and calculated the implied impulse response function for the level of these variables.²¹

In the case of U. S. real *GNP*, the $-2\ln$ likelihood is within the range between 699.55 (ARMA(3, 2)) and 689.42 (ARMA(0, 1)), and all of the impulse responses of the models show the effect of one unit of innovation on the output is greater than one and persistent to 80 horizon periods (quarters), except the impulse responses of model ARMA (1,3), where the responses are less than one after 16 horizon

²⁰ Campbell and Mankiw [5] use this approach to identify the nature of U. S. real *GNP* in the period from 1947:1 to 1985:4.

²¹ We are grateful to John Y. Campbell and N. Gregory Mankiw for use of their programs. The initial value of estimation is given by the estimates of RATS package.

periods. The Akaike Criterion²² shows that AR(2) model is an appropriate parsimonious model to represent the generating mechanism of U. S. real *GNP* in the period from 1961 to 1987. The estimates of this model are: $\phi_1 = .229^*$, $\phi_2 = .185^*$, where ϕ_1 and ϕ_2 are parameters of AR(2) (* indicates that the *t*-statistic is significant at the 5% level). The Schwarz Criterion²³ shows that AR(1) model is an appropriate parsimonious model to represent the generating mechanism of the U. S. real *GNP* in the period from 1961 to 1987. The estimates of this model are: $\phi_1 = .281^{**}$, where ϕ_1 is a parameter of AR(1) (** indicates that the *t*-statistic is significant at the 1% significance level).

The output impulse responses of AR(2) model and AR(1) model to one unit of innovation are listed in Table 5. The impulse responses of AR(2) model are greater than one from the first quarter and settle at 1.71 from sixteen quarters, remaining there even at eighty quarters. The impulse responses of AR(1) model are greater than one from the first quarter and settle at 1.39 from four quarters, remaining there even at eighty quarters. Both AR(2) and AR(1) models' impulse responses indicate that one unit of innovation causes more than one unit of impulse response and has a persistent effect on the U. S. real *GNP*. Therefore, the trend component plays a more important role than the cycle component in the fluctuations of U. S. real *GNP* in the period from 1961 to 1987. The result is consistent with Campbell and Mankiw's empirical result [5].

Table 5: Impulse Responses of U. S. real *GNP*

Model	Horizon (Quarter)							
	1	2	4	8	16	20	40	80
AR(2)	1.23	1.47	1.63	1.70	1.71	1.71	1.71	1.71
(Akaike Criterion)	(.10)	(.14)	(.23)	(.30)	(.31)	(.31)	(.31)	(.31)
AR(1)	1.28	1.36	1.39	1.39	1.39	1.39	1.39	1.39
(Schwarz Criterion)	(.09)	(.15)	(.18)	(.18)	(.18)	(.18)	(.18)	(.18)

Note: The numbers in parentheses are standard errors of forecast.

²² The Akaike Criterion [1] is to choose a model which maximize $2\ln L - 2k$, where $\ln L$ is log likelihood, k is the number of parameters.

²³ The Schwarz Criterion [29] is to choose a model which maximize $2\ln L - k\ln T$, where $\ln L$ is log likelihood, k is the number of parameters, T is the number of observations.

In the twelve models of Taiwan's real *GNP*, the $2\ln$ likelihood lie between 511.92 (ARMA(3, 3)) and 415.2 (ARMA(1, 0)). Both the Akaike Criterion and the Schwarz Criterion show that the ARMA(3, 3) is the most appropriate model to represent the generating mechanism of Taiwan's real *GNP* for the period from 1961 to 1987. The estimates of this model are: $\phi_1 = -.962^{**}$, $\phi_2 = -.946^{**}$, $\phi_3 = -.963^{**}$, $\theta_1 = .748^{**}$, $\theta_2 = .551^{**}$, $\theta_3 = .543^{**}$, where ϕ_1 , ϕ_2 , and ϕ_3 are parameters of AR(3), θ_1 , θ_2 , and θ_3 are parameters of MA(3) (** indicates that the t -statistic is significant at the 1% level).

The impulse responses to one unit of innovation for this model is reported in Table 6. According to the t -statistics, we find that the impulse responses of this model increase from less than one to equal to one from four quarters and remain there to eighty quarters.²⁴ Although the impulse responses are not above one, the effect of innovations on Taiwan's real *GNP* is persistent. Therefore, the hypothesis that the trend component plays a more important role than the cycle component in the fluctuations of Taiwan's real *GNP* can be accepted.

The $2\ln$ likelihood of Taiwan's real industrial production ranges from 1447.47 (ARMA(2, 3)) to 318.50 (ARMA(1, 0)). Both the Akaike Criterion and the Schwarz Criterion show that the ARMA(2, 3) is the most appropriate parsimonious model to represent the generating mechanism of Taiwan's real industrial production in the period from 1961 to 1987. The estimates of this model are: $\phi_1 = .546^{**}$, $\phi_2 = .454^{**}$, $\theta_1 = .09$, $\theta_2 = -.827^{**}$, $\theta_3 = -.275^{**}$, where ϕ_1 and ϕ_2 are parameters of AR(2), θ_1 , θ_2 , and θ_3 are parameters of MA(3) (** indicates that the t -statistic is significant at the 1% level). The impulse responses of this model to one unit of innovation is reported in Table 6. The table shows that one unit of innovation produces greater than one unit of impulse responses from the first quarter and the impulse responses steadily increase to 1.96 at eighty quarters. This indicates that the fluctuations of Taiwan's real industrial production are persistent rather than transitory.

The estimates of the above structural model and the unrestricted ARMA models show that the nature of U. S. real output fluctuations is consistent with the stochastic trend model in the period 1961 to 1987. Also, we cannot reject the hypothesis that the trend component plays a more important role than the

²⁴ The t -statistics cannot reject the hypothesis that the impulse responses are equal to unity for the horizons 4, 8, 16, 20, 40, and 80 at the 5% significance level.

cycle component in Taiwan's real output fluctuations.²⁵

Table 6: Impulse Responses of Taiwan's Real GNP and Real Industrial Production

Model	Horizon (Quarter)							
	1	2	4	8	16	20	40	80
ARMA (3, 3) (GNP)	.79 (.10)	.60 (.11)	.98 (.02)	.96 (.03)	.93 (.06)	.91 (.07)	.84 (.11)	.76 (.13)
ARMA (2, 3) (Industrial Production)	1.64 (.08)	1.61 (.09)	1.63 (.09)	1.65 (.10)	1.68 (.12)	1.70 (.14)	1.78 (.23)	1.96 (.42)

Note: The numbers in parentheses are standard errors of forecast.

5. Synchronization of Output Changes

This section deals with the synchronization of output fluctuations between Taiwan and the U. S. First, the fluctuations of annual growth rate of real *GNP* between Taiwan and the U. S. will be compared. Second, the OLS model and the ARIMA model will be used to decompose the annual real *GNP* data (from 1951 to 1987) of Taiwan and the U. S. into the trend and the cycle components. This is useful in helping us to understand the relationship of output fluctuations between Taiwan and the U. S.

From Figure 1, we can see that there is a strong synchronization of annual growth rate of real *GNP* (or growth cycle) between Taiwan and the U. S. Except for the year immediately following the two oil crises (1973 and 1979), no serious economic recession was found in Taiwan's economy.

In order to further compare the pattern of business cycle between Taiwan and the U. S., the traditional linear ordinary least square is used to decompose Taiwan's and U. S. real *GNP* data into the trend and the cycle components through trend analysis. Those shown in equations (12) and (13) are the results of this analysis for Taiwan and the U. S. respectively.

$$Y_t = 11.7 + .086 T, \quad (12)$$

(686.4) (110.7)

²⁵ We ever calculated Cochrane's limiting variance ratio [9] by the sample autocorrelation of first-difference log real output variables to the 80 lagged periods. We find that all of variance ratio approach zero for U. S. real *GNP*, Taiwan's real *GNP* and real industrial production. This indicates that output fluctuations are temporary not permanent for these three variables. The U. S. result is consistent with Cochrane's empirical study for U. S. real per capita *GNP* in the period from 1869 to 1986.

$$R^2 = .997, DW = .31,$$

$$X_t = 8.5 + .37 T - .0002T^2, \quad (13)$$

(493.7) (17.7) (3.5)

$$R^2 = .99, DW = .52,$$

where Y_t is Taiwan's annual log real *GNP*, X_t is the U. S. annual log real *GNP*, T is the linear time trend, T^2 is the square of the linear time trend, R^2 is the coefficient of determination, DW is the Durbin-Watson statistic, and the numbers in parentheses are t -statistics.

The fitted values of equations (12) and (13) are the trend component of real *GNP*, the residuals of equations (12) and (13) are the cycle component of real *GNP*. For a cross-country comparison of output fluctuations, the cycle component is divided by the trend component to get the relative deviation of real *GNP*. The relative deviations derived by the OLS model for Taiwan's and U. S. real *GNP* are shown in Figure 2.

Beveridge and Nelson [2] present a new approach to decompose economic time series into the trend and the cycle components. Suppose Y_t is a stochastic trend series which is stationary in first difference, then $y_t = Y_t - Y_{t-1}$ can be represented by an ARMA process. The Wold decomposition theorem states that any stationary stochastic variable, y_t , can be expressed by a moving average process, possibly of infinite order. That is

$$y_t = \mu + \epsilon_t + \lambda_1 \epsilon_{t-1} + \dots, \quad (14)$$

where μ is the deterministic long-run mean of the y series, the λ_i are constant, and the ϵ 's are uncorrelated random innovations with mean zero and variance σ^2 .

Equation (14) implies that y_{t+k} , $k \geq 1$, can be forecasted by the ϵ_{t-i} , $i=0, \dots, \infty$. That is

$$\hat{y}_t(k) = \mu + \phi_0 \epsilon_t + \phi_1 \epsilon_{t-1} + \dots, \quad (15)$$

where $\hat{y}_t(k)$ is the forecasted y_{t+k} .

The conditional expectation of Y_{t+h} , $h \geq 1$, given knowledge of all Y 's up to time t is the sum of Y_t and the forecasted $\hat{y}_t(k)$, $k = 1$ to h . That is

$$E(Y_{t+h} | I_t) = Y_t + \hat{y}_t(1) + \cdots + \hat{y}_t(h), \quad (16)$$

where I_t is the information set of all Y 's up to time t .

Substitute equation (15) into equation (16), the conditional expectation of Y_{t+h} can be expressed as the sum of Y_t and the innovation of ϵ_{t-i} . Mathematically, it can be expressed as

$$E(Y_{t+h} | I_t) = h\mu + Y_t + \theta_0\epsilon_t + \theta_1\epsilon_{t-1} + \cdots. \quad (17)$$

The trend component of Y_t is called the "permanent component" of Y_t . According to Friedman's permanent income theory [12], the unobserved permanent income can be measured in terms of past observed income. Muth [20] shows that the exponentially weighted moving average is an appropriate measure of permanent income if the permanent income follows a random walk process and the transitory income is independently distributed and independent of innovations in permanent income. Suppose the trend component follows a random walk with drift process, then the trend component is the conditional expectation of the series adjusted for its mean rate of change, hence the trend component of Y_t , \tilde{Y}_t , is

$$\tilde{Y}_t = Y_t + \theta_0\epsilon_t + \theta_1\epsilon_{t-1} + \cdots. \quad (18)$$

The cycle component of Y_t , C_t , is

$$C_t = \tilde{Y}_t - Y_t = \theta_0\epsilon_t + \theta_1\epsilon_{t-1} + \cdots. \quad (19)$$

Beveridge and Nelson suggest a two-step procedure for measuring the cycle component:

- (1) Specify and estimate an ARMA model for the first differences of the non-stationary series of interest.
- (2) Evaluate the cycle component using a practical equivalent of equation (19).

We can rewrite equation (14) as

$$Y_t - \mu = \epsilon_t + \lambda_1\epsilon_{t-1} + \cdots. \quad (20)$$

In empirical analysis, equation (20) is equivalent to equation (19). Hence, we can specify a moving average model for the first difference of real *GNP*, y_t , and approximate the cycle component of real *GNP* by subtracting the deterministic term, μ , from the fitted y_t .

According to this ARIMA model decomposition approach, we fit first difference of U. S. real *GNP* by a MA(9) model

$$\begin{aligned} y_t &= .03 + (1 - .34 B^9)u_{y,t}, \\ (9.9) \quad & (1.8) \end{aligned} \quad (21)$$

$DW = 1.9, Q(17) = 5.9,$

where B is backshift operator, $u_{y,t}$ is uncorrelated random innovation with mean zero and variance σ^2 .

The first difference of Taiwan's real *GNP* is fitted by a MA(4) model

$$\begin{aligned} y_t &= .088 + (1 + .74 B - .30 B^4)u_{y,t}, \\ (12.9) \quad & (6.0) \quad (2.1) \end{aligned} \quad (22)$$

$DW = 2.0, Q(16) = 11.3.$

The t -statistics and Q -statistics both show that equations (21) and (22) are well specified. The cycle component of U. S. real *GNP* is calculated by subtracting the fitted value of equation (21) from the deterministic term ($=.03$). The cycle component of Taiwan's real *GNP* is calculated by subtracting the fitted value of equation (22) from the deterministic term ($=.088$). The relative deviations derived by the ARIMA model for Taiwan's and U. S. real *GNP* are shown in Figure 3.

The numerical values of the growth cycles, the relative deviation derived by the OLS model, and the relative deviation derived by the ARIMA model for Taiwan's and U. S. real *GNP* are listed in Table 7 and 8 respectively. From those tables we find that the cycle components derived by the ARIMA model are much smaller than the cycle components derived by the OLS model (except 1986 and 1987 for Taiwan, and 1984 to 1987 for the U. S.).²⁶ This finding is consistent with the

²⁶ From equation (21), we know that the fitted first difference of U. S. real *GNP* is: $\hat{y}_t = .03 + u_{y,t} - .34u_{y,t-9}$, hence the cycle component of U. S. real *GNP* is: $\hat{y}_t - .03 = u_{y,t} - .34u_{y,t-9}$. From equation (22), we know that the fitted first difference of Taiwan's real *GNP* is: $\hat{y}_t = .088 + u_{y,t} + .74u_{y,t-1} - .3u_{y,t-4}$, hence the cycle component of Taiwan's real *GNP* is: $\hat{y}_t - .088 = u_{y,t} + .74u_{y,t-1} - .3u_{y,t-4}$. From 1983 onward, Taiwan's and U. S.

theoretical implication that in the stochastic trend model, the trend component plays a more important role than the cycle component in the output fluctuations.

The business cycles relationship in terms of change in real *GNP* can be estimated by either OLS or ARIMA approach. The output cycles of Taiwan's and U. S. real *GNP* derived by the OLS model are shown graphically in Figure 2; derived by the ARIMA model are shown graphically in Figure 3. The comparison of the output cycles derived by the OLS with by the ARIMA model (or comparing the output cycles derived from the deterministic trend model with the stochastic trend model) are shown graphically in Figure 4 and 5 for Taiwan's and U. S. real *GNP* respectively.

Although the output cycles are derived by the different approaches, Figure 2 and 3 show that the two countries' derived output cycles show a clear similarity. Both Figure 4 and 5 show that the output cycles derived by the OLS model have larger magnitude but lower frequency, the output cycles derived by the ARIMA model have smaller magnitude but higher frequency.

Is Figure 2 or Figure 3 more appropriate to represent the output cycles relationship between Taiwan and the U. S.? By comparing Figure 6 with Figure 7, we can see that the relative deviation of Taiwan's real *GNP* derived by the ARIMA model looks more consistent with the fluctuations of growth rate than that derived by the OLS model. In addition, by the Dickey-Fuller test, the hypothesis that Taiwan's annual real *GNP* follows a stochastic trend process can be accepted at the 10% significance level.²⁷ Therefore, from the theoretical point of view, the relative deviation derived by the ARIMA model seems to be more appropriate than the relative deviation derived by the OLS model in representing Taiwan's output cycles.

For the U. S., comparing the relative deviations derived by the OLS model and the ARIMA model with growth cycles (Figure 8 and 9), cannot determine

economies are in a situation of more stable and prosperous than before 1983, this implies that economic innovations after 1983 are smaller than economic innovations before 1983. That is after 1983, in equation (21) $u_{y,t}$ will much smaller than $u_{y,t-9}$; in equation (22) $u_{y,t}$ and $u_{y,t-1}$ will much smaller than $u_{y,t-4}$. This may explain why the relative deviations derived by the ARIMA model for Taiwan's and U. S. real *GNP* become bigger (in absolute value), in recent years.

²⁷ We test the evolution of Taiwan annual logarithmic real *GNP* from 1952 to 1987 by the Dickey-Fuller test. The results are (all the symbols are the same as Table 1 to 3): $\hat{\tau}_T = -1.67^*$, $\hat{\tau}_\mu = -.53^*$, $\hat{\tau} = 17.74$, $\Phi_3 = 1.36^*$, $\Phi_2 = 1.45^*$, $\Phi_1 = .78^*$, where * means that the null hypothesis of the stochastic trend model is accepted at the 10% significance level.

immediately which is the more appropriate one to represent the U. S. output cycles. But, with the Dickey-Fuller test, we have accepted the hypothesis that the U. S. annual real *GNP* follows a random walk process.²⁸ Therefore, the relative deviation derived by the ARIMA model should be used to represent the U. S. output cycles.²⁹

Since the hypothesis that the evolutions of Taiwan's and U. S. annual real *GNP* follow a stochastic trend process cannot be rejected, the relative deviations derived by the ARIMA model are hence more appropriate than the relative deviations derived by the conventional OLS model to represent both countries' output cycles. Therefore, Figure 3 rather than Figure 2 should be used to represent the output cycles relationship between Taiwan and the U. S. in the period from 1952 to 1987.

6. Conclusion

The conventional proposition is that the output evolution follows a deterministic trend process. In contrast, Nelson and Plosser consider that the output evolution is a stochastic trend process. In this paper, we use Dickey and Fuller's τ and likelihood ratio tests, and Phillips and Perron's nonparametric test to test the three different unit root models for the output evolutions in Taiwan and the U. S. Our finding confirms that in so far as Taiwan and the U. S. are concerned, the hypothesis that the evolution of output follows a stochastic trend process cannot be rejected.

Through the testing of the structural model and the ARIMA model, we also found that the trend component plays a more important role than the cycle component in the U. S. output fluctuations. This is consistent with the nature of output fluctuations attributed to the stochastic trend model. Also, we cannot reject the hypothesis that the trend component plays a more important role than the cycle component in Taiwan's output fluctuations.

²⁸ We test the evolution of U. S. annual logarithmic real *GNP* from 1952 to 1987 by the Dickey-Fuller test. The results are (all the symbols are the same as Table 1 to 3): $\hat{\tau}_\tau = -1.8^*$, $\hat{\tau}_\mu = -.50^*$, $\hat{\tau} = 7.5$, $\Phi_3 = 1.88^*$, $\Phi_2 = 1.51^*$, $\Phi_1 = .35$, where * means that the null hypothesis of the stochastic trend model is accepted at the 10% significance level.

²⁹ Beveridge and Nelson [2] use the ARIMA model to derive business cycles of the postwar U. S. economy, they find that by the new approach, the dating of cyclical episodes tends to lead the traditional NBER dating.

Finally, both the OLS and the ARIMA models are used to derive the output cycles of Taiwan and the U. S. We find that the output cycles derived by the OLS model have larger magnitudes but lower frequencies than the output cycles derived by the ARIMA model.

Although the visual observation cannot determine immediately whether the relative deviations derived by the OLS model or the ARIMA model is more appropriate one to represent both countries' output cycles. But, referring to the theoretical background, the relative deviations derived by the new ARIMA approach rather than by the conventional OLS approach should be used to represent the output cycles relationship between Taiwan and the U. S. in the period from 1952 to 1987.

The Synchronization of Output Fluctuations between Taiwan and the United States

Table 7 Fluctuations of Taiwan's real GNP (Unit: Proportion)

Period	Growth Rate	Relative Deviation Derived by the OLS Model	Relative Deviation Derived by the ARIMA Model
1952	.1205	.0042	—
1953	.0932	.0044	—
1954	.0957	.0048	—
1955	.0809	.0041	—
1956	.0550	.0014	— .0020
1957	.0729	.00005	— .0002
1958	.0657	— .0018	— .0013
1959	.0774	— .0027	.0001
1960	.0646	— .0046	— .0006
1961	.0683	— .0062	— .0007
1962	.0785	— .0070	— .0006
1963	.0937	— .0067	.0001
1964	.1230	— .0043	.0005
1965	.1102	— .0029	.0016
1966	.0900	— .0029	— .0001
1967	.1056	— .0018	— .00004
1968	.0907	— .0017	.0002
1969	.0900	— .0017	— .0001
1970	.1127	— .0002	.00000009
1971	.1289	.0024	.0007
1972	.1331	.0053	.0014
1973	.1282	.0077	.0010
1974	.0112	.0022	.0006
1975	.0427	— .0010	— .0050
1976	.1349	.0019	.0009
1977	.1005	.0025	.0010
1978	.1390	.0056	.0015
1979	.0846	.0053	.0006
1980	.0713	.0040	— .0013
1981	.0571	.0018	.0001
1982	.0330	— .0019	— .0022
1983	.0788	— .0026	— .0009
1984	.1052	— .0017	.00005
1985	.0508	— .0042	.0013
1986	.1164	— .0025	— .0024
1987	.1104	— .0013	— .0027

Note: — is the period allowed for the lagged of MA.

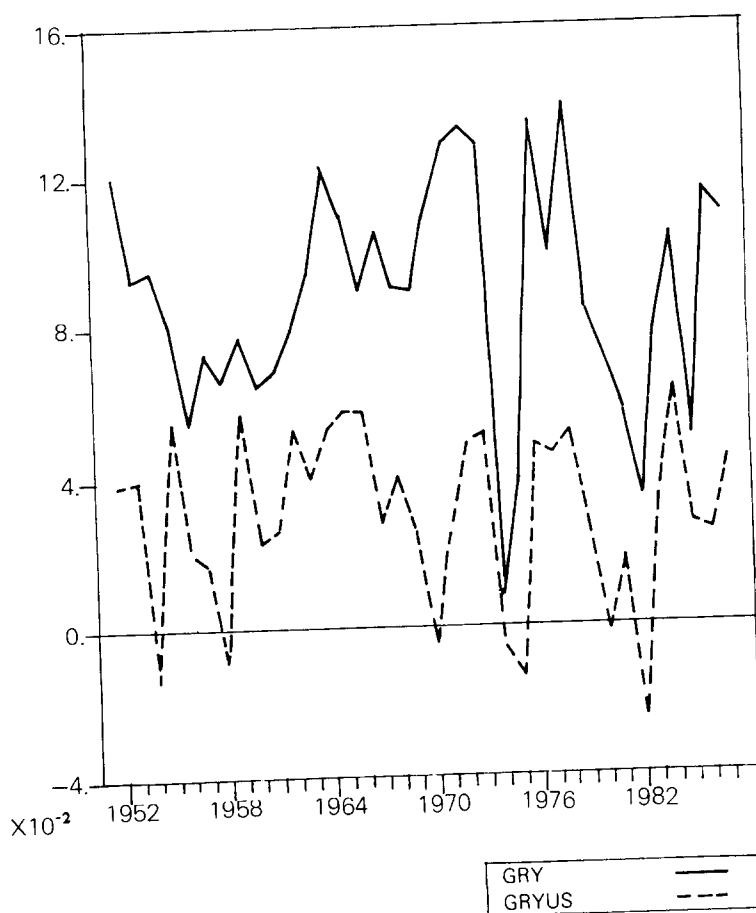
Table 8: Fluctuations of U. S. real GNP (Unit: Proportion)

Period	Growth Rate	Relative Deviation Derived by the OLS Model	Relative Deviation Derived by the ARIMA Model
1952	.390	.0046	—
1953	.0400	.0049	—
1954	-.0133	-.0008	—
1955	.0556	.0014	—
1956	.0206	-.0003	—
1957	.0167	-.0024	—
1958	-.0077	-.0071	—
1959	.0584	-.0045	—
1960	.0222	-.0058	—
1961	.0261	-.0066	-.0003
1962	.0531	-.0045	-.0004
1963	.0411	-.0036	.0017
1964	.0534	-.0014	-.0009
1965	.0579	.0014	.0004
1966	.0579	.0041	.0005
1967	.0285	.0038	.0014
1968	.0415	.0049	-.0010
1969	.0244	.0042	.0003
1970	-.0029	.0006	.0001
1971	.0284	.0004	-.0009
1972	.0498	.0025	.0002
1973	.0519	.0049	-.0011
1974	-.0054	.0012	-.0008
1975	-.0126	-.0031	-.0008
1976	.0489	-.0010	.0005
1977	.0467	.0010	-.0007
1978	.0529	.0036	.0003
1979	.0248	.0034	.0012
1980	-.0016	.0004	-.0002
1981	.0193	-.0003	-.0006
1982	-.0255	-.0057	-.0011
1983	.0357	-.0046	.0010
1984	.0643	-.0007	.0013
1985	.0273	-.0004	-.0004
1986	.0250	-.0003	-.0008
1987	.0440	.0017	-.0035

Note: — is the period allowed for the lagged of MA.

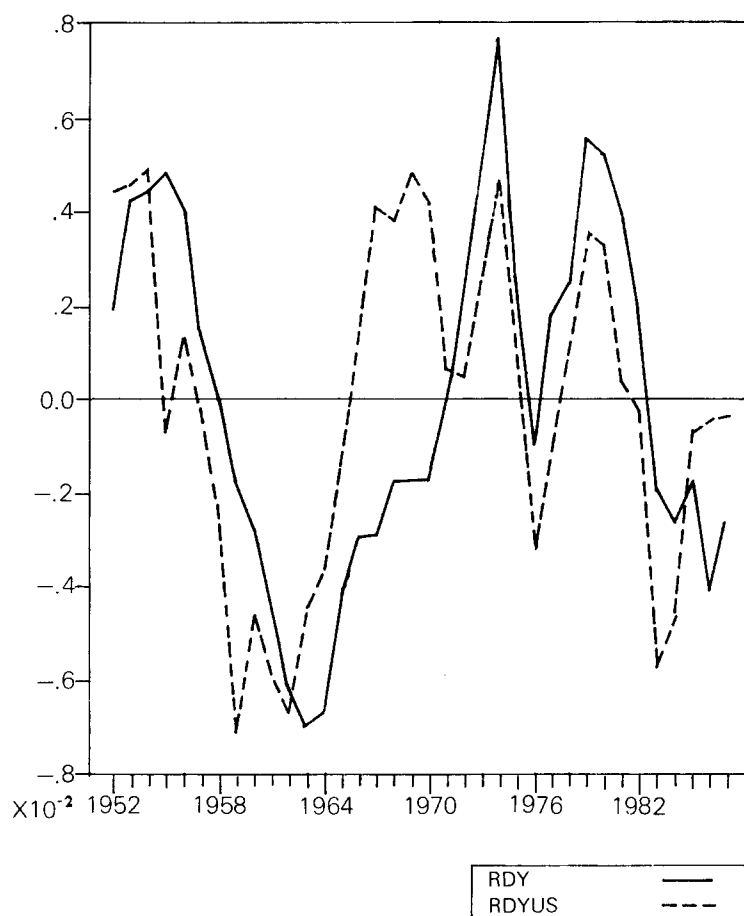
The Synchronization of Output Fluctuations between Taiwan and the United States

Figure 1: Synchronization of Growth Cycles of Real GNP between Taiwan and the U. S.



Note: GRY is the growth cycle of Taiwan's real GNP.
GRYUS is the growth cycle of U.S. real GNP.

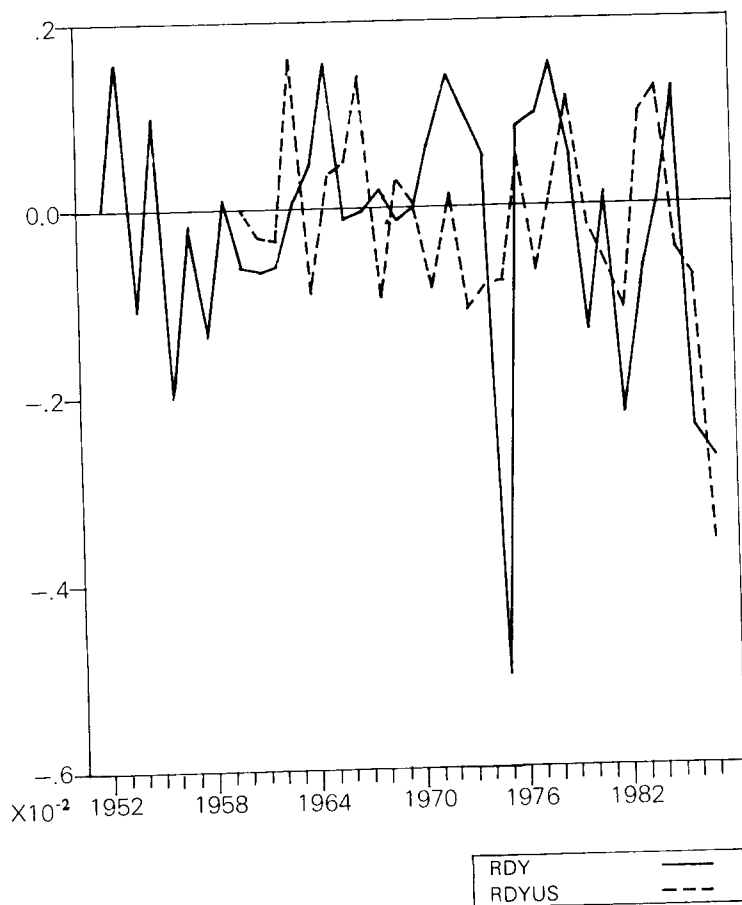
Figure 2: Relative Deviations of Taiwan's and the U. S. Real GNP Derived by the OLS Model



Note: RDY is the relative deviation of Taiwan's real GNP.
RDYUS is the relative deviation of U.S. real GNP.

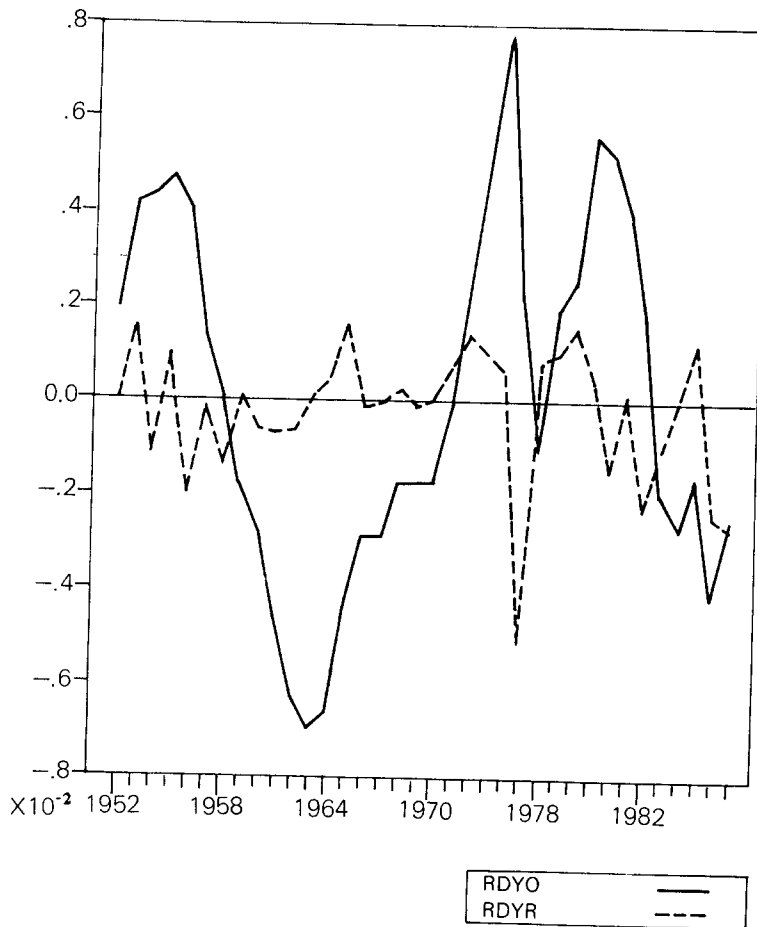
The Synchronization of Output Fluctuations between Taiwan and the United States

Figure 3: Relative Deviations of Taiwan's and the U. S. Real GNP Derived by the ARIMA Model



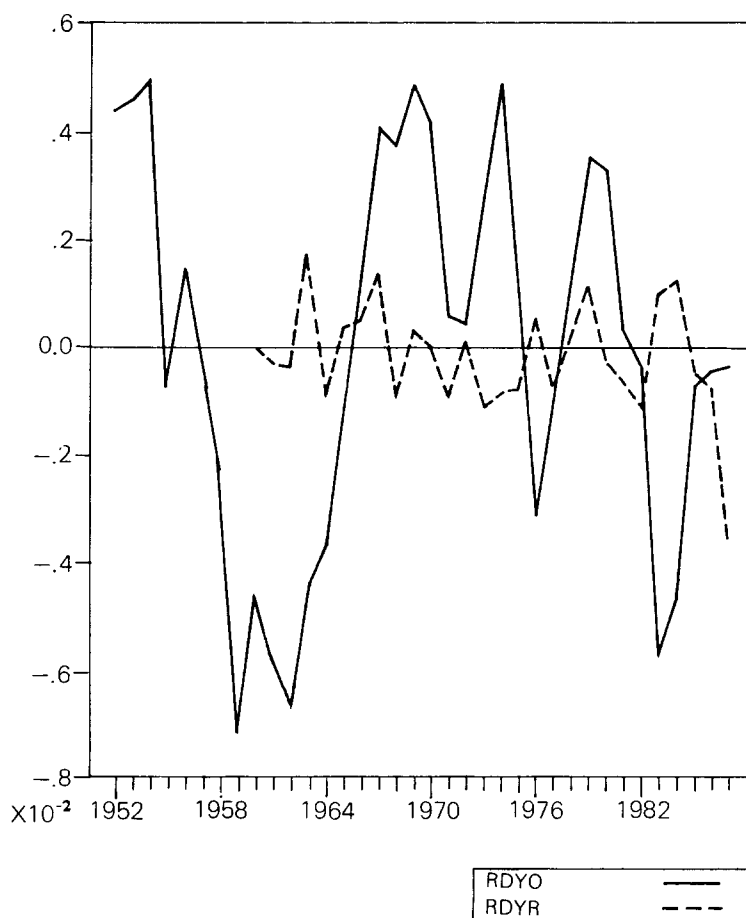
Note: RDY is the relative deviation of Taiwan's real GNP.
RDYUS is the relative deviation of U.S. real GNP.

Figure 4: Relative Deviations of Taiwan's Real GNP Derived by the OLS Model and the ARIMA Model



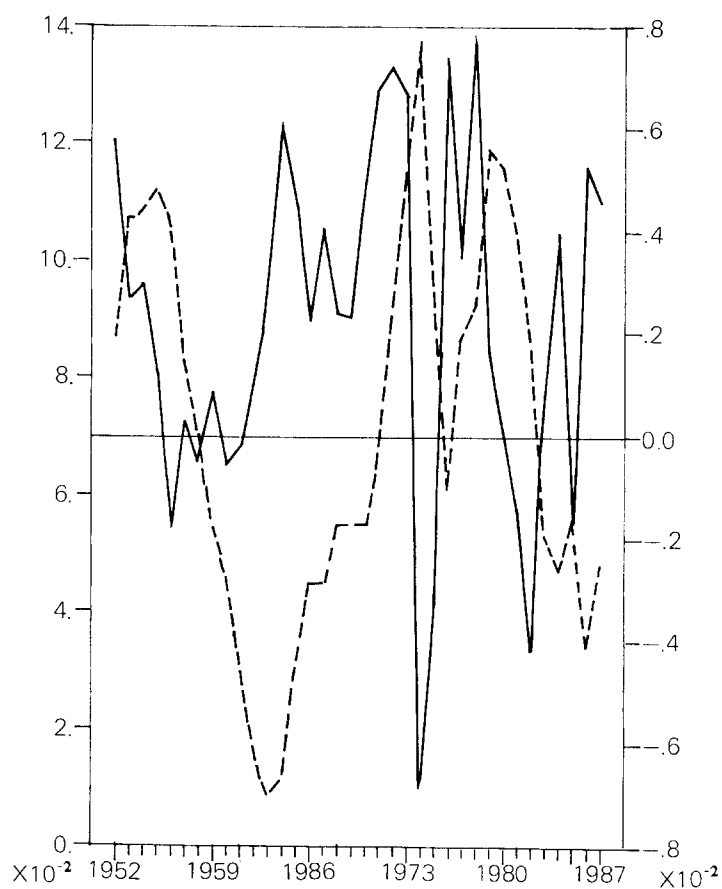
Note: RDYO is the relative deviation derived by the OLS model.
RDYA is the relative deviation derived by the ARIMA model.

Figure 5: Relative Deviations of U. S. Real GNP Derived by the OLS Model and the ARIMA Model



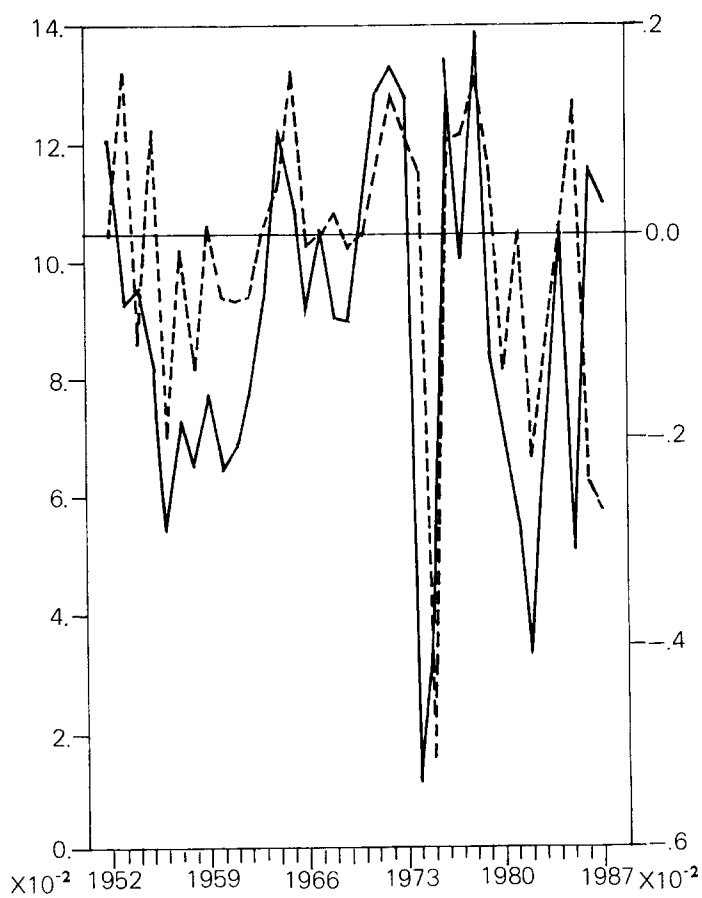
Note: RDYO is the relative deviation derived by the OLS model.
RDYR is the relative deviation derived by the ARIMA model.

Figure 6: Growth Cycle and Relative Deviation Derived by the OLS Model of Taiwan's Real GNP



Note: The solid line is the growth cycle.
The dashed line is the relative deviation derived by the OLS model.

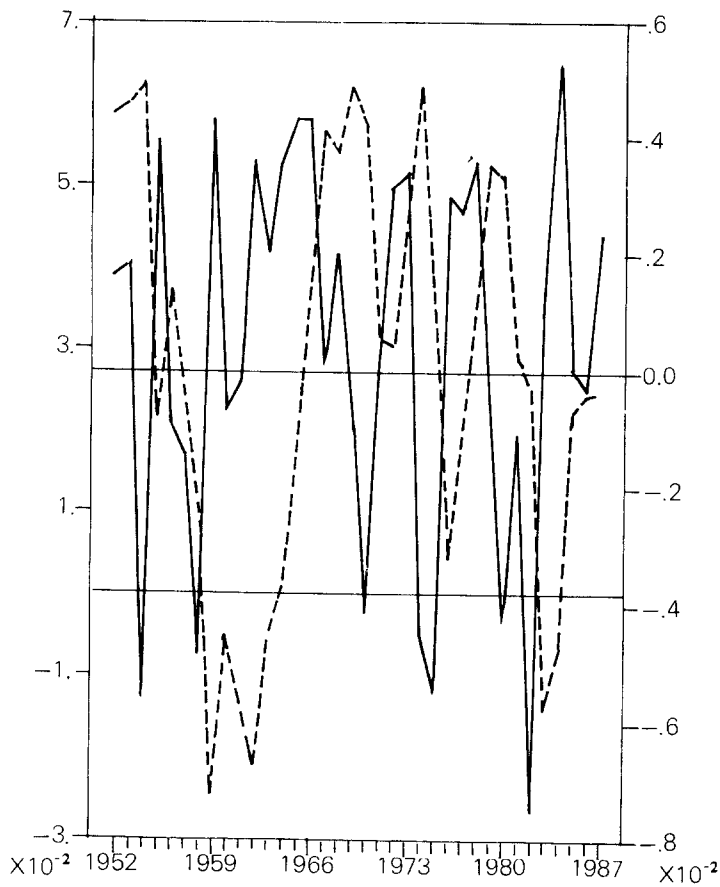
Figure 7: Growth Cycle and Relative Deviation Derived by the ARIMA Model of Taiwan's Real GNP



Note: The solid line is the growth cycle.

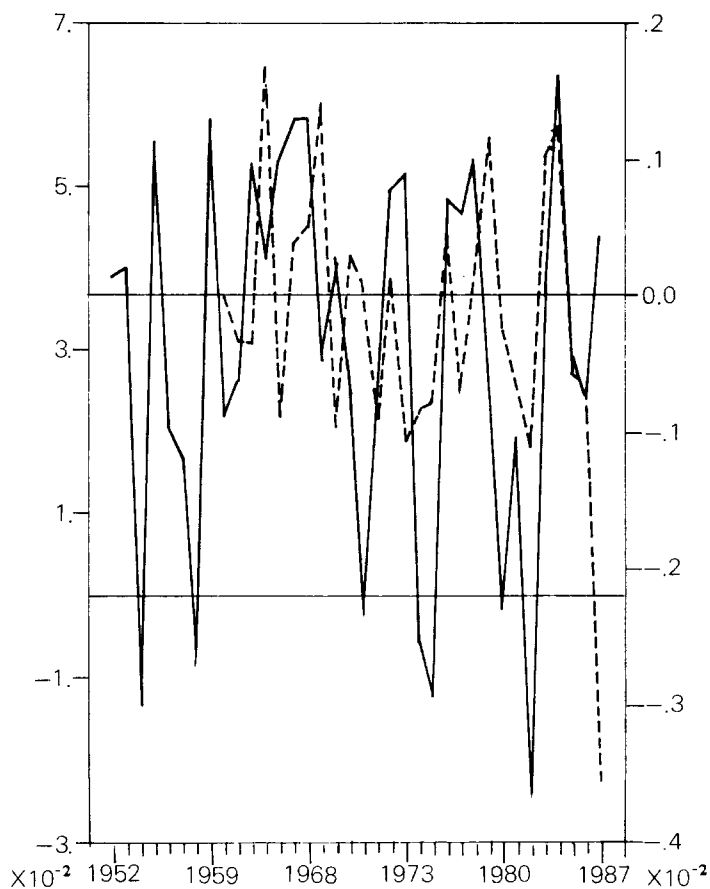
The dashed line is the relative deviation derived by the ARIMA model.

Figure 8: Growth Cycle and Relative Deviation Derived by the OLS Model of U. S. Real GNP



Note: The solid line is the growth cycle.
The dashed line is the relative deviation derived by the OLS model.

Figure 9: Growth Cycle and Relative Deviation Derived by the ARIMA Model of U. S. Real GNP



Note: The solid line is the growth cycle.
The dashed line is the relative deviation derived by the ARIMA model.

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