

行政院國家科學委員會專題研究計畫成果報告

線星圖的特徵

Characterization of Linear Substar Graphs

計畫編號：NSC 88-2115-M-004-002

執行期限：民國 87 年 8 月 1 日至民國 88 年 7 月 31 日止

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## 一、中文摘要

在這計畫中，我們探討圖的交集代表之參數。

在一圖形之交集代表中，每一點都能從一毛蟲 (caterpillar) 中分配到至多  $t$  個子星 (substar)，我們稱此代表為  $t$ -星代表。我們稱此最小的  $t$  使得此圖形有一  $t$ -星代表為此圖的線星圖。

此計畫的主要目標為確立線星圖，即線星數為 1 之圖的特徵，我們希望能找出此種圖所不能包含的子圖。

此外，我也希望能找出在何種狀況下，圖的線星圖數會小於或等於它的線段數。

關鍵詞：交集代表，星，毛蟲，線段數

## Abstract

In this project, we study intersection representation parameters for graphs.

We introduce linear star number of a graph  $G$ , which is the minimum  $t$  such that  $G$  is the intersection graph of unions of  $t$  substars of a host tree that is a caterpillar.

Our main goal is to characterize graphs with linear star number 1 which are called linear star graphs by providing forbidden subgraphs.

We will also study the condition in which the linear star number of a graph is at most its interval number.

**Keywords:** intersection representation, star, caterpillar, interval number.

## 二、緣由與目的

Special representations of graphs can provide compact encoding of graphs and can provide data structures that permit rapid solution of problems that are different for graphs in general.

In this project, we consider special types of representations using intersections of sets. An intersection representation of a graph  $G$  is a function  $f$  that maps  $V(G)$  into a collection of sets, such that vertices  $u, v$  are adjacent if and only if  $f(u)$  and  $f(v)$  intersect. When we limit the types of sets that may be used in a representation, we obtain special families of graphs.

The sets may, for example, be intervals on the real line, in which case the resulting graph is an interval graph. They have been studied in [1] and [2]. More generally, the sets may be unions of  $t$  intervals (call  $t$ -internals). The minimum  $t$  such that a graph  $G$  has a  $t$ -interval intersection representation is its interval number  $i(G)$ . Basic results on interval number are summarized in [8].

We can use a few discrete intervals as subpaths of a host path. We can generalize this by letting the sets used to represent the vertices be subtrees of a host tree. It is well-known ([3], [6] and [7]) that the graphs obtainable as intersection graphs of subtrees of a tree are precisely the chordal graphs, defined to be those having no chordless cycles. If we consider the substars of a host tree by analogy with interval number, we have star number of  $G$ . And if the host tree is a caterpillar, we introduce linear star number of  $G$ . Some preliminary results on star number and structural characterizations of star graphs are in [4] and [5]. So, we would like to get a forbidden subgraph characterization of linear substar graphs.

### 三、結果與討論

We define a *big star*  $s$  with  $k$  beams to be a rooted tree in which the root has  $k$  children and each child has exactly one child. And we let  $b(s)$  to be the number of beams.

We can prove that a graph has a big

star  $s$  with  $b(s) \geq 4$  as an induced subgraph can not be a linear substar graph.

We have now 15 nonisomorphic forbidden subgraphs of linear star graphs but they all have big stars with  $2 \leq b(s) \leq 3$  as their subgraphs.

### 四、計畫成果自評

We solved part of the problems in our proposal. These results of this project and some other results of the author will be submitted to some journal.

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