## 行政院國家科學委員會專題研究計畫 成果報告

## 隨機數位網路上最佳路徑選擇之研究(I)

計畫類別: 個別型計畫

計畫編號: NSC94-2213-E-004-003-

執行期間: 94年08月01日至95年07月31日

執行單位: 國立政治大學應用數學學系

計畫主持人: 陸行

報告類型: 精簡報告

處理方式:本計畫可公開查詢

中華民國95年9月18日

計畫類別:□v個別型計畫 □整合型計畫

計畫編號:NSC 94-2213-E-004-003

執行期間: 94年8月1日至 95年7月31日

計畫主持人:陸行

共同主持人:

本成果報告包括以下應繳交之附件:
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

執行單位:政治大學應用數學系

中 華 民 國 95 年 8 月 15 日

## 行政院國家科學委員會專題研究計畫成果報告 隨機數位網路上最佳路徑選擇之研究(I) Optimal Path Selection on Stochastic Networks

計畫編號: NSC 94-2213-E-004-003 執行期限: 94年08月01日至95年07月31日

Abstract. We propose a precomputation-based scheme which offers a Pareto optimal solution to the network optimization problem. This scheme is performed by means of a two-phase procedure. The first phase precomputes paths for a wide range of possible constraints, while the second phase just needs to select a adequate path through a simple procedure. That is, the first phase prepares a database which enables to identify a suitable path upon each connection request through the second phase. We propose a bandwidth allocation model with nonlinear utility functions. The objective of the problem is to provide a proportionally fair treatment of all the competing classes. We also study fairness in allocating bandwidth for modern communication networks.

Keywords: Bandwidth Allocation, QoS Routing,

#### 1 Introduction

We consider the operational processes involved in the efficient set-up and usage of a network. The three main components of these processes are designing which links to develop to meet certain connectivity requirements, determining how much capacity to put on the links to serve all traffic demands, and choosing which paths to use for the various traffic streams to meet demand without violating capacity restrictions on links.

In this report, we focus on the precomputation perspective of QoS routing [8]. We deal with the problem of dimensioning bandwidth for elastic data appli-

cations in packet-switched communication networks, which can be considered as a multiple-objective optimization model. We focus on allocating resources with proportional fairness and finding a routing scheme on All-IP communication networks. We present an approach for the fair resource allocation problem and QoS routing in All-IP networks that offer multiple services to users. The basic function of QoS routing [2] is to select a path that is likely to be able to meet the QoS requirements. Users' satisfaction is summarized by means of their achievement functions, and each user is allowed to request more than one type of service. The objective of the optimization problem is to determine the amount of required bandwidth for each class to maximize the sum of the users' satisfaction.

# 2 Bandwidth Allocation Policies

Consider a directed network topology G=(V,E), where V and E denote the set of nodes and the set of links in the network respectively. There are m (different) QoS classes in this network. Given the maximal possible capacity  $U_e$  of each link  $e \in E$ . Suppose, for each link e, the cost consists of delay  $d_e$  and the purchasing cost  $\kappa_e$  of bandwidth. In this network, there are m different classes which have their own QoS requirement. In each class i, every connection is allocated the same bandwidth  $\theta_i$  and has the same QoS requirement. Suppose each connection is deliv-

ered between the same source o and destination d in this (core) network. Under a limited available budget B, we want to allocate the bandwidth in order to provide each class with maximal possible QoS.

Kelly et al. [3] advocated proportional fairness characterized by  $\log(\theta_i)$ . This log utility function is strictly concave. The proportional fair bandwidth allocation is determined by the following objective function:

Maximize 
$$\sum_{i \in I} K_i \log(K_i \theta_i). \tag{1}$$

Determining the maximizer of (1) can be done explicitly for simple networks.

In [6], an allocation policy called minimum potential delay is proposed with utility functions (2). The minimal potential delay allocation is characterized as the following objective function:

Maximize 
$$\sum_{i \in I} -\frac{K_i}{\theta_i}.$$
 (2)

For any given number  $\alpha$  ( $\alpha > 0$ ,  $\alpha \neq 1$ ), the class of  $(w, \alpha)$ - proportionally fair bandwidth allocation [7] is characterized as the following objective function:

Maximize 
$$\sum_{i \in I} w_i K_i^{\alpha} \frac{(K_i \theta_i)^{1-\alpha}}{1-\alpha}, \qquad (3)$$

where  $w_i$ ,  $i \in I$ , are fixed parameters.

# 3 A Precomputation Scheme for Network Optimization

First, we transform the different measurements onto a normalized scale by using the concept of achievement functions [4]. Depending on the specified aspiration and reservation levels,  $a_i$  and  $r_i$ , respectively, we construct our achievement function of  $\theta_i$  as follows [5]:

$$f_i(\theta_i) = \log_{\alpha_i} \frac{\theta_i}{r_i}$$
, where  $\alpha_i = \frac{a_i}{r_i}$ . (4)

When using the achievement function (4) interpreted as a measure of QoS on All-IP networks, we can formulate the mathematical model of the fair bandwidth allocation.

$$\max \qquad \sum_{i=1}^{m} w_i f_i(\theta_i) \tag{5}$$

s. t. 
$$\sum_{e \in E} \sum_{i=1}^{m} \sum_{i=1}^{K_i} \kappa_e A_{i,j}(e) \le B$$
 (6)

$$\sum_{i=1}^{m} \sum_{j=1}^{K_i} A_{i,j}(e) \le U_e \tag{7}$$

$$A_{i,j}(e) - M \cdot \chi_{i,j}(e) \le 0 \tag{8}$$

$$\theta_i - A_{i,j}(e) \le M(1 - \chi_{i,j}(e))$$
 (9)

$$A_{i,j}(e) - \theta_i \le M(1 - \chi_{i,j}(e))$$
 (10)

$$\theta_i \ge b_i \tag{11}$$

$$\sum_{e \in E_o} A_{i,j}(e) = \theta_i \tag{12}$$

$$\sum_{e \in E_{\nu}^{in}} A_{i,j}(e) = \sum_{e \in E_{\nu}^{out}} A_{i,j}(e)$$
 (13)

$$\sum_{e \in E_d} A_{i,j}(e) = \theta_i \tag{14}$$

$$A_{i,j}(e) \ge 0 \tag{15}$$

$$\theta_i > 0 \tag{16}$$

$$\chi_{i,j}(e) = 0 \text{ or } 1$$
 (17)

where  $w_i \in (0,1)$  is given for each class i and  $\sum_{i=1}^{m} w_i = 1$ .

We have the budget constraint (6) due to the limited budget on network planning. The constraint (7) says that the aggregate bandwidth of all connections at any link does not exceed the capacity. Constraints (9), (10), and (11) show that every connection in the same class uses the same bandwidth and has the same bandwidth requirement. Constraints (12), (13), and (14) express the node conservation relations indicating that flow in equals flow out for every connection j in class i. Although  $A_{i,j}(e)$  are continuous variables, constraints (12)-(14) are standard flow conservation constraints with the help of constraints (8)-(10). Continuous decision variables and binary variables must be nonnegative in constraints (15)-(17). From the optimization of this precomputation scheme (the first phase), we have the pareto optimal bandwidth allocation and a routing database. Next, using the output of the first phase, we will formulate a routing scheme with end-to-end QoS guarantees (the second phase).

### 4 Model Solutions

Using the mathematical model, given a limited available budget B, we can get the optimal solutions  $A_{i,j}^*(e)$  and  $\theta_i^*$  which represent the optimal bandwidth allocation for each link e and for each connection of class i. We find the bandwidths  $K_i\theta_i^*$  allocated to each class i. Moreover, we also attain the maximal bandwidth  $R_{i,e}$  which the link e can offer for class i. That is,

$$R_{i,e} = \sum_{i=1}^{K_i} A_{i,j}^*(e). \tag{18}$$

Bandwidth are allocated along less expensive paths that connect the origin o and the destination d. After the optimization of the above precomputation scheme, we can obtain the optimal path p from the source o to the destination d. The optimal path p may not be unique. Let P denote the set of all optimal paths, the routing database, obtained from the execution of the first phase. That is,  $P = \{p|p \text{ is the optimal path from } o \text{ to } d\}$ . The set P includes the inexpensive routes from the source to the destination on the network.

If, for each class i, the bandwidth allocated to each optimal path p is  $\theta_{i,p} \geq 0$ , then

$$\sum_{p \in P} \theta_{i,p} = K_i \theta_i^* \tag{19}$$

and

$$0 \le \sum_{i=1}^{m} \theta_{i,p} \le \min_{e \in p} U_e. \tag{20}$$

A link e is called bottleneck link if the right equality holds in (20).

### 5 A Routing Scheme with Endto-End QoS Guarantees

From the optimization in the first phase, we have a network G = (V, E'), where V is the original set of nodes and E' is the subset of links used for each end-to-end path  $p \in P$ . In each class i, a connection should be routed through some path p between the source and destination nodes. We shall denote by n(p) the number of links of a path p. When a connection of class i is routed over a path p with a bandwidth  $\theta_i^*$ , the following end-to-end delay D(p) applies [1]:

$$D(p) = \frac{n(p) \cdot \sigma_i}{\theta_i^*} + \sum_{e \in p} d_e.$$
 (21)

A path p between o and d is feasible, for a connection of class i, if  $D(p) \leq D_i$ . A connection of class i is feasible if it has a feasible path.

For a path p, we denote by  $A_p$  the residual bandwidth of its bottleneck link, that is,

$$A_p = \min_{e \in p} (R_{i,e} - \theta_i^*).$$
 (22)

The problem is to find an optimal path p that maximizes  $A_p$  from the routing database P (obtained by the first phase.) This routing scheme distributes the connection among the paths so as to avoid overloaded links

For each class i, we give the following scheme (the second phase) of the routing with End-to-End QoS guarantees.

maximize 
$$A_p$$
  
subject to  $A_p \leq R_{i,e} - \theta_i^*, \forall e \in p$   
 $D(p) \leq D_i$   
 $p \in P.$  (23)

The optimization goals of this scheme is to enhance the performance of IP traffic while utilizing the bandwidth on All-IP networks economically. This QoS routing is to make more efficient use of bandwidth on the network.

#### 6 Conclusions

We present an approach for the fair bandwidth allocation and QoS routing in All-IP networks. Our approach is performed by means of a two-phase procedure. The first phase is executed in advance and

its purpose is to precompute solutions as a database for later usage. The second phase selects one of the solutions (precomputed at the first phase) by performing a few additional computations. The purpose of the second phase is to promptly provide an adequate solution when connections arrive. Users' utility functions are summarized by means of achievement functions. Using the bandwidth allocation model, we can find a Pareto optimal allocation of bandwidth on the network under a limited available budget, and this allocation can provide the so-called proportional fairness to every class. That is, this allocation can provide the similar satisfaction to each user in all classes.

### 參考書目

- [1] Atov, I., Tran, H. T., and Harris, R. J.: OPQR-G: Algorithm for efficient QoS partition and routing in multiservice IP Networks. Computer Communications 28 (2005) 1987–1996
- [2] Guérin, R. A., Orda, A.: QoS routing in networks with inaccurate information: Theory and algorithms. IEEE/ACM Transactions on Networking 7 (3) (1999) 350–364
- [3] Kelly, F. P., Maulloo, A. K., Tan, D. K. H.: Rate control for communication networks: Shadow prices, proportional fairness and stability. Journal of the Operational Research Society 49 (1998) 237–252
- [4] Luh, H. and Wang, C. H.: Mathematical Models of Pareto Optimal Path Selection on All-IP Networks. Proceedings of The First Sino-International Symposium on Probability, Statistics, and Quantitative Management (2004) 185– 197
- [5] Luh, H., Wang, C. H.: Proportional bandwidth allocation for unicasting in All-IP networks. Proceedings of the 2nd Sino-International Symposium on Probability, Statistics, and Quantitative Management (2005) 111–130

- [6] L., Massoulié, Roberts, J.: Bandwidth sharing: Objectives and algorithms. IEEE/ACM Transactions on Networking 10 (3) (2002) 320–328
- [7] Mo, J., Walrand, J.: Fair end-to-end windowbased congestion control. IEEE/ACM Transactions on Networking 8 (5) (2000) 556–567
- [8] Orda, A., Sprintson, A.: Precomputation schemes for QoS routing. IEEE/ACM Transactions on Networking 11 (4) (2003) 578-591
- [9] Paganini, F., Wang, Z., Doyle, J. C., Low, S. H.: Congestion control for high performance, stability, and fairness in general networks. IEEE/ACM Transactions on Networking 13 (1) (2005) 43–56
- [10] Wang, C. H. and Luh, H.: Fair Budget Allocation of Precomputation in All-IP Networks. in: Proceedings of IFORS International Triennial Conference, Honolulu, Hawail, page 105, July 2005.
- [11] Wang, C. H. and Luh, H.: Network Dimensioning Problems of Applying Achievement Functions. Operations Research and Its Applications, Lecture Notes in Operations Research 6 (2006) 35–59
- [12] Wang, C. H. and Luh, H.: A Precomputation-Based Scheme for QoS Routing and Fair Bandwidth Allocation. in: Proceedings of the 13th annual IEEE International Conference on High Performance Computing (HiPC 2006), Bangalore, India, December 18-21, 2006.
- [13] Wu, H., Jia, X., He, Y., Huang, C.: Bandwidth-guaranteed QoS routing of multiple parallel paths in CDMA/TDMA ad hoc wireless networks. International Journal of Communication Systems 18 (2005) 803–816