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半純函數之分解與固定點

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半純函數之分解與固定點

On the factorization and fix-point of meromorpic functions

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On the factorization and fix-point of meromorpic functions

Abstract

In 1925, Nevanlinna published his remarkable paper on the value distribution of meromorphic functions. Since, then, the first and second fundamental theorems [3] become an important tool in the studying of meromorphic functions. In this report, we will discuss the studying of the factorization and the shared value problems of meromorphic functions.

Keywords: factorization, shared values, meromorphic function, primeness and pseudo -primeness.

First we review some definitions.

Given a meromorphic functions $F(z)$. If $F(z) = f \circ g(z)$, where $f(z)$ is meromorphic and g is entire (or $f(z)$ is rational and g is meromorphic), then the expression $F(z) = f \circ g(z)$ is called a factorization of $F(z)$. We say that $F(z)$ is prime (pseudo-prime) if every factorization $F(z) = f \circ g(z)$ implies that either $f(z)$ or $g(z)$ is bilinear (f is rational or g is polynomial).

See [3] for a further study of factorization of meromorphic functions.

We say that two meromorphic functions f and g share the value a, where $a \in C_\infty$, provided that $f(z) = a$ if, and only if $g(z) = a$. We will state whether a shared value is by CM (counting multiplicities), by IM (ignoring multiplicities), or by DM (by different multiplicities at one or more points).

In 1929, Nevanlinna proved the following two well-known theorems.

Theorem 1[4]. *If two nonconstant meromorphic functions f and g share five values IM, then* $f \equiv g$.

Theorem 2[4]. *If two distinct nonconstant meromorphic functions f and g share four value aj , 1 ≤ j ≤ 4,* CM*, then f is a M¨obius transformation of g, two of the shared values, say* a_1 *and* a_2*, must be Picard exceptional values and the cross ratio* $(a_1, a_2, a_3, a_4) = -1$.

In 1979 and 1983, G. Gundersen generalized Theorem 2 by proving the following two results.

Theorem 3[1]. *If two distinct nonconstant meromorphic functions f and g share three vaules* CM *and share a fourth value* IM*, then f and g share all four values* CM*, hence, the conclusions of Theorem 2 hold.*

Theorem 4[2]. *If two distinct nonconstant meromorphic functions f and g share two values* CM *and the other two values* IM*, then f and g share all four values* CM*, hence, the conclusions of Theorem 2 hold.*

Now, a natural question arises, namely, what happen if two meromorphic functions *f* and *g* share four values a_j, $1 \le j \le 4$, IM, and *f* and *g* share a_1 CM and a_j, $2 \le j \le 4$, IM? Also, what are the relations of two meromorphic functions *f* and *g* sharing four values DM?

So far, the above two questions are still open.

From Theorem 1 to Theorem 4, we may ask, from the other point of view, the following question: Let *f* and *g* be two nonconstant meromorphic functions and share four value IM. Then *f*(*z*) is prime (resp., pseudo-prime) if, and only if *g*(*z*) is prime (resp., pseudo-prime).

In the remaining, we will study an example given by G. Gundersen [1] to see some hint about the problem.

Example 5 *Let b be a nonzero complex number and h*(*z*) *be a nonconstant entire function. Define*

$$f(z) = \frac{e^{h(z)+b}}{(e^{h(z)} - b)^2}, \quad g(z) = \frac{(e^{h(z)} + b)^2}{8b^2(e^{h(z)} - b)}.$$

Then, obviously, f and g have the following properties:

1. f and g share 0, ∞, $\frac{1}{b}$ *and* $-\frac{1}{8b}$ *DM at every points*

2. f is not a M¨obius transformation of g

3. None of the shared valued are Picard exceptional values of f (resp., g)

4. The cross ratio of any permutations of 0, ∞, $\frac{1}{b}$ *and* $-\frac{1}{8b}$ *does not equal to* -1

5. There exist rational functions R(*z*), *S*(*z*) *and an entire function α*(*z*) *such that*

$$f(z) = R \circ \alpha(z) \text{ and } g(z) = S \circ \alpha(z).$$

In fact, $R(z) = \frac{z}{z-2b}$, $S(z) = \frac{z^2}{8b^2(z-2b)}$, and $\alpha(z) = e^{h(z)} + b$

The example shows that f and g share four values DM and satisfy property (5). So one may ask further:

If two meromorphic functions f and g share four values DM, do they satisfy property (5)?

All these problems listed above are found during the study of value distribution of meromorphic functions last year, we hope they can be answered in the coming year.

References

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