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An Empirical Study of Mortality Models in Taiwan

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Abstract

There has been a significant increase in the life expectancies of the Taiwanese population after the end of Second World War. Like in many developed countries, due to the prolonging life expectancy and lower fertility rates, the aging population has now become a major policy concern in Taiwan. The search for feasible methods for modeling the future mortality changes has become a popular issue in Taiwan. The Lee-Carter (LC) model, the reduction factor (RF) model and the age-period-cohort (APC) model are three frequently used methods for modeling future mortality dynamics. In this paper, we introduce these three models and discuss their respective pros and cons. We carry out an empirical study using these models based on Taiwan mortality experience. In addition, we make a comparison analysis of different models with different mortality experience in Japan, England and Wales, and the US.

I. Introduction

Mortality Improvement and Population Aging in Taiwan. Mortality modeling has received much attention in recent years as a result of the significant increases in the population life expectancy experienced by many countries in the last century and because this trend shows no signs of slowing down in the near future. Take the life expectancies in the US as an example. While the increases have been more significant before the 1960's, still, the increases remain consistent since 1960. As shown in Figure 1, the life expectancies of US male and female in 2006 were approximately 75 and 81 years, respectively.

In Taiwan, the significant increases started at the end of the Second World War. The increments of increase in the life expectancies were dramatic before the 1970s and became stable from 1980 (Figure 2). In particular, the increments in male life expectancies have been slightly smaller than those in female's. Life expectancies still increase at approximately 0.2 year annually, and the present life expectancies in Taiwan become similar to those in the US.

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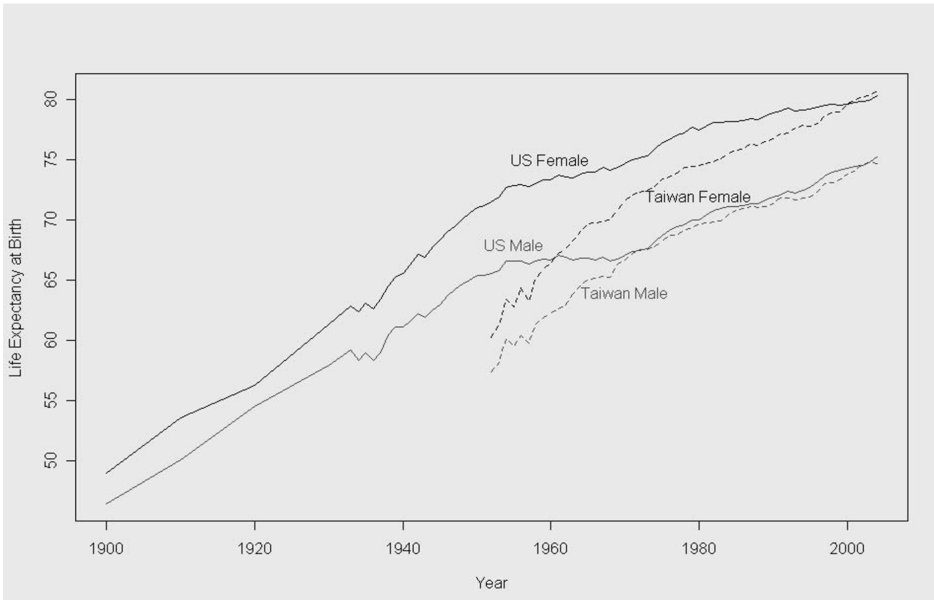
Figure 1: Life Expectancy at Birth in the US and Taiwan



Source: Human Mortality Database and Ministry of Interior, Taiwan

Figure 2: Increments in Life Expectancy of Taiwan Male and Female

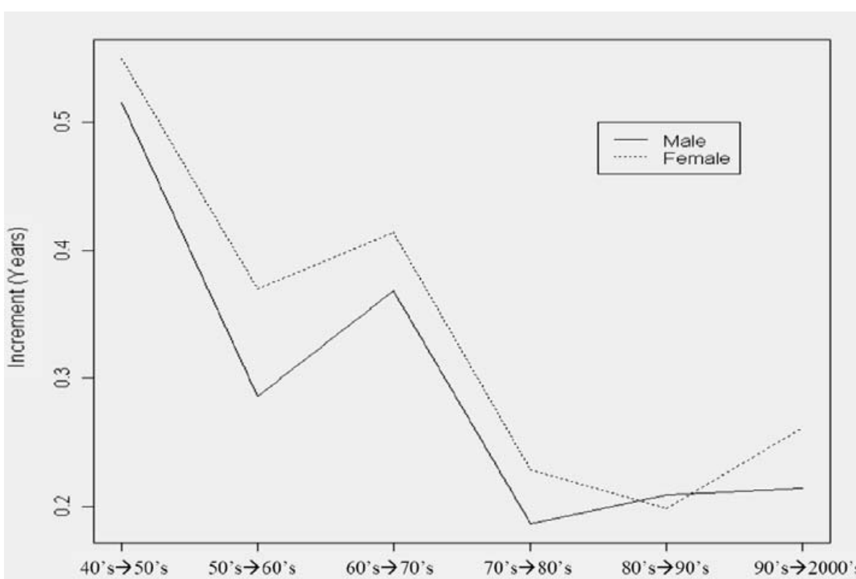


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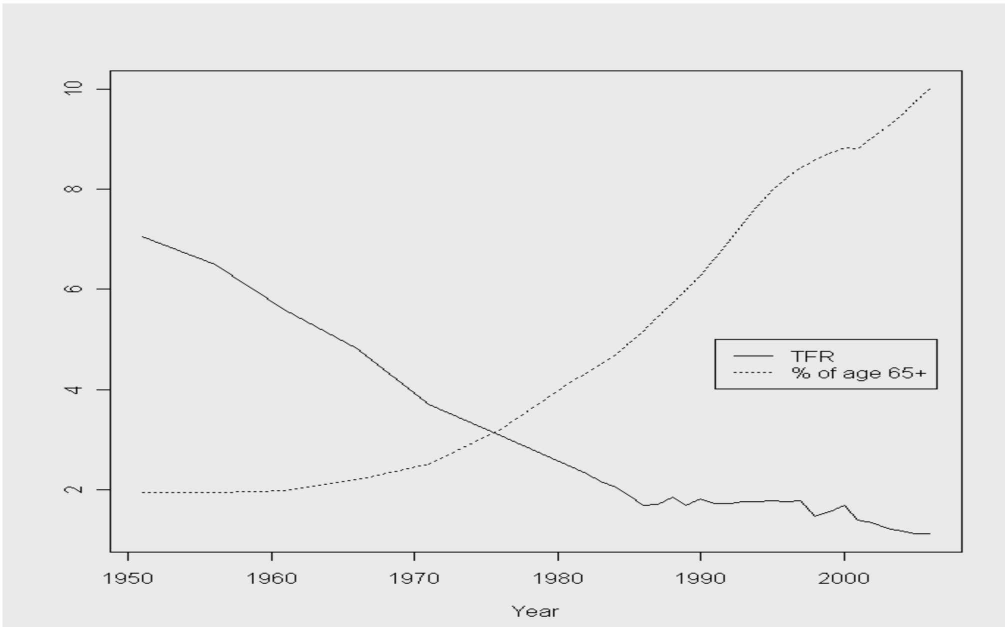
Figure 3: TFR and the Proportion of People Aged 65 and over in Taiwan



Thus, Taiwan can now be treated as a developed country with respect to the value of life expectancy. For example, the life expectancies of Taiwan males and females in 2006 were approximately 75 and 81 years, which are almost identical to those in the US.

The prolonging life expectancy and rapidly reducing fertility rates are two key phenomena of the *demographic transformation*, which has occurred in many developed countries. Taiwan has also been experiencing a rapid reduction in fertility rates, coupled with decreases in mortality rates. The total fertility rate (TFR) – a rough approximation of the number of children born by a female in Taiwan – was 7.0 in the 1950s and had since decreased to less than 1.2 in 2004. The reductions in mortality and fertility rates have together contributed to the problem of the aging population. Figure 3 shows the TFR and the proportion of people ages 65 and over (i.e., the elderly). The proportion of the elderly seems to be strongly correlated to the TFR and passed 7 percent -- the threshold for “aging population” as defined by the United Nations – in 1994. The proportion was over 10 percent in 2006. This rapid trend of the aging population in Taiwan is expected to continue and the proportion of the elderly will probably surpass the 20 percent mark within 20 years.

The elderly population is expected to consume more resources than the younger population, and have a higher risk of becoming sick or disabled. The National Insurance Bureau of Taiwan reports that the average annual medical expenditure for the elderly was about 4.4 times that of people between the ages 0-64 in 2000. For example, the proportion of disabled persons was 9.12 percent of the elderly population or about 16 times higher than that of those aged 0-64 (0.57 percent), according to the population census in 2000. Therefore, the prolonged life expectancy would create challenges to personal financial planning. In particular, we are interested in the issue of whether the retirement assets would eventually become insufficient, i.e., the longevity risk.

Impacts of Longevity Risk and Solutions. Increased life expectancy has a great impact on the operation of annuity providers and pension funds. Traditionally, actuaries have used a fixed and deterministic mortality assumption to price and reserve for life insurance policies. If the future life expectancy is longer than the expected life expectancy, it indicates that there is a risk of underestimating insurance premiums for life annuity policies and the insurer may go into insolvency. Many studies note that mortality risk may cause substantial losses if handled improperly. Refer to Marceau and Gaillardetz (1999), Milevsky and Promislov (2001) Olivieri and Pitacco (2003), Wilkie et al. (2003), Blake (2007) and Bauer and Weber (2007) for specific examples.

The solutions for dealing with longevity risk are widely discussed in recent years. Securitizations of longevity risk, such as survivor bonds and survivor swaps, have been proposed and studied by Blake and Burrows (2001), Dowd (2003), Blake et al. (2006a and 2006b), Cairns(2006a), Lin and Cox (2005), Cox and Lin (2006), Dowd et al. (2006) and Denuit et al. (2007).

Similar to the concept of survivor swap, natural hedging is another strategy by which an insurer can hedge longevity risks internally between its own business products (life insurance and annuity). Cox and Lin (2007) suggested that while natural hedging can be applied, it may be too expensive to be effective in the context of internal life insurance and annuity products. Wang et al. (2007) used the concept of immunization to propose the natural hedging strategy for hedging the longevity risk for the life insurer.

The analysis of the longevity issues and finding the solutions in the insurance and pension field may require the use of a stochastic mortality model. The studies mentioned above make use of a dynamic mortality model to deal with mortality risk. Therefore, the use of stochastic mortality models to manage mortality risk has become an important tool for actuarial professionals. Many stochastic models have been proposed and developed, and we summarize them in the next section.

Development on Mortality Studies. Mortality risk has been a point of interest in the insurance industry and therefore, this is not the first time that mortality risk is being analyzed in an extended manner. Among all stochastic mortality models, the Lee-Carter (LC) model, proposed by Lee and Carter in 1992, is one of the most popular choices. The LC model is easy to implement and outperforms other models with respect to prediction errors, as evidenced by several empirical studies such as Lee and Nault (1993), Lee and Rofman (1994), Koissi et al. (2006), Melnikov and Romaniuk (2006). Various modifications of the LC model have also been proposed to attain a broader interpretation (Brouhns et al., 2002; and Renshaw and Haberman, 2003). In the actuarial profession, the Continuous Mortality Investigation Bureau (CMIB, 2006) in Britain has even suggested the use of the LC model – in lieu of the reduction factor (RF) model it previously proposed – as a means to compute stochastic mortality.

Since Renshaw and Haberman (2006) proposed adding a cohort effect to the LC model, there has been a growing trend of modifying the LC model by adding cohort effect in mortality models, such as Currie (2006) and Cairns et al. (2007). Cairns et al. (2006b) introduced a mortality model that differs from previous models by assuming a functional relationship between mortality rates across ages. Cairns et al. (2007) further carried out a comparison analysis for eight modified LC models based on England & Wales and the US mortality experiences. Continuous-time framework is another way of modeling stochastic mortality (e.g., Milevsky and Promislow, 2001; Dahl, 2004; Dahl and Moller, 2006; Miltersen and Persson, 2005; Luciano and Vigna, 2005; Biffis, 2005; and Schrager, 2006).

In this study, we examine mortality patterns in Asia (particularly, Taiwan and Japan) and compare the analytical results with those of selected Western countries (i.e., the US and

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England & Wales). We employ the Lee-Carter (LC) model, the reduction factor (RF) model and the age-period-cohort (APC) model for the analysis. In the remaining part of the paper, we introduce the three mortality models, discuss their pros and cons, apply them to data from the Human Mortality Database (HMD) of Taiwan, and detail the empirical results generated. We also discuss the limitations of our research and provide suggestions for further studies.

II. Mortality Models

Lee-Carter (LC) Model. Introduced by Lee and Carter (1992), this model addresses the age-specific mortality on a central death rate. That is, we can denote $m_{x,t}$ the central death rate for a person at age x at time t such that:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \qquad (1)$$

where parameter α_x describes the average age-specific mortality and κ_t represents the general mortality level. In this equation, the decline in mortality at age x is captured by β_x and the mortality level κ_t is usually a linear function in time. The term $\varepsilon_{x,t}$ denotes the deviation of the model from the observed log-central death rate; it should be white noise with a zero mean and relatively small variance (Lee, 2000).

The parameter estimates can be derived from matrix operations, such as the singular value decomposition (SVD). Equivalently, by applying the constraints $\Sigma t\ \kappa_t = 0$ and $\Sigma x\ \beta x = 1$, we can find the estimate of parameter α_x being the average log-central death rate over time t or $\hat{\alpha}_x = \sum_{t=t_1}^{t_1+T-1} \ln(m_{x,t})/T$ where t_1 is the starting year and T is the number of years in the data. The parameters α_x and β_x are functions of age x and do not change with time, and the parameter κ_t is a linear function of time. By modeling the values of κ_t as a time series, we can project the future mortality. Also, if there are missing values, an approximation method and modifications (Wilmoth, 1993) can be used for parameter estimation.

The LC model contains relatively few parameters and provides fairly good estimates and predictions of the observed mortality rates in many countries, such as the US, Japan and Chile. As a result, the LC model has gained significant attention and popularity. However, the parameter assumption in the LC model does not always make sense. For example, the parameters (βx's) are not always constant in many empirical studies. The infant and children mortality improved the most in past and now senior mortality is improving more rapidly. (This is exactly the case in many developed countries like the Great Britain, Japan and the US.) In other words, the assumption that the reduction rates β_x*'s* are constant is questionable empirically. Forecasts of future mortality rates based on the constancy assumption in the LC model need to be modified.

Because κ_t is usually a linear function of time t in Equation (1), the mortality rates for all ages eventually go to 0. In other words, the limiting mortality rates for all ages are assumed to be 0 in the LC model. Together with the assumption that β_x remains constant, the LC model restricts the pattern and magnitude of mortality improvement. Empirical results from previous studies show that when the LC model is used, the estimation and prediction errors for different age groups might differ significantly (e.g., Yue, 2002).

Several modifications have been proposed to cope with the limitations of the LC model. For example, the reduction shift of ages for different time periods can be treated as a "cohort" effect and introducing a cohort effect into the LC model has become a popular approach. The original LC model nearly combines the age effect and the interaction of age and time, so possible modifications create additional terms related to the cohort effect. For example, Hyndman and Ullah (2007) proposed adding more interaction terms for age and time, such that:

$$\ln(m_{x,t}) = \alpha_x + \sum_{j=1}^{J} \beta_x(j)\ \kappa_t(j) + \varepsilon_{x,t}, \tag{2}$$

where $\beta_x(j)\ \kappa_t(j)$ is the jth interaction term between age and time, j = 1, 2, …, J. They also proposed using a principal component (PC) decomposition to solve for the paired parameters ($\beta_t(j),\ \kappa_t(j)$). The idea behind this approach is similar to that proposed by Bell (1997), according to which the LC model displays similar behavior for both one PC and two PCs.

Reduction Factor Model. The Reduction Factor (RF) model was widely used in the actuarial practice for projecting mortality in the UK. In the model, the projected probabilities of death for persons aged x at time t ($q_{x,t}$) are based on the current mortality rates ($q_{x,0}$) and the reduction factor. The probabilities can be expressed as:

$$q_{x,t} = q_{x,0} \cdot RF(x,t), \tag{3}$$

where $RF(x,t)$ denotes the reduction factor of age x for the mortality rate projected to future year t.

The reduction factor adopted by CMIB (1999 and 2006) for pensioners takes the following form:

$$RF(x,t) = \alpha(x) + [1-\alpha(x)][1-f(x)]^{t/20}, \tag{4}$$

where:

$$\alpha(x) = \begin{cases} c & x < 60 \\ \dfrac{110-x}{110-60}c + \dfrac{x-60}{110-60} & 60 \le x \le 110 \\ 1 & x > 110 \end{cases}$$

$$f(x) = \begin{cases} h & x < 60 \\ \dfrac{110-x}{110-60}h + \dfrac{x-60}{110-60}k & 60 \le x \le 110 \\ k & x > 110 \end{cases}$$

with c = 0.13, h = 0.55 and k = 0.29, respectively.

The calculation of the reduction factors is based on a 20-year time span and the mortality experience uses the UK pension annuity data from 1991 to 1994. Note that, unlike in the LC model, the limiting mortality rates for age x in the RF model is $\alpha(x)$ which does not necessarily have to be 0.

The Society of Actuaries (SOA) proposed a similar method in projecting mortality rates. The projected probabilities of death are determined using the improvement factor on the 1994 mortality rates, which is defined as:

$$q_{x,t} = q_{x,1994} \cdot (1 - AA_x)^{t-1994} \quad . \tag{5}$$

This model is simple and easy to implement once the reduction rate is decided. However, unlike the limiting value of $q_{x,t}$ is $\alpha(x)$ in the RF model, the limiting mortality rates will be 0 for all ages if $AA_x > 0$.

Although the RF model relaxes the restriction for the limiting values, the LC model still achieves a better fit. Yue, et al., (2008) found that the forms of $\alpha(x)$ and $f(x)$ play an important role in giving a better model fit, and the limiting age is relatively unimportant. The original forms suggested by the CMIB can be further modified. We used cubic spline (as a function of age) to replace the original forms of $\alpha(x)$ and $f(x)$ suggested by the CMIB, and we found that the fitting errors were reduced by about 50 percent. Therefore, for the rest of this study, we use the modification of the original RF model to compare with the LC model.

Age-Period-Cohort Model. The mortality projected by Age-Period-Cohort (APC) model considers a cohort effect in addition to an age and period effect. Suppose that the number of deaths of people aged x at year t ($O_{x,t}$) follows a Poisson distribution, given the number of people aged x at year t ($N_{x,t}$), the log mortality rate for a person age x at year t is expressed as:

$$Y_{x,t} = \log(\frac{O_{x,t}}{N_{x,t}}) = \mu + \alpha_x + \beta_t + \gamma_k + \varepsilon_{x,t} \quad , \tag{6}$$

where α_x is the age effect, β_t is the period effect and γ_k is the cohort effect with $k = t - x$. Empirically, $\sum_x \alpha_x = \sum_t \beta_t = \sum_k \gamma_k = 0$ is usually assumed to avoid the confounding problem in the parameter estimation.

Because the birth time k is equal to the current time t minus the age x (i.e., $k = t - x$), the parameters in the APC model are linearly dependent. Therefore, the parameter estimates in the APC model are not unique; that is, there still exists the problem of identification. Several methods were proposed to deal with this problem. For instance, Carstensen and Keiding (2005) suggested a sequential approach, which ranks the importance of parameters such as age, period, and cohort and then estimates the parameters according to this order. Fu (2008) proved that it is possible to derive unique parameter estimates and proposed an approach called the Intrinsic Estimator (IE). One of the computing advantages of using the IE approach is that the parameter estimates and their variances can be derived through principal component regression.

It should be noted that the APC model was first proposed in the studies of epidemiology and designed as a tool of descriptive statistics. In other words, there are no structural forms suggested for parameters in the APC model yet. It will be difficult to project future mortality rates using the APC model if the estimated parameters do not show obvious patterns (such as linear in time). Also, there are no complete empirical comparisons of the APC model to other mortality models.

III. Empirical Analysis of the Three Models

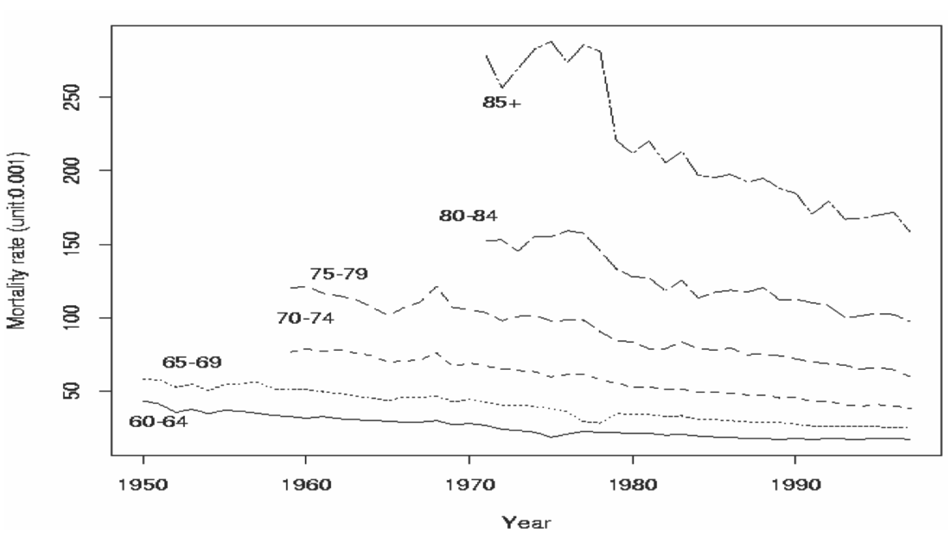
In this section, we first evaluate the LC, modified RF and APC models using the Taiwan mortality data. We then make a comparison of mortality experience from four countries (Taiwan, Japan, the US and England & Wales). The mortality data of Japan, the US and England & Wales are obtained from the HMD Database while the data for Taiwan is obtained from the Ministry of Interior of Taiwan. We use the 5-year age group data or a total of 20 age groups of 0-4, 5-9, … and 95-99.

For the evaluation of the performances of the three models, we consider the mean absolute percentage error (MAPE). The MAPE measument is defined as follows:[1]

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|Y_i - \hat{Y}_i\right|}{Y_i}\times 100\% \text{ ,} \tag{7}$$

where Y_i and $\hat{Y}_i$ are the observed and estimated values, and *n* is the number of observations. We examine the estimates of the parameters for all three mortality models in the next section. Because the estimates have similar patterns, we use the results from the Taiwan data as a demonstration.

Figure 4: Highest Age Group Recorded in Taiwan (1950 -2000)



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[1] A frequently used criterion of the MAPE values is as following:

MAPE	<10%	10%~20%	20%~50%	>50%
Efficiency	Excellent	Good	Reasonable	Bad

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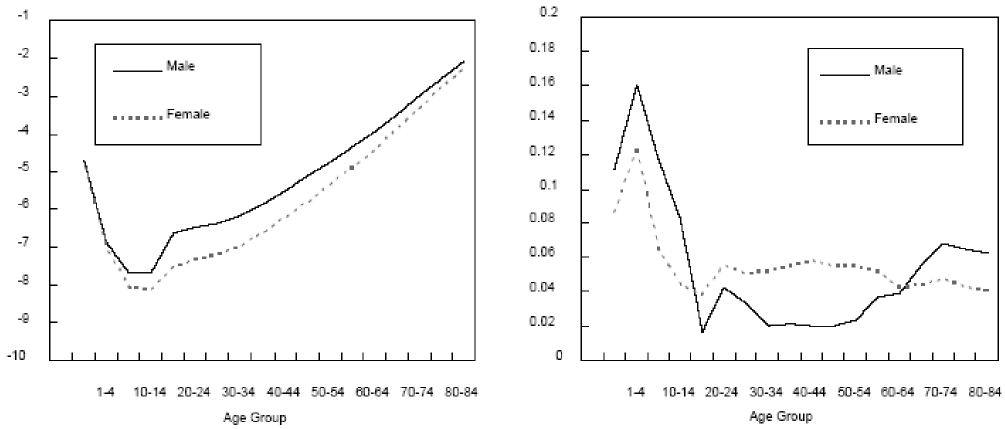
Figure 5: α_x (Left) and β_x (Right) Estimates of the LC Model (Taiwan, 1971-2000)



Parameter Estimates of Three Models Based on Taiwan Data. We will discuss the estimates of parameters for all three models first, following by the comparison of fitting errors in the next subsection. The highest recorded age groups in Taiwan vary with time, and the numbers of observations available for different age groups are not equal (Figure 4). Given that the original SVD can not be used for parameter estimation, we use the approximation method. The estimates of parameters α_x and β_x are given in Figure 5. The parameter α_x reflects the average mortality values and the estimates for the male population are larger than those for the female population. Because the parameter k_t is usually a linear function of time, the parameter β_x can be treated as the reduction rates in mortality improvement and the younger and older age groups have the largest improvements.

Similar to many previous studies, the estimates of parameter κ_t in Taiwan look like linear functions of time for both male and female (see Figure 6). Nonetheless, the slope of the linear function is steeper for the females, indicating that the female has larger mortality improvement given the same value of β_x .

The estimated reduction factors in the RF model are shown in Figure 7. It is obvious that the original linear function assumption of $\alpha(x)$ does not fit well in the Taiwan case, and this is the reason why we introduce a cubic spline fit for $\alpha(x)$. In the following discussion, we use the modified RF model, instead of the original RF model, for model comparisons.

Note that, unlike in the LC and RF models, it requires more observations to check the cohort effect in the APC model. Thus, the data used in the Taiwan data is between years 1961-2005. Figure 8 shows the estimates of parameters in the APC model, using the IE approach. The result of age effect is similar to α_x in the LC model. The result of period effect does look like a linear function of time after 1960 but is close to constant between 1900 and 1960. The result of cohort effect behaves like κ_t in the LC model and the slope for the females is steeper.

Figure 6: κ_t Estimates of the LC Model (Taiwan, 1971-2000)

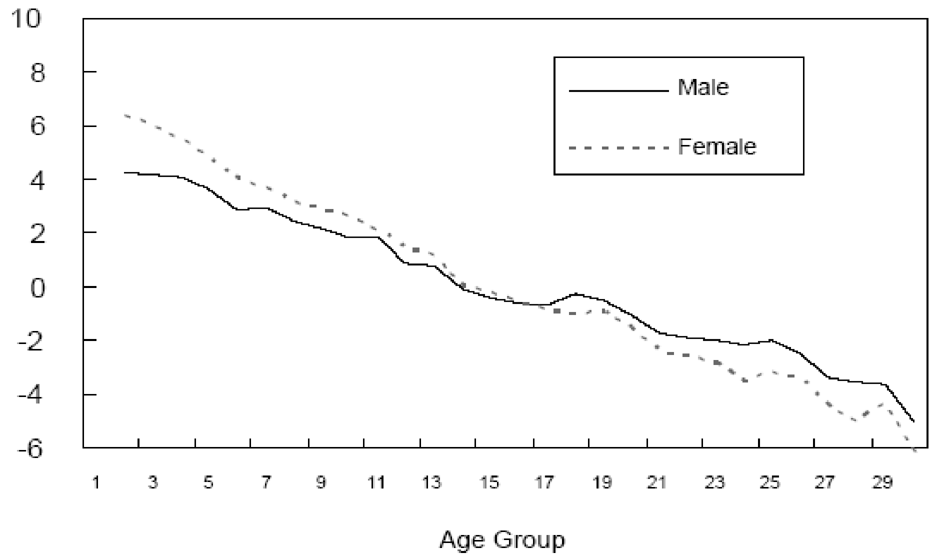
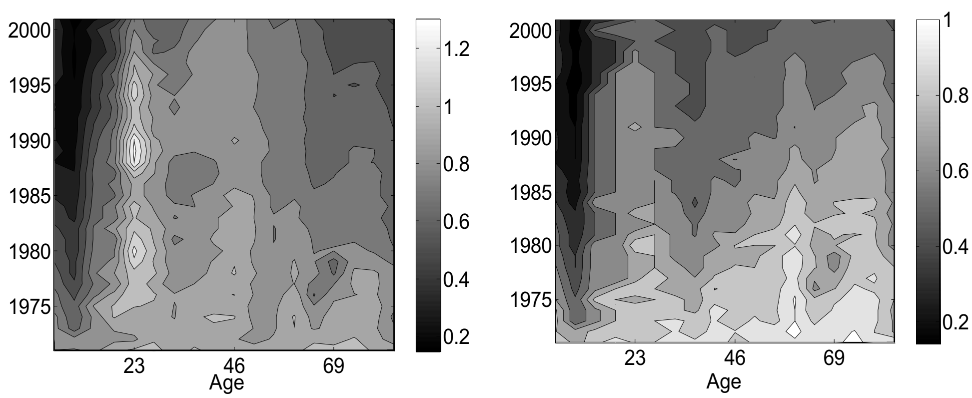


Figure 7: Reduction Factor of Male (Left) and Female (Right) (Taiwan, 1972-2000)



The parameter estimates of these three models for other countries are similar to those in the Taiwan case. In all three models, the age effect is the most obvious and the cohort effect can be fitted as a linear function of time, but it is not easy to find a functional form for the period effect. Also, judging from the estimates of period and cohort effects in the APC model (Figure 8), there might exist some "interaction" effect, which means that the mortality improvement can be different. In other words, the constant assumption in the LC model is likely to be questionable. We return to this issue in the final section.

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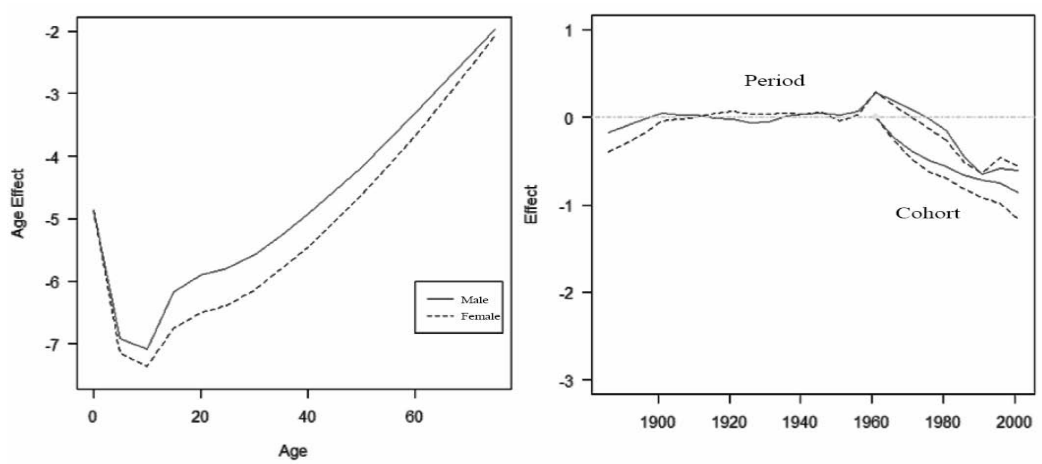
Figure 8: Age, Period and Cohort Effects in APC Model (Taiwan, 1961-2005)



Empirical Comparisons of Model Fit from Different Countries. In this section, we use empirical data to compare the model fit of three mortality models. Since the CMIB has studied the UK data for more than 20 years, we only look at the data from Taiwan, Japan and the US. Table 1 lists the MAPE of all three models. The RF model is the modified RF model. There are two APC models in the table: one is the sequential approach and the other is the IE approach. The LC model is the best among three models and it also contains the fewest number of parameters. Although the MAPEs of the APC model is closer to those of the LC model, the number of parameters used in the APC model is approximately 50 percent greater than that in the LC model. Also, as mentioned in the previous section, there are no functional forms for the parameters in the APC model and it is not easy to implement prediction. Thus, we only consider the LC and RF models in the following comparison of predictions.

Table 1: MAPE of Model Fitting (Three Countries, 1971-2003)

	Taiwan		Japan		US	
	Male	Female	Male	Female	Male	Female
LC	5.49	5.69	4.38	4.02	4.01	2.98
RF (Modified)	12.13	9.93	12.56	13.28	11.55	5.95
APC (1)	7.68	9.44	6.52	7.74	4.10	3.33
APC (2)	8.05	10.09	7.10	8.30	4.20	3.70

Table 2: MAPE of Model Prediction (Taiwan, 1971-2000 vs. 2001-2004)

Estimation	Male	Female
RF(CMI)	27.23	24.80
RF (1)	20.00	13.17
RF (2)	16.39	10.21
LC	12.73	9.42

We use the Taiwan data to evaluate the prediction accuracy. The data from the time period 1971-2000 are used as “training” data” (in obtaining the parameter estimates) while the data from years 2001-2004 are treated as “testing” data” (or pretending that they are the future data to be observed). Table 2 shows the prediction errors of original RF model, two modified RF models and the LC model. The two modified RF models are the modifications of $\alpha(x)$ and $f(x)$, using the quadratic and cubic spline functions, respectively. Similar to the results in Table 1, the LC model again out-performs the RF models with respect to the prediction errors. Our empirical results can be used to explain why the CMIB officially decided to adapt the LC model for mortality projection and stop using the RF model it originally proposed.

IV. Conclusions and Discussions

Increased longevity has become a common phenomenon in the world. Most of the premium calculations today are “period” based, i.e., mortality rates are fixed and based on empirical experiences of past claims. Hence, should the mortality rates decline faster than expected, the premiums will be under-estimated and the insurance companies will be in danger of insolvency. Using the stochastic mortality models to calculate premiums is one of the possible solutions, and the UK and Germany have adopted this idea more than 10 years ago. The problem can be solved if the stochastic mortality models can capture the pattern of mortality improvement.

In this study, we compared three frequently used stochastic mortality models and investigated whether these models have good empirical fits using data from Taiwan, Japan, the UK and the US. The results of estimation are consistent with those of the prediction. The LC model outperforms both the RF (original and modified) and APC models and still possesses the advantages of having fewer parameters and ease of computation. This can explain the popularity of the LC model in many countries.

Nonetheless, there is still room for further modification to the LC model. Adopting the frame work of APC model, age, period and cohort are three main effects. The LC model is a model with main effect of age and an interaction term of age plus period. Figure 8 and other empirical studies (e.g. Booth et al., 2002) suggest that there may exist other main effect (such as period) and interaction terms (such as age plus cohort). For example, Cairns et al. (2007) considered the original LC model and seven modified models, adding other main effect or interaction terms. Using the data from England & Wales and the US, Cairns et al. (2007) found that two of the modified LC models have better model fits.

Following the approach of Bell (1997), we also conducted principal component analysis (PCA) for the mortality rates. We found that we can extract at least two principal components from most countries, and these principal components behave like linear functions of time, similar to κ_t.

Table 3: MAPE of Model Fit for the LC model (Four Countries, 1971-2004)

	Method	Taiwan	Japan	England and Welch	US
Male	Lee-Carter	5.63%	7.04%	5.73%	4.47%
	PCA	0.85%	1.71%	1.31%	1.45%
Female	Lee-Carter	5.11%	9.35%	5.54%	3.50%
	PCA	1.02%	1.70%	1.12%	1.07%

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In other words, the idea of PCA modification is similar to adding more interaction terms of age plus period. Table 3 are the MAPE of the model fit for the LC model and the PCA model, based on the data from Taiwan, Japan, the UK and the US. The PCA model has better fit in all four countries, for both the male and female. This result is similar to those in Cairns et al. (2007) and indicates that adding extra main effect or interaction terms is a possible way of modifying the LC model. The results in Table 3 are findings from the "training" data, and we plan to work for "testing" data and apply this model to data from other countries.

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