Does External Habit Formation Help Increase Real Exchange Rate Persistence? An Analytical Characterization

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Abstract

The objective of this paper is to investigate the effect of external habit formation on real exchange rate persistence in a two-country dynamic stochastic general equilibrium (DSGE) model. The purchasing power parity (PPP) puzzle states that high persistence and volatility of the real exchange rate found in empirical studies can not be captured in a single model with conventional specifications. Part of the puzzle is resolved through a DSGE model with nominal rigidity: the model generates enough real exchange rate volatility but not enough persistence. Habit formation was suggested as an additional factor contributing to real exchange rate persistence. In this paper, we focus on the consumption externalities caused by external habit formation to understand how habit stocks affect real exchange rates. The analytical solutions show that external habit formation decreases the persistence of real exchange rates. The main reason is that the externality caused by external habit formation has an opposite effect on the real exchange rate to that of real shocks. Alternative specifications of the habit-related parameters do not reverse the negative effect of external habit formation on real exchange rate persistence.

Keywords: Habit Formation, Real Exchange Rate, Purchasing Power Parity Puzzle

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1. Introduction

One of the primary puzzles in international macroeconomics is the purchasing power parity puzzle. The extensive proposition of purchasing power parity addresses that the relative inflation rates between the two countries shall be reflected in nominal exchange rate movements. Based on this assertion, the real exchange rate shall be constant. However, empirical studies show that the real exchange rate exhibits high persistence and high volatility. The difficulty in reconciling these two properties of the real exchange rate in a theoretical model is called the purchasing power parity puzzle. Earlier empirical findings have been surveyed by Rogoff (1996), and more recent efforts devoted to this issue with alternative approaches include Chen and Rogoff (2003) and Murray and Papell (2005).

Despite the many attempts in the literature to explain this puzzle, such as Bergin and Feenstra (2002), it remains unresolved. The most well-known paper explaining real exchange rate persistence and volatility is Chari, Kehoe and McGrattan (2002). They use a dynamic stochastic general equilibrium (DSGE) model and successfully generate both the volatility and persistence of the real exchange rate. The authors' calibration results show that the real exchange rate in the DSGE model is approximately four to five times more volatile than GDP, as observed from the OECD data. However, the persistence of the real exchange rate ranges from 0.63 to 0.68 per quarter, smaller than the 0.83 found empirically. Long persistence, close to the empirical finding, can be obtained only when a nominal rigidity of 2 to 3 years is assumed. Requiring such a high degree of nominal rigidity is unreasonable; that the model fails to generate enough persistence in the real exchange rate remains a "persistence" puzzle.

Habit formation was suggested as one of the possible solutions to the persistence puzzle. Because, in a DSGE model the real exchange rate is determined by the consumption ratio across countries when there are complete asset markets of tradable securities, factors that can increase persistence in consumption may help reinforce the real exchange rate persistence. Of these factors, habit formation has been considered as the primary one.

The importance of time non-separable utility functions has been recognized in various empirical studies. In the literature, there are two types of habit formation specifications: internal and external. The specification of internal habit formation is the "habit persistence" commonly indicated in the literature. The notion asserts that consumers take the accumulation of their own past consumptions as the reference stock and do not want their current consumption to deviate too much from past consumption habits. The persistence of consumption and most of the real variables are increased accordingly. In addition to its wide applications to growth and macroeconomics related literature, habit formation has important implications in the evaluation of monetary policies, as noted in Fuhrer (2000). While monetary policy evaluation is based on welfare, namely, expected utility, the specification of utility function matters because it determines how the variables in the economy react to monetary shocks. Fuhrer (2000) finds that a model with habit persistence can generate more hump-shaped responses of output to monetary shocks, closer to the ones observed in the data. A subsequent study, Amato and Laubach (2004), also supports this finding.

On the other hand, the specification of external habit formation states that consumers want to "catch up" with others and thus take the accumulation of past average consumption in the economy as the reference stock. This is known as "catching up with the Joneses" in the literature. As a result, the higher consumption of each consumer imposes an external effect on others. The importance of consumption externality has been recognized in economics, as noted in Liu and Turnovsky (2005). The consumption externality is most commonly used in the asset pricing literature, such as Campbell and Cochrane (1999) and Abel (1999). Consumers with habit formation preferences do not like drastic changes in consumption, thus they require a higher risk premium to hold risky assets that may result in a sharp decline in future consumption.

The objective of this paper is to investigate the effect of habit formation on real exchange rate persistence in a DSGE model with nominal rigidity. The introduction of habit formation to DSGE models is not new. While DSGE models have been widely used in international macroeconomics, many recent studies adopt habit formation in the utility function as a more general specification of the utility function, in an attempt to increase the persistence of real variables. However, Justiniano and Preston (2004), among others, find that habit formation does not increase real exchange rate persistence, even though habit formation does improve the fitting of the model to the data. Although this finding is pervasive, no one has yet provided an explanation.

In this paper, we adopt an analytically tractable model to characterize the role of habit formation in international macroeconomics, which is implicit in a generalized DSGE model where the derivation of analytical solutions is not feasible. Our model is similar to a standard DSGE model in general, but differs in some important respects. Because we do not intend to solve the purchasing power parity puzzle, but instead focus on habit formation’s effect on the real exchange rate, we make several assumptions to facilitate the derivation of analytical solutions. Based on the analytical solutions, we can shed light on the role of externalities caused by the habit stocks in the economy, how the external effects are passed across countries and how they affect the real exchange rate and its persistence.

To do so, first, we take external habit formation, rather than internal habit persistence. Second, instead of staggered pricing, the nominal rigidity is on wages, which are preset one period ahead. In this model, the deviation from the purchasing power parity is not from the nominal rigidity, but from the nontradable goods. With this specification, we can simplify both analytical and algebraic complications. However, with the complete asset markets, the Cobb-Douglas utility function and the sticky wage, the real exchange rate is forced to be one if the initial debt is zero. To assure the movements of the real exchange rate, we need to impose the condition that the initial security is nonzero. Third, the logarithmic form of the money demand utility function leads to constant nominal interest rates and isolates the nominal exchange rate movements from habit formation changes. This simplified case provides us with the intuition concerning habit formation's effect on the real exchange rate through relative wage. Last, we assume that labor is the only production input. Technology shocks are autocorrelated, but the nominal shocks are not. Therefore, real shocks are the primary source of persistence, in addition to habit formation, in this model. The analysis is centered on the tradeoff between the effect of habit formation and productivity shocks on relative wage.

The results show that external habit formation decreases real exchange rate persistence, while increasing the persistence of real variables. The negative effect is mainly caused by the adverse effect of habit formation on the level of real exchange rate to that of past productivity shocks. Although the results may strongly depend on the assumptions we make, the intuition can be carried without losing generality. Without staggered pricing, external habit formation does not influence real exchange rate volatility.

In addition to the discussion based on analytical solutions, we also conduct calibrations to examine, numerically, the effects of habit formation on the AR(1) coefficient of the real exchange rate. The results show that the real exchange rate persistence declines with the increase in the importance of habit formation and the adjustment speed of the habit stock accumulation. In spite of different treatments of the habit-related parameters, habit formation's negative effect on the real exchange rate cannot be reversed.

The structure of this paper is as follows. In Section 2 and Section 3, respectively, we present the specification of the basic model and the analytical solutions derived to demonstrate the effects of the productivity shocks and external habit formation on wages. In Section 3, we provide the intuition behind the results. Section 4 then presents a sensitivity analysis based on calibration, while Section 5 conducts alternative specifications of the parameters. Finally, in Section 6, we present our conclusions and discuss areas for future research.

2. The model

2.1 Production

There are two countries, home and foreign. Each country produces both tradable and nontradable goods. Y_H and Y_N denote traded and nontraded goods j produced in home country, while Y_F and Y_N^* denote traded and nontraded goods in foreign country. There is a continuum of varieties for each type of goods. Home goods are indexed from $j \in (0,1)$ while foreign goods are indexed from $j \in (1,2)$. Labor is the only input in the production process. Traded and nontraded goods j are produced according to the production functions:

$$Y_{H,t}(j) = A_t \left[\int_0^1 L_{H,t}(j,i)^{(\phi-1)/\phi} di \right]^{\phi/(\phi-1)} \tag{1a}$$

$$Y_{N,t}(j) = A_t \left[\int_0^1 L_{N,t}(j,i)^{(\phi-1)/\phi} di \right]^{\phi/(\phi-1)} \tag{1b}$$

$L(j,i)$ is the demand for labor type i in the production of good j. ϕ is the elasticity of substitution among labors in the production. A_t represents the autocorrelated country specific productivity shock.

As described, the labor market is a monopolistic competition market. Wage stickiness is the primary source of nominal rigidity and each worker has a certain market power to preset the wage. The composite wage, W_t, is derived from the cost minimization of producing one unit of product:

$$W_t = \left[\int_0^1 W_t(i)^{1-\phi} di \right]^{1/(1-\phi)} \tag{2}$$

Thus, the labor demand for labor i in the production of good j can be obtained:

$$L_t(j,i) = \left[\frac{W_t(i)}{W_t} \right]^{-\phi} \frac{Y_t(j)}{A_t} \tag{3}$$

Due to wage stickiness, labor demand determines the labor employment in the labor market. Productivity shocks have direct effects on the labor employment, because the same amount of goods can be produced by fewer workers when a positive productivity shock occurs. Thus, the marginal cost for the home firm is W_t / A_t, from the production function in Eq. (1). The logarithm of home productivity shock, indicated by lower-case a_t, evolves according to the following AR(1) process:

$$a_t = (1-\rho_a) a + \rho_a a_{t-1} + \varepsilon_{a,t}, \;\; \varepsilon_{a,t} \sim N(0,\sigma_a^2) \tag{4a}$$

Foreign production functions and wages are in the parallel way and indicated by asterisks. Foreign productivity shock behaves analogously:

$$a_t^* = (1-\rho_a^*) a^* + \rho_a^* a_{t-1}^* + \varepsilon_{a,t}^*, \;\; \varepsilon_{a,t}^* \sim N(0,\sigma_a^{*2}) \tag{4b}$$

And the foreign marginal cost is W_t^* / A_t^*.

2.2 Consumption

Consumers in each country consume both home and foreign produced tradable goods and domestic nontradable goods. For each individual i in the home country, the composite consumption index is constructed in the Cobb-Douglas form:[1]

$$C = \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \tag{5a}$$

Where C_T and C_N are the indices of tradable and nontradable consumptions. The tradable consumption index is composed of equal shares of home and foreign tradable goods, C_H and C_F:

$$C_T = 2C_H^{1/2} C_F^{1/2} \tag{5b}$$

The composite consumption index for each type of goods is in the CES form:

$$C_H = \left[\int_0^1 C_T(j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}, \quad C_F = \left[\int_1^2 C_T(j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)}, \quad C_N = \left[\int_0^1 C_N(j)^{(\theta-1)/\theta} dj\right]^{\theta/(\theta-1)} \tag{5c}$$

θ is the price elasticity among goods. The aggregate price index for the composite consumption is

$$P = P_T^{\gamma} P_N^{1-\gamma}, \text{ where } P_T = P_H^{1/2} P_F^{1/2} \tag{6a}$$

where

[1] With this specification of the composite consumption index, the elasticity of substitution between each type of good is equal to one. As noted in Obstfeld and Rogoff (1998), the current account will be equal to zero when the initial debt is assumed zero, even though there are asset markets to share risks across countries. This point will be discussed in more detail when we talk about the market clearing conditions.

$$P_H = \left[\int_{i=0}^{1} P_T(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},\ P_F = \left[\int_{i=1}^{2} P_T(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},\ P_N = \left[\int_{i=0}^{1} P_N(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{6b}$$

According to the composite consumption and price indices, the home demand for each commodity is:

$$C_T(h) = \left[\frac{P_T(h)}{P_H} \right]^{-\theta} C_H,\ C_T(f) = \left[\frac{P_T(f)}{P_F} \right]^{-\theta} C_F,\ C_N(h) = \left[\frac{P_N(h)}{P_N} \right]^{-\theta} C_N \tag{7a}$$

Home demand functions for home and foreign tradable goods are

$$C_H = \frac{1}{2} \left[\frac{P_H}{P_T} \right]^{-1} C_T,\ C_F = \frac{1}{2} \left[\frac{P_F}{P_T} \right]^{-1} C_T \tag{7b}$$

and the tradable and nontradable good demands are:

$$C_T = \gamma \left[\frac{P_T}{P} \right]^{-1} C,\ C_N = (1-\gamma) \left[\frac{P_N}{P} \right]^{-1} C \tag{7c}$$

With this composite consumption index, total spending of tradable goods takes the share of γ in the total expenditure, PC. The consumption and price indices in the foreign country are in the analogous form with asterisks.

2.3 Optimization

Consumers face the intertemporal maximization problem as follows:

$$\max\ E_t \sum_{t=0}^{\infty} \sum_{s_{t+1}} \beta^t \left(\frac{1}{1-\rho} \left(\frac{C^i(s_t)}{\left(H(s_t)\right)^{\alpha}} \right)^{1-\rho} + \chi \ln \left(\frac{M^i(s_t)}{P_t(s_t)} \right) - \frac{\kappa}{\upsilon} \left(L^i(s_t) \right)^{\upsilon} \right) \tag{8a}$$

$$\text{s.t.} \quad \begin{aligned} & P(s_t)C^i(s_t)+M^i(s_t)+\sum_{s_{t+1}}B(s_{t+1}|s_t)D^i(s_{t+1}) \\ & \leq P(s_t)W(i,s_t)L^i(s_t)+M^i(s_{t-1})+D^i(s_t)+\int_0^1[\prod_H(j,s_t)+\prod_N(j,s_t)]dj+P(s_t)T(s_t) \end{aligned} \tag{8b}$$

An individual i obtains the utility from the consumption of composite good, C_t^i, real money holding and the disutility from labor supply. β measures the subjective discount factor, and $\rho > 0$ denotes the curvature of the utility. We assume that money demand enters the utility function in a logarithmic form for analytical simplicity. This is a special case of the general CRRA utility function form, under which closed-form solutions of money demand and the nominal exchange rate arise.[2] This thus permits us to abstract the discussion of the real exchange rate from the nominal exchange rate but to focus on relative price.[3] υ indicates the inverse of labor supply elasticity. That $\upsilon \geq 1$ assures a positive elasticity of labor supply. χ and κ are positive constants determining the utility of real money holdings and the disutility caused by labor supplied.

The primary variation of this model is habit formation on consumption, H_t, in the utility function, which turns the utility function into a time non-separable form. Here, we adopt external habit formation, which is known as “catching up with the Joneses” in literature. With external habit formation, consumers take the accumulated past average consumption of the entire economy as the reference habit stock. The main property of external habit formation is the consumption externality of each individual’s consumption on the consumptions of others. An increase in the consumption of one consumer in the previous period raises the external habit stock in the economy and increases the marginal utility of consumption for all other consumers in this period. Thus, all consumers have the incentive to increase current

[2] See Obstfeld and Rogoff (1998), Devereux and Engel (1998) and Appendix 1.A.
[3] This is the special case in Devereux and Engel (1998) as noted below.

consumption to "catch up" with the aggregate consumption level when $\rho > 1$ and $\alpha > 0$. However, from the individual perspective, the impact of one's consumption on the aggregate consumption is negligible, thus individual consumers do not take into account this external effect in the optimization. As shown below, the external effects are spilled over to another country through international trades in the two-country model.

An alternative characterization of consumption externality, as in Gali (1994), is called "keeping up with the Joneses". Current average consumption, instead of the past, is taken as the reference stock; that is, $H_t = \overline{C}_t$ and consumption behaviors may vary with the sign of α. When α is positive, consumers are jealous and want to "keep up" with the Joneses, and thus greater aggregate consumption induces higher consumption. On the other hand, when α is negative, the higher aggregate consumption decreases both the marginal utility and consumption, in which case consumers tend to "run away" from the Joneses. This occurs because people admire the consumption of others. The following analysis is primarily based on the "catching with the Joneses" case with positive α, as most studies are. The extensive analysis of negative α is conducted by calibration.

The consumption utility is characterized as the ratio of current consumption to the habit stock, with α as the importance of habit stock on the utility of consumption, which lies between 0 and 1.[4] When $\alpha = 0$, the habit stock is irrelevant to the decision making, and thus the utility function collapses to the conventional specification. Moreover, when $\rho = 1$, even though habit formation remains in the utility function, it does not affect the marginal utility of

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[4] We follow the specification in Abel (1990) of taking the ratio of consumption to habit stock. This specification has a drawback in that the utility function is not guaranteed to be concave; thus the optimal consumption derived from the first-order condition is not guaranteed optimal. Other studies, such as Campbell and Cochrane (1999) and Ljungqvist and Uhlig (2000), take the difference between current consumption and habit stock to avoid this problem. Moreover, the alternative specification does not require $\rho > 1$ to assure the positive effect of habit stock on the marginal utility of an additional unit of consumption.

consumption. To ensure the positive effect from the habit stock on the marginal utility of consumption, we assume $\rho > 1$.[5]

The habit stock, the accumulation of past consumptions, adjusts with time. Let the external habit stock H_t evolve with past aggregate consumption in the following process, similar to the one specified in Alvarez-Cuadrado, Monterio and Turnovsky (2004):

$$\frac{H_t}{H_{t-1}} = \left(\frac{\overline{C_{t-1}}}{H_{t-1}} \right)^{\lambda} \tag{9}$$

$\overline{C_{t-1}}$ represents the average consumption of the economy at the period $t-1$. λ is the adjustment speed of habit stock accumulation, which lies between 0 and 1. Higher λ implies that the adjustment of habit stock accumulation falls more heavily on the latest consumption. If $\lambda = 1$, $H_t = \overline{C_{t-1}}$ and the habit stock in the economy depends on the aggregate consumption from the previous period only.

We assume that asset markets are complete; that is, there is a complete set of state-contingent securities $D(s_{t+1})$ [6] with the price $B(s_{t+1} | s_t)$. The household, as a worker, receives wages from her labor supplied and, as a firm, collects profits from production. $P_t T_t$ is the per capita lump-sum transfer from the government, financed by the creation of money. The government's budget constraint is as follows:

$$P_t T_t = M_t - M_{t-1}$$

Foreign consumers face the same optimization problem:

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[5] In most studies, such as Gali (1994) and Ljungqvist and Uhlig (2000), they do not require $\rho > 1$. Here, we follow Amato and Laubach (2004) to assure the positive effect of habit stock on marginal utility.

[6] Because all the exogenous shocks are lognormal, there will be a continuum of states. The specification of discrete states in the budget constraint can be extended to continuous states directly.

$$\max\ E_t \sum_{t=0}^{\infty} \sum_{s_{t+1}} \beta^t \left(\frac{1}{1-\rho^*} \left(\frac{C^{*i}(s_t)}{\left(H^*(s_t)\right)^{\alpha^*}} \right)^{1-\rho^*} + \chi^* \ln \left(\frac{M^{*i}(s_t)}{P_t^*(s_t)} \right) - \frac{\kappa^*}{\upsilon^*} \left(L^{*i}(s_t) \right)^{\upsilon^*} \right) \quad (10a)$$

s.t.

$$P^*(s_t)C^{*i}(s_t) + M^{*i}(s_t) + \sum_{s_{t+1}} \varsigma_t(s_t) B(s_{t+1}|s_t) D^{*i}(s_{t+1})$$
$$\leq P^*(s_t)W^*(i,s_t)L^{*i}(s_t) + M^{*i}(s_{t-1}) + D^{*i}(s_t) + \int_0^1 [\Pi^*_{H,t}(j) + \Pi^*_{N,t}(j)]dj + P^*(s_t)T^*(s_t) \quad (10b)$$

Where $\varsigma(s_t)$ is the nominal exchange rate in the state s_t at period t, defined as the home currency price of one foreign currency. The foreign habit stock is accumulated in an analogous process:

$$\frac{H_t^*}{H_{t-1}^*} = \left(\frac{\overline{C_{t-1}^*}}{H_{t-1}^*} \right)^{\lambda^*} \quad (11)$$

And the foreign government budget constraint is balanced in a similar form:

$$P_t^* T_t^* = M_t^* - M_{t-1}^*$$

Both the home and foreign money supplies are random walks and their growth rates are lognormally distributed as follows:[7]

$$M_t = \mu_t M_{t-1},\ \eta_t = \eta + \varepsilon_{\eta,t},\ \varepsilon_{\eta,t} \sim N(0,\sigma_\eta^2) \quad (12a)$$

$$M_t^* = \mu_t^* M_{t-1}^*,\ \eta_t^* = \eta^* + \varepsilon_{\eta,t}^*,\ \varepsilon_t^* \sim N(0,\sigma_{\eta^*}^2) \quad (12b)$$

where $\eta = \log \mu$. Monetary authorities in these two countries act independently and neither reacts to shocks. η and η^* are the means of η_t and η_t^*. The distributions of the money supply growth rates are assumed as not autocorrelated and $i.i.d$.

[7] As noted in Devereux and Engel (1998), lognormally distributed money supplies are not necessary for the derivation of close-form solutions. Close-form solution can be obtained by assuming normal distribution of money supplies.

2.3.1 First-Order Conditions

In each period, consumers choose the consumption, money holding and state-contingent securities to maximize the utility. The first-order conditions of each individual i are as follows:

$$\frac{\chi}{M_t^i} = \frac{C_t^{i-\rho} H_t^{\alpha(\rho-1)}}{P_t} - E_t \beta \frac{C_{t+1}^{i\ -\rho} H_{t+1}^{\alpha(\rho-1)}}{P_{t+1}} \tag{13a}$$

$$B\left(s_{t+1} \middle| s_t\right) = \beta \pi\left(s_{t+1} \middle| s_t\right) \frac{C^i\left(s_{t+1}\right)^{-\rho} H\left(s_{t+1}\right)^{\alpha(\rho-1)}}{C^i\left(s_t\right)^{-\rho} H\left(s_t\right)^{\alpha(\rho-1)}} \frac{P\left(s_t\right)}{P\left(s_{t+1}\right)} \tag{13b}$$

Eq. (13a) is the condition with respect to the money held intertemporally that consumers equate with the benefit and cost from the holding of one more currency. Rewriting this equation, we can obtain the Euler equation:

$$\frac{M_t^i}{P_t^i} C_t^{i-\rho} H_t^{\alpha(\rho-1)} = \chi \frac{1+i_t}{i_t} \tag{14}$$

where $1/\left(1+i_t\right) = \sum_{s_{t+1}} B\left(s_{t+1} \middle| s_t\right)$. The derivation is in Appendix I.A. From this equation, we know that the nominal interest rate is determined by the change in consumptions across periods, instead of the level. Thus it will be smoother than in the conventional model.

Eq. (13b) describes the condition for consumers to determine the holdings of state-contingent securities. $\pi\left(s_{t+1} \middle| s_t\right)$ is defined as $\pi\left(s_{t+1}\right)/\pi\left(s_t\right)$, indicating the probability of the occurrence of state s_{t+1} given the current state s_t. When $\alpha=0$ or $\rho=1$, habit formation has no effect on the consumption and these two conditions collapse to the conventional ones. Foreign consumers solve the same problem and parallel first-order conditions arise:

$$\frac{\chi^*}{M_t^{*i}} = \frac{C_t^{*i-\rho} H_t^{*\alpha^*(\rho^*-1)}}{P_t^*} - E_t \beta^* \frac{C_{t+1}^{*i\ -\rho} H_{t+1}^{*\ \alpha^*(\rho^*-1)}}{P_{t+1}^*} \tag{15a}$$

$$B\left(s_{t+1}\middle|s_t\right)=\beta^*\pi\left(s_{t+1}\middle|s_t\right)\frac{C^{*i}\left(s_{t+1}\right)^{-\rho^*}H^*\left(s_{t+1}\right)^{\alpha^*\left(\rho^*-1\right)}}{C^{*i}\left(s_t\right)^{-\rho^*}H^*\left(s_t\right)^{\alpha^*\left(\rho^*-1\right)}}\frac{\varsigma\left(s_t\right)}{\varsigma\left(s_{t+1}\right)}\frac{P^*\left(s_t\right)}{P^*\left(s_{t+1}\right)} \quad (15b)$$

2.3.2 Money Demand Function

As pointed out by Obstfeld and Rogoff (1998) and Devereux and Engel (1998), the logarithmic utility function yields the closed-form solution of the money demand function from the money market equilibrium condition, Eq. (13a). In Appendix I.A, we show that consumption can be written as

$$\frac{M_t}{P_t}=\frac{\chi\mu}{\mu-\beta e^{\sigma_\eta^2/2}}C_t^{\rho}H_t^{\alpha(1-\rho)} \quad (16a)$$

The main difference between this equation and those in the papers mentioned above is the dependence of the money demand on the habit stock, not just on current consumption. When $\alpha=0$ or $\rho=1$, the money market equilibrium condition is insulated from habit formation.[8, 9] From this equation, given the money supply and price, the habit stock has a positive effect on consumption. Moreover, the habit stock does not influence consumption's responses to shocks.

[8] The result comes from the assumption of the logarithmic form of the money demand utility function. Obstfeld and Rogoff (1998) use a general CRRA form of utility function:

$$\frac{1}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{\frac{1}{1-\varepsilon}}$$

where $1/\varepsilon$ is the curvature. When $\varepsilon=1$, the form collapses to the one we adopt in this paper. In the more general form case, we are not able to obtain closed-form solutions, but the approximation around nonstochastic steady-state variables is needed. The logarithmic utility function and the complete asset markets, equation (11), lead to the simplified behavior of the nominal exchange rates as shown below.

[9] In Bouakez (2003), by taking a general specification of money demand utility function, shows that real exchange rate persistence decreases with the value of the parameter ε. The highest value of the real exchange rate persistence is reached when $\varepsilon=1$.

Because individuals are symmetric in each country, $C_t^i = C_t = \overline{C_t}$. i will be dropped throughout the paper and $\overline{C_t}$ in habit stock accumulation will be replaced by C_t .

While this study focuses on the persistence, this equation provides us with the information on how the economy reacts to shocks. Combining Eq. (16a) with Eq. (14), we can see that the nominal interest rate shall remain constant. In the face of a positive nominal shock, Eq. (16a) shows that both current consumption and price levels increase. (Note however that the adjustments of the prices to the monetary expansion are primarily on the nominal exchange rate movements since wages are predetermined). While the real interest rate must drop with the higher current consumption, the higher current price level increases the expected future price level. With the logarithmic money demand utility function, the movements of the real and nominal interest rates offset with each other, as pointed out by Devereux and Engel (1998).

The impact of a positive productivity shock will be a bit different. While a technological innovation occurs on the production side, it affects the good demand by lowering the prices. As shown below in the optimal pricing, a positive real shock leads to lower prices and induces greater consumptions. The lower price from the current positive productivity shock leads to lower expectation to future price level, while the expected consumption growth shall accompany lower real interest rate. Again, these two effects offset with each other and thus the nominal interest rate is constant. The constant nominal interest rate remains in the presence of the habit stock. The effect of the habit stock on the real interest rate will be offset by its effect on the price movements, mainly on wages.

The foreign money demand function follows analogously:

$$\frac{M_t^*}{P_t^*} = \frac{\chi^* \mu^*}{\mu^* - \beta^* e^{\sigma_{\eta^*}^2/2}} C_t^{*\rho} H_t^{*\alpha(1-\rho)} \tag{16b}$$

2.3.3 The Risk-Sharing Condition and the Nominal Exchange Rate

With the complete asset markets, from Eq. (14b) and (15b), we are able to derive the risk-sharing condition as follows:

$$Q_t = \frac{\varsigma_t P_t^*}{P_t} = \frac{C_t^{\rho} H_t^{\alpha(1-\rho)}}{C_t^{*\rho^*} H_t^{*\alpha^*(1-\rho^*)}} = \frac{\left(\frac{C_t}{H_t^{\alpha}}\right)^{\rho-1} C_t}{\left(\frac{C_t^*}{H_t^{*\,\alpha^*}}\right)^{\rho^*-1} C_t^*} \tag{17}$$

Q_t is defined as the level of the real exchange rate. Again, when $\alpha=0$ or $\rho=1$, this risk-sharing condition collapses to the standard one without habit formation. Moreover, in the case of $\lambda=1$, $H_t = C_{t-1}$. The far right term in Eq. (17) shows that the real exchange rate determines both the level and growth rate of consumption across countries.

This risk-sharing condition shows that the ratio of the marginal utility of consumption moves along with the real exchange rate. The marginal utilities of consumption across countries will be equalized if the purchasing power parity holds and thus consumptions across countries shall be identical every period. As a result, even though habit formation is present, consumptions are identical from the initial state. However, the purchasing power parity in this model does not hold in general. The deviation of the real exchange rate from one is primarily driven by the presence of nontradable goods, given that the initial debt is not zero as discussed below.

Throughout this paper, we assume that all the parameters are identical between countries, except for being asymmetric in α and λ in the further analyses by calibration. With the identical coefficients and by substituting Eq. (16a) and (16b) into the risk-sharing condition, we can obtain the nominal exchange rate as the ratio of money supply:

$$\varsigma_t = \frac{M_t}{M_t^*} \tag{18}$$

From this equation, we know that the nominal exchange rate does not overshoot in reacting to nominal shocks. In spite of external habit formation, this result is same as the logarithmic case in the standard utility function specification in Devereux and Engel (1998). As discussed, the impact of a positive nominal shock will be exactly reflected in the nominal exchange rate movements, causing the permanent depreciation of home currency. This simple form of the nominal exchange rate implies that the effects of habit formation on the real exchange rate result, in their entirety, from their effects on relative price, or relative wage in this model.

2.4 Preset Wage

Each household faces the labor demand and determines the wage one period ahead to optimize her expected utility. Wage is determined to equalize the expected return from the utility of consumption because of higher wage and the expected loss measured by the disutility from greater labor supply induced by higher wage. [10] Preset wage is expressed as

$$W_t = \frac{\kappa\phi}{\phi - 1} H_t^{\alpha(1-\rho)} \frac{E_{t-1}\left\{L_t^{\upsilon}\right\}}{E_{t-1}\left\{\frac{L_t}{P_t}(C_t)^{-\rho}\right\}} \tag{19}$$

$\phi / (\phi - 1)$ is the constant markup from constant elasticity of labor demand. The wage obtained here is almost the same as the one in Obstfeld and Rogoff (2000) but differs in the

[10] The effects of sticky wages as nominal rigidity on the economy may not change the persistence in the economy qualitatively, but can affect it quantitatively. Anderson (1998) states that sticky wage would produce higher output persistence. However, in this paper, we are trying to discuss the effect of habit formation on the real exchange rate qualitatively. This issue may not be relevant in a qualitative sense. With sticky wage, marginal cost is sensitive to current productivity shocks and thus productivity shock would have a greater effect on the level of prices and real exchange rate than the sticky price case. But the qualitative effect from habit formation on the real exchange rate would be negative in both cases.

way that external habit formation enters the equation because of its effect on the expected marginal utility of consumption. Since habit stock is known at time $t-1$, it lies outside the expectation operator. Foreign wage is in the analogous form.

2.5 Optimal Pricing

Since the price elasticity of the demand for each product is constant at θ, the markup over the marginal cost of a monopolistic competition firm is also constant and the optimal pricing for each type of good is as follows:

$$P_{N,t} = P_{H,t} = \left(\frac{\theta}{\theta-1}\right)\frac{W_t}{A_t},\; P^*_{N,t} = P^*_{F,t} = \left(\frac{\theta^*}{\theta^*-1}\right)\frac{W^*_t}{A^*_t} \tag{20}$$

$$P^*_{H,t} = \frac{1}{\varsigma_t}\left(\frac{\theta^*}{\theta^*-1}\right)\frac{W_t}{A_t} = \frac{1}{\varsigma_t}P_{H,t},\; P_{F,t} = \varsigma_t\left(\frac{\theta}{\theta-1}\right)\frac{W^*_t}{A^*_t} = \varsigma_t P^*_{F,t}$$

In this model, firms are allowed to charge different prices for the goods sold in different markets, which is known as pricing to market (PTM) in the literature. However, if $\theta = \theta^*$, $P_H = \varsigma_t P^*_H$, thus the law of one price holds. Even though the nominal exchange rate variation has complete pass-through onto the prices for tradable goods sold outside the producers' countries, the real exchange rate deviates from unity due to nontradable goods. Substituting optimal pricing in terms of wages in Eq. (6a) and (20), the real exchange rate Q_t can be written as:

$$Q_t = \frac{\varsigma_t P^*_t}{P_t} = \left(\frac{\varsigma_t W^*_t}{W_t}\frac{A_t}{A^*_t}\right)^{1-\gamma} \tag{21}$$

This equation discloses the reason that we use the sticky wage with nontradable goods, instead of sticky price, to examine real exchange rate movements. In the model, the purchasing power parity holds in a model with tradable goods only, while the law of one price holds. In the literature, the factor that drives the deviation of real exchange rate from one is primarily the local-currency price (LCP), where the preset prices for goods sold in home and foreign countries are denominated in consumers' currency respectively and may differ from each other.[11] The detailed discussions about the alternative nominal rigidity are in Hwang (2006). Using Eq. (21) and the risk-sharing condition, Eq. (17), the labor employment can be significantly simplified.

Similarly, the terms of trade can be written as:

$$\tau_t = \frac{\varsigma_t P^*_{F,t}}{P_{H,t}} = \frac{\varsigma_t W^*_t}{W_t}\frac{A_t}{A^*_t} \tag{22}$$

2.6 Market Clearing Conditions

In equilibrium, good markets have to clear:

$$C_{H,t} + C^*_{H,t} = Y_{H,t}, \quad C_{N,t} = Y_{N,t} \tag{23}$$

where $Y_{H,t}$ and $Y_{N,t}$ are the productions of home tradable and nontradable goods, respectively. By substituting the demands for tradable goods in Eq. (7) into the commodity market equilibrium condition in expenditure, we can obtain the following condition:

[11] With the LCP, we have to conduct the study with four prices; that is, prices for home and foreign tradable goods in both countries. This will significantly increase algebraic loads. Devereux and Engel (2000) and Obstfeld (2004) obtain the closed-form solutions under LCP by simplifying the model with perfectly elastic labor supplies, namely, $\upsilon=1$. This is not feasible in a study on habit formation, because the effect of habit formation disappears when $\upsilon=1$, as shown below.

$$P_{H,t}Y_{H,t} = P^*_{H,t}Y^*_{H,t} = \frac{1}{2}P_{T,t}C_{T,t} + \frac{1}{2}\varsigma_t P^*_{T,t}C^*_{T,t}$$

Thus, home and foreign tradable incomes are equal. As Obstfeld and Rogoff (1998, 2000) point out, consumers with Cobb-Douglas preferences and isoelastic utility function for the composite consumption good have no incentive to conduct security trades, if the initial debt is zero. As a result, the international asset market is redundant.

However, from Eq. (5a), we know that the expenditure of nontradable goods takes a fixed share, $1-\gamma$, in the total expenditure. Because the expenditure and income will be identical for the nontradable goods in each country, the following condition should hold:

$$P_{N,t}C_{N,t} = P_{N,t}Y_{N,t} = (1-\gamma)P_tC_t$$

If the current account is zero, the national income identity should follow:

$$P_tC_t = P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} \tag{24}$$

Thus, the home traded income shall also be a fixed proportion of the total expenditure:

$$P_HY_H = \gamma PC$$

When this holds, the market clearing condition, Eq. (23), implies that the real exchange rate will determine relative consumption in the global economy:

$$\frac{\xi_t P^*_t}{P_t} = \frac{C_t}{C^*_t} \tag{25}$$

Combined with the risk-sharing condition, Eq. (17), for complete asset markets in a model without habit formation, consumptions have to be identical across countries. This implies that the purchasing power parity must hold. [12] That is, the unity real exchange rate is required for both goods market equilibrium and zero asset transaction in the asset market equilibrium, even

[12] In Obstfeld and Rogoff (2000), there is no asset market. Relative consumption is solely determined by the real exchange rate, as in Eq. (24), but does not necessarily equalize across countries.

though there are nontradable goods. When consumptions, thus the intertemporal elasticity of substitution, for every possible state, are equal, there is no need to share risks via international asset transactions.

Therefore, to examine the deviation of the real exchange rate from the purchasing power parity, we assume nonzero initial debt, $D_0 \neq 0$. Given this condition, shocks may induce consumers to trade state-contingent securities across countries and Eq. (24) does not necessarily hold in equality. Substituting the good demand functions, Eq. (7) and the production function Eq. (1) into Eq. (23), the market clearing condition can be written as:

$$A_t L_t = Y_t = \frac{\gamma}{2}\frac{P_t C_t}{P_{H,t}} + \frac{\gamma}{2}\frac{P_t^* C_t^*}{P_{H,t}^*} + \left(1-\gamma\right)\frac{P_t C_t}{P_{N,t}} \tag{26a}$$

$$A_t^* L_t^* = Y_t^* = \frac{\gamma}{2}\frac{P_t C_t}{P_{F,t}} + \frac{\gamma}{2}\frac{P_t^* C_t^*}{P_{F,t}^*} + \left(1-\gamma\right)\frac{P_t^* C_t^*}{P_{N,t}^*} \tag{26b}$$

Using the optimal pricing in terms of wages in Eq. (20) and the price index in Eq. (6a), we can rewrite the equations above as:

$$A_t L_t = \left(1-\frac{\gamma}{2}\right)\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{\gamma/2} C_t + \frac{\gamma}{2}\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{1-\gamma/2} C_t^* \tag{27a}$$

$$A_t^* L_t^* = \frac{\gamma}{2}\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{\gamma/2-1} C_t + \left(1-\frac{\gamma}{2}\right)\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{-\gamma/2} C_t^* \tag{27a}$$

From Eq. (21), we know that the term in the bracket is simply the terms of trade, Eq. (22). Therefore, Eq. (27) shows how the terms of trade movements affect productions. The home production increases when the home terms of trade worsens, because both home and foreign demands for home goods are greater.

As we discuss later, we will apply log-linearization to these two equations after substituting the real exchange rate from the risk-sharing condition to derive the explicit log-linear labor employment in terms of consumption and habit stock. Using log-linearization will do no harm to this study, as noted below.

3. Analytical Solutions

3.1 Labor Employment

To derive analytical solutions and examine the effects of habit formation on real exchange rate persistence, we assume that these two countries are exactly symmetric so that $\alpha = \alpha^*$ and $\lambda = \lambda^*$. Substituting consumption from Eq. (16a) into the home wage in Eq. (19), the predetermined home nominal wage can be written as:

$$W_t = \frac{\kappa\phi}{\phi - 1} \frac{E_{t-1}\left\{L_t^{\upsilon}\right\}}{E_{t-1}\left\{L_t M_t^{-1}\right\}}$$

Since all the exogenous variables are lognormally distributed, wages can be written in logs:

$$w_t = \ln \frac{\kappa\phi\left(\mu - \beta e^{\sigma_\eta^2/2}\right)}{\chi\mu\left(\phi - 1\right)} + \left(\upsilon - 1\right) E_{t-1} l_t + E_{t-1} m_t + \Lambda_w \tag{28a}$$

$$w_t^* = \ln \frac{\kappa\phi\left(\mu - \beta e^{\sigma_\eta^2/2}\right)}{\chi\mu\left(\phi - 1\right)} + \left(\upsilon - 1\right) E_{t-1} l_t^* + E_{t-1} m_t^* + \Lambda_w^* \tag{28b}$$

where $\Lambda_t = \sigma_{lm} + \frac{\left(\upsilon^2 - 1\right)}{2}\sigma_l^2 - \frac{1}{2}\sigma_m^2$, $\Lambda_t^* = \sigma_{l^* m^*} + \frac{\left(\upsilon^2 - 1\right)}{2}\sigma_l^{*2} - \frac{1}{2}\sigma_m^{*2}$. Lower-case letters indicate the logs of these variables. Wages are determined by expected labor demand, expected money stock and the correlation and volatility terms, incorporating all the moments of exogenous shocks. From this equation, we can see that workers tend to preset higher nominal wages in reaction to uncertainties, characterized by the moment terms. The moment terms captured by lognormal distribution are the primary advantages of the model but are neglected in the linear approximation, as noted by Obstfeld and Rogoff (2000). Here, however, we pay the least attention to the covariance and volatilities for two reasons. First, these two terms are irrelevant to persistence in the economy. Second, the identical Λ_w and Λ_w^* in a symmetric global economy will be cancelled out and absent from the real exchange rate which is determined by relative economic condition across countries.

From Eq. (28), we know that the habit stock affects wages mainly through its effect on the labor demand. However, when $\upsilon = 1$, habit formation's effects disappear. From the market clearing condition in Eq. (27), we can obtain the labor employment. Since Eq. (27) is not a log-linearized equation, to derive the closed-form solutions explicitly, we substitute the real exchange rate in terms of consumption from the risk-sharing condition, Eq. (17), and then apply log-linearization to this equation around the steady state. The steady state is defined as a non-stochastic state where all the shocks are in their mean values.[13] Moreover, since the current account is zero, consumptions are identical across countries, or $\bar{C} = \bar{C}^* = \bar{H} = \bar{H}^*$. As shown in Appendix I.B, the logarithms of labor employment from the log-linearization approximation are:

$$l_t = \frac{(2-\gamma)(\gamma\rho+1-\gamma)}{2(1-\gamma)} c_t + \frac{\gamma\rho(\gamma-2)+\gamma(1-\gamma)}{2(1-\gamma)} c_t^* + \frac{\alpha\gamma(1-\rho)(2-\gamma)}{2(1-\gamma)} h_t + \frac{\alpha\gamma(\rho-1)(2-\gamma)}{2(1-\gamma)} h_t^* - a_t \quad (29a)$$

$$l_t^* = \frac{\gamma\rho(\gamma-2)+\gamma(1-\gamma)}{2(1-\gamma)} c_t + \frac{(2-\gamma)(\gamma\rho+1-\gamma)}{2(1-\gamma)} c_t^* + \frac{\alpha\gamma(\rho-1)(2-\gamma)}{2(1-\gamma)} h_t + \frac{\alpha\gamma(1-\rho)(2-\gamma)}{2(1-\gamma)} h_t^* - a_t^* \quad (29b)$$

These two equations above are assured to be real numbers, because we had imposed the condition that $\gamma \neq 1$ in order to induce real exchange rate movements.

[13] The definition of the steady state is the same as the one used in Benigno (2004).

Not surprisingly, this equation shows that consumption demands increase domestic labor employment. Without real exchange rate movements, home consumption demand shall also increase the labor demand in the foreign country. However, the risk-sharing condition states that greater home consumption, relative to foreign consumption, is associated with home real depreciation, which lures the global demand away from foreign commodities. Therefore, home consumption demands tend to decrease employment abroad. These two equations show how shocks and external effects of habit stocks are shared between countries.

The following discussion of habit formation's influence on the real exchange rate will center on the trade-off between the effects of habit stock and past productivity shocks on relative wage, because autocorrelated productivity shocks and habit stocks are the primary causes of persistence in the economy and are the primary past information referred to by firms in the expectation. Their contributions to the persistence will interact in the wage predetermination. Although Eq. (28) and (29) show the channel that exogenous shocks and habit stocks are passed across periods, they are not closed-form solutions. To examine their effects exactly, we need to obtain the closed-form solution by considering the money clearing condition in Eq. (16).

3.2 Level of Real Exchange Rate

As shown in Eq. (21), the real exchange rate is determined by relative wage and shock. Therefore, to study the real exchange rate, we may simply take the relative terms across countries to describe the real exchange rate. Taking the difference between Eq. (29a) and (29b), relative employment is shown as:

$$l_t - l_t^* = \frac{\gamma\rho(2-\gamma)+(1-\gamma)^2}{1-\gamma}\left(c_t - c_t^*\right) + \frac{\alpha\gamma(1-\rho)(2-\gamma)}{1-\gamma}\left(h_t - h_t^*\right) - \left(a_t - a_t^*\right)$$

Combining the expectation of relative labor employment from the equation above, the nominal exchange rate from Eq. (18), the logarithm of the money market clearing condition and the difference in wages from Eq. (28) yields relative wage in terms of relative habit stock and exogenous shocks:

$$w_t - w_t^* = d_{w1}\left(h_t - h_t^*\right) + d_{w2}\left(a_{t-1} - a_{t-1}^*\right) + \left(m_{t-1} - m_{t-1}^*\right) \tag{30}$$

where

$$d_{w1} = \frac{\alpha(1-\gamma)(\rho-1)(\upsilon-1)}{\rho+(\upsilon-1)\left[\gamma\rho(2-\gamma)+(1-\gamma)^2\right]} > 0, \qquad d_{w2} = -\frac{(\rho-1)(\upsilon-1)(1-\gamma)^2\rho_a}{\rho+(\upsilon-1)\left[\gamma\rho(2-\gamma)+(1-\gamma)^2\right]} < 0.$$

Given $\Lambda_w = \Lambda_w^*$, these terms are absent in relative wage. With the direct effect of labor employment on wages, Eq. (30) reflects the relative employment. Note that the effects of habit formation on the labor employment and on all other variables as shown below, are governed by the importance of habit formation in the utility function α. When $\alpha = 0$, the utility depends on current consumption solely. From this equation, we can see that relative productivity shock and habit stock engender the reverse effects on relative wage when $\alpha > 0$.

Proposition 1 *When* $0 < \alpha \leq 1$, *relative habit stock affects the level of the real exchange rate in the opposite way that relative real shock does.*

Proof

From Eq. (30) and the log of Eq. (21), the real exchange rate can be written as:

$$q_t = d_{q1}\left(h_t - h_t^*\right) + d_{q2}\rho_a\left(a_{t-1} - a_{t-1}^*\right) + (1-\gamma)\left[\left(\varepsilon_{a,t} - \varepsilon_{a,t}^*\right) + \left(\eta_t - \eta_t^*\right)\right] \tag{31}$$

where $d_{q0} = \rho\left[\upsilon - (\upsilon - 1)(1-\gamma)^2\right] + (1-\gamma)^2(\upsilon - 1) > 0$, $d_{q1} = \frac{-\alpha(\rho - 1)(\upsilon - 1)(1-\gamma)^2}{d_{q0}} < 0$,

$d_{q2} = \frac{\upsilon\rho(1-\gamma)}{d_{q0}} > 0$. Relative habit stock, past real shock, current real and nominal shocks are the major determinants of the real exchange rate. All the shocks and consumption externalities are passed across countries. When $\alpha > 0$, relative habit stock places a negative effect on relative wage, which results in a negative effect on the real exchange rate.

The effects of relative habit stock and real shock on the real exchange rate are in the opposite sign. This is primarily from the trade-off between the effects of relative habit stock and productivity shock on relative wage. While higher past home productivity shock, relative to foreign productivity shock, decreases home wage, home habit stock leads to higher home wage. The lower wage from the positive home real shock will reinforce the increase in home consumption, but this effect is relatively small. As a result, relative real shock causes the home real depreciation and relative habit stock acts to offset this effect. The effect of habit formation increases with the importance of habit formation, α. The reverse effects of relative habit stock and productivity shock on the real exchange rate have the important implications for the persistence.

The magnitudes of the effects are based on the share of tradable goods in the composite consumption γ, the intertemporal substitution ρ and the elasticity of labor supply υ. With nontradable goods, shocks impact the home economy by $1-\gamma/2$, while in the foreign economy by $\gamma/2$. $1-\gamma$ thus describes the difference between the impacts on the home and foreign countries, and shocks fall more heavily on the domestic country with the presence of nontradable goods. When there is no nontradable good, $\gamma = 1$ and the impacts on these two countries are identical. As mentioned above, nontradable good is the main factor that causes the deviation from the purchasing power parity in this model. Even if there is no asset market, the purchasing power parity shall hold in the economy with only tradable goods due to the holding of the law of one price.[14] Workers with lower elasticity of labor supply, higher υ, tend to preset a higher wage, leading to greater real exchange rate movements.

Since we assume that nominal shocks are not autocorrelated, the relative past money supply in relative wage is offset by nominal exchange rate movements. As a result, the past nominal shocks do not appear in the Eq. (31). As shown, both nominal and productivity shocks account for the volatility of real exchange rate. However, the magnitude of volatility is governed only by the share of tradable goods, γ, which determines how shocks are passed across countries. Habit formation does not influence real exchange rate volatility due to certain assumptions of this model, as explained below.

3.3 Persistence of Real Exchange Rate

The preceding subsection shows the effect of relative habit stock, $h_t - h_t^*$, on the level of the real exchange rate. In this subsection, we characterize the information embodied in habit stocks to shed light on how relative external habit stock affects the persistence of the real exchange rate. In contrast with Fuhrer (2000), who characterizes the change in consumption across time in a model with internal habit formation, we characterize relative habit formation as the accumulation of past relative shocks. Because the habit stock contains the past

[14] The result that the habit formation has a positive effect on home wages is because of the asset market structure and the logarithm form of the money demand function, which isolates the movement of the nominal exchange rate from any real variables in the economy. If the asset market structure is modified, for example, the asset market is absent as specified in Obstfeld and Rogoff (2000), the nominal exchange rate is not determined from the risk-sharing condition, but determined by the relative wage income across countries. In that case, habit formation will lead to greater nominal exchange rate movement. Higher home habit formation enlarges the depreciation of home currency and thus the world demand moves toward home goods. As a result, the negative effect of home habit formation on foreign consumption demand will dominate compared to its positive effect on home consumption toward foreign goods and will result in a decrease in foreign labor demand. In this case, foreign habit formation will have a negative effect on the home wage instead.

information, it crucially influences the persistence of the variables in the economy. While habit formation increases the persistence of real variables, as described by the habit formation literature, it does not increase the real exchange rate persistence as shown below or in the calibration.

Proposition 2 *When* $0<\alpha\leq 1$, *habit formation reduces the persistence of the real exchange rate.*

Proof

Rewrite the risk-sharing condition (17) as

$$c_t - c_t^* = \frac{\alpha(\rho-1)}{\rho}(h_t - h_t^*) + \frac{1}{\rho}q_t \tag{32}$$

Relative current consumption embodies the information of current habit stock and current real exchange rate. Let $\overline{C_{t-1}} = C_{t-1}$ due to identical consumers and take the logarithm of the habit accumulation path from Eq. (11) and the foreign analogy, then the difference between home and foreign habit stocks evolves with

$$h_t - h_t^* = \lambda\left(c_{t-1} - c_{t-1}^*\right) + \left(1-\lambda\right)\left(h_{t-1} - h_{t-1}^*\right) \tag{33}$$

Remember that λ is the adjustment speed of the reference stock accumulation. From this equation, relative habit stock is a weighted average of past relative consumption and past habit stock. Substituting this equation recursively back into the initial date, relative habit stock is simply the accumulation of relative past consumptions, which are determined by past shocks. Therefore, when the past shocks are accrued in the habit stock, all past information is passed through the external effect on current consumption. External habit formation has created an extra avenue to carry past shocks and thus influence the persistence of macroeconomic variables.

Substitute $c_{t-1} - c^*_{t-1}$ by taking one period ahead from Eq. (32) into Eq. (33); thus relative habit stock becomes

$$h_t - h^*_t = \lambda\left(c_{t-1} - c^*_{t-1}\right) + \left(1-\lambda\right)\left(h_{t-1} - h^*_{t-1}\right) = \frac{\lambda}{\rho} q_{t-1} + \left(\left(1-\lambda\right) + \frac{\alpha\left(\rho-1\right)}{\rho}\right)\left(h_{t-1} - h^*_{t-1}\right) \qquad (34)$$

As shown, relative habit stock embodies the lagged real exchange rate. Substitute Eq. (34) into the real exchange rate in Eq. (32):

$$\begin{aligned} q_t = {} & d_{q1}\left(\frac{\lambda}{\rho} q_{t-1} + \left(\left(1-\lambda\right) + \frac{\alpha\left(\rho-1\right)}{\rho}\right)\left(h_{t-1} - h^*_{t-1}\right)\right) + d_{q2}\rho_a\left(a_{t-1} - a^*_{t-1}\right) \\ & + \left(1-\gamma\right)\left(\varepsilon_{a,t} - \varepsilon^*_{a,t} + \ln \eta_t - \ln \eta^*_t\right) \end{aligned} \qquad (35)$$

When $\alpha > 0$, $d_{q1} < 0$, while $d_{q2} > 0$. Because habit formation acts in the opposite way to past real shocks do, the persistence of the real exchange rate caused by relative real shocks is diminished by the external habit stock. Thus, relative habit stock will have a negative effect on the real exchange rate persistence.

The interaction between productivity shocks and habit stocks can be summarized here. In the period $t-1$, a positive technology shock decreases the wage. The consumption and the habit stock are increased accordingly. These two changes from a technology shock will influence wages and consumption in the next period. While the wage for the period t is decreased further due to the expected higher productivity, greater habit stock raises the wage. The lower nominal wage in period $t-1$ appears to be followed by lower wage in period t, leading to the persistence of wage. Thus the additional impact of the productivity shock on the wage, through habit stock, dampens the direct effect, the primary cause of the wage persistence.

Note that we shall not think of Eq. (30) and (31) as closed-form solutions, although the habit stock is treated as an exogenous variable to the extent that it is known one period ahead. However, the habit stock is composed of past consumptions, which are the endogenous variables determined from the preceding periods. With the closed-form solution of consumption given below, we are able to characterize the information contained in the consumption habit more closely.

3.4 Consumption

The role of habit stocks in the open economy can be more elusive from the closed-form solution of consumption. Substituting relative wage in Eq. (30) into the relative optimal price and the money market equilibrium condition, relative consumption can be written as

$$c_t - c_t^* = d_{c1}\left(h_t - h_t^*\right) + d_{c2}\rho_a\left(a_{t-1} - a_{t-1}^*\right) + \frac{1-\gamma}{\rho}\left[\left(\varepsilon_{a,t} - \varepsilon_{a,t}^*\right) + \left(\eta_t - \eta_t^*\right)\right] \qquad (36)$$

Where $d_{c1} = \frac{\alpha(\rho-1)\left[1+\gamma(\upsilon-1)(2-\gamma)\right]}{d_{q0}} > 0$, $d_{c2} = \frac{\upsilon(1-\gamma)}{d_{q0}} > 0$. This equation shows that consumptions are determined by external habit stocks and exogenous shocks. Both relative habit stock and productivity shock have positive effects on relative consumption. While relative habit stock decreases the real exchange rate persistence, it reinforces the persistence of consumption. Because relative output is determined by consumption demand, relative habit stock will also increase the persistence of output. This is consistent with the findings in Justiniano and Preston (2004) that habit formation does help increase the persistence of real variables, but is not significant for nominal price or the real exchange rate persistence.

Current shocks also increase consumptions via lowered prices. As mentioned above, the price adjustments in reaction to monetary shocks rely on the flexible nominal exchange rate. A positive home nominal shock leads to home currency depreciation, and thus benefits both the home export and consumption.

Substituting relative consumption for the period $t-1$ from Eq. (36) into the habit stock adjustment process in Eq. (9), relative habit stock can be written as the accumulation of all past shocks back to the initial state:

$$h_t - h_t^* = \lambda B^{t-1}\left(c_0 - c_0^*\right) + \lambda d_{c2}\rho_a \sum_{j=0}^{t-2} B^j \left(a_{t-2-j} - a_{t-2-j}^*\right) + \frac{\lambda(1-\gamma)}{\rho}\sum_{j=0}^{t-2} B^j \left[\left(\varepsilon_{a,t-1-j} - \varepsilon_{a,t-1-j}^*\right) + \left(\eta_{t-1-j} - \eta_{t-1-j}^*\right)\right] \tag{37}$$

where $B = \lambda d_{c1} + (1-\lambda) > 0$ and $\lambda\left(c_0 - c_0^*\right) = h_1 - h_t^*$, given that $h_0 - h_0^* = 0$. The derivation is in Appendix I.C. Note that c_0 does not equal c_0^*, as we have imposed a condition that the initial debt is not zero. This equation discloses the information contained in the relative habit stock, the composition of past productivity and nominal shocks. Due to the positive effects of shocks on consumptions, all the past shocks are accumulated in a positive way. α and λ , the adjustment speed which determines the weight that more recent shocks take in the current habit stock, are the primary parameters that govern the habit stock process. It is clear that past shocks have stronger impacts on relative habit stock with α , whereas the influence of λ faces the trade-off. Taking the first-order derivative of λB^j with respect to λ , where j represents the periods at which the shocks are ahead of the current period:

$$\frac{\partial \lambda B^j}{\partial \lambda} = 1 + \lambda\left(d_{c1} - 1\right)\left(j + 1\right)$$

The sign of this term depends on the size of j . It is positive for small j , while negative for greater j . Because a higher λ implies a higher weight of recent shocks, the effect of recent shocks on the current habit stock shall increase with λ . The first-order derivative stays positive as long as $j < \left(1-\lambda\left(1-d_{c1}\right)\right)/\left(\lambda\left(1-d_{c1}\right)\right)$. When $\alpha = 0$ and $\lambda \neq 0$, although the habit stock keeps accumulating, it has no influence on the current economy. On the other hand, when $\lambda = 0$, regardless of the value of α, relative habit stock stays zero, as there is no such dynamic accumulation process of the habit stock.

Substituting Eq. (37) into Eq. (31), the real exchange rate can be written as:

$$q_t = d_{q1}\left\{\lambda d_{c2}\rho_a \sum_{j=0}^{t-2} B^j\left(a_{t-2-j}-a^*_{t-2-j}\right)+\frac{\lambda(1-\gamma)}{\rho}\sum_{j=0}^{t-2} B^j\left[\left(\varepsilon_{a,t-1-j}-\varepsilon^*_{a,t-1-j}\right)+\left(\eta_{t-1-j}-\eta^*_{t-1-j}\right)\right]\right\}$$
$$+d_{q2}\rho_a\left(a_{t-1}-a^*_{t-1}\right)+(1-\gamma)\left(\varepsilon_{a,t}-\varepsilon^*_{a,t}+\ln\eta_t-\ln\eta^*_t\right) \tag{38}$$

This equation clearly shows that past shocks involved in the habit stock counteract the effect of past productivity shocks because of the negative d_{q1}.

In one of the special cases in Devereux and Engel (1998), similar to the specification in this paper, they pointed out that money is neutral in the long run, leaving no real effect one period after the nominal shock hits the economy. In our model, we can see that money is neutral in the long run if $\alpha=0$. However, money is not neutral even in the very long run when $\alpha\neq 0$. Through reference stocks, past nominal shocks affect current and future real variables in the economy.

As noted above in the wage predetermination, nominal exchange rate movements play a critical role in determining the effects of external habit stock on the real exchange rate. In this model, due to the complete asset market structure and the logarithmic form of the money demand utility function, the nominal exchange rate is simply the difference between money supplies across countries. While it is insulated from the movement of habit formation, it fails to offset the negative effect of relative habit formation on real exchange rate persistence.

The volatility of the real exchange rate is another important aspect addressed in the purchasing power parity puzzle. From the closed-form solutions, we see that external habit formation are independent from current shocks and do not reduce real exchange rate volatility. Several reasons account for this result. First, the logarithmic utility function relevant to money demand neglects one effect of habit formation. As shown in Hwang (2006), the interest elasticity of money demand governs the reactions of consumptions and other variables, such as the nominal exchange rate and interest rate, to shocks. When the interest elasticity is not unity, the consumption adjustments do not completely offset the effects of productivity shocks on prices and leave part of the adjustment job in the flexible nominal exchange rate. In that case, habit formation reduces the responses of consumption to productivity shocks. This effect is absent in this model due to the unity of interest elasticity of money demand.

The second factor is the assumption of external habit formation. If habit formation is internal instead, each consumer takes her own past consumptions as the reference stock and will consider not only the current consumption utility, but also the impact of current consumption on future utilities. Thus consumers will lessen their reactions to current shocks in order to smooth out the consumption path. Therefore, with the internal habit formation, we may expect lower real exchange rate volatilities than in the external habit formation case.

The third reason is the omission of staggered pricing, which are used in most of recent dynamic stochastic general equilibrium studies. All the wages were preset one period ahead and none react to current shocks. According to the Calvo-type staggered pricing, a certain proportion of workers in the market reset their wages in response to current shocks. With habit formation, they may react less to current shocks and thus habit formation may affect the volatility and persistence of wage, as well as other real variables in the model.[15]

As pointed out by Rogoff (1996), the volatility of real exchange rate is four times more volatile than that of consumption. In this model, we can see that the ratio of the standard deviation of real exchange rate to that of consumption is $1/\rho$. This is same as what is shown in Chari, Kehoe and McGratten (2000). While $\rho > 1$, indeed, the real exchange rate is more volatile than consumption. As reported in Chari, Kehoe and McGratten (2002), the ratio of the standard deviation of the real exchange rate to consumption is around 5.25.[16] Thus, in the following calibrations, we use 5 as the benchmark value for ρ.[17]

4 Sensitivity Analysis

The analytical solution for the real exchange rate derived above is not exactly in the first-order autocorrelation form. Therefore, to assess the effects of parameters on real exchange rate persistence, we run calibrations for the AR(1) coefficient of real exchange rate against different values of habit-related parameters, the importance of habit formation α, and the adjustment speed of habit stock accumulation λ. Since the quantitative measure of habit formation's influence and productivity shocks are determined by values of parameters,

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[15] Jung (2004) examines how external habit formation affects the performance of a model with staggered pricing. He finds that external habit formation does decrease the volatility of real variables at all frequencies of nominal rigidity. This results from the interaction of external habit formation with the staggered pricing.

[16] In Chari, Kehoe and McGratten (2002), the ratio of the standard deviation of real exchange rate to output is 4.36, while the ratio of the standard deviation of consumption to output is 0.83, from the data.

[17] Note that the coefficient of the intertemporal substitution has been changed in the utility function with habit formation and the way that habit formation enters the utility function as a ratio to current consumption. While $1/\rho$ is the short-run intertemporal substitution, $1/(\alpha + \rho(1-\alpha))$, is the long-run intertemporal substitution. See Alvarez-Cuadrado, Monterio and Turnovsky (2004) and Carroll, Overland and Weil (2000).

including productivity shock persistence, ρ_a , elasticity of intertemporal substitution, ρ , elasticity of labor supply, υ , and the openness of the economy, γ , we also conduct the sensitivity examination of habit formation's effect to these parameters. Keep in mind, we do not intend to solve the purchasing power parity puzzle by generating a high value for the AR(1) coefficient of the real exchange rate, but instead we simply are interested in how the effect of habit formation on real exchange rate persistence varies with the habit related parameters.

Before proceeding to the calibration, we need to specify values for the common parameters that are our main concern. As shown in analytical solutions, some parameters are not relevant to real exchange rate movements, but they are needed for calibrations. These parameters, listed in Table 1, are chosen according to conventional specifications and empirical findings.

Table 1: Specifications of Parameters

	Parameters
Production	$\phi = 2$
Preference	$\theta = 10,\ \beta = 0.99,\ \chi = 1,\ \kappa = 1$
Mean of shocks	$\eta = 0,\ a = 0,\ \sigma_\eta^2 = 1,\ \sigma_a^2 = 1$
Initial state	$a_0 = a_0^* = 1, c_0 - c_0^* = 1, h_0 - h_0^* = 0.$

As mentioned above, in order to get the real exchange rate deviation from the purchasing power parity, we need to assume nonzero initial debt. Thus, in the calibration, we let the initial productivity shocks be as one. As a result, the total production in the global economy is two, given the population in each country is normalized to one. The initial consumptions are not equal, but differ by one, which allows the transactions of tradable state-contingent bonds across countries. The initial habit stocks are assumed identical across countries.

Because the productivity shock persistence is the primary determinant of real exchange rate persistence, ρ_a would be the most critical factor among all. In the calibrations, we take 0.95 as the benchmark value for ρ_a, following the work of Chari, Kehoe and McGratten (2000). Furthermore, we also consider the values of ρ_a as 0.9 and 0.85 for sensitivity studies.

As explained above, we adopt $\rho = 5$ as the benchmark value to reproduce the relative volatility of real exchange rate to consumption, which is close to the data reported by Chari, Kehoe and McGratten (2002). However, we are also interested in the sensitivity of habit formation's effect on the real exchange rate persistence to the value of ρ. Under the restriction of ρ as greater than one and to assure a positive effect from habit formation on the marginal utility of consumption, we also examine real exchange rate persistence with alternative values, 3 and 1.5.

According to Bouakez, Cardia and Ruge-Murcia (2002), the elasticity of labor supply is close to 1.6. Thus, the benchmark value of 2.6 is chosen for υ. Since labor supply is rather inelastic in empirical findings, we also consider the case that $\upsilon = 6$. While the home bias in goods consumption is represented by $1-\gamma/2$ in this model, we let $\gamma = 0.3$, implying home-bias consumption to be 0.85, as suggested by the U.S data from Bergin and Feenstra (2001).

4.1 The Importance of Habit Formation: α

In this section, we run calibrations for $\rho(q)$, the AR(1) coefficient of the real exchange rate, against different values of α, given $\lambda = 1$. It is shown in the next section that $\rho(q)$ decreases with λ, which implies that habit formation's effect increases with λ. Therefore, we take the highest value for λ to signify the effect of external habit formation on $\rho(q)$.[18]

All the graphs in Figure 1 show a clearly declining trend of $\rho(q)$ with the increase in α, regardless of the parameter values. The negative effect of relative habit stock gets more significant when habit formation becomes more important in the utility of consumption. Figure 1 (a) shows that ρ_a plays a significant role in the real exchange rate persistence. The highest value of $\rho(q)$ reported is about 0.85, when $\rho_a = 0.95$ and $\alpha = 0$. The value is lower than 0.97 as reported by Rogoff (1995).[19] The primary reason is the rather low degree of nominal rigidities. In general, a decrease in ρ_a by 5% decreases $\rho(q)$ by approximately 8% for all levels of α. Figure 1 (b) shows that the greater value of ρ leads to higher $\rho(q)$. However,

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[18] The value of λ is taken as 0.2 and 0.8 in the studies of Carrol et. al (1997) and Fuhrer (2000).

[19] Chari, Kehoe and McGratten (2002) use the H-P filtered data, which would suggest lower persistence. We compare our results with the value reported by Rogoff (1995) based on level of data.

this effect is not that drastic. $\rho(q)$ remains at very close values when the value of ρ changes from 5 to 3, and so does the habit formation's effect.

In Figure 1 (c), higher γ lowers $\rho(q)$ as shown in the analytical solutions. When the global economy becomes more open, more productivity shocks and external effects are shared across countries, causing less difference between countries and smaller real exchange rate movements. Higher values of α dampen this effect and thus changing γ does not cause much difference. On the other hand, higher γ, lower $1-\gamma$, eliminates α 's effect on $\rho(q)$. When $\gamma = 0.8$, $\rho(q)$ stays around the same level regardless of the level of α. The results in Figure 1 (d) suggest a rather considerable role of υ in habit formation's effect on the real exchange rate persistence. The gap between the two trend lines diminishes with α, implying that the higher value of υ reinforces the effect of habit formation on $\rho(q)$. This conforms with the closed-form solutions, which suggests that υ plays a crucial role in habit formation's effect on real exchange rate movements, though it does not affect the level of $\rho(q)$ critically. In summary, ρ_a and γ are the most substantial determinaters for the magnitude of the real exchange rate persistence, while γ and υ dictate habit formation's effect on the real exchange rate persistence.

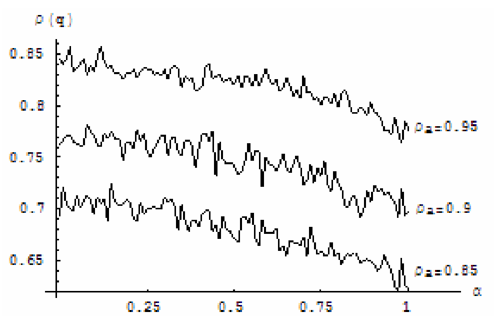


Figure 1 (a): ρ_a

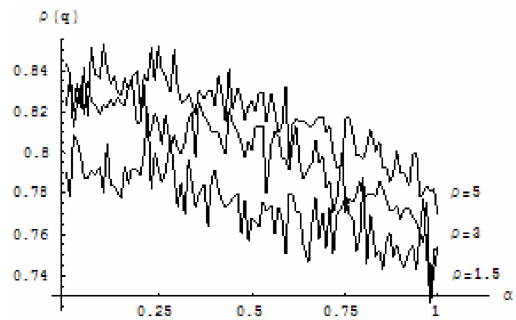


Figure 1 (b): ρ

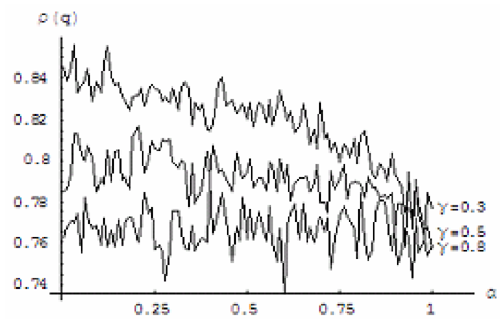


Figure 1 (c): γ

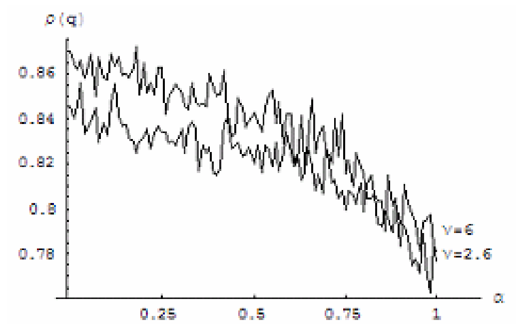


Figure 1 (d): υ

Figure 1: AR(1) Coefficient of Real Exchange Rate, $\rho(q)$, with Different Values of α

4.2 The Adjustment Speed of Habit Stock Accumulation: λ

In Figure 2, we fix the value of α at 1 to examine how real exchange rate persistence, $\rho(q)$, varies with the value of λ. As shown in Figure 2, we can also see the declining trend in $\rho(q)$ when λ grows. As discussed above in analytical solutions, the more rapid adjustment speeds imply a higher weight of recent shocks in the habit stock, which have greater impacts on the present. This thus magnifies the effect of external habit stock on real exchange rate persistence. However, lower ρ_a significantly diminishes the effect of habit formation. $\rho(q)$ stays at approximately the same level when ρ_a gets smaller than 0.9.

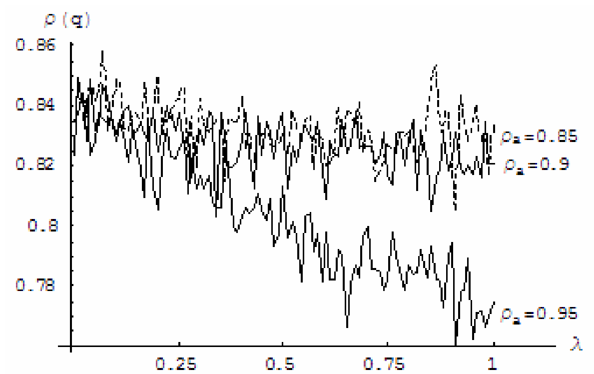


Figure 2: AR(1) Coefficient of Real Exchange Rate, $\rho(q)$, with Different Values of λ

While external habit formation's effect rises with the increase in α and λ under all parameter values, we are also interested in the interaction between α and λ. From Figure 3, we can see that the effect of α almost disappears for small values of λ. Its effect becomes more significant when λ is large and vice versa. This is because a share of λ in the shock that occurs in the previous period is carried to subsequent periods with a weight α on the utility. Thus α and λ reinforce each other. However, from the graph on the right-hand side, the effect of λ declines significantly with the decrease in α. As pointed out in the discussion of Eq. (37), after all, α governs the importance of habit formation. Even if the habit stock keeps accumulating when $\lambda \neq 0$, it is not effective when α is small.

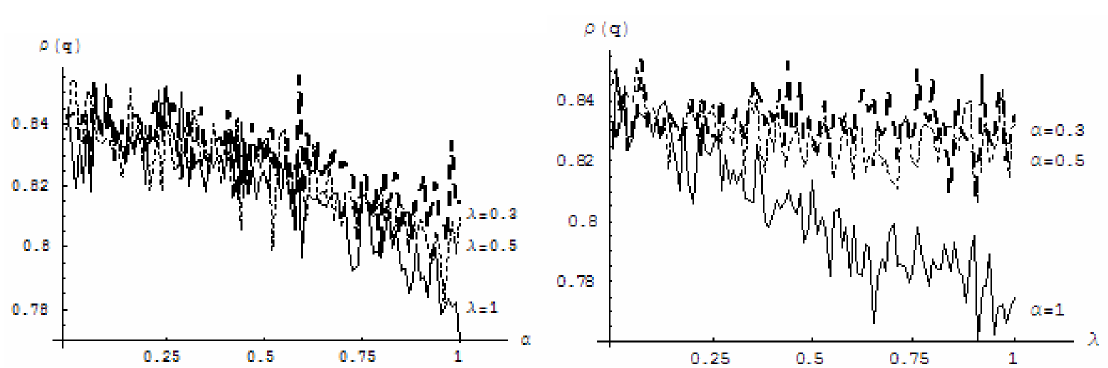


Figure 3: The interaction of α and λ in the AR(1) coefficient of real exchange rate, $\rho(q)$

5 Alternative Specifications of α and λ

5.1 Negative Consumption Externality: $\alpha<0$

Thus far, all the discussions have been based on a positive α, which suggests the "jealousy" inherent in consumers' behavior. However, as mentioned in Gali (1994), consumers may admire the aggregate habit stock, rather than being jealous. In this case, $\alpha < 0$. The negative external effect on the marginal utility of consumption motivates consumers to lower the consumption. With the negative externality, Eq. (31) and Eq. (37) show that it is likely that both the level and persistence of real exchange rate are increased by relative habit stock when the coefficient governing the effect of relative habit stock turns positive. However, the calibration results in Figure 4 suggest that the negative externality does not alter real exchange rate persistence at all. For each ρ_a, $\rho(q)$ remains approximately the same, at the level occurring when $\alpha = 0$ in Figure 1 (a). Thus, although the negative externality eliminates the negative effect of relative habit stock on real exchange rate persistence, it fails to strengthen the persistence.

An explanation can be provided by Eq. (37). While the negative α leads to a positive coefficient for relative habit stock in the real exchange rate, it turns the information carried by relative habit stock, lying in the bracket, negative. As a result, the negative and positive effects offset each other, leaving no impact on real exchange rate persistence.

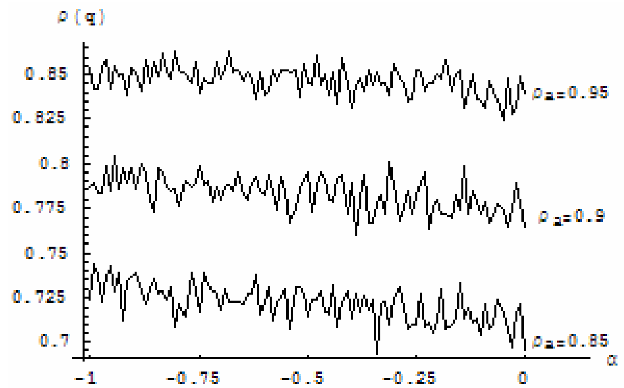


Figure 4: AR(1) Coefficient of Real Exchange Rate, $\rho(q)$, with Negative α

5.2 Asymmetry Across Countries

We have been assuming the symmetric parameter values in the two countries. However, Benigno (2004) emphasizes the contribution of the asymmetry in the price rigidity across countries to real exchange rate persistence. He finds that the asymmetry drives relative price persistence,[20] and thus significantly increases real exchange rate persistence. Based on his findings, we would like to shed light on the asymmetry between α and α^*, λ and λ^*, to see if the negative effect of habit formation on $\rho(q)$ can be reversed.

5.2.1 Relative Importance of Habit Formation

Divide α and α^* into two parts: the average value of α, $\bar{\alpha}$, and the idiosyncratic part in α across countries, $\Delta\alpha$, where $\bar{\alpha} = (\alpha + \alpha^*)/2$ and $\Delta\alpha = (\alpha - \alpha^*)/2$. $\bar{\alpha}$ determines the level of α, similar to the role of α in the symmetric case, while $\Delta\alpha$ represents the discrepancy

[20] The relative nominal rigidity he takes is $\alpha^H - \alpha^F$, where home has greater nominal rigidities than the foreign country.

between the countries. Thus α can be represented by, $\alpha = \bar{\alpha} + \Delta\alpha$ and $\alpha^* = \bar{\alpha} - \Delta\alpha$. The relative habit stock can be written as:

$$\alpha h_t - \alpha^* h_t^* = \bar{\alpha}\left(h_t - h_t^*\right) + \Delta\alpha\left(h_t + h_t^*\right) \tag{39}$$

We examine $\rho(q)$ with different values for $\bar{\alpha}$ and $\Delta\alpha$, as shown in Figure 5 (a) where $\bar{\alpha}$ is fixed at 0.5 and $\Delta\alpha$ varies from 0.1 to 0.5. The result suggests, there is no clear pattern showing that $\rho(q)$ moves with the values of $\Delta\alpha$, but fluctuate slightly around 0.8, approximately the level reported in Figure 1 (a) for $\rho_a = 0.95$ and $\alpha = 0.5$. (When $\Delta\alpha = 0$ in this case, it implies $\alpha = \alpha^* = 0.5$ as in the Figure 1) Therefore, the asymmetry across countries in α fails to offset the negative effects caused by relative habit formation. Note that, the value of $\rho(q)$ stays approximately the same level as reported here when $\alpha^* > \alpha$.

Figure 5 (b) shows the values of $\rho(q)$ under different values of $\bar{\alpha}$, with $\Delta\alpha$ fixed. Since $\rho(q)$ does not vary with $\Delta\alpha$ from above, the choice of $\Delta\alpha$ is rather flexible. We thus specify $\Delta\alpha = 0.1$ and $\bar{\alpha}$ ranging from 0.1 to 0.9. From this graph, $\rho(q)$ does decline with the increase in $\bar{\alpha}$, as in Figure 1 (a). Therefore, this numerical evaluation suggests that, only the level of α can affect $\rho(q)$, but not the difference.

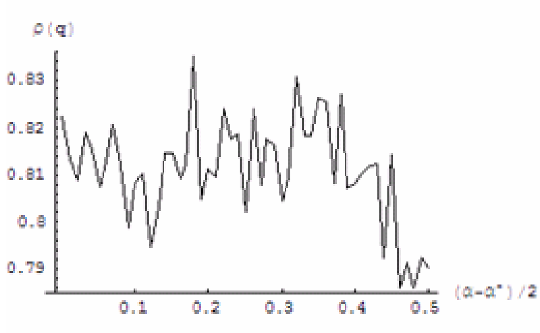


Figure 5 (a): $\Delta\alpha$

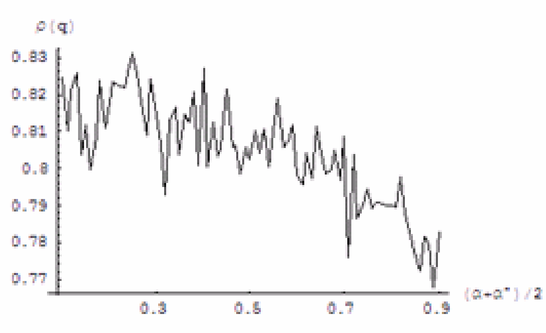


Figure 5 (b): $\overline{\alpha}$

Figure 5: AR(1) Coefficient of Real Exchange Rate, $\rho(q)$, with Asymmetric α

5.2.2 Relative Adjustment Speed of Habit Stock

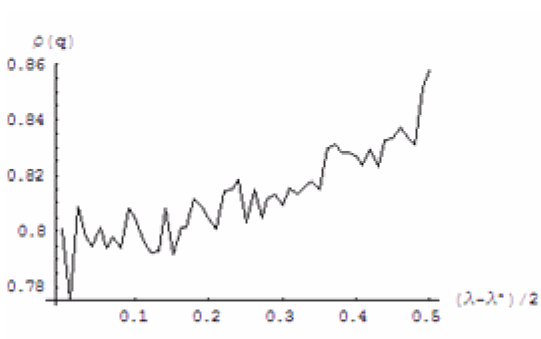
The asymmetry in λ is another interesting issue to study. Similarly, let $\lambda = \overline{\lambda} + \Delta\lambda$, $\lambda^* = \overline{\lambda} - \Delta\lambda$, where $\overline{\lambda}$ is the average value of λ, which is $\left(\lambda + \lambda^*\right)/2$ and $\Delta\lambda$ represents the asymmetry, $\left(\lambda - \lambda^*\right)/2$. From the habit stock accumulation process, the relative habit stock can be written as:

$$h_t - h_t^* = \overline{\lambda}\left(c_{t-1} - c_{t-1}^*\right) + \left(1 - \overline{\lambda}\right)\left(h_{t-1} - h_{t-1}^*\right) + \Delta\lambda\left[\left(c_{t-1} + c_{t-1}^*\right) - \left(h_{t-1} + h_{t-1}^*\right)\right] \qquad (40)$$

The first two terms are similar to the relative habit stock in the symmetric case as shown in Eq. (33), while the far right term arises because of the asymmetric λ. It will disappear when $\lambda = \lambda^*$.

In the calibration in Figure 6 (a), $\overline{\lambda}$ is fixed at 0.5 and $\Delta\lambda$ ranges from 0.1 to 0.5. The result shows that $\rho(q)$ moves approximately from 0.78 to 0.85 when $\Delta\lambda$ increases from 0.1 to 0.5. Since $\rho(q)$ is about 0.78 in Figure 2 when $\lambda = 0.5$, we know that the asymmetry in λ does strengthen $\rho(q)$, but fails to raise $\rho(q)$ above 0.85 as the case $\lambda = 0$ reported in

Figure 2. In Figure 6 (b), we fix $\Delta\lambda$ at 0.1 and the range of $\overline{\lambda}$ is specified between 0.1 and 0.9. $\rho(q)$ follows approximately the same declining trend with the increase in λ as in the Figure 2. Again, the asymmetry in λ does not contribute to the real exchange rate persistence, while the level diminishes the persistence. In summary, the numerical assessments in this section show that alternative specifications of α and λ can not reverse the negative effect of habit formation.



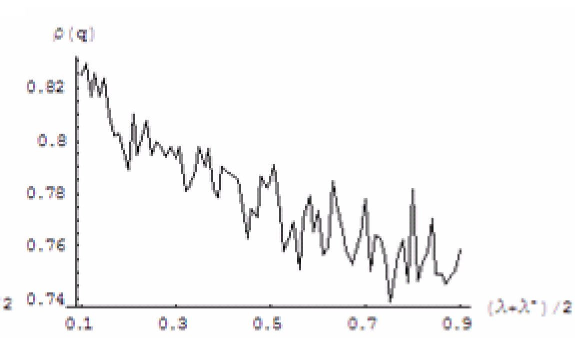


Figure 6 (a): $\Delta\lambda$ Figure 6 (b): $\overline{\lambda}$

Figure 6: AR(1) Coefficient of Real Exchange Rate, $\rho(q)$, with Asymmetric λ

6 Conclusion

We employ a highly tractable DSGE model to demonstrate the effect of external habit formation on the real exchange rate. While habit formation was considered to be effective in enhancing real exchange rate persistence by increasing consumption persistence, the results in this study have shown that external habit formation imposes a negative effect on real exchange rate persistence. This result conforms to recent empirical findings. The analytical solutions in the model explicate the intuition.

The main reason that the real exchange rate persistence is lowered by habit formation is the trade-off between the effects of habit stocks and past real shocks on wages. Because of the serial correlation of the productivity shock, a positive productivity shock, which decreases current wages, will lead to lower wages in the following period. At the same time, the productivity shock also increases the contemporaneous consumption and the habit stock for the next period. The increase in the habit stock raises the expected consumption demands and results in higher wages, which offsets the effect of real shocks on wages. The trade-off will be passed onto the real exchange rate, which is primarily determined by relative wage. The effect of habit stock, which contains the information on past real shocks and is the opposite of the effect of real shocks on the level of real exchange rates, dampens the real exchange rate persistence, as primarily determined by autocorrelated real shocks.

Calibration results show that real exchange rate persistence significantly decreases with the importance of habit formation and the adjustment speed of habit stock accumulation. We also consider alternative cases with the negative externality of external habit formation, asymmetries in the importance of habit formation and the adjustment speed of habit stock accumulation across countries. The numerical assessments suggest that neither the negative externality nor the asymmetries can increase real exchange rate persistence beyond the level in the case without habit formation.

The results in this paper are based on several assumptions. In this model, wage stickiness is the main source of nominal rigidity. The optimal pricing strategy of firms will vary inversely with current productivity shocks, and thus real shocks tend to have higher impact on prices than the case with price stickiness. However, habit formation may still reduce real exchange rate persistence in a price stickiness model, because prices will be preset higher with the greater expected consumption due to greater habit stock. Moreover, without staggered pricing, we may have neglected the interaction between habit formation and the degree of the nominal stickiness.

The key assumptions that may cause a major difference are the asset market structure and the logarithmic money demand function, which determine the movement of nominal exchange rates. These two assumptions insulate nominal exchange rates from real shocks and habit stock. If either of these two assumptions is relaxed, changes in habit formation due to past shocks will lead to current nominal exchange rate movements. This may have important implications for habit formation's effect on the real exchange rate.

Another interesting issue to be explored is alternative monetary policies. If we use the interest rate rule, as specified in Obstfeld (2004) and Benigno (2004), instead of the control over money supply, as the instrument of central banks' policies, external habit formation may incur different effects on real exchange rate persistence depending on how the past shocks are passed.

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APPENDIX:

Appendix A: The First-Order Conditions

(i) The Euler Equation:

The first-order conditions of each individual i are as follows:

$$\frac{\chi}{M^i(s_t)} = \frac{C^{i-\rho}(s_t)H^{\alpha(\rho-1)}(s_t)}{P(s_t)} - E_t\beta\frac{C^{i-\rho}(s_{t+1})H^{\alpha(\rho-1)}(s_{t+1})}{P(s_{t+1})} \quad (13a)$$

$$B(s_{t+1}|s_t) = \beta\pi(s_{t+1}|s_t)\frac{C^i(s_{t+1})^{-\rho}H(s_{t+1})^{\alpha(\rho-1)}}{C^i(s_t)^{-\rho}H(s_t)^{\alpha(\rho-1)}}\frac{P(s_t)}{P(s_{t+1})} \quad (13b)$$

Eq. (13a) can be written as

$$\frac{\chi P(s_t)C^{i\rho}(s_t)H^{\alpha(1-\rho)}(s_t)}{M^i(s_t)} = 1 - \beta E_t\frac{C^{i-\rho}(s_{t+1})H^{\alpha(\rho-1)}(s_{t+1})P(s_t)}{C^{i-\rho}(s_t)H^{\alpha(\rho-1)}(s_t)P(s_{t+1})} \quad (A.1)$$

Writing Eq. (13b) for each possible state in the next period, s_{t+1}, and taking the sum across states, we are able to obtain the following condition:

$$\sum_{s_{t+1}} B(s_{t+1}|s_t) = \beta E_t\frac{C_{t+1}^{-\rho}H_{t+1}^{\alpha(\rho-1)}}{C_t^{-\rho}H_t^{\alpha(\rho-1)}}\frac{P_t}{P_{t+1}} \quad (A.2)$$

If there is only one risk-free bond, the condition will become:

$$\frac{1}{1+i_t} = \beta E_t\frac{C_{t+1}^{-\rho}H_{t+1}^{\alpha(\rho-1)}}{C_t^{-\rho}H_t^{\alpha(\rho-1)}}\frac{P_t}{P_{t+1}} \quad (A.3)$$

where i_t is the risk-free nominal interest rate. Thus, let $\sum_{s_{t+1}} B(s_{t+1}|s_t) = \frac{1}{1+i_t}$, Eq. (A.1) can be written as:

$$\frac{M_t^i}{P_t^i} = \chi C_t^{i\rho} H_t^{\alpha(1-\rho)} \frac{1+i(s_t)}{i(s_t)} \tag{14}$$

(ii) The Closed-Form Solution to the Money Demand Function under the Logarithmic Utility Function

As shown in Obstfeld and Rogoff (1998), the first order condition in Eq. (13a),

$$\frac{\chi}{M_t^i} = \frac{C_t^{i-\rho} H_t^{i\alpha(\rho-1)}}{P_t} - E_t \beta \frac{C_{t+1}^{i\ -\rho} H_{t+1}^{i\ \ \alpha(\rho-1)}}{P_{t+1}},$$

can be solved explicitly. This equation can be written as:

$$\begin{aligned} 1 &= \chi \frac{P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} + \beta \frac{\chi P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} E_t \frac{M_t C_{t+1}^{-\rho} H_{t+1}^{\alpha(\rho-1)}}{\chi P_{t+1}} \\ &= \chi \frac{P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} + \beta \frac{\chi P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} E_t \left(\frac{M_t}{M_{t+1}} \frac{M_{t+1}}{\chi P_{t+1} C_{t+1}^{\rho} H_{t+1}^{\alpha(1-\rho)}}\right) \end{aligned} \tag{A.4}$$

The path of home money supply is specified as $M_t = \mu_t M_{t-1}$ where $\eta_t = \eta + \varepsilon_{\eta,t}$, $\varepsilon_{\eta,t} \sim N(0, \sigma_\eta^2)$, $\eta_t = \log \mu_t$ in Eq. (12a). If

$$\frac{\chi P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} = \varphi$$

which is a constant over time, given the assumption that the consumption process is stationary. Thus the equation above can be written as:

$$1 = \chi \frac{P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} + \beta E_t \left(\frac{M_t}{M_{t+1}}\right) \tag{A.5}$$

Substitute the money supply into the equation, then

$$\chi \frac{P_t C_t^{\rho} H_t^{\alpha(1-\rho)}}{M_t} = 1 - \frac{\beta E_t (e^{-\varepsilon_{\eta,t+1}})}{\mu}$$

Since $e^{\varepsilon_{\eta,t+1}}$ follows lognormal distribution, $E_t e^{-\varepsilon_{\eta,t+1}} = \exp\left(\sigma_\eta^2/2\right)$. The following relation shall hold:

$$\frac{M_t}{P_t} = \frac{\chi\mu}{\mu - \beta e^{\sigma_\eta^2/2}} C_t^\rho H_t^{\alpha(1-\rho)} \tag{16a}$$

(iii) The Risk-Sharing Condition

Rearrange the equation above, we are able to obtain Eq. (16a) :

$$C_t^\rho H_t^{\alpha(1-\rho)} = \frac{\mu - \beta e^{\sigma_\eta^2/2}}{\chi\mu} \frac{M_t}{P_t}$$

From Eq. (13b) and (14b) in complete asset markets, we can obtain the following equation:

$$\frac{C(s_t)^{-\rho} H(s_t)^{\alpha(\rho-1)}}{P(s_t)} \frac{P(s_{t-1})}{C(s_{t-1})^{-\rho} H(s_{t-1})^{\alpha(\rho-1)}} = \frac{C^*(s_t)^{-\rho^*} H^*(s_t)^{\alpha^*(\rho^*-1)}}{\varsigma(s_t) P^*(s_t)} \frac{\varsigma(s_{t-1}) P^*(s_{t-1})}{C^*(s_{t-1})^{-\rho^*} H^*(s_{t-1})^{\alpha^*(\rho^*-1)}}$$

By substituting the preceding equation recursively back to the initial period, the international risk-sharing condition can be written as

$$\frac{\varsigma(s_t) P^*(s_t)}{P(s_t)}$$
$$= \left[\frac{\varsigma(s_0) P^*(s_0)}{P(s_0)} \frac{C(s_0)^{-\rho} H(s_0)^{\alpha(\rho-1)}}{C^*(s_0)^{-\rho^*} H^*(s_0)^{\alpha^*(\rho^*-1)}} \right] \frac{C^*(s_t)^{-\rho^*} H^*(s_t)^{\alpha^*(\rho^*-1)}}{C(s_t)^{-\rho} H(s_t)^{\alpha(\rho-1)}} = \vartheta \frac{C^*(s_t)^{-\rho^*} H^*(s_t)^{\alpha^*(\rho^*-1)}}{C(s_t)^{-\rho} H(s_t)^{\alpha(\rho-1)}}$$

Where

$$\vartheta = \left[\frac{\varsigma(s_0) P^*(s_0)}{P(s_0)} \frac{C(s_0)^{-\rho} H(s_0)^{\alpha(\rho-1)}}{C^*(s_0)^{-\rho} H^*(s_0)^{\alpha(\rho-1)}} \right]$$

This represents the variables in the initial states. Although we assume the initial states in these two countries are not identical, we can still make the assumption that ϑ=1.

Appendix B: Log-linearization of Labor Employment

The market clearing conditions in Eq. (26) are as follows:

$$A_t L_t = \left(1-\frac{\gamma}{2}\right)\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{\gamma/2} C_t + \frac{\gamma}{2}\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{1-\gamma/2} C_t^* \qquad \text{(26a)}$$

$$A_t^* L_t^* = \frac{\gamma}{2}\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{\gamma/2-1} C_t + \left(1-\frac{\gamma}{2}\right)\left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{-\gamma/2} C_t^* \qquad \text{(26a)}$$

From the risk-sharing condition, we know that the real exchange rate equals:

$$Q_t = \left(\frac{\varsigma_t W_t^* / A_t^*}{W_t / A_t}\right)^{1-\gamma} = \frac{C_t^{\rho} H_t^{\alpha(1-\rho)}}{C_t^{*\rho} H_t^{*\alpha(1-\rho)}} \qquad \text{(A.6)}$$

Substitute $\left(\varsigma_t W_t^* / A_t^*\right) / \left(W_t / A_t\right)$ in terms of consumption from Eq. (A.6) into Eq. (26) such that productions are simply functions of consumptions. Thus the productions can be written as:

$$A_t L_t = \left(1-\frac{\gamma}{2}\right)\left(C_t^{\frac{\gamma\rho+2(1-\gamma)}{2(1-\gamma)}} C_t^{*\frac{-\gamma\rho}{2(1-\gamma)}} H_t^{\frac{\alpha\gamma(1-\rho)}{2(1-\gamma)}} H_t^{*\frac{-\alpha\gamma(1-\rho)}{2(1-\gamma)}}\right) + \frac{\gamma}{2}\left(C_t^{\frac{\rho(2-\gamma)}{2(1-\gamma)}} C_t^{*\frac{-\rho(2-\gamma)+2(1-\gamma)}{2(1-\gamma)}} H_t^{\frac{\alpha(2-\gamma)(1-\rho)}{2(1-\gamma)}} H_t^{*\frac{-\alpha(2-\gamma)(1-\rho)}{2(1-\gamma)}}\right) \qquad \text{(A.7a)}$$

$$A_t^* L_t^* = \frac{\gamma}{2}\left(C_t^{\frac{\rho(\gamma-2)+2(1-\gamma)}{2(1-\gamma)}} C_t^{*\frac{-\rho(\gamma-2)}{2(1-\gamma)}} H_t^{\frac{\alpha(\gamma-2)(1-\rho)}{2(1-\gamma)}} H_t^{*\frac{-\alpha(\gamma-2)(1-\rho)}{2(1-\gamma)}}\right) + \left(1-\frac{\gamma}{2}\right)\left(C_t^{\frac{-\rho}{2(1-\gamma)}} C_t^{*\frac{-\gamma\rho+2(1-\gamma)}{2(1-\gamma)}} H_t^{\frac{-\alpha\gamma(1-\rho)}{2(1-\gamma)}} H_t^{*\frac{\alpha\gamma(1-\rho)}{2(1-\gamma)}}\right) \qquad \text{(A.7b)}$$

Take the log-linearization to the right-hand side of these two equations around the steady state, $\bar{C} = \bar{C}^* = \bar{H} = \bar{H}^*$, then we can write the labor employment as in Eq. (29):

$$l_t = \frac{(2-\gamma)(\gamma\rho+1-\gamma)}{2(1-\gamma)} c_t + \frac{\gamma\rho(\gamma-2)+\gamma(1-\gamma)}{2(1-\gamma)} c_t^* + \frac{\alpha\gamma(1-\rho)(2-\gamma)}{2(1-\gamma)} h_t + \frac{\alpha\gamma(\rho-1)(2-\gamma)}{2(1-\gamma)} h_t^* - a_t \qquad \text{(29a)}$$

$$l_t^* = \frac{\gamma\rho(\gamma-2)+\gamma(1-\gamma)}{2(1-\gamma)} c_t + \frac{(2-\gamma)(\gamma\rho+1-\gamma)}{2(1-\gamma)} c_t^* + \frac{\alpha\gamma(\rho-1)(2-\gamma)}{2(1-\gamma)} h_t + \frac{\alpha\gamma(1-\rho)(2-\gamma)}{2(1-\gamma)} h_t^* - a_t^* \quad (29b)$$

Substitute the money market clearing condition into the expected relative employment by taking the difference between the expectations of these two equations (or the risk-sharing condition) to write the relative employment in terms of relative wage:

$$E_{t-1}\left(l_t - l_t^*\right) = \frac{\gamma\rho(2-\gamma)+(1-\gamma)^2}{\rho}\left(w_t - w_t^*\right) + \frac{\alpha(\rho-1)(1-\gamma)}{\rho}\left(h_t - h_t^*\right) + \frac{(1-\rho)(1-\gamma)^2}{\rho}\rho_a\left(a_{t-1} - a_{t-1}^*\right)$$
$$+ \frac{\gamma\rho(2-\gamma)+(1-\gamma)^2}{\rho} E_{t-1}\left(m_t - m_t^*\right) \quad (A.8)$$

Combining Eq. (A.8) and the relative wage from the difference between Eq. (28a) and (28b), we can solve relative wage as in Eq. (30):

$$w_t - w_t^* = d_{w1}\left(h_t - h_t^*\right) + d_{w2}\left(a_{t-1} - a_{t-1}^*\right) + \left(m_{t-1} - m_{t-1}^*\right) \quad (30)$$

where d_{w1} and d_{w2} are defined as in the text.

Appendix C: The Information Contained in the Habit Stock

From the habit stock adjustment process in Eq. (9), relative habit stock can be written as

$$h_t - h_t^* = \lambda\left(c_{t-1} - c_{t-1}^*\right) + (1-\lambda)\left(h_{t-1} - h_{t-1}^*\right)$$

Substitute the closed-form solution of relative consumption from Eq. (36), then the equation above can be rewritten as the difference equation of relative habit stock:

$$h_t - h_t^* = \left(\lambda d_{c1} + (1-\lambda)\right)\left(h_{t-1} - h_{t-1}^*\right) + \lambda d_{c2}\rho_a\left(a_{t-2} - a_{t-2}^*\right) + \frac{\lambda(1-\gamma)}{\rho}\left[\left(\varepsilon_{a,t-1} - \varepsilon_{a.t-1}^*\right) + \left(\eta_{a,t-1} - \eta_{a.t-1}^*\right)\right] \quad (A.9)$$

Thus we can substitute the relative habit stock recursively back to the first period:

$$h_t - h_t^* = B^{t-1}\left(h_1 - h_1^*\right) + \lambda d_{c2}\rho_a \sum_{j=0}^{t-2} B^j \left(a_{t-2-j} - a_{t-2-j}^*\right) + \frac{\lambda(1-\gamma)}{\rho}\sum_{j=0}^{t-2} B^j \left[\left(\varepsilon_{a,t-1-j} - \varepsilon_{a.t-1-j}^*\right) + \left(\eta_{a,t-1-j} - \eta_{a.t-1-j}^*\right)\right] \tag{A.10}$$

where $B = \lambda d_{c1} + 1 - \lambda < 1$ and $h_1 - h_1^* = \lambda\left(c_0 - c_0^*\right) + (1-\lambda)\left(h_0 - h_0^*\right)$. Assume $h_0 = h_0^*$ but $c_0 \neq c_0^*$, then $h_1 - h_1^* = \lambda\left(c_0 - c_0^*\right)$. Thus, relative habit stock can be presented as in Eq. (37):

$$h_t - h_t^* = \lambda B^{t-1}\left(c_0 - c_0^*\right) + \lambda d_{c2}\rho_a \sum_{j=0}^{t-2} B^j \left(a_{t-2-j} - a_{t-2-j}^*\right) + \frac{\lambda(1-\gamma)}{\rho}\sum_{j=0}^{t-2} B^j \left[\left(\varepsilon_{a,t-1-j} - \varepsilon_{a.t-1-j}^*\right) + \left(\eta_{a,t-1-j} - \eta_{a.t-1-j}^*\right)\right] \tag{37}$$