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Path-dependent processes and the emergence of the rank size rule

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Abstract. Many things in the natural world consist of an ever-larger number of ever-smaller pieces. This is called a fractal, which implies both the power law and rank size rule. Various models have been applied to explain the power law or Zipf's law in the distribution of city size. Gibrat's law proposes general and neat interpretations for this regularity in a city distribution, but the homogeneity assumption in Gibrat's law shows a disregard of the agglomeration effect that is essential in economic interpretation. The purpose of this paper is to examine the relation between the feature of increasing returns in the dynamic growth process and the property of power law in the static limiting distribution. We apply the path-dependent processes in Authur (2000) called *nonlinear Polya processes* to analyze the relation between the feature of agglomeration in the path-dependent processes and rank-size relations in the limiting distributions. The simulation result shows that the growth process with a diminishing returns' agglomeration economy or a bounded increasing returns' agglomeration economy converges to a stable limiting distribution with a constant expected proportion. On the contrary, the growth process with an unbounded increasing returns' agglomeration economy could generate a fractal kind of limiting distribution with a time variant expected value. The unbounded increasing returns' agglomeration economy is the necessary condition to generate the rank size rule in the limiting distribution. Given the assumption of agglomeration economies and robust evidence of Zipf's in city distribution, our result suggests that agglomeration benefits increase without a ceiling as residents are added to the city. The increase of the diseconomies of agglomeration (congestion, pollution, crime, etc.) is not too severe to confine the limiting level of the net agglomeration effect.

JEL classification: R120

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1. Introduction

It is widely recognized that the size distribution of cities is surprisingly well described by Zipf's law across countries with various economic structures and histories. Zipf's law, which is a special case of the power law, essentially characterizes the size distribution of cities. The general power law not only appears in cities distribution, but also in other subjects. Shiode and Batty (2000) show that the most mature domains with the most pages follow the power law; moreover, Adamic (2001) shows that the distribution of the number of AOL users' visits to various sites in 1997 fits the power law. Distribution, which follows the power law, is a part of the family of fractal.

Different models have been applied to explain the power law or the special case of Zipf's law (see e.g., Losch 1954; Hoover 1954; Beckman 1958; Simon 1955; Simon and Bonini 1958; Fujita et al. 1999; Gabaix 1999; Solomon et al. 2000, 1998, 1996; Ferdinando Semboloni 2001). Simon (1955) proposes a stochastic model of the growth process to explain the Pareto kind distribution of firms' size. The stochastic model is formalized into Gibrat's law of proportionate effect. Gabaix (1999) proposes Gibrat's law as an explanation of Zipf's law based on a probabilistic process. He finds that homogeneous growth processes in cities could lead the distribution to converge into the Zipf pattern. Solomon et al. (2000, 1998, 1996) study the dynamical systems whose sizes evolve according to multiplicative stochastic rules. They show that the dynamics and the way in which the minimal size is enforced are crucial for obtaining power laws and the particular values of their exponents. Ferdinando Semboloni (2001) proposes a model based on agents with dycotomic goals to show how a rank-size distribution can be generated. Their works propose general and neat interpretations for this regularity in a city distribution; however, the homogeneity assumption of growth processes in Gibrat's law and the random number assumption of the multiplicative stochastic rules in Solomon's work show a disregard of the agglomeration effect that is essential in economic interpretation.

The distribution of cities and the distribution of website users are different subjects; however, the dynamic generating processes in both cases may contain certain features that could result in a similar limiting distribution. The effect of agglomeration economies is indispensable in the formation and growth of a city; nevertheless, it is nearly absent in studies about the Zipf pattern or power law of the limiting distribution. The underlying mechanism of the agglomeration economies in urban economics is analogous to the feature of positive feedback in the increasing returns. The purpose of this paper is to investigate the possible underlying relation between agglomeration economies and the rank size relation in the limiting distribution of city size. In general, this study will examine the relation between the feature of increasing returns in the dynamic growth process and the property of power law in the static limiting distribution. We analyze and simulate Authur's (2000) model based on the general Polya processes with a path-dependent property to examine the rank-size relations in the limiting distributions and to examine the features that could generate the power law. In Sect. 2, we introduce the fractal distribution and increasing returns. The proposed path dependent stochastic model from Authur (2000) is described and discussed in Sect. 3. In Sect. 4, simulation results are presented and concluding remarks are formulated in Sect. 5.

2. Fractal distribution and increasing returns

2.1. Fractal distribution and the power law

The assumption of normality implies that data can be meaningfully characterized by a constant mean and variance. However, much of nature does not contain a unique mean and variance and is not "normal". Many distributions in the natural world consist of an ever-larger number of ever-smaller pieces. This is called a fractal, which can be an object in space or a process in time. This fractal system has been observed in various fields, such as in the physical, biomedical, and social sciences (Bunde and Havlin 1994; Liebovitch 1998; Bassingthwaighte et al. 1994; Lannaccone and Khokha 1995; Dewey 1997; Batty and Longley 1994; Peters 1994). For example, fractal systems have shown up in the timing of heart attacks, blood vessels of the circulatory system, the surfaces of proteins, durations of consecutive breaths, the distribution of cities, and the number of users visiting various websites.

A general distribution function of the fractal system has the power function form:

$$y = f(x) = Ax^{-\alpha}, \tag{1}$$

where $f(x)$ is the probability density function (PDF) of x; this can be transformed to

$$\log(y) = \log(A) - \alpha\log(x). \tag{2}$$

This Eq. (2) explains the essential feature of the fractal distribution that it is a straight line with a negative slope on a plot of the log [PDF(x)] versus log(x), which is called the power law. A fractal from a process in time could be characterized by a parameter, α, which measures the relative number of smaller values compared to the large values. An example of fractal distribution is shown in Fig. 1.

In a fractal distribution, the population mean is not defined since the sample mean does not converge to a constant. Both the mean and variance of a fractal distribution depend on the amount of data analyzed, and consequently, the average number and variance can no longer characterize data in fractal systems. Different from the normal distribution, fractal distribution is defined by the linearity of the power law form of the PDF, and the corresponding slope characterizes the fractal distribution. A constant slope α implies a constant size elasticity of PDF, ε_{fx}.

$$\alpha = \varepsilon_{fx} = \lim_{\Delta x \to 0} \frac{\Delta f(x)/f(x)}{\Delta x/x}. \tag{3}$$

In a fractal distribution, the percentage change in the size's PDF due to a percentage change in its size does not vary by size.

The Pareto distribution shows the probability that a value is greater than or equal to a certain value, which is given in terms of the cumulative distribution function (CDF). A power law distribution has the following Pareto distribution:

$$P[X > x] = \left(\frac{A}{\alpha - 1}\right)x^{-(\alpha-1)} = \left(\frac{A}{\alpha - 1}\right)x^{-\beta}, \tag{4}$$

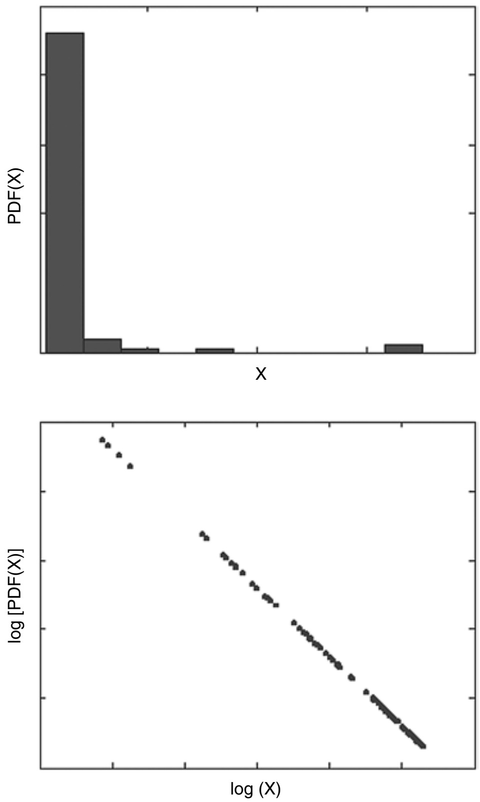

Fig. 1. Fractal distribution

where $\beta = \alpha - 1$. The cumulative distribution function could be interpreted as the rank of size x; thus, the Pareto distribution in (4) implies that the rank of the largest occurrence for size x is inversely proportional to size x with a constant exponent. This is called the rank size rule:

$$Rank = B * Size^{-\beta} \tag{5}$$

$$\log(Rank) = \log(B) - \beta \log(Size). \tag{6}$$

The rank size rule becomes Zipf's law when the exponent $\beta = 1$.

Data overall from a fractal system is defined by the form called the power law and is characterized by its slope. In addition, the data also fulfills the Pareto law and the rank size rule. If the exponent in the rank size rule (5) equals one, then the data also fits Zipf's law. In short, fractal distribution implies both the power law and rank size rule.

2.2. Increasing returns and agglomeration economies

Both the equilibrium and optimal solution in conventional economic theory are derived from the assumption of diminishing returns. Diminishing returns imply stabilization and a single equilibrium point for an economy. In many parts of an economy, unstabilized forces do appear. Arthur (1984) conducts work on the problem of increasing returns in an economy and mentions that western economies have undergone a transformation from the processing of resources to the processing of information. He states that the resource-based part of an economy for the most part appears to have diminishing returns, while the knowledge-based economy is largely subject to increasing returns. This is similar to the attribute of positive feedback in the information economy. The high fixed costs and low marginal costs of production information lead to supply-side economies of scale; in addition, the network externalities and positive feedback of information products lead to demand-side economies of scale (see Shapiro and Varian 1999). Positive feedback makes large networks become bigger; this is the feature of increasing returns in the demand side.

The underlying mechanisms of economic behavior have shifted from diminishing returns to increasing returns, and increasing returns, driven by self-reinforcement and positive feedback, generate not only an equilibrium, but also instability. The evolution process of increasing returns is non-predictable, locked-in, and historically dependent. It is modeled as dynamic and non-linear rather than static and deterministic. The growth of cities is essentially characterized by the effect of agglomeration economies. The agglomerative economies, which emerge both among firms and residents, include positive and negative externalities caused by firms (residents) locating close to one another. Firms cluster to decrease their production costs (agglomeration economies in production), or increase their production revenue (agglomeration economies in marketing). Similar to the urbanization economies in production, the gathering of residents allows the realization of scale economies in the provision of business service and public services.

Higher accessibilities to the work opportunities in the large cities also favor residents' interest. Residents cluster to increase their utility of living in the site. The mechanism of agglomeration economies is analogous to the positive feedback and the network externalities in the demand side of information economy. Similar to the network externalities in the demand side of an information economy, agglomeration externalities make large cities become even bigger. The feature of increasing returns of the formation of a city shows in the demand side. A growth process with an increasing returns feature is proposed to explain the distribution of city size in the following section.

3. The nonlinear path-dependent Polya processes

The model applied in this paper is developed and introduced in Arthur (1984, 2000); it is based on a class of path-dependent stochastic processes called *nonlinear Polya processes*. This path-dependent process is adequate in interpreting the features of positive feedback and agglomeration externalities. The long-run limiting behavior of this nonlinear Polya-type, path-dependent process is examined to investigate the possible relation between the dynamic increasing returns process and the static fractal distribution.

Assume residents decide on locating in one of N possible cities in the region. Let $s_t^i (i = 1, \ldots, N)$ describe the city size for each city at time t; and $x_t^i (i = 1, \ldots, N)$ describes the proportion of population of city i in the region at time t. Assume the benefits, $r_j^i (i = 1, \ldots, N)$, of resident j for locating in city

i, consist of two components: geographical benefit and the agglomeration benefit.

$$r^i_j = q^i_j + g(x^i), \tag{7}$$

where q^i_j is the geographical benefit to resident j for locating in site i; and $g(x^i)$ represents the agglomeration benefit of resident in site i. The location attractiveness due to geographical considerations is independent of the current location's shares.

The economies of agglomeration in the location choice showed by the positive agglomeration benefit of the resident ($g'(x^i) > 0$) occurs if the utility of a resident increases as the location's shares (relative size), x^i_t, increases. The agglomeration benefit $g(x^i)$ is the external benefit resulting from the scale of the entire urban economy: the gathering of residents allows the realization of scale economies in the provision of business services and public services; also, higher accessibilities to the work opportunities in the large cities favor residents' interest. On the other hand, the diseconomies of agglomeration in the location choice (such as congestion, air pollution, and crime) are shown by the negative agglomeration benefit of residents ($g'(x^i) < 0$).

Assume that the geographical benefit is not resident specific (the homogeneity in tastes of the geographical benefit). The probability that the next resident prefers site i over all other sites is:

$$p^i = \Pr\mathrm{ob}\{[q^i + g(x^i)] > [q^j + g(x^j)] \quad all \quad j \neq i\}. \tag{8}$$

Consequently, given the time invariant geographical benefit, q^i, the probabilities of the locational choice for city i at time t, $p^i_t(x^i_t)$, depends on the current location's shares, x^i_t. The change of size at city i follows the dynamic process:

$$s^i_{t+1} = s^i_t + z^i_t(x^i_t), \quad i = 1, \ldots, N, \tag{9}$$

where

$$z^i_t = \begin{cases} 1 & with \quad probability \quad p^i_t(x^i_t) \\ 0 & with \quad probability \quad 1 - p^i_t(x^i_t) \end{cases},$$

$$E[z^i_t] = p^i_t(x^i_t) \quad Var[z^i_t] = E[(z^i_t)^2] = p^i_t(x^i_t).$$

Consequently,

$$s^i_t = s^i_0 + z^i_1 + z^i_2 + \cdots + z^i_{t-1}. \tag{10}$$

Each random variable, z^i_t, has an expected value, $p^i_t(x^i_t)$, which is a function of the current proportion rather than a time invariant constant.

$$E[s^i_t] = s^i_0 + p^i_1(x^i_1) + p^i_2(x^i_2) + \cdots + p^i_{t-1}(x^i_{t-1}). \tag{11}$$

$$Var[s^i_t] = \sum_{k=1}^{t-1} p^i_k(x^i_k) + 2\sum_{k \neq l}^{t-1} Cov(z^i_k, z^i_l). \tag{12}$$

Both the strong law of large numbers and the central limit theorem cannot be applied in this general Polya process, as the limiting size proportion does not exist. Both the mean and variance of city size actually varies by time, and the expected value of city size is not defined. In addition, according to Eq. (9), the evolution of the relative city size at city i is:

$$
\begin{aligned}
x_{t+1}^{i} &= x_{t}^{i} + \frac{1}{(w+t)}[z_{t}^{i}(x_{t}^{i}) - x_{t}^{i}] \\
&= x_{t}^{i} + \frac{1}{(w+t)}[p_{t}^{i}(x_{t}^{i}) - x_{t}^{i}] + \frac{1}{(w+t)}u_{t}^{i}(x_{t}^{i}), \quad i = 1, ..., N,
\end{aligned} \tag{13}
$$

where $w = \sum_i s_1^i$, which is the total population initially; and the disturbance term $u_t^i(x_t^i) = z_t^i(x_t^i) - p_t^i(x_t^i)$ is with zero conditional expectation. This path-dependent process consists of a determinate part, $x_t^i + \frac{1}{(w+t)}[p_t^i(x_t^i) - x_t^i]$; and a perturbation part, $\frac{1}{(w+t)}u_t^i(x_t^i)$. The determinate part includes the preceding proportion and the difference between the probability and the preceding proportion. In addition, the expected motion of the locational share depends on the determinate part, which contains the choice probability function.

$$
E[x_{t+1}^{i}|x_{t}^{i}] = x_{t}^{i} + \frac{1}{(w+t)}[p_{t}^{i}(x_{t}^{i}) - x_{t}^{i}]. \tag{14}
$$

The feature of the location choice probability function, $p_t^i(x_t^i)$, essentially characterizes the limiting proportion. In addition, the expected motion tends to be directed by the term $[p_t^i(x_t^i) - x_t^i]$ in the determinate part. A positive term would drive the expected motion to grow.

Case 1. If there are no economies or diseconomies of agglomeration in the location choice ($g(x^i) \equiv 0$ in (7)), which means that the location benefit is independent of the location's share, then the location choice probability function depends only on the predetermined geographical attributes, $p_t^i(q^i)$. The vector of the limiting proportion of N cities in the region is just the vector of the fixed location specific probability, $p_t^i(q^i)(i = 1, \ldots, N)$, determined by the given geographical attributes. The location share hence tends to converge to a single equilibrium point

Case 2. There are economies of agglomeration in the location choice ($g(x^i) \neq 0$ in (7)), and the choice probability equals the current proportion ($p_t^i(x_t^i) = x_t^i$). This is called the standard Polya process. The determinate part in (13) disappears, and the perturbation part dominates the motion. It is therefore proved that the vector of limiting expected proportions tends to be a fixed vector with a probability of one. (Polya 1931)

Case 3. There are economies and diseconomies of agglomeration in the location choice. The probability function, $p_t^i(x_t^i)$, is assumed to be non-linear. The stochastic process (13) with a non-linear probability function is called a non-linear Polya process (Arthur 2000). In the case of the non-linear Polya process, a negative first derivative of the probability function characterizes a diminishing returns process (such as Fig. 2(e)), $p_t^i(x_t^i) < 0$; the limiting expected proportion converges to a single equilibrium point, $\overline{x^i}$. Thus, the limiting corresponding probability is time invariant, $p^i(\overline{x^i})$. As a result, the limiting proportion of city i is $p^i(\overline{x^i})$, according to the strong law of large numbers. Assuming the probability function is not city specific, the limiting proportion is a constant, $p^1(\overline{x^1}) = ... = p^N(\overline{x^N}) = p(\overline{x})$. The mean of the size proportion is defined.

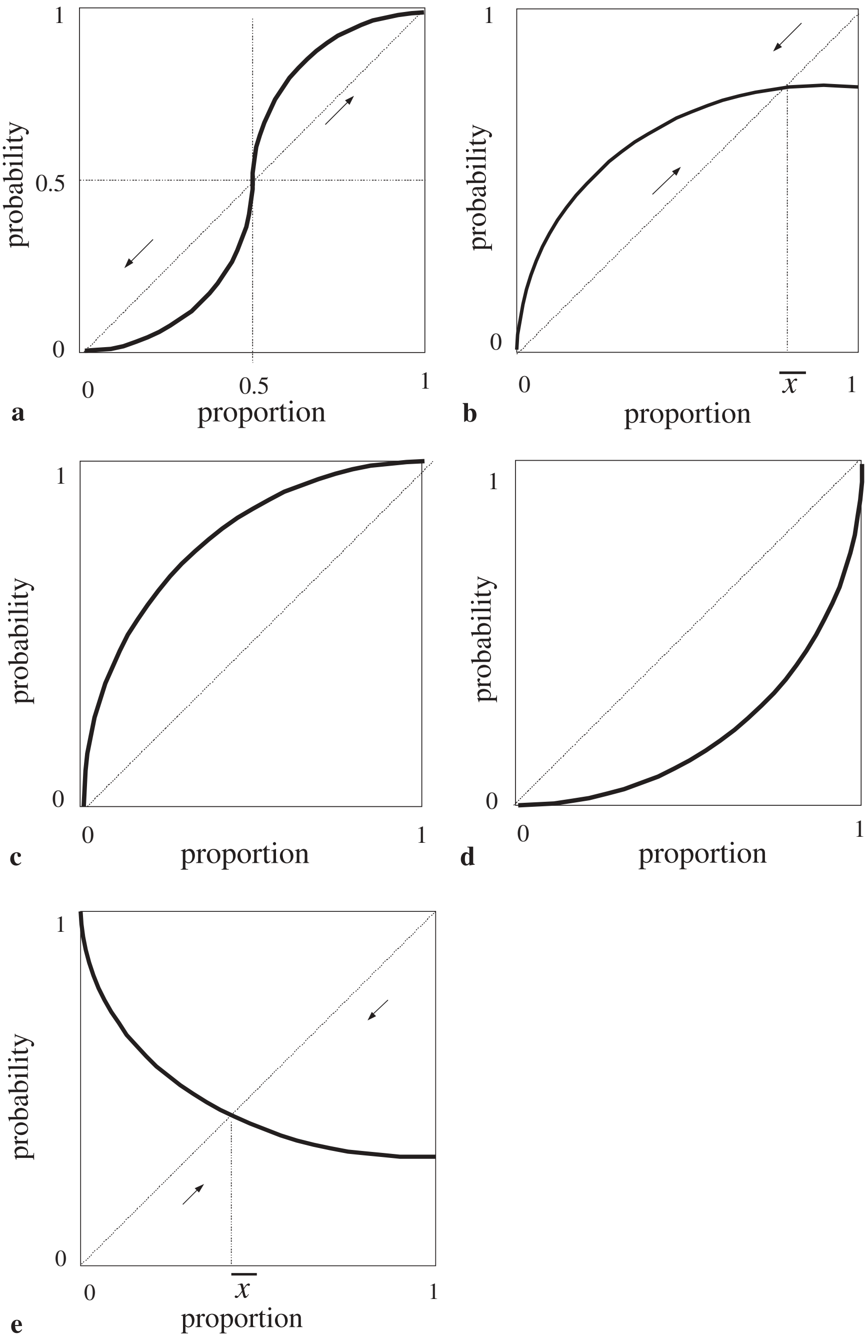

Fig. 2 a–e. Probability functions assumed in the Polya process. **a** Increasing returns (Unbounded agglomeration economies); **b** Increasing returns (Bounded agglomeration economies); **c** Increasing returns (Unbounded agglomeration economies); **d** Increasing returns (Unbounded agglomeration economies); **e** Diminishing returns

A positive first derivative of the probability function refers to an increasing returns process, $p_t^i(x_t^i) > 0$; the tendency of the city size proportion is to be attracted toward one or several fixed points depending on the functional form of the probability function.

The feature of positive feedback in the increasing returns process consists of two major categories: bounded agglomeration economies and unbounded agglomeration economies. Agglomeration economies are unbounded if the agglomeration benefits increase without ceiling as residents are added to a city (that is, if the function $g(x^i)$ is monotonically increasing without upper bound). This situation prevails when the problems of congestion, pollution, and crime compared to the advantages of agglomeration are not too severe to limit the net effect of agglomeration economies.

Increasing returns with bounded agglomeration economies (Fig. 2b), the net effect of economies and diseconomies of agglomeration with a ceiling, can be presented by a diminishing increasing returns of the probability function ($p_t^{i'}(x_t^i) > 0$, and $p_t^{i''}(x_t^i) < 0$). This situation prevails when the problem of congestion, pollution, and crime is becoming much more severe than the advantages of agglomeration as the size of a city increases; it results in a diminishing increase of the net effect of agglomeration economies with an upper limit. The limiting expected proportion converges to a single equilibrium point. Consequently, similar to the case of the diminishing returns process discussed above, the limiting proportion is a constant both in time and site under the condition of the homogeneous probability function, and hence the mean of size proportion is defined.

Increasing returns with unbounded agglomeration economies (Fig. 2a, c, d) are presented by increasing returns of the probability function without a ceiling ($p_t^{i'}(x_t^i) > 0$). The relative forces and interaction between economies and diseconomies of agglomeration characterize the shape of the net agglomeration economies as bounded or unbounded. Under this situation, the limiting expected proportion may not converge to a single equilibrium point, and a fixed expected proportion may not exist. It depends on the feature of agglomeration economies.

The question that we are interested in this paper is to ask whether these various possible equilibrium states, according to the different features of the path-dependent process, could characterize their limiting distribution? Moreover, is the limiting fractal distribution associated with certain features of the dynamic stochastic process?

4. Simulation

We simulate the proposed nonlinear path-dependent Polya process in Sect. 4 in order to analyze the asymptotic distribution properties of particular classes of stochastic equations, especially increasing returns. In this model one resident is added into the region at each time; the probabilities of an addition to a city depend on their current proportions. The functional form of the probability function is essential in characterizing both the growth process and its limiting distribution. Both cases of increasing returns and decreasing returns are simulated in this section.

4.1. Increasing returns with unbounded agglomeration economies: Function (a)

A Polya process given the probability function (a) in Fig. 2 characterizes increasing returns with unbounded agglomeration economies. The larger the size proportion in the region is, the higher the probability will be that the city will grow. Furthermore, as the city size proportion passes one half of the region, the probability that this city will grow is greater than 0.5 at a diminishing rate, showing a tendency toward 0 or 1. The effect of a change in relative size is highest when the choice probabilities indicate a high degree of uncertainty regarding the choice; as the choice becomes more certain, the effect of a given change in an observed variable lessens.

Assume a region of 50 cities starts with a uniformly-distributed city size; the simulating dynamic processes for all cities after 3000 iterations are shown in Fig. 3. The relative size distributions at different numbers of iterations (t) are shown in Fig. 4. This states that as the time increases, the location shares distribute closer to the fractal distribution. This distribution tendency can be

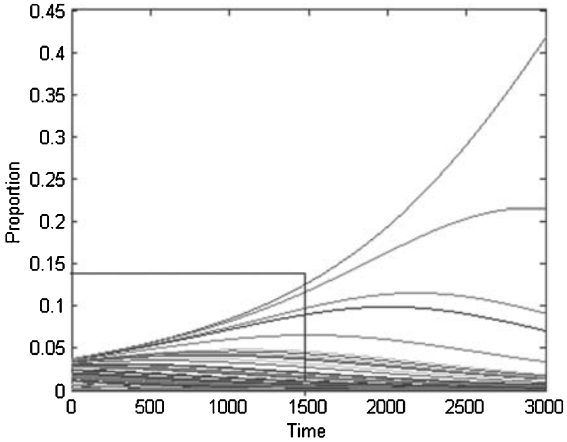

Fig. 3.1. The dynamic process of function (a): 50 cities after 3000 iterations starting from uniform distribution

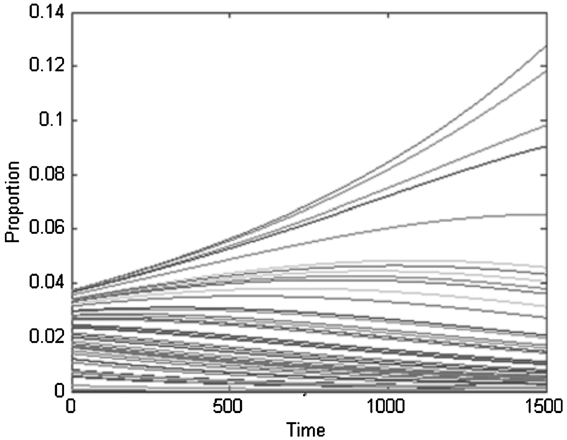

Fig. 3.2. The dynamic process of function (a): 50 cities after 1500 iterations starting from uniform distribution

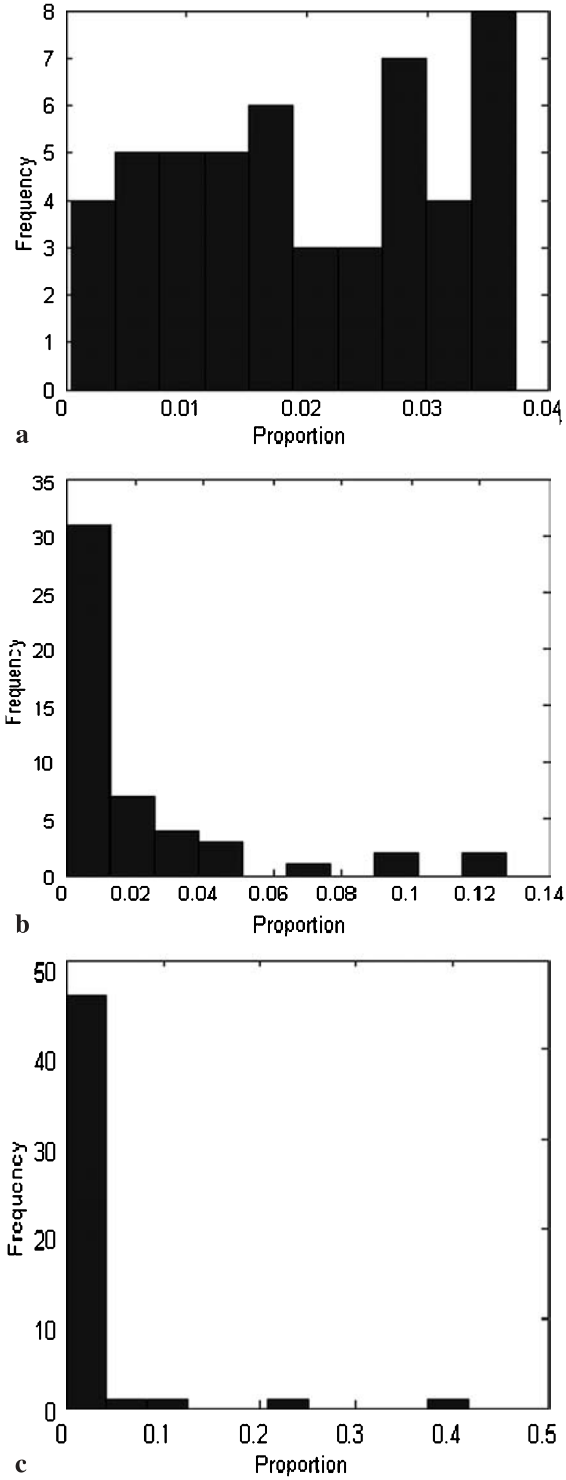

Fig. 4. a–c. The frequency distribution of proportion. **a** t = 1; **b** t = 1500; **c** t = 3000

observed in Fig. 5. The plot of log(Rank) versus log(Proportion) tends to be linear, which is the rank size rule.

Table 1 lists the estimated slopes and R-square of the plots in Fig. 5.2. As the number of iterations increases, the R-square value goes up, while the absolute value of the estimated slope decreases. A smaller absolute value of the estimated slope represents a more diversely-distributed city size in the region. The diminishing tendency of the absolute value of the slope and the increasing linearity of the log(Rank) versus log(Proportion) plot is consistent with the experimental city distribution.

4.2. Increasing returns with bounded agglomeration economies: Function (b)

The probability function shown in Fig. 2b characterizes increasing returns for a diminishing increasing rate. It reveals a tendency toward a fixed point $\bar{x}$. The simulating dynamic process of the region with 15 cities after 3000 iterations is produced in Fig. 6.1. The expected size proportion of all cities in the region

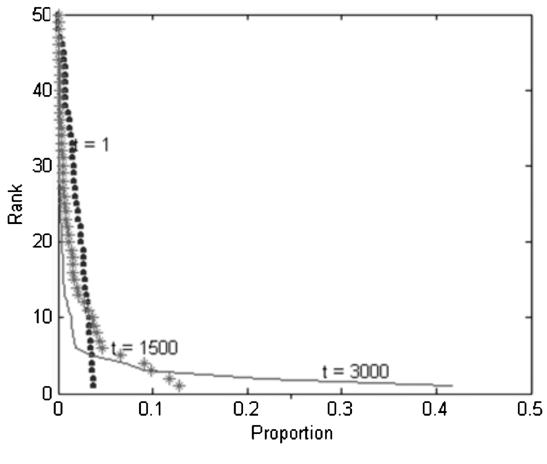

Fig. 5.1. The plot of proportion and rank of function (a)

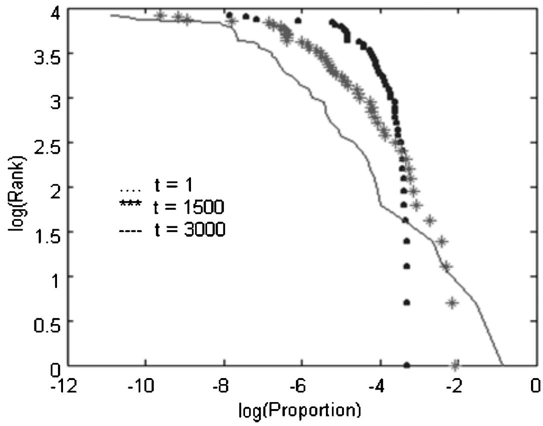

Fig. 5.2. The plot of log(Proportion) and log(Rank) of function (a)

Table 1. The regression result of the plot of the log(Rank) versus the log(Proportion) of function (a)

Iterations	Estimated slope	R-square
200	−0.5358	0.5072
400	−0.5129	0.5625
600	−0.4950	0.6098
800	−0.4805	0.6508
1000	−0.4683	0.6870
1200	−0.4579	0.7190
1400	−0.4488	0.7474
1600	−0.4408	0.7727
1800	−0.4336	0.7950
2000	−0.4271	0.8145
2200	−0.4211	0.8314
2400	−0.4157	0.8460
2600	−0.4108	0.8584
2800	−0.4063	0.8688
3000	−0.4023	0.8774

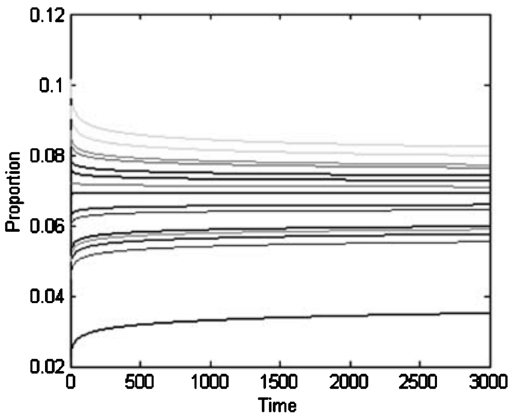

Fig. 6.1. The dynamic processes of probability function (b): 15 cities after 3000 iterations starting from uniform distribution

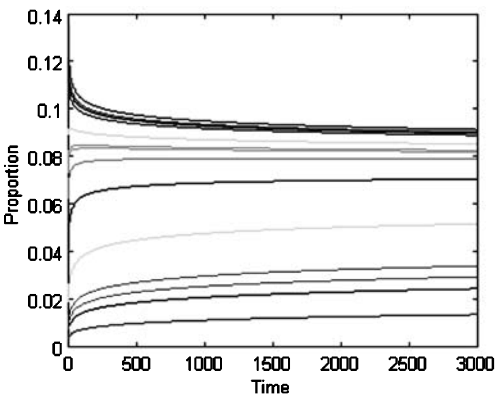

Fig. 6.2. The dynamic processes of probability function (c): 15 cities after 3000 iterations starting from uniform distribution

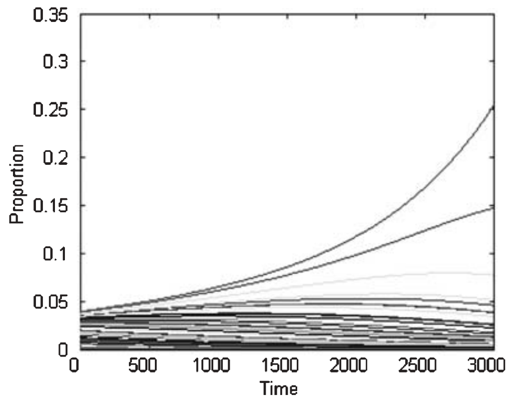

Fig. 6.3. The dynamic processes of probability function (d): 50 cities after 3000 iterations starting from uniform distribution

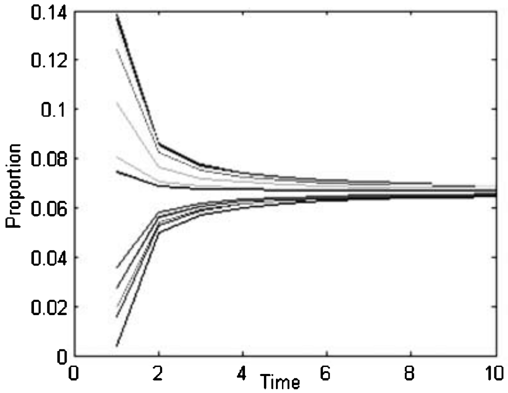

Fig. 6.4. The dynamic processes of probability function (e): 15 cities after 10 iterations starting from uniform distribution

tends to converge to a stable ratio. Moreover, the expected value of the limiting location does exist, which is very different from the fractal distribution.

4.3. Increasing returns with unbounded agglomeration economies: Function (c)

The probability function shown in Fig. 2c characterizes diminishing increasing returns, and it shows a tendency toward 1. The simulating dynamic process of the region with 15 cities after 3000 iterations is displayed in Fig. 6.2. The expected size proportion converges to a stable point. Similar to the case of function (b), the expected value of the limiting location shares does exist.

4.4. Increasing returns with unbounded agglomeration economies: Function (d)

The probability function shown in Fig. 2d characterizes rising increasing returns, and shows a tendency toward 0. The simulating dynamic process of the region with 50 cities after 3000 iterations is shown in Fig. 6.3. It is similar to the dynamic process of function (a), which offers a tendency toward a fractal distribution.

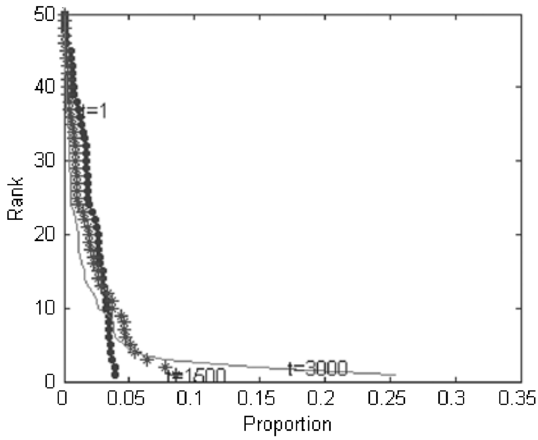

Fig. 7.1. The plot of proportion and rank of function (d)

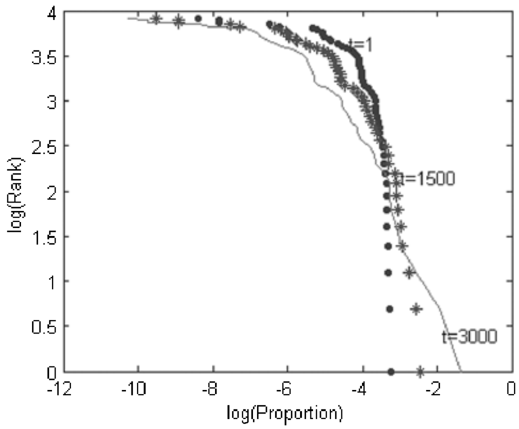

Fig. 7.2. The plot of log(Proportion) and log(Rank) of function (d)

4.5. Diminishing returns: Function (e)

The probability function shown in Fig. 2e characterizes diminishing returns. It appears to have a tendency toward a fixed point $\bar{x}$. The simulating dynamic process of the region with 15 cities after 10 iterations is displayed in Fig. 6.4. The expected size proportion converges much faster than the process of function (b) to a stable point. Consequently, the expected value of the limiting location shares does exist, and the limiting location shares do not distribute as a fractal.

The simulation results show that the dynamic process of diminishing returns and increasing returns with bounded agglomeration economies tend to converge to a stable point; there also exists a fixed expected locational pattern of proportions. By contrast, in most cases of increasing returns with unbounded agglomeration economies, a constant expected proportion does not exist, and the limiting distribution of the location shares tends to be fractal and displays the power law. It implies that the unbounded agglomeration economy may generate limiting fractal distribution; this is the necessary condition to generate the rank size rule (power law) in the limiting distribution.

5. Concluding remarks

The character and the simulation results of the proposed path-dependent processes are concluded as follows. If the benefits from agglomeration economies in a residents' location benefit are absent, then the size distribution depends only on the geographical benefit that does not contain positive feedback and path-dependent properties. Given the geographical endowment in the region and the assumption of homogeneous residents' location preferences, residents cluster according to the given geographical benefits. On the other hand, if residents' location tastes are heterogeneous, then the distribution of the city size is more dispersed than in the homogeneous case. Both the size evolution and limiting distribution tend to be stable and predictable. In the general case where agglomeration economies are present, the probability of attracting new residents depends upon any past addition, so that the standard strong law is not usable and the size evolution is historically dependent and non-predictable. The limiting distribution is closely related to the feature of the dynamic evolution process, especially the mechanism of the agglomeration economies.

The simulation results from the general Polya processes in Authur (2000) show that if the addition of residents confers a net benefit on a location, then under upper limit-bounded agglomeration economies, the dynamic process for each city's share tends to converge to a stable point and there exists a fixed expected value in the limiting distribution. On the other hand, if the addition of residents confers a net benefit on a location, then under no upper limit-unbounded agglomeration economies, the dynamic process of each city's share may tend to diverge to a fractal distribution that follows the power law. The expected location share is time variant and undefined. The unbounded agglomeration economy therein may result in a fractal limiting distribution; it is the necessary condition to generate the rank size rule in the limiting distribution. Given the assumption of agglomeration economies and robust evidence of Zipf's in city distribution, our result suggests the presence of the unbounded agglomeration economies for the benefit of residents' location. That is, agglomeration benefits increase without a ceiling as residents are added to the city; the increase of the diseconomies of agglomeration (congestion, pollution, crime, etc.) along with the growth of city size is not too severe to confine the limiting level of the net agglomeration effect.

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