The Interaction of Natural Science Models In Spatial Interaction Behavior

Hsin-Ping Chen

Associate Professor

Department of Economics

National Chengchi University

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Abstract

This paper serves three purposes. First, it gives a systematic review of interactions between some natural science concepts and regional science phenomena in both static and dynamic states. Second, it aims to understand why non-linear feature is crucial in the emergence of chaotic behavior. What role does "non-linear" play in a chaotic dynamic system? And finally, simulating the non-linear dynamic system to observe its features. This review shows that maximum entropy concept can be applied in the spatial interaction model, and result in a gravity type model; based on this gravity model, a logit discrete choice model is followed; consequently, a dynamic logit model will generate a logistic type growth model. It shows that these biological or physical based models are correlated and correspond to regional phenomena. From optimal entropy to generated dynamic logit model, they are vertically related. Horizontally, each natural science model interprets certain regional science phenomenon. Simulation results show that non-linear dynamic system is not only able to perform all regular trajectories of linear dynamic system, but also perform non-periodic irregular motion patterns given different initial conditions. The chaotic systems do not cause different irregular trajectories given the same initial conditions and parameter values. The "stochastic" term in describing chaotic behavior refers to its unpredictable and random time series path. Also, non-periodic evolution is extremely sensitive depending on the initial conditions. Non-linear is the necessary condition for the emergence of chaos; the level of parameter value is the sufficient condition for chaotic dynamic system.

Key words: entropy, gravity, and chaos.

1. INTRODUCTION

Natural science focuses on analyzing characteristics and interactions relations of particles or certain objects; similarly, research in social science mainly deals with the behavior and interactions of human beings. Human beings are part of the nature world; it is very likely to apply natural science theories to the analysis in social science fields, especially in economics. Medawar (1969) had refereed the analogies between different sciences, called reduction scheme, in the deterministic worldview as the followings: physics → chemistry → biology → economics. Economics studies issues of human behaviors using the most systematic and scientific method within social science. Similar to biology, the inner nature and constitution of research objects in economics, as well as outer form are constantly changing. Among fields in economics, regional and urban economics has applied several models and theories in biology or physics to describe and explain phenomena in regional science both in static and dynamic states.

Recent studies have shown that in static states, gravity theory in physics is interpreted as the prototype of spatial interaction model in regional science. Entropy theory is applied to the spatial interaction analysis and optimal control in macro level as well. In the dynamic analysis, the ecologically based logistic growth model and prey-predator model are largely utilized as urban growth model and spatial competition model. Moreover, the complex and irreversible characters of chaos, which is mainly inherited from the nonlinear dynamic feature, has been implied to interpret the evaluations of dynamic spatial systems. Most articles study each theory separately or partially regarding their applications in regional science; there is few researches reviewing the complete sets of natural science models, which have been applied to the regional studies.

The purpose of this paper is first to have a systematic and complete review of the relations within natural science theories and also between natural and regional science phenomena in both static and dynamic states. Secondly, to understand why the non-linear features is so crucial in the emergence of chaotic behavior. What role does "non-linear feature" play in the chaotic dynamic system? And finally, to observe the features of non-linear dynamic system through simulations of the simple May type logistic equation. This paper is organized as follows. Section 2 presents some statically natural science models and theories that have been greatly applied in explaining spatial interactions in regional studies. The inter-relations and their implications of entropy concept, gravity model and discrete logit model are introduced. Section 3 states the dynamically biological-based models applied in regional aspects. Discussion and simulations of the special features of non-linear dynamic system, especially its chaotic behavior, will be presented in Section 4. Conclusion is in Section 5.

2. STATIC NATURAL SCIENCE MODELS:

The major difference between regional economics and other fields in economics is that "where" the economic activities are held is concerned in regional economics. That is, idea of location is the distinguished point in the research of urban and regional economics. Location theory is essential in the analysis of spatial structure and emergence of centers; flows of units within spatial system are followed by location choice of each units. Thus, spatial interactions, which are consequences of units' location choices, are fundamental in analyzing spatial structure.

Entropy Theory

The entropy concept originates essentially from thermos-dynamics; it has a probabilistic background related to the statistical distribution of events in an uncertain situation. It mainly describes the most probable configuration of particles in a closed physical system. Since flows in spatial systems also have various configurations, it is conceivable to relate entropy concept to spatial interactions. Spatial interactions of units in a system may have large variety of arrangements given total outflow, total inflow and travel budget constraints. The basic idea is that the statistically most probable configuration can be identified by maximizing entropy, the number of possible arrangements given numbers of inflow and outflow of each location in system.

The following simple model will explain this concept. Assume a spatial system composed of origin and destination points with uncertain flows T_{ij}, which is the degree of spatial interaction from i to j (e.g., migration flows, trade flows, telephone calls etc.). The following additivity conditions need to be held:

$$\sum_j T_{ij} = O_i \tag{1}$$

and

$$\sum_i T_{ij} = D_j \tag{2}$$

where O_i stands for total outflows from zone i, and D_j stands for total inflows to zone j. The following consistency condition should also hold:

$$\sum_i \sum_j T_{ij} = \sum_i O_i = \sum_j D_j = T \tag{3}$$

Moreover, the spatial system has an upper limit on its total transport cost budget, and has a fixed unit transport cost c_{ij} between i and j (e.g. transport cost per entity from i to j). The travel budget C is expressed as follows:

$$\sum_i \sum_j c_{ij} T_{ij} = C \qquad (4)$$

Given the constraint equation (1)-(4), maximize the objective function: the entropy concept. Maximized entropy represents the most probable arrangement of spatial distribution of trips in the system. This can be formulated as follows:

$$w(T_{ij}) = \frac{T_{ij}!}{\prod_i \prod_j T_{ij}!} \qquad (5)$$

The maximum value of $w(T_{ij})$ is the maximum number of assignments of individual units to an origin-destination matrix. The optimal question is to maximize logarithm of (5) subject to additivity conditions (1), (2) and transport cost budget (4). From the first order condition, the following optimal flow T_{ij} is derived:

$$T_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij}) \qquad (6)$$

where $A_i = \{\sum_j B_j D_j \exp(-\beta c_{ij})\}^{-1}$ and $B_j = \{\sum_i A_i O_i \exp(-\beta c_{ij})\}^{-1}$ (7)

Parameter β is the Lagrange multiplier in the transport cost condition (4); it represents the marginal change in the optimal value of $\ln w(T_{ij})$ (e.g. logarithm of the number of possible states of the system) respect to a change in transport budget C. Parameter c_{ij}, introduced in (4), is the transport cost from i to j per entity; it is proportional to the distance between j and i. The term $-\beta c_{ij}$ represents the reduction in total numbers of possible states (the objective function of the optimal problem) induced from transport cost between j and i. The term $\exp(-\beta c_{ij})$ is therefore called the distance friction function. A_i and B_j are balancing factors.

It can be proved that the second-order condition is also satisfied at this optimal solution. The optimal flow (6) between zone i and j is a function of total outflows from i, total inflows to j, distance friction function $\exp(-\beta c_{ij})$, and balancing factors. Total outflows O_i represent a push force, and total inflow D_j is a pull force in the spatial interaction. The farther the distance between these two zones, the less flows between these two zones will occur. The function of this optimal flow appears corresponding to the idea of gravity theory, which will be explained in the following section.

The main idea behind the constrained maximization question concerned in this entropy type issue is to find a formula of interaction flow that gives the largest combination of individual assignment given the outflow and inflow constraints. In other words, this chosen formula of flow makes the arrangement of spatial distribution of trips most possible comparing to other type of flow.

As we understand well that the economic driving forces of the location decision of each individual agents are to maximize their utility or profit functions subject to their corresponding constraints. This level of concerns is not expressed in the entropy model. This entropy model mainly explains the aggregate interaction behavior from the nature driving force point of view; the economic consideration of each individual is not explained or clearly expressed in this model. However, not expressing it clearly does not mean it is not exist or has been changed in the spatial interaction subject. The optimal problems of both aggregate level and individual agents are not mutually exclusive.

This model does not only explain the planning problem; it explains the general spatial interaction problem including migration flows, trade flows, phone calls and so on. In stead of thinking the maximized objective function as the planner's object to plan the field, it is more suitable to think it as the nature driving force behind the space.

Gravity Model

The classical gravity theory is formed by Newton's law from physics, which states that the attraction force a_{ij} between two objects i and j is proportional to their respective masses m_i and m_j, and inversely proportional to the squared distance, d_{ij}^2, between these objects. The law of gravitational attraction states:

$a_{ij} = \alpha m_i m_j d_{ij}^{-2}$, Where α is a constant. (8)

Ravenstein (1885) first attempted to apply this gravity concept to study migration flows between English cities. Following, Young (1924), Reilly (1931), Stewart (1941) and Zipf (1949) had developed this idea in analyzing farm population migration, trade flows etc. The general form applied in their spatial interaction studies is mainly derived from the gravity model (8):

$$T_{ij} = \alpha \overline{O}_i \overline{D}_j d_{ij}^{-2}, \qquad (9)$$

Which states that the flows (migration) from i to j , T_{ij}, is proportional to the stock variable (population) of origin and destination zones ($\overline{O}_i$ and $\overline{D}_j$), and inversely proportional to the squared distance, d_{ij}^2. The population of origin zone i ,$\overline{O}_i$, is a push force toward flow T_{ij} ; similarly, the population of destination zone j ,$\overline{D}_j$, is a pull force toward flow T_{ij} . Formula (9) is the very beginning spatial interaction model based on the physics entropy model. A simple general formula is followed:

$$T_{ij} = A_i B_j \overline{O}_i \overline{D}_j F_{ij}, \qquad (10)$$

where $A_i = (\sum_j B_j \overline{D}_j F_{ij})^{-1}$ and $B_j = (\sum_i A_i \overline{O}_i F_{ij})^{-1}$ (11)

Parameter α is replaced by two balancing factors A_i and B_j; they are interpreted as a measure of accessibility of one zone with respect to other zones (see Kirby, 1970). F_{ij} is interpreted as the distance friction function, which is inversely proportional to a function of distance between i and j. The flows between two zones, i and j, is not only proportional to the absolute stock variables of two zones, but also influenced by the relative attractive or repulsion in the system (this part is captured by the balancing factors).

The optimal flow derived from maximum entropy in spatial interaction model corresponds to the gravity model in Newtonian physics. Entropy theory offers a theoretical reason for the source of gravity model in spatial interaction problem.

Gravity model explains the aggregate spatial interaction behavior that is based on the location choice of each individual agent. To achieve the optimal stage is the motivation for each agent to locate. For household, to search a higher utility level subject to the budget constraint is the major economic consideration; and for firm, to maximize profit might be the driving force behind the location decision. These micro level considerations or motivations of each individual agent's location choice are the source of spatial interaction, but not clearly expressed in this aggregate gravity type model. However, from Chen (1996), these microeconomics optimum motivations of both household and firm do lead to a similarly gravity type aggregate interaction behavior. In another words, gravity type interaction model implies the optimal motivation of each individual agent in the space.

Summarily, each individual agent's optimum motivation is the major driving force of the spatial interaction; the aggregate of each individual agent's location choice behavior presents the aggregate type gravity model character. The maximization of total flow combinations implied by the entropy theory is not the reason or driving force of the spatial interaction flow. It offers another point of view of gravity type interaction behavior; it shows that the gravity type interaction behavior has the largest possibilities of individual combination (Fujita and Smith (1990), Isard and Bramhall (1990), Sen and Smith (1995)).

Discrete Choice Model

Follow from the derived maximum entropy gravity type flow from i to j (6), the probability of a destination choice from i to j is as follows:

$$P_{ij} = \frac{T_{ij}}{\overline{O}_i} = A_i B_j \overline{D}_j \exp(-\beta c_{ij}) = \frac{B_j \overline{D}_j \exp(-\beta c_{ij})}{\sum_j B_j \overline{D}_j \exp(-\beta c_{ij})} \tag{12}$$

The above probability is based on macro foundations and maximum entropy concept; it corresponds to gravity model. In addition, it is also a formula of multinomial logit models (MNL) in discrete choice models (DCM), which assume household i (i =1,...,n) chooses alternative j (j = 1,..., m) to maximize his utility V_i. According to the assumption and formula in MNL, the probability that household i chose alternative j is as follows:

$$P_{ij} = \frac{\exp(V_{ij})}{\sum_j \exp(V_{ij})} \tag{13}$$

where V_{ij} is the utility of household i consuming alternative j. This derived probability is based on micro foundation (maximum personal utility). Its formula is the same as the probability based on maximum entropy and gravity model above (12). From this point of view, equation (12) is based on both macro and micro foundations, and it is an insightful application of physics, ecology and probability theory into spatial interaction model of regional science.

3.DYNAMIC MODEL

In the static analysis in regional science, location theory is essential in analyzing spatial structure; consequently, spatial interaction model deduced from location choice is one of the major issues in the researches of regional science. When the time variable is considered in the field, we would focus on another interesting subject: growth and evolution of the system. There are different kinds of dynamic models used in the spatial interaction (Cochrane (1992), Nijkamp and Poot (1987), Nijkamp and Reggiani (1988)), we introduce the classical ones as follows.

Logistic Growth Model (Ecologically-Based Model)

The logistic growth model is a non-linear first-order difference equation. In biology, it represents the growth of biological populations. The main idea of this model is to capture the tendency of the growth of certain variable (biological populations). The population grows from one generation to the next when it is small in population size, and it dwindles when population is large. That is the feature of non-linear iteration function with monotonically decreasing slope from positive to negative. It was first developed by Verhulst (1838), and has been extensively discussed by May (1976).

$$X_{t+1} = aX_t(1-X_t) \tag{14}$$

This is so-called logistic equation or Verhulst dynamics while the parameter a is the growth parameter reflecting the maximum rate of increase of time-dependent variable x. The existence condition for the equation is that $0 \le x \le 1$ and $0 < a < 4$. Even though this first-order difference equations have extremely simple and deterministic mathematics form; it can exhibit a surprisingly rich evolution spectrums including stable points, stable cycles, bifurcation, periodically stochastic map and chaos phenomena. This interesting feature has attracted attentions and been extensively discussed by May and other authors regarding many fascinating problems, for instance, Baker and Gollub (1990), Baumol and Benhabib (1989), and Kelsey (1988).

The essential feature of this logistic equation is that for $a > a^{*}$, a critical value, a cycle of period 3 appears (e.g. a population value reiterates every third generation). As a cycle of period 3 appears, it follows that cycles of every period are possible to appear. Furthermore, both periodic and periodic trajectories exist at this stage and all trajectories are highly sensitive dependence on initial conditions (SDIC).This SDIC feature makes that evolution path unpredictable and irreversible; those are essential characteristics of chaos. Overall, the interesting point of this logistic equation is that given this simple deterministic first-order difference equation, stochastic, unpredictable and irreversible trajectories are possible to appear. As May (1976) stated "it is a simple mathematical model with very complicated dynamics".1

Dynamic logit model.

Equation (12) represents the probability of flow from i to j; which is clearly the standard of multinomial logit model with maximum entropy and gravity foundation. Before we extend it from static states to dynamic states, I would like to add the time variable into the model:

$$P_{ij,t} = \frac{B_{j,t}\overline{D}_{j,t}\exp(-\beta c_{ij,t})}{\sum_{j} B_{j,t}\overline{D}_{j,t}\exp(-\beta c_{ij,t})} \tag{15}$$

where $c_{ij,t}$ represents the distance between i and j at time t. Equation (15) can

[1] See Lorenz (1989) for detail.

be easily transformed into simple form (by assuming $B_{j,t}\overline{D}_{j,t} = 1$, and $-\beta c_{ij,t} = u_{j,t}$, and interpreting $u_{j,t}$ as utility):

$$P_{j,t} = \frac{\exp(u_{j,t})}{\sum_n \exp(u_{n,t})} \tag{16}$$

Then the change of $P_{j,t}$ with respect to time t is:

$$\frac{dP_{j,t}}{dt} = \dot{P}_{j,t} = \frac{d}{dt}\left[\frac{\exp(u_{j,t})}{\sum_n \exp(u_{n,t})}\right]$$

$$\dot{P}_j = \dot{u}_j P_j(1-P_j) - P_j \sum_{n \neq j} \dot{u}_n P_n \tag{17}$$

where the symbol t is omitted for simplicity. The term $\dot{u}_j$ represents the change of utility by time; it is assumed to be a constant γ. Expression (17) is a system of Lotka-Volterra type, and the first term at the right-hand side corresponds to the logistic growth of population P_j. The second term at the right-hand side represents interaction effects among population; since the value of the second term is extremely small and we will focus on the analysis of logistic equation (the first term), it is omitted in the following study. The final expression of (17) becomes:

$$\dot{P} = \gamma P_j(1-P_j) \tag{18}$$

Furthermore, Nijkamp and Reggiani (1991) transform equation (18) into discrete type model as follows:

$$P_{j,t+1} - P_{j,t} = \gamma P_{j,t}(1-P_{j,t}) \tag{19}$$

$$P_{j,t+1} = NP_{j,t}\left(1 - \frac{N-1}{N} P_{j,t}\right) \tag{20}$$

where $N=\gamma$, and the feasible range of probability, $P_{j,t}$, is between zero and one. Nijkamp and Reggiani called equation (20) degenerated logit model, which is close to the previous Verhulst dynamic equation (14), and showed it has similar dynamic features as Verhulst dynamic equation. It is stable given certain range of parameter value and become irregularly chaotic as parameter value above a critical value (Nijkamp and Reggiani (1991)). Overall, in spatial interaction system, maximum entropy will derive the gravity type model, and from the gravity model, a logit discrete choice model is deduced; finally, the dynamic logit model will generate the Verhulst type logistic model.

Spatial Competition Model: Prey-Predator Model (Ecologically-Based Model)

The prey-predator model is another ecologically based dynamic model, which has been discussed and applied quite a lot. Lotka (1925) originally developed this model as a tool for studying competitive behavior between different species (one prey and one predator). It is also called Volterra-Lotka model and has a general form as follows:

$$\dot{x} = f_1(x,y),$$
$$\dot{y} = f_2(x,y), \tag{21}$$

The functional forms f_1 and f_2 are nonlinear; this model describes non-linear differential equations.

There are two species x and y competing with each other. One species, the prey, is constrained in its growth by the presence of a predator; in other hand, the predator will grow with the growth of prey (see Lotka (1920)). This model was first applied in economics by Goodwin (1967) to describe the dynamic interaction in employment rate and workers' income share. Nijkamp and Reggiani (1990a) used this concept in studying transport flows and workplaces.

This is a simple general expression of this kind model. The specific interaction formula is different according to the specific interaction character. The simplest formula is the linear type. The purpose in this section is to introduce the general idea of this spatial competition model, we do not present the specific model here.

Both logistic growth model and prey-predator model are non-linear differential models. One of the important features of these non-linear differential models is that the stochastic evolution trajectories are possible to be generated according to this deterministic model. Besides, the trajectories based on these models are highly sensitively depending on the initial conditions (initial values of variables and parameters); this special property may lead those trajectories to be unpredictable and irreversible. Consequently, evolution of these types is called chaos.

4.CHAOS AND NONLINEAR DYNAMICS

In recent year, a great many articles and books have discussed the phenomena and features of chaos. The interesting feature of a chaotic system is that the originally completely deterministic system can lead to a stochastic evolution given critical initial conditions. The stochastic features of this determined-form system are totally self-generated, not come from exogenous fluctuations. Furthermore, the generated stochastic trajectories are irreversible and unpredictable. The system's evolution will achieve some equilibrium points given some range of parameter values; it will turn to periodic cycles of some order or even become periodic situations all depending on the initial setting of parameter values. Why would a deterministic system generate such unpredictable evaluations? The condition is that this system should be non-linear to have the chaotic features. In this section, we investigate what is the critical difference between linear and non-linear dynamic system in terms of resulting evolution.

Linear Versus Non-Linear First Order Dynamics

Linear dynamics.

We will first take a brief review about linear dynamic system both in continuous and discrete cases. Assume the general form of continuous dynamic system in vector notation as follows:

$$\dot{X} = AX(t) + c, \quad X \in R^n, \; t \in R, \tag{22}$$

with $\dot{X} = dX(t)/dt$, parameter t represents time, A is the n×n matrix, and c is an n-dimensional column vector of constants. Since constant vector c is not crucial in determining the tendency of trajectories, for simplicity sake, we consider the case n=2 and c=0. The solution is:

$$x_i(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \; , \; i = 1,2 \tag{23}$$

Where A_1 and A_2 are arbitrary constants, and λ_1 and λ_2 are eigenvalues of matrix A. The tendency of the solved time path (23) mainly depends on the value of λ_1 and λ_2. When the values of λ_i are real numbers (complex), the trajectory of the system monotonically approaches a node or a saddle point (oscillations to focus). When both λ_i are positive (negative), the trajectory is divergent (convergent). Since λ_1 and λ_2 are eigenvalues of matrix A, values of λ_i are mainly determined by matrix A. The components in matrix A are constant in the linear model; the trajectory will either converge or diverges in systematically monotonic or oscillatory ways. As a result, a linear dynamic system (22) has a deterministic form, and its evolution is deterministic too.

The general form of a discrete dynamic system in vector notation is assumed as follows:

$$X_{t+1} = AX_t + c, \quad t \in Z, \tag{24}$$

where t represents time, A is the n×n matrix, and c is an n-dimensional column vector of constants. We also consider the case n=2 and c=0. The solution is:

$$x_t^i = A_1\lambda_1^t + A_2\lambda_2^t \text{ , } i = 1,2 \tag{25}$$

where A_1 and A_2 are also arbitrary constants, and λ_1 and λ_2 are eigenvalues of matrix A. Similar to the continuous case, the features of the trajectory totally depend on the values of λ. Again, the value of is determined by the matrix A. All components in matrix A are constants due to the "linear" feature of this system. The property of the system's trajectory (24) is determined given the initial conditions and coefficient values. Similar to the continuous case, it is a deterministic system and leads to a deterministic evolution.

Non-linear dynamics.

In linear case, both continuous and discrete types have similar conclusions that the tendency of trajectories are mainly based on matrix A. Since matrix A are constants, the feature of system is determined given the initial values and coefficients values. The general form of the continuous non-linear dynamics case in vector forms is as follows:

$$\dot{X} = f(X), \quad X \in R^n, \tag{26}$$

Take the Taylor expansion of this nonlinear function f (x) at a fixed point x^* to analyze the local behavior in the neighborhood of point x^*. We only consider the first two terms of the Taylor expansion and drop all remaining terms:

$$\dot{X} \cong f(X^*) + J_{|X=X^*}(X - X^*) = J_{|X=X^*}(X - X^*), \tag{27}$$

where $f(X^*)$ is equal to zero since x^* is a fixed point , and $J_{|X=X^*}$ is the Jacobian matrix of partial derivatives at x^* as follows:

$$J_{|X=X^*} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \tag{28}$$

where f_i is the i th nonlinear equation in the system (26). The role this Jacobian matrix plays in nonlinear dynamic system is very similar to the part that matrix A plays in the linear dynamic system. As we know, the value of the elements in matrix A are the key in determining the features of trajectories; similarly, so does the values of the elements in Jacobian matrix. In a linear case, the elements in Jacobian matrix are all constants; however, in the nonlinear case, at least one element in Jacobian matrix is the function of system's variables. That is, in the nonlinear case, the components of Jacobian matrix are not all constants throughout the system and time path; the feature of evolution will not be consistent through time since the values of system variables varies by time. At time 1, the trajectory may be monotonically convergence to a fixed point; at the next period, it might change into dampened oscillations due to the change of variables' values in Jacobian matrix. This nonlinear dynamic system is possible to diverge to a fixed point, to periodic trajectories or even periodic trajectories given different parameter values and initial conditions.

Chaotic System

In a linear dynamic systems, no matter its evolution is stable or unstable, convergence or divergence, monotone or oscillation, the trajectory will always be deterministic and systematic following a certain route. However, in the non-linear case, due to the elements in Jacobian matrix are composed of system variable, which vary as time changes, the tendency of evolution is possible to change each time period. When the dynamic behavior becomes extremely irregular and unpredictable, it is called chaotic behavior or deterministic chaos. Meanwhile, a non-linear dynamic system does not necessarily lead to chaos phenomenon; the key is the value of system parameters. Take the simple first-order difference equation: logistic equation (14) for example. Its dynamic system possesses a stable fixed point when the value of parameter a is less than a certain value a_1. As parameter a has value above a_1 and under a critical value a^*, the system possesses a stable orbit; if parameter a has value above a^*, the dynamic system shows chaotic behavior and no stable orbit exists.

Level of parameter value is critical in the emergence of chaos phenomena. In the generated dynamic logit model (20), parameter N is assumed to be sum of γ and one, and parameter γ represents the marginal utility with respect to time. If this marginal utility level is higher than some critical value, the dynamic system will become chaotic. In general, the parameter in generated dynamic logit system represents the marginal value of the objective function with respect to time; therefore, the marginal value is the major reason whether the system is chaotic or not. To sum up, "non-linear" feature is the necessary condition for a dynamic system appearing chaos; the level of parameter value is the sufficient condition for chaos.

The chaotic model discussed and simulated in the next section is the simplest one, which only expresses the dynamic evolution on one location. For the more complicate ones like the space competition model introduced in section 3, which denotes the spatial interaction behavior between or among regions. It would lead to a much more complicate dynamic evolution (Cochrane (1992), and Nijkamp and

Reggiani (1990b)).

Simulation

A nonlinear dynamic model is capable to perform the chaotic behavior, but it does not need to. Given different initial conditions (initial variables and parameter values), evolution of a nonlinear dynamic system is possible to have monotone trajectories leading to a fixed point (monotone convergence or divergence), periodic cycles (converging, diverging or steady oscillations), or even irregularly non-periodic trajectories (chaos). For a better understanding of a non-linear dynamic model, especially its chaotic behavior, we simulate the well known May type logistic equation (14) in this section. May type logistic equation is a very simple and popular nonlinear dynamic sample. Many researches have simulated this first-order difference equation to investigate features of chaos from different aspects due to its simple form and rich spectrum in chaotic behavior (see Nijkamp and Reggiani (1990b), Baker and Gollub (1990)). Even though they have pointed out many special features of chaotic behavior, there are a few points need to be clarified. We will do so and show the major three categories of non-linear dynamic system in the following simulations.

$$X_{t+1} = aX_t(1 - X_t) \tag{14}$$

We run 100 time periods (t=100) in each experiment assuming different value of parameter a and the initial value of variable X_t .

Regular motion patterns.

Assume initial value $X_0 = .1$ and parameter $a = 1$ ($a = 3.1$). The result is presented in Figure 1(Figure 2). Simulated pattern presented in Figure 1 is a monotonic convergent trajectory to a fix point; Figure 2 shows two period steady cycle. Both these two patterns are regular and can be performed by a linear dynamic system. This non-linear dynamic system can also perform other kinds of regular patterns as linear dynamic system given different values of X_0 and a. Due to the space constraints, we will not present them here.

Irregular motion pattern.

Assume the initial value of X_0 =.1 and parameter $a = 3.9$. The simulation result is in Figure 3, which shows irregular motions and non-periodic trajectory. If each time path is shown by points instead of connecting lines in Figure 3, we can clearer observe the distribution of each time path. Figure 4 shows the points of each time path corresponding to Figure 3, and Figure 5 is the points of each time path corresponding to Figure 2 for comparison. In Figure 4, each time path distributes rather randomly than following some certain trace as in Figure 5. All variable values from zero to one (the range of variable) will be visited by the trajectory with a more or less equal probability. This is where the important describing term: "stochastic" coming from. We run this simulation several times given the same initial and parameter values, and the resulting irregular time series all have the same traces. This result shows that the stochastic feature of this irregular pattern does not mean the system will have different evolution path given the same initial conditions. The term "stochastic" refers to the motion pattern having time series similar to stochastic process (as Figure 4). We also call this stochastic feature as "unpredictable" since there is no systematic trace to follow as in Figure 5.

Another very important characteristic of chaotic behavior is SDIC (sensitive dependence on initial conditions). For comparison, we simulate all three patterns in Figure 1, 2 and 3 based on their original parameter values and slightly different initial variable values X_0 =.10001. The results show that after changing the initial value .00001 more and keep the parameter value the same, the monotonic convergent trajectory (Figure 1) still remains a very similar path (monotonic convergence); the correlation coefficient of the old time series in Figure 1 and the new time series after a slightly change in initial value is one. The plot of these two trajectories is shown in Figure 6; it presents a straight line and almost overlaps the 45-degree line. Besides, a very small change of the initial value (.00001) in the periodic pattern in Figure 2 also leads to the same two-period time series with correlation coefficient equals one. Results of both regular patterns show that the regular motion patterns are not sensitive to the initial conditions.

We do the same experiment on the non-periodic motion pattern in Figure 3, and the simulating result is also non-periodic and irregular. Plot of the original time series (Figure 3) and the new time series (X_0 =.10001) is presented in Figure 7. It shows that original and new time series are almost uncorrelated; their correlation coefficient is .19. A very small change (.00001) in the initial variable leads to a totally different irregular trajectories and time path. This simulation result stresses its SDIC feature.

A bifurcation diagrams of the Logistic dynamic system (14) is presented in Figure 8. The y-axis is the limit points of X_t, and the x-axis is the value of parameter a within range (2.5, 4). This graph shows how dynamic evolution is affected by parameter value. X_t leads to a fixed point when a is less than 3; it becomes two-period cycle when a is larger than 3. As a is getting larger, more than two periods cycles appear and finally become randomly distributed within its range (0,1); the phenomena of those randomly distributed limit values is so called complex or chaos behavior. Overall, given different parameter value, this first-order difference equation would lead to different evolution type: regular and irregular (chaos). The irregular motion pattern has determined system form but stochastic time path; they are unpredictable and extremely sensitive to the initial conditions.

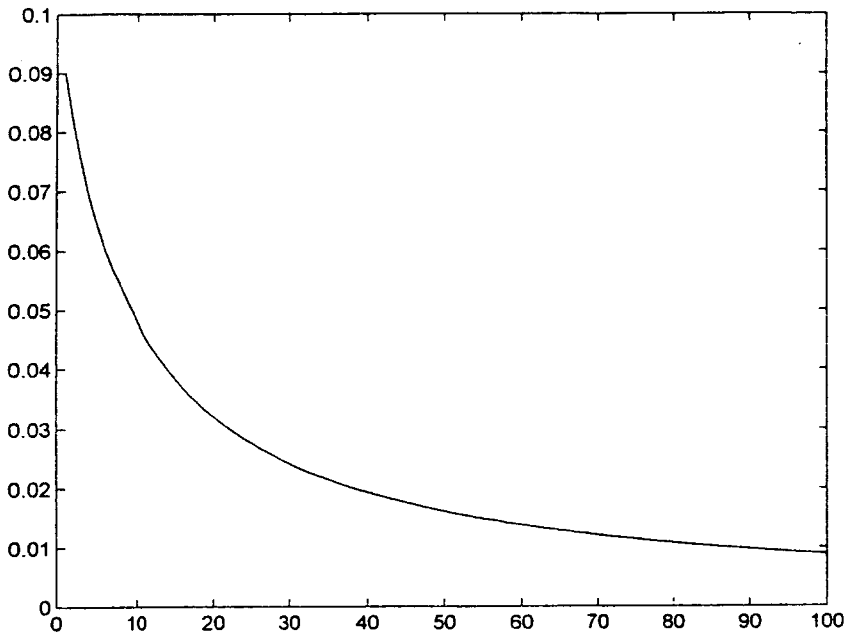

FIGURE 1. Logistic Model for $X_0 = .1$ and $a = 1$ with y - axis = X_t; x - axis = t (time)

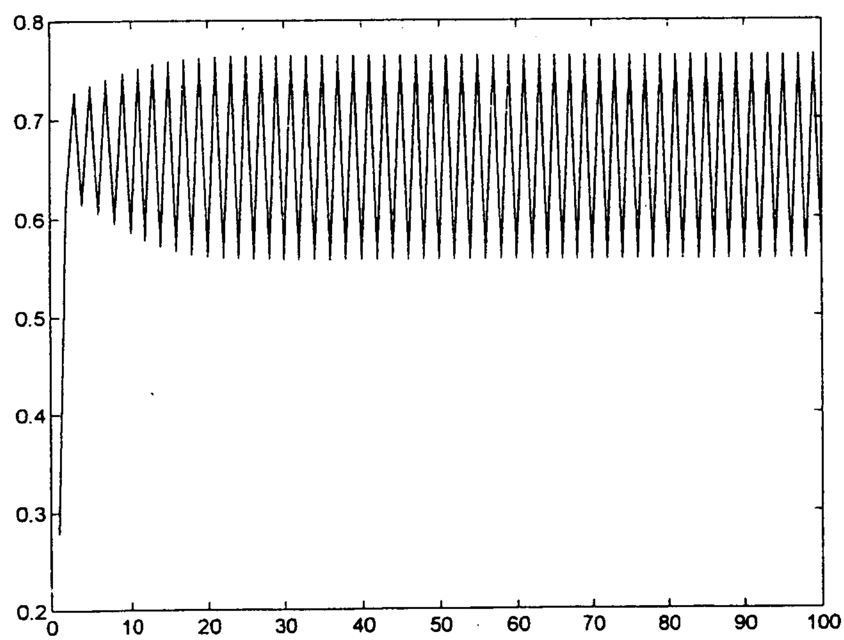

FIGURE 2. Logistic Model for $X_0 = .1$ and $a = 3.1$ with y - axis = X_t; x - axis = t (time)

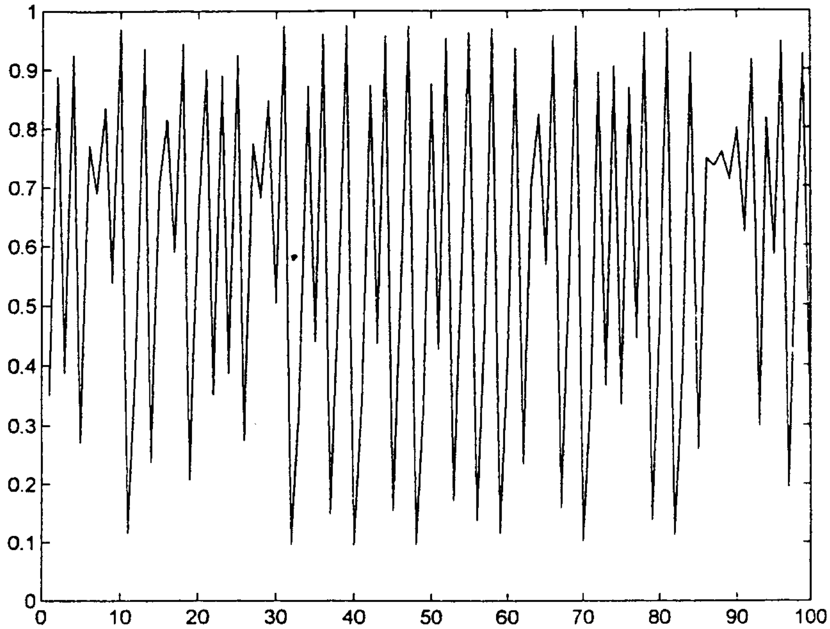

FIGURE 3. Logistic Model for $X_0 = .1$ and $a = 3.9$ with y - axis = X_t; x – axis = t (time)

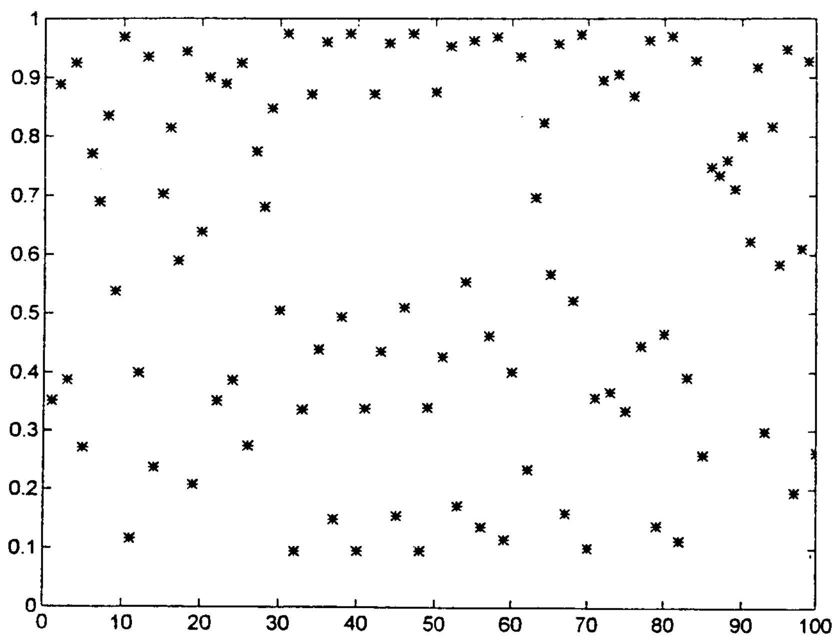

FIGURE 4. Logistic Model for $X_0 = .1$ and $a = 3.9$ (by points) with y - axis = X_t; x – axis = t (time)

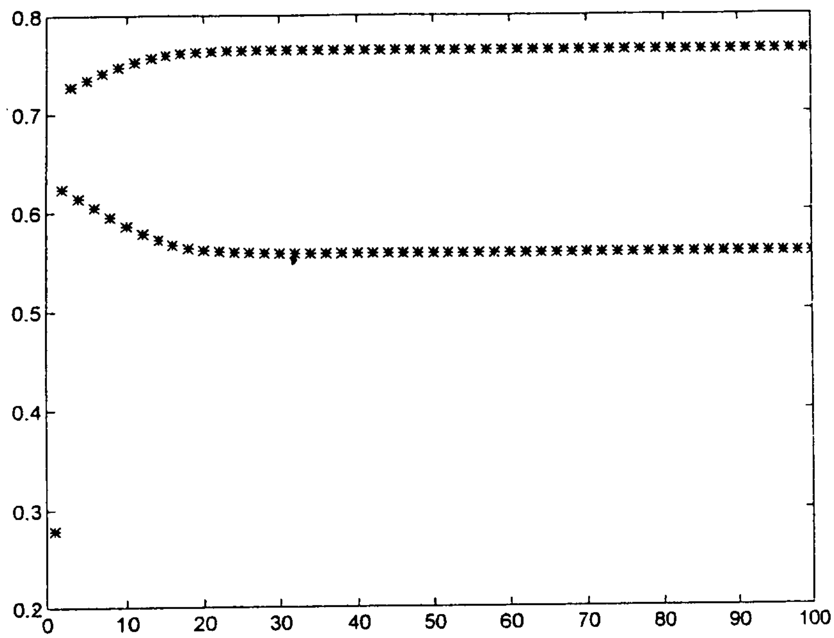

FIGURE 5. Logistic Model for $X_0 = .1$ and $a = 3.1$ (by points) with y - axis = X_t ; x - axis = t (time)

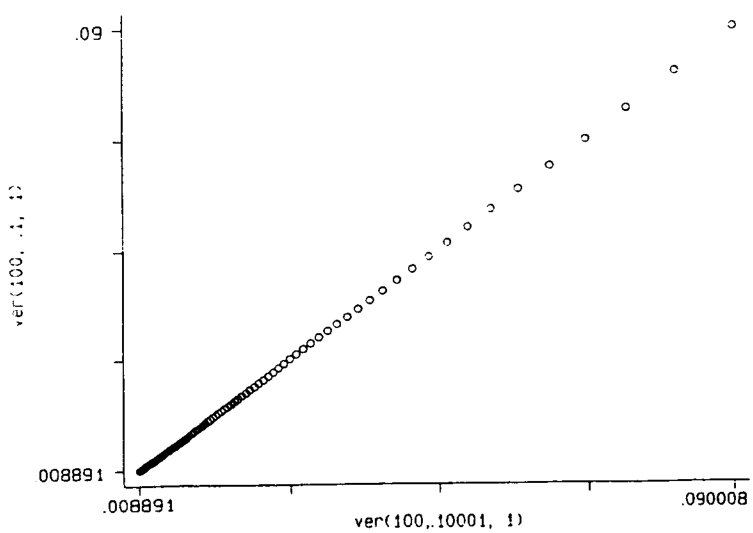

FIGURE 6. Plot of Time Series X_t (as in Fig. 1) and New Time Series Z_t ($X_0 = .10001$ and $a = 1$) with y - axis = X_t ; x - axis = Z_t

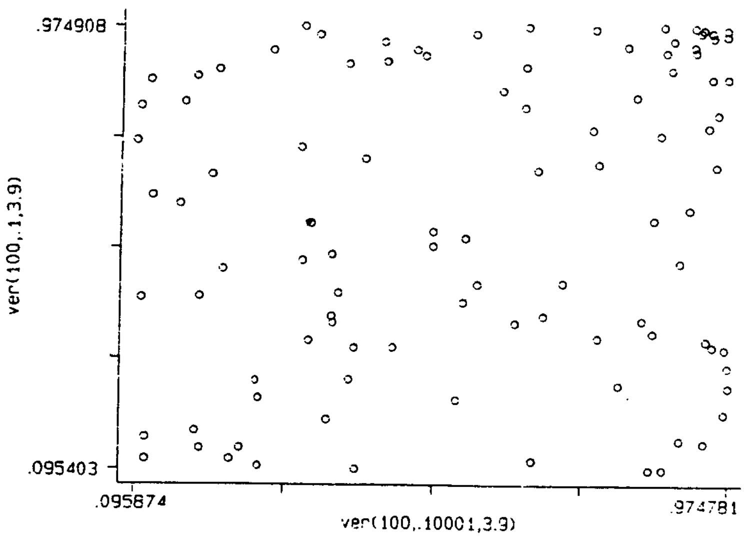

FIGURE 7. Plot of Time Series X_t (as in Fig.3) and New Time Series Z_t (X_0 = .10001 and a = 3.9) with y-axis = X_t; x-axis = Z_t

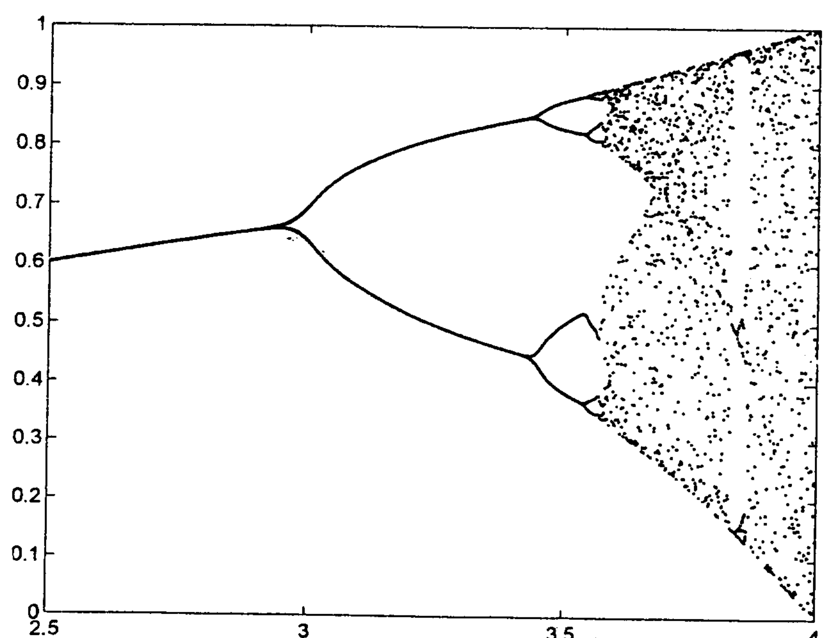

FIGURE 8. Bifurcation Diagrams of Logistic Equation ($2.5 \leq a \leq 4$) with y-axis = X_t; x-axis = a

5. CONCLUSION

This article reviews the application of models in natural science in the spatial analysis of regional science. We have a complete understanding about the inter-relations between several popular models in physics and biology; and also understand the regional interpretation of each model in natural science. From optimal entropy to generated dynamic logit model, all are vertically correlated; horizontally, each natural science model also corresponds to regional science phenomenon. Besides, we also find that non-linear feature is the necessary condition for the emergence of chaos, and the parameter value is the sufficient condition for the existence of chaotic dynamic system. Finally, we observe the feature of non-linear dynamic system by simulating the simple first-order difference equation: May type logistic model. The simulation results show that non-linear dynamic system is able to perform all evolutionary trajectories in linear dynamic system, which include fixed point or periodic terms. It also performs non-periodic irregular motion patterns that are beyond the limit of linear dynamic system. All these different outcomes are based on the initial variable values and parameter values. The non-periodic trajectory is extremely sensitive to the initial conditions and has stochastic time series path. These irregular non-periodic trajectories are unpredictable, irreversible, stochastic and with SDIC features; those are the special features of chaotic behavior.

Due to the unpredictable feature of the chaotic behaviors resulted from non-linear dynamic systems, they are much later applied in researches in social science and economics than the linear dynamic system. However, its rich dynamic spectrums have attracted increasing attentions from researchers. On the other hand, research on the causes and consequences of interaction among human activities and natural environment is an interesting and challenging issue. More investigating research works still need to be done in these topics both in theoretical and empirical respect.

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The Interaction of Natural Science Models in Spatial Interaction Behavior

自然科學模型在空間交互行為分析之應用

陳　心　蘋

政大經濟系副教授

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摘　要

本文簡要系統地介紹區域科學裏空間交互行爲分析中常被應用的自然科學模型之間縱向與橫向的相互關係。包括靜態的熱力學之 Entropy 概念與重力定理，以及動態的生態基礎成長模型、logit 模型和空間競爭模型間的相關性與在區域科學上的應用。最後並探討前述動態模型中之混沌特性與非線性之相關。

關鍵字：熱力學、重力定理、混沌。