# 1 Introduction

In 1929, R. Nevanlinna [1] proved several remarkable results in the theory of value distributions, namely, two non-constant meromorphic functions share five distinct values must be identical; if two non-constant meromorphic functions share four values CM, then one of them must be a Möbius transformation of the other. Thereafter, the field of value distribution received more interesting, leading to the growing research of the theory for decades.

Nevanlinna's results say that five values determine a meromorphic function uniquely, and if two meromorphic functions $f$ and $g$ share four values CM, then they are closely related. Now, a natural question arises, namely, what happen if $f$ and $g$ share four values IM? In 1979 and 1983, G. G. Gundersen [3, 4] proved that if two non-constant meromorphic functions $f$ and $g$ share either three values CM and the other one IM, or two values CM and other two IM, then $f$ and $g$ share four values CM, in particular, $f$ is a Möbius transformation of $g$. The remaining case, namely, $f$ and $g$ share one values CM and the other three IM, is still open.

Besides, we can also obtain some uniqueness results by imposing some conditions on meromorphic functions. In this thesis, we study that if two meromorphic functions satisfying some functional equation, then they are closely related. Also, if two meromorphic functions share four small functions IM and have few poles, then they are in some particular forms.

The thesis contains five sections. In section 1, we give some introduction, and review the basic theory of value distribution, especially, the Nevalinna's first and second fundamental theorem, in section 2. In section 3, we list some uniqueness results on rational functions sharing values. Also, we study the uniqueness results of two meromorphic functions satisfying some functional equation in section 4. Finally, in section 5, we study two meromorphic functions with few poles sharing four small functions.