3 Rational Functions Sharing Values

In this section, we recall some basic facts about the uniqueness of rational functions. In fact, the following results are well-known whose proofs can be found in [5].

Theorem 3.1 *Let f and g be non-constant polynomials and a be a finite complex number. If f and g share a CM, then there exists a non-zero constant K such that*

$$f - a = K(g - a).$$

Corollary 3.2 *Let f and g be non-constant polynomials and a be a finite complex number. If f and g share a CM, and if there exists a point z_0 such that $f(z_0) = g(z_0) \neq a$, then $f = g$.*

Theorem 3.3 *Let f and g be non-constant polynomials and a, b be distinct finite complex numbers. If f and g share a and b IM, then $f = g$.*

Lemma 3.4 *Let f and g be non-constant rational functions. If f and g share 0, ∞ CM, then there exists a non-zero constant K such that $f = Kg$.*

Theorem 3.5 *Let f and g be non-constant rational functions. If f and g share distinct values a, b CM, then there exists a non-zero constant K satisfying*

$$\frac{f-a}{f-b} = K\frac{g-a}{g-b}.$$

Corollary 3.6 *Let f and g be non-constant rational functions. If f and g share distinct values a, b CM, and if there exists a point z_0 such that $f(z_0) = g(z_0) \notin \{a, b\}$, then $f = g$.*

Theorem 3.7 *Let f and g be non-constant rational functions. If f and g share distinct values a_1, a_2, a_3 and a_4 IM, then $f = g$.*