1 Introduction

Classically, we all use the ordinary absolute value to measure the distance of rational numbers, and, regard a Cauchy sequence as a single point to get the completion of the rational number field $\mathbb{Q}$, so the real number field $\mathbb{R}$ is obtained. Therefore, a natural question is to ask: is there another way to measure the distance between two rational numbers? The answer is affirmative, namely, the p-adic distance on the rational number field. Apply the some procedure from $\mathbb{Q}$ to $\mathbb{R}$, we get the p-adic number field $\mathbb{Q}_p$.

In this thesis, we will study the general theory of valuations on fields, the topology induced by a valuation, the completion of a valuation and the p-adic analysis on the p-adic number fields, and obtain some new phenomena and examples.

The thesis contains seven sections. In section 2, we review some properties of valuations and give a general procedure to construct valuations on some interesting fields. The topological properties of a valuated field and its difference from the usual Euclidean topology on $\mathbb{R}$ are given in section 3. Given a valuation on a field, we can always get its completion, the detail is discussed in section 4. Several equivalent conditions of equivalence between two valuation are proved in section 5. Finally, in section 6 and 7, we study some elementary p-adic analysis on the p-adic number fields, especially, the p-adic sequence and series, and obtain some new results and examples.