Chapter 3
Research Method

3.1 Framework

3.1.1 Research Design

A two-phase study was designed to explore the feasibility of RM in aging societies from both supply and demand aspect. In the aspect of the borrowers, the study conducted a survey. A questionnaire was designed for the middle-aged homeowners in Taiwan regarding their opinions and attitudes towards the feasibility of RM. The definition of the middle-aged homeowners was further designated to be those from the range 30-60 years of age in 2008. The questionnaires are conducted through the Binary Logistic Regression Model to explore the respondents' willingness to apply for RM and relative factors.

As for the lenders' concerns, a simulation model and a RM pricing model are established in this study to indicate how much the disbursement the RM lenders could offer. Under the break-even hypothesis, the disbursement is determined by computing the LTV in RM. So, the study attempts to compute how much the LTV the RM lenders could offer with different RM payment programs. Furthermore, this study examines whether the IRR_{-RM} in RM could meet the basic needs for the elders in Taiwan.

3.1.2 Research Subject

This study assumes that the middle-aged homeowners are the likely targets for the introduction of RM and could be the most eligible beneficiaries if RM is available in the coming decades. Accordingly, the participants of this research were middle-aged homeowners, selected from a stratified random sample of middle-aged homeowners in Taiwan of four groups: the Northern Area (including Taipei City, Taipei County, Taoyuan County, Hsinchu City, Hsinchu County, and Keelung City), the Central Area (including Miaoli County, Taichung County, Taichung City, Changhua County, and Nantou County), the Southern Area (including Kaohsiung City, Kaohsiung County, Yunlin County, Chiayi City, Chiayi County, Tainan City, Tainan County, and Pingtung County), and the Eastern Area (including Yilan County, Hualien County, and Taitung County). The survey was conducted according to the proportion of the quarterly number of household in these four areas in Taiwan[8]: 487 cases in the northern area, 219 cases in the central area, 344 cases in the southern area, and 50 cases in the eastern area.

Data collection took one month (08/07/2008-09/07/2008). Data were collected through interview (27.3%), mail (27.3%), and e-mail (45.5%). Among the 1,100 Taiwan middle-aged adults, 478 (43%) completed all phases of the survey, among with 70 (6.4%) cases were discarded due to the missing values. Therefore, the valid sample of the study is 396 respondents (α =95%, d =4.92%).

3.2 Method 1: Questionnaire

The questionnaire used in the survey contained multiple choice questionnaires and fill in items; the former comprised the greater part of the survey questionnaire (see Appendix 1). The participants were asked to fill out a questionnaire which consisted of 33 items and was divided into five main parts, as follows:

Current circumstance of house

The respondents were asked to fill in the house age, the house value, how many houses they own, and whether if they rent their house.

Financial situation

The respondents were asked to answer about their financial status including asset holding, income, employment, and wealth.

Career planning

Two questions were designed to search for the planning for the respondents after retiring and the attitude toward inheritance.

[8] The source of the quarterly number of households in Taiwan in the second quarter of 2008 is the Department of Statistics, Ministry of the Interior.
(web site: http://housing.cpami.gov.tw/house/default.aspx , last scanning by Nov. 19, 2008)

Willingness

In this part, respondents were firstly introduced the concept of RM, then they were asked for the intention of applying for reverse mortgages.

Demographic information

The demographic characteristics such as gender, age, address, marital status, career, number of children, and education level were asked in this section.

3.3 Method 2: Binary Logistic Regression Model

3.3.1 Model Design

According to Gujarati, D. N. (2003), the probability density function of the binary logistic regression model could be written as follows:

$$f(z) = \left| \frac{\exp[(z-\mu)/\tau]}{c[1+\exp\{(z-\mu)/\tau\}]} \right|, \quad -\infty < z < \infty \tag{1}$$

The cumulative distribution function (CDF) is

$$P = \int_{-\infty}^{x} f(z)dz = \frac{\exp[(x-\mu)/\tau]}{1+\exp[(x-\mu)/\tau]} \tag{2}$$

Replacing the following items, $\beta_0 = -\mu/\tau$ and $\beta_0 = 1/\tau$. The CDF is then

$$P \equiv \frac{\exp[\beta_0 + \beta_1 x]}{1+\exp[\beta_0 + \beta_1 x]} \tag{3}$$

The logistic regression model takes the following form:

$$\pi_i = E(Y_i) = P_i = \frac{1}{1+e^{-f(x)}} = \frac{e^{f(x)}}{1+e^{f(x)}} \tag{4}$$

$$Z_i = f(x) = \alpha + \beta_1 X_{i1} + \beta_2 X_{21} + \ldots\ldots + \beta_k X_{ik} \tag{5}$$

P_i is the probability of the willingness to apply for reverse mortgage from (4); and $(1-P_i)$ is the probability of unwillingness to apply for reverse mortgage, or

$$1-\pi_i = 1-E(Y_i) = 1-P_i = 1-\frac{1}{1+e^{-(\alpha+\beta_1 X_{i1}+\beta_2 X_{i2}+\ldots\ldots+\beta_k X_{ik})}} = 1-\frac{1}{1+e^{-Z_i}} = \frac{1}{1+e^{Z_i}} \tag{6}$$

Therefore, the odds ratio was:

$$\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right)=\frac{P_i}{1-P_i}=\frac{1+e^{Z_i}}{1+e^{-Z_i}}=e^{Z_i}=e^{(\alpha+\beta_1 X_{i1}+\beta_2 X_{i2}+......+\beta_k X_{ik})} \quad (7)$$

Taking the natural log of (7), we can obtain the following Logistic Equation L_i,

$$L_i=Z_i=\ln\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right)=\ln\left(\frac{P_i}{1-P_i}\right)=\alpha+\beta_1 X_{i1}+\beta_2 X_{i2}+......+\beta_k X_{ik} \quad (8)$$

3.3.2 Definition of Variables

The variables in the logistic regression model are described below: first, the dependence variable in the study is set as binary. In the questionnaire, respondents were asked whether or not they want to apply for RM. This study notes the answer "no" as "0" and "yes" as "1." Second, this study developed 18 independence variables and divides them into four groups:

Panel A: Demographic Variables

Demographic Variables included three continuous variables (age, education level, and number of children) and three dummy variables (gender, career, and marital status).

Panel B: Real Estate Variables

Housing age, value of house, location of house, leasing house or not, having second house or not were obtained in this panel. The value of the house was estimated by means of 6-point scale. Moreover, the locations of house were classified into four categories of areas.

Panel C: Asset and Income Variables

Three dummy variables were formed in this panel: the financial assets were measured by asking respondents the major type of their asset (excluding the home equity). This study divides the major asset type into two parts and assumes people holding the stocks, bonds or funds as their major property are more risk-taking than people holding the cash, deposit, or gold. Furthermore, if respondents have income (whether the income came from full-time job, part-time job, or retirement pension), they are classified into employment level; denoted as "1". One continuous variable was the income, indicating the average household income per month.

Panel D: Subjective Perception Variables

Wealth variable was rated on a 5-point scale ranging from 1=poor to 5=wealthy. Two dummy variables were used for bequests and if they live alone, which indicated the respondent's subjective perception for bequest and retirement plan.

3.4 Method 3: Reverse Mortgage Pricing Model

3.4.1 Assumption

Un-Independence of Housing Value and Interest Rate

The most important risks for lender providing the reverse mortgage are the house value risk and the interest risk. To estimate the extent that the LTV lenders could offer in the RM, it is critical to realize the fluctuation of the house value and the interest rate. This study simulated these two risks according to the following assumptions: (1) the housing return follows geometric Brownian motion process; and (2) the risk-free interest rate follows the Cox-Ingersoll-Ross (CIR) model[9]. However, we argue that the asset price, particularly in property is characterized by abrupt and unanticipated large changes known as "jumps" because of the shocks such as deregulated plot ratio, key zones for development, redevelopment of old region, and some restrictive or encouraging policy. Furthermore, the financial crisis will drag down the housing price. Accordingly, this study presents the housing geometric Brownian motion process with a mean revering jump-diffusion processes, which is a generalization of the standard Merton (1976) model[10].

Moreover, we noticed that the fluctuation of the house value and the interest rate are not independent in reality. Therefore we assume that the motion process of the housing value and the interest rate are correlated with one another. This study estimates the correlation coefficient between the housing value and the loan rate. According to the assumption above, the house value and the interest rate simulation model could be defined as follows:

[9] We developed the model presented by Steven (2004), "Stochastic Calculus for Finance 2, Continuous-time models," Springer, P. 468-470.

[10] See: Merton (1976), "Option pricing When Underlying stock returns are discontinuous," Journal of Financial Economics, P.3.

$$
\begin{cases} dr(t) = k(\theta - r(t))dt + \sigma_r \sqrt{r(t)} dZ_r(t) \\ \dfrac{dH(t)}{H(t)} = (r(t) - \delta_H)dt + \sigma_H dZ_H(t) + d(J(t) - \lambda_H \beta t) \end{cases}
$$

$$
Cov(dZ_r(t), dZ_H(t)) = \rho_{rH} dt
$$

$$
J(t) = \sum_{i=1}^{N(t)} (Y_i - 1) \tag{9}
$$

$$
N(t) \sim P(\lambda_H t)
$$

$$
\ln Y_i \sim N(\theta_N, \sigma_Y^{\ 2})
$$

Where

$dr(t) =$ the differentials of interest rate at some future time t following CIR model;

$k =$ the mean-reverting intensity of interest rate;

$\theta =$ the long-term mean-reverting level of the interest rate;

$dt =$ the differentials of time;

$\sigma_r =$ the volatility of risk-free rate;

$dZ =$ the Wiener process with the normal distribution with a mean of 0 and a standard deviation of 1;

$dH =$ the differentials of house value;

$\delta_H =$ the regular expenditure rate of housing;

$\sigma_H =$ the volatility of house value;

$\rho_{rH} =$ Correlation coefficient of house value and interest rate;

$d(J(t) - \lambda_H \beta t) =$ the jump process, where the $J(t)$ is the jump with Poisson distribution;

$Y_i =$ the jump with log-normal distribution: $\ln Y_i \sim N(\theta_N, \sigma_Y^{\ 2})$

$\lambda_H =$ the jump frequency, which is the average number of jumps per year;

$\sigma_Y =$ jump volatility, which is the standard deviation of the proportional jump.

Independence of Loan Terminations

After modeling the fluctuation of the house value and interest rate, the study supposes the RM loan only terminates when the borrower decease. That is, the loan termination in this study is only determined by the demise of borrowers, which is independent from the fluctuation of the interest rate and the housing price. The RM mortgage will not default during the loan term.

This study adopts the mortality of borrowers from the Taiwan Standard Ordinary Experience Mortality (2002 TSO). Once the loan terminated, this study assumes that the lenders will sell the house to repay the mortgage debt.

Break-even Program

This study suggests the RM pricing model is conducted under the break-even program. The present value of the expected losses of the RMs should to be less than or equal to the present value of the expected premium of RMs. If the RM is insured by the insurance agency, they should insure the expect value of the mortgage insurance premium could cover the expected losses. On the other side, if the lenders provide the RM without the insurance program, they should charge the risk premium rate on the loan to incorporate the possible losses in RM.

3.4.2 Model Design

This study derives the RM pricing model to determine the loan-to-value ratio in the RM under the break-even hypothesis based on Szymanoskis' (1994) HECM model. However, the HECM model assumes the RM is under the mortgage insurance program by Fannie Mae and Freddie Mac in the U.S. For the RM provider, the fundamental condition is that the present value of the expected losses on a pool of RMs equals the present value of the expected premium of RM. We thus modified the original model as the following equation:

$$\sum_{t=0}^{\infty} E\left[L(t)\right](1+i)^{-t} \leq \sum_{t=0}^{\infty} E\left[P(t)\right](1+i)^{-t} \tag{10}$$

Where

$E[\ ] =$ the expected value operator;

$L(t) =$ the loss incurred in period t;

$i =$ the periodic discount rate; and

$P(t) =$ the mortgage risk premium or the mortgage insurance premium, which is the scheduled premium collected in period t.

This study simulates the RM mortgage termination for 10,000 times to obtain the distribution of the loss and the mortgage premium. Further, the study replaces the expected value operator with the Conditional Tail Expectation (CTE) to reflect the practical uses in the insurance company. Thus, the RM pricing model in this study is desctribed as follows:

$$CTE_{90}\left[L(t)\cdot(1+i)^{-t}\right] \le CTE_{90}\left[P(t)\cdot(1+i)^{-t}\right] \qquad (11)$$

Where

$CTE_{90}[\ ] =$ the conditional tail expectation operator, which is under the 90% level.

In the equation (11), the expected value of the $P(t)$ is related to the loan survival probability, which is dependent on the borrower's age. For the RM under the mortgage insurance program, the $P(t)$ is combination of the initial insurance premium and annual insurance premium, who respectively charge from the capped property value and the mortgage balance. And the mortgage balance at time t is:

$$BAL(t) = BAL(t-1) + Interest\,(t) + P(t) \qquad (12)$$

For illustration, the demonstration, this study assumes the initial insurance premium is 2% of the capped property value, and the annual insurance premium is 0.5% of the outstanding balance. For the RM without insurance program, the lenders should assume the high risk and consequently charge high risk premium. The current study supposes the interest premium rate is 2%.

On the left-side of the expression (11), the study estimates the expected loss value of $L(t)$ by computing the CTE_{90} of the mortgage loss. The CTE of the mortgage loss could be estimate through the simulation result of the loan termination, housing price, and the interest rate. Yet, the loss only occurs when a loan is terminated, with the housing value is lower than the mortgage balance.

Thus,

$$L(t) = \left(BAL(t) - H(t)\right)$$
$$When, \qquad (13)$$
$$\mathrm{BAL(t)} > H(t)$$

Where

$BAL(t) =$ the outstanding of the RM when the loan termination at time t;

$H(t) =$ the house selling price at time t.

According to the fundamental relationship between the expected losses and the expected mortgage premium, the extent LTV that the lender could offer in the RM can be calculated. This study further computes the IRR_{-RM} of different RM programs in different main regions of Taiwan with different income. We can represent the above rule of RM model with the flow chart shown as Figure 3.1.

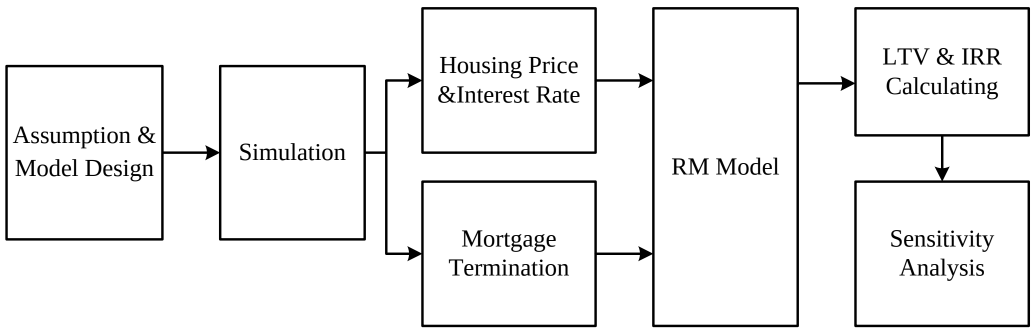

Figure 3.1 Flow chart of the RM Model

3.4.3 Parameter Setting

The housing prices and the interest rate are simulated 10,000 times. The parameters in the simulation model are shown in Table 3.1 and Table 3.2. The process of the parameter estimated in the study as follows:

First, the parameters in the interest rate simulation model are calculated by using the Ornstein-Uhlenbeck process in Vasicek Model[11]. The data of the interest rate adopted is the one-year deposit rate in Taiwan Post Office Deposits. There are 276 data ranging from 1986 January to 2008 December.

Table 3.1 Parameters in the process of simulation

Items	Parameters
Simulation times	**10,000**
Increments (dt)	**1**
Parameters in the Interest Rate Model	
mean reversion speed of interest rate (k)	0.0480
average long-term interest rate (θ)	0.0251
standard deviation of risk-free rate (σ_r)	0.0104
initial risk-free rate (r_0)	2.50%
Parameters in the Housing Value Model	
regular expenditure rate of housing (δ_H)	0.001
jump frequency (λ_H)	1
expected value of the jumps (θ_H)	0
jump volatility (σ_Y)	0.02

Second, the parameters in housing price model, including the average housing price (H_0), the standard deviation of house value (σ_H), the correlation coefficient of the housing price, and the interest rate (ρ_{rH}) are all computed from the housing transaction data and the one-year deposit rate in the Post

[11] If the Vasicek Model is form as: $dr = a(b-r)dt + \sigma dZ$, the $AR(1)$ could be estimated as: $r_t = b(1-\exp(-a)) + \exp(-a)r_{t-1} + \sigma\left[\frac{1-\exp(-2a)}{2a}\right]^{0.5} \varepsilon_t$.

Office. The housing transaction data is collected by this study from an anonymous large real estate agency in Taiwan. The data ranges from 1997 January to 2008 December, with 66,540 cases in nine Taiwan administrative regions, ie., Taipei City, Taipei County, Taoyuan County, Hsinchu City, Taichung City, Tainan City, Kaohsiung City, Yilan City, and Keelung City. The jump factor in this study we assume the jump frequency (λ_H) is one time a year with the 0.02 volatility (σ_Y) and the expect value of the jumps (θ_H) is zero.

As shown in Table 3.2, it is noted that there are different parameters in the housing price models of different regions in Taiwan. According to the housing transaction data, Taipei city and Taipei county have higher housing prices with a lower standard deviation (σ_H =0.0491 & 0.039).

Table 3.2 Parameters in the Housing Value Model

	Average Housing Value (H_0)	Standard Deviation of House Value (σ_H)	Correlation Coefficient of House Value and Interest Rate (ρ_{rH})
Taipei City	11,912,343	0.0491	-0.1638
Taipei County	5,940,344	0.0390	-0.1309
Taoyuan County	3,540,344	0.0622	0.1899
Hsinchu City	4,292,867	0.0572	-0.0163
Taichung City	4,403,122	0.0979	-0.1380
Tainan City	3,242,146	0.0834	-0.1044
Kaohsiung City	3,604,704	0.0529	-0.1333
Yilan City	3,446,524	0.1043	-0.1907
Keelung City	2,748,612	0.1048	-0.3141

Table 3.3 presents the parameters in RM pricing model. The mortgage premium has distinct structure under different type of the RM, with or without the insurance program.

Table 3.3 Parameters in the RM Pricing Model

Items	Parameters
Simulation times	**10,000**
Increments (*dt* **)**	**1**
Mortgage premium rate (within insurance program)	
initial insurance premium (% of property value)	2.0%
annual insurance premium (% of outstanding balance)	0.5%
loan rate (plus in Risk-free Rate)	0.5%
Mortgage premium rate (without insurance program)	
risk premium rate (% plus in Risk-free Rate)	2.0%