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EMERGENT COMPLEXITY IN AGENT-BASED COMPUTATIONAL ECONOMICS

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Abstract. In this paper, we shall review two kinds of emergent complexity in agent-based computational economics (ACE). The first kind is based on the complex systems initiated in the 1980s or even earlier by mathematicians and physicists, whereas the second kind is based on the idea of complex adaptive systems composed of autonomous agents, for which many representative works have been collected in Rosser. For the latter, we shall go further to examine the two elements which have just recently been incorporated in agent-based economic modelling, namely, *intelligence* and *modularity*. This augmentation leads to the development of *neurocognitive software agents*, which can guide the generation and direction of future ACE studies with a multistage 'brain/behaviour-to-emergency-to-brain/behaviour' approach.

Keywords. Agent-based computational economics; Autonomous agents; Computational intelligence; Emergent complexity; Intelligence; Modularity

1. Motivation and Introduction

1.1 *Emergence in an Integrated Framework*

If we were to regard economics as being studied in an interdisciplinary scientific context, then it would be amazing to perceive how economics has constantly expanded and become intertwined with other old and new disciplines. The expansion does not just refer to the enlargement of the application domains, as Gary Becker already pointed out a long while ago (Becker, 1981, 1996), but it also denotes the consolidation of the foundations of economics via the enrichment contributed by other disciplines. Many of these kinds of interdisciplinary studies have been conducted so superbly that they have led to the award of a Nobel Prize. Among them, the three which concern us the most are Herbert Simon's and Daniel Kahneman's contributions on the *behavioural, cognitive and psychological foundations* of economics (Simon, 1997; Kahneman, 2003), Thomas Schelling's pioneering piece on the *agent-based foundation* of economics (Schelling, 1978), and Vernon Smith's *experimental foundation* of economics (Smith, 2006).

This enduring interdisciplinary trend has to some extent changed our own perception of the status of economics in the social sciences. Economics, which used to be regarded as having a very prestigious position or even being the Queen in the social sciences, has now become a much more friendly civilian who also humbly learns from and interacts with other members of the social sciences. Nonetheless, this humble position does not lead to a deterioration in the substance of this discipline as a science; quite on the contrary, it enhances this substance. One of the best manifestations of this is the recent integration of the following four branches of economics: *behavioural economics*, *neuroeconomics*, *experimental economics* and *agent-based computational economics (ACE)*.[1] The essence of the research network of the four is an integration of *human agents* and *software agents* in economics research, and this integration becomes increasingly active when it has elements and ideas that are constantly being imported from psychology, cognitive science, physics and neural science.

This augmented integration not only makes us better equipped to look into how human beings actually behave and why they behave in that way, but it also advances our understanding and predictions of the possible social consequences arising from these behavioural elements. For the latter, we are inquiring into how these behavioural elements can contribute to the *emergent complexity* that appears at a higher or aggregate level. The *sum* of these four pillars makes such emergent complexity differ from each of the four with some promising synergy effects.

First, although behavioural economics and neuroeconomics enable us to know the cognitive biases and possible neural mechanisms of these behavioural constraints, they normally do not move further to see whether these biases can have non-trivial aggregate effects.[2] This so-called *emergent phenomenon argument* is one major argument against neuroeconomics as well as behavioural economics (Clithero, Tankersley, and Huettel, 2008). Second, although experimental economics can help us observe the aggregate outcomes or policy/design effectiveness directly from human behaviour, it normally downplays the possible influences of the cognitive and psychological attributes of subjects, and may even assume these effects to be noises crossing different experiments (Frederick, 2005). This ignorance is partially due to the fact that cognitive, psychological or even cultural attributes of human subjects are costly to know and are difficult to control. As for the last restriction, ACE can provide a relaxation by using *software agents* and can generate the emergent complexity from these software agents (Duffy, 2006). However, it must work with the other three pillars to ensure that the design of the software agents is appropriate via some formal procedure, such as the *Turing test* (Arifovic, McKelvey, and Pevnitskaya, 2006). Therefore, by putting the four together, we can move further to explore the emergent complexity of the cognitive, psychological and cultural characteristics of economic agents.

1.2 *Organization of the Paper*

The remainder of this paper is organized as follow. In Section 2, we provide a brief review of the complex system which gained its popularity in the 1980s in both physics and economics. Its extensive applications to agent-based economic and financial models in the 1990s has generated a major class of ACE models called the *N*-type models (Chen, 2008). This complex system, however, is composed of only *simple and homogeneous agents*, which means that this system is not far away from the *particle system* in physics, and in many regards cannot accommodate the needs of the integrated framework mentioned above. We, therefore, starting from Section 3, review the other class of complex systems, which is also known as the class of *complex adaptive systems* (CAS) (Arthur, Surlauf, and Lane, 1997; Miller and Page, 2007). One essential ingredient of the CAS is the *autonomous agent*.

The second part of the paper introduces two new elements which have recently been introduced to ACE. The first element concerns the human nature (neurocognitive aspect) of the *autonomous agents*. The role of intelligence or cognitive capacity has been recently studied in the context of experimental economics. Section 4 provides a review of the development of this literature. Nevertheless, the counterpart work in ACE is rather lacking, and so Section 5 highlights some initial progress, pointing out possible avenues for future research. Section 6 introduces the second new element, namely, *modularity*, and is followed by the concluding remarks which are given in Section 7.

2. Agent-based Models with Simple Agents

The conventional complex systems are readily demonstrated by Thomas Schelling's segregation model (Schelling, 1978), John Conway's *Game of Life* (Gardner, 1970) and Stephen Wolfram's Cellular Automata (Wolfram, 1986). As simplifications of the more complex von Neumann's self replicating automata (von Neumann, 1966), these agent-based systems nicely demonstrate what the *emergent properties* are, in particular, in the spirit of *unpredictability*.[3] The essence of these complex systems is simply to show how agents with simple and even homogeneous rules can together generate very complex patterns which are difficult to predict. However, not all behavioural rules will lead to emergent complexity. Therefore, one has to carefully choose the behavioural rules that lead to complex and unpredictable patterns.[4]

Given its brevity, this complex system paradigm has been powerfully applied to a number of economic systems, and the most successful and attractive one is probably the *agent-based financial market*. If simple behavioural rules can generate complex patterns, then it is not implausible that the entire financial market complexity can be generated by very simple and almost homogeneous financial agents. Li and Rosser (2004) and many others adopted this approach in modelling financial markets. In these kinds of models, the behavioural rules of financial agents are governed by very simple discrete choice models with only a few alternatives, say, two or three, which are also known as *N*-type models. It has been shown that this simple setting is already sufficient for generating various complex financial dynamics, such as volatility clustering, fat tails, long memory, bubbles and crashes.[5]

However, the main issue is that the agents in these systems are all simple. There is no feedback from the systems to the agents. The emergent complexity is only demonstrated for outside observers of the systems. The agents within the systems are, however, not conscious of the emergent complexity. Neither will they do anything about it, not to mention learning, adapting or discovering. By taking the familiar *fundamentalist–chartist model* as an example, regardless of what kinds of patterns appear in the aggregate dynamics, what our fundamentalists or chartists can do is simply follow a very static reverting or extrapolating form of behaviour. The only allowed learning or adaptive behaviour is manifested through their discrete choice model which generates the switching behaviour between the two possible behavioural alternatives. Therefore, agents are very passive or even idle.

3. Agent-based Models with Autonomous Agents

3.1 *Autonomous Agents*

In contrast to the agent-based models with simple agents, the other class of agent-based models replaces the simple agents with *autonomous agents*. The autonomous agent is the agent who is able to behave (to think, to learn, to adapt and to make strategic plans) with a set of specifications and rules which are given initially, and are fixed with no further intervention. The use of autonomous agents is, in one way or the other, connected to the notion of *bounded rationality*, initiated by Herbert Simon.

To build autonomous agents, agent-based computational economists need to employ existing algorithms or develop new algorithms which can enable agents to behave with a certain degree of autonomy. For example, among many alternatives, genetic algorithms and genetic programming (GP) are two of the most popular. The distinguishing feature delivered by these tools is that it allows the agents to learn and to discover on their own, and hence it enriches our study of the emergent complexity not only at the macro level, but also at the micro level. Moreover, it provides us with a more vivid demonstration of the macro–micro relationship.

3.2 *Emergent Novelties*

To illustrate the kind of emergent complexities addressed in the agent-based models with autonomous agents, we shall compare the evolution of their micro-structure with that of the models with simple agents. We shall in particular provide this illustration using examples from agent-based financial markets, namely, the *market fraction hypothesis* versus the *dinosaur hypothesis*.

The *market fraction hypothesis* (MFH) is associated with agent-based financial models with *simple agents*, in particular, the famous fundamentalist–chartist model (Kirman, 1993; Brock and Hommes, 1998; He and Li, 2008). The market fraction refers to the fraction of each type of agent at a certain time. Market fractions co-evolve with the asset price dynamics. The MFH is then composed of two parts, namely, the short-run one and the long-run one. The short-run one basically says that most of the time the market fraction is distant from the equal share, say, 50% in the case of fundamentalists and chartists, or 33% if contrarians are also included. Between zero and one, the market fraction can exhibit large fluctuations. In other words, the entropy of the market fraction rarely comes close to its maximum. The long-term MFH, however, says that, if we take lone enough, the fractions of each type of trader are approximately equal. Hence, in the long-run, the performances of fundamentalists and chartists are equally good; neither can drive the other out of the market.

Although the MFH is an interesting abstraction of the complex dynamic market microstructure and can even be tested econometrically, it fails to capture one essential dimension of markets, namely, *novelty*. From time to time, only a fixed number of rules is available. The aggregate dynamics will not generated new rules or behaviours, hence the further feedback cycling between micro and macro is limited.

The *dinosaur hypothesis* is associated with agent-based financial models with *autonomous agents* (Arthur, 1992; Chen and Yeh, 2001). The use of the metaphor 'dinosaur' implies that each rule, no matter how powerful or how popular it has been, will eventually be driven out by the market. This kind of emergent complexity is not shared by the former models with simple agents.

These two classes of agent-based models are simultaneously used by economists, and their advantages and disadvantages are also well discussed in the literature (Tesfatsion and Judd, 2006). We do not intend to defend either of them except to indicate that autonomous agents can be more useful when agent-based models are placed in the interdisciplinary framework that we outlined in Section 1.1, in particular, in terms of its integration with modern behavioural and neural experiments in economics. We shall come to this point in the next few sections.

4. Intelligence in Experimental Economics

It is probably only very recently that experimental economists started to realize that traditional experiments with human subjects are not as tightly controlled as we might think. Figure 1 indicates a deliberate selection process of experimental subjects based on their heterogeneity in terms of intelligence, personality and cultural backgrounds. These three dimensions of human factors have recently received increasing treatment among experimental economists.

In this section, we shall briefly review the literature which connects *parameterized intelligence* to behavioural experiments in economics. We shall categorize the literature based on how intelligence is introduced as a key parameter in the respective experiments, or, alternatively, which aspect of intelligence is involved in the decisions of the associated experiments. A number of cognitive tasks stand out, namely, the *depth of reasoning*, *judgements* and *cooperation*.

Before we proceed further, maybe it is necessary to make one point clear. Measuring personal traits, including cognitive ability, personality and culture, is not a straightforward job. Various measures have been developed over time and they were and are under constantly reviewed. Criticisms, debates and some associated controversial issues are extensively available.[6] Taking them into account will be beyond the scope of the paper. Here, we simply acknowledge these possible limitations or constraints to which this section may be subject to. This section is simply to point out the recent trend and attempt to bring the dimensions of intelligence into the studies of experimental economics, which is more generally

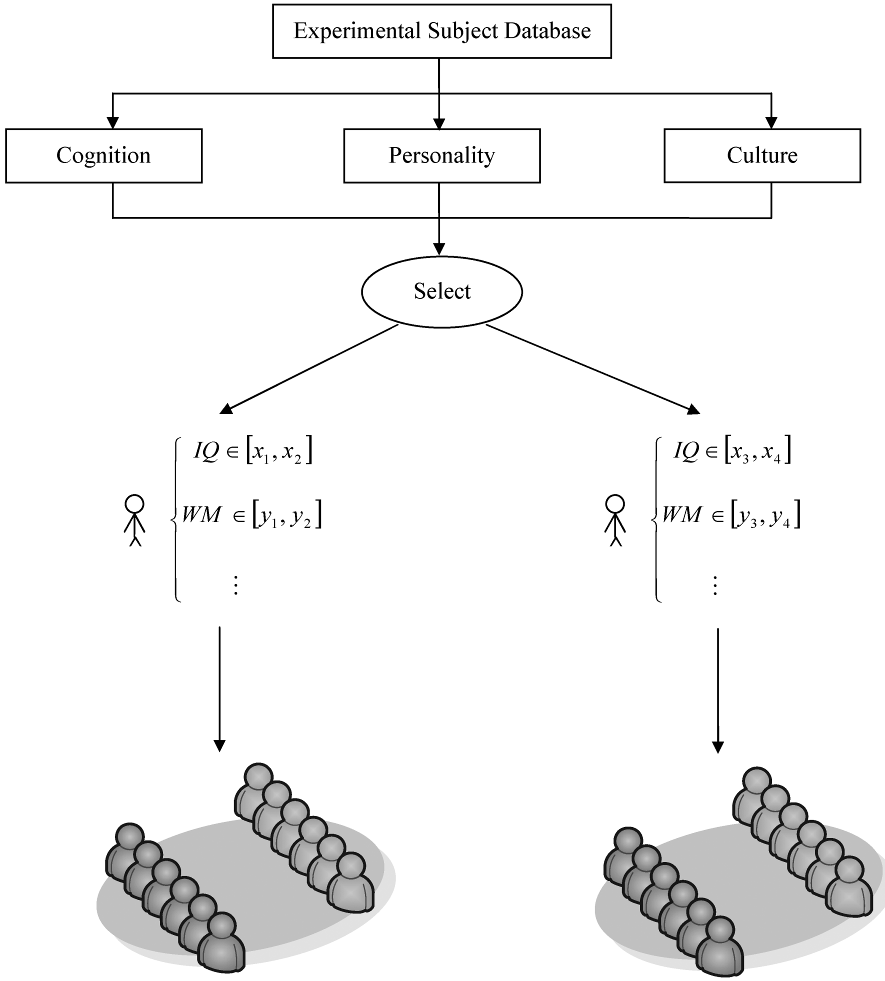

Figure 1. Behavioural Experimental Economics.

Notes: A database storing experimental subjects' cognitive, personality and cultural attributes is established. Designers can then select subjects based on their needs. For example, as indicated above, two experiments are conducted to test the significance of intelligence effects. Experimental subjects are selected based on the required range of IQ, working memory, etc.

sketched in Figure 1. Given the complexities of the associated measure, this attempt is naturally facing several potential challenges and fundamental questions.

4.1 *Depth of Reasoning*

Among all aspects of decision making, *complexity* seems to be the most natural connection to human intelligence. Obviously, complex problem solving requires intelligence. However, not all experiments can give rise to a natural measure of the complexity with regard to the elicited decision making. Exceptions exist only in a few experiments, particularly those based on the notion of *iterated dominance*, such as the dirty face game (Littlewood, 1953), and Keynes' beauty context game (Nagel, 1998), etc. The interesting feature of these games is that it allows us to develop a *step-by-step* computation in the acquisition of a dominant strategy, and hence develop a complexity measure in the spirit of computational theory.

In the iterated dominance game, the number of steps required to eliminate all dominated strategies is also called the *depth of reasoning*. How deliberate a strategic behaviour is can then be connected to this depth. Differentiating the complexity of games based on the depth of reasoning or steps of iterated reasoning is not new (Camerer, 2003); however, the involvement of the intelligence variable in experimental games was absent until very recent. For example, in a *beauty contest game* (Nagel, 1998), Ohtsubo and Rapoport (2006) found a positive relationship between the depth of reasoning and the intelligence measure known as the imposing memory task.

4.2 *Judgements and Learning*

Of course, not all games have an iterated-dominance structure, yet intelligence remains important in other contexts. *Judgemental forecasting* can be an example. Whether the ability to make a good judgement can be related to subjects' cognitive capacity becomes another issue of interest for experimental economists. Casari, Ham, and Kagel (2007) introduced intelligence variables into the *common value auction* experiment (Kagel and Levin, 2002), and found that cognitive ability as measured by SAT (Scholastic Aptitude Test)/ACT (American College Testing) scores matters in terms of avoiding the *winner's curse* (Kagel and Levin, 1986). It is also found that bidders with below median composite SAT/ACT scores, on average, suffer from a winner's curse even as experienced bidders.

4.3 *Cooperation, Generosity and Fairness*

Intelligence involved in the depth of reasoning or judgemental forecasting mainly concerns the correlation between intelligence and *individual performance*. What may be equally, or even more, important is the correlation between intelligence and *social performance*. The study of this correlation is important because there is already a pile of studies indicating the positive impact of intelligence on a country's economic performance, and another pile of studies showing that trust and social capital are essential elements of economic development (Landes, 2000; Francois and Zabojnik, 2005). Therefore, to connect these two piles of empirical studies, it is natural to ask: would intelligence facilitate the formation of cooperation, social capital and trust? Alternatively, as posed by Jones (2008), *are smart groups more cooperative*?

Segal and Hershberger (1999) is the first study which examines how intelligence quotient (IQ) can affect cooperation. They report how pairs of identical and fraternal twins play a repeated prisoner's dilemma game. Their results indicate that pairs scoring higher in terms of IQ were more likely to be *mutually cooperative* than pairs scoring lower in terms of IQ. Jones (2008) corroborates this result using university students as experimental subjects. It is found that students cooperate 5–8% more often for every 100-point increase in the school's average SAT score. Following Axelrod (1984), he argued that *patience* and *perceptiveness* are two important personal traits which promote cooperation, and smarter people, as many other empirical studies have verified, are more patient and more perceptive (Benjamin and Shapiro, 2005; Frederick, 2005).

In addition to the test based on parametric intelligence, there are a number of studies that examine the effects of intelligence by manipulating cognitive loading through taxing short-term memory capacity. In a *dictator game*, Cornelissen, Dewitte, and Warlop (2007) found that under a higher cognitive load 'dictator' tends to offer more. In their study, short-term memory is manipulated by asking the subjects to memorize a string of eight numbers.

5. Intelligence in Agent-based Computational Economics

Despite the increasing tendency to make intelligence an explicit control variable in experimental economics and to explore its emergent outcomes, agent-based computational economics, which is normally claimed as the software counterpart of experimental economics, has paid almost no attention to this development. On the contrary, from a pure engineering viewpoint, there is a tendency to make software agents as smart as possible, and usually *equally smart*. This design principle, therefore, obviously contradicts our understanding of human agents. If the society of software agents cannot reasonably reflect the dispersion of human intelligence, then any resultant social simulation would be of little help to us in gaining insights into the emergent complexities, such as the perplexing relationship between IQ and social development (Lynn and Vanhanen, 2002; Lynn, 2006). Therefore, designing software agents with heterogeneous intelligence is the next step in exploring the emergent complexities of ACE.

5.1 *Agent-based Models with Heterogeneous Working Memories*

Chen, Zeng, and Yu (2008) have probably developed the first agent-based model to tackle this issue. In the context of the agent-based double auction market, they used GP to model agents' adaptive behaviour. This way of modelling is not new; however, they no longer assume that agents are equally smart; instead, following the series of experiments which provided evidence of the importance of heterogeneity in subjects' short-term memory capacity (Cornelissen, Dewitte, and Warlop, 2007), they manipulated one control parameter of GP so that the agents' 'working memory capacity' can be 'born' differently. The parameter which they manipulated was the *population size*.

GP is a population-based algorithm, which can implement parallel processing. Hence, the size of the population will directly determine the capability of parallel processing. On the other hand, the human's short-term memory capacity is frequently tested based on the number of the cognitive tasks which human beings can simultaneously process. Dual tasks have been used in hundreds of psychological experiments to measure the attentional demands of different mental activities (Pashler, 1998). Hence, the population size seems to be an appropriate choice with regard to mimicking the working memory capacity of human agents.[7] A smaller population size, therefore, corresponds to a smaller working memory capacity, whereas a larger population size corresponds to a larger working memory capacity. In this way, a market composed of agents with different working memory capacity is introduced.

Chen, Zeng, and Yu (2008) then simulated this agent-based double auction market, and examined the emergent properties at both the macro level (the market performance) and the micro level (the individual performance). At the macro level, they run a regression of market efficiency on the population size (a proxy for the working memory capacity). It is found that working memory capacity has a positive and significant impact on the market efficiency, which is measured by the sum of the realized consumer's and producer's surpluses. Hence, the same institutional arrangement when applied to a population of agents with different intelligence may have different results in terms of market efficiency. The more intelligent group will perform better than the less intelligent one. Of course, this result can still be crude, and require more extensive tests, but the point here is how agent-based simulation by incorporating the intelligence variable can be more communicative with field data (Weede and Kampf, 2002; Jones and Schneider, 2006; Ram, 2007).

In addition to the aggregate outcome, they also compared the strategies learned from agents with different working memory capacity. Because all the strategies learned from GP have their LISP (list programming) structure, they can be depicted as a parse tree. This tree structure gives us a simple measure of the complexity for any acquired strategies based on the sizes of the trees. Chen, Zeng, and Yu (2008) then analyse the relation between the complexity of profitable strategies learned by the agents and their associated working memory capacity. They find that some strategies which are more complex but also more profitable had never been found by agents with a capacity of 10, but could quite frequently be found by agents with a capacity of 50. Further analysis of these strategies shows that additional capacity facilitates the combinatorial operation which agents need to cook up with more complex and profitable strategies. In a sense, this result extends the findings of Ohtsubo and Rapoport (2006) to a more complex situation, namely, a double auction game.

A more challenging part of their work is to examine the co-evolutionary dynamics when competing agents become equally smarter. This brings us closer to the situation discussed in Section 4.3. Chen, Zeng, and Yu (2008) show that, even in a competing situation like the double auction game, pairs of smarter agents can figure out a way to cooperate so as to create a win-win situation, whereas this collaboration is not shared by pairs of less smarter agents.

Altogether, agent-based modelling with proper incorporation of the essential characteristics of human agents can make itself a proper toolbox to enhance our

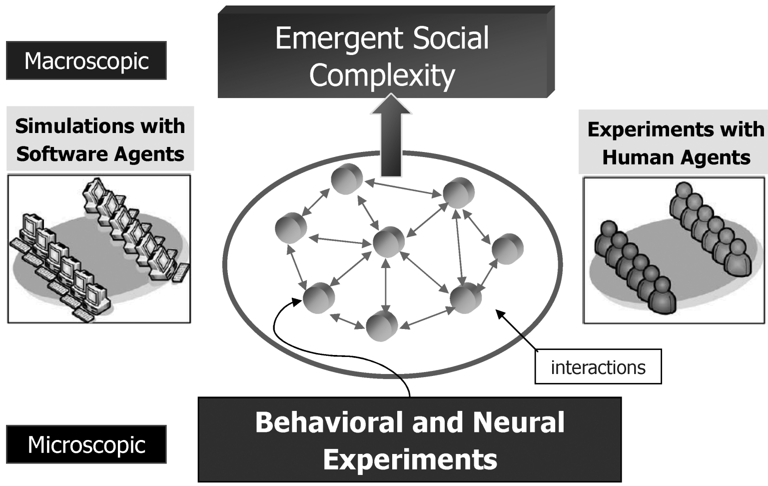

Figure 2. ACE and Experimental Economics: Building a Bridge by Human-Like Designs of Software Agents.

understanding of the emergent outcomes from human experiments or from field data. The entire picture is provided in Figure 2.

5.2 *Intelligence and Learning Algorithms*

We have mentioned the recent experimental economics that has focused on the intelligence effect. By parameterizing intelligence using short-term memory, Casari, Ham, and Kagel (2007) introduced intelligence variables into the *common value auction* experiment (Kagel and Levin, 2002), and found that cognitive ability as measured by SAT/ACT scores matters in terms of avoiding the *winner's curse* (Kagel and Levin, 1986). Casari, Ham, and Kagel (2007)'s result has important implications for agent-based economic modelling and agent engineering, because they show that intelligence not only influences agents' static performance, but also their *learning dynamics*. So, even though the experiments are repeatedly conducted with the same subjects, their performance may not converge or may only converge at a slow rate.

In agent-based modelling, this phenomenon was first addressed in Feltovich (2005). Feltovich (2005) shows that if decision makers learn via a specific version of reinforcement learning, their behaviour typically changes only *very slowly*, and persistent mistakes are likely. Feltovich (2005) pointed out the difference between slow learning and no learning. Although Feltovich (2005) did not make explicit reference to short-term memory capacity, his manipulation of reinforcement learning can, in a sense, be interpreted as a search for software agents with lower short-term memory capacity.

In Feltovich (2005)'s case, the cognitive capacity of software agents is manipulated with the same learning algorithm, namely, reinforcement learning. Nevertheless, reinforcement learning has been frequently compared with other learning algorithms in the agent-based modelling of games, for example, belief-based learning and genetic algorithms (Duffy, 2006). Therefore, here comes another issue, i.e. instead of manipulating the control parameters of the same algorithm, be it GP (Chen, Zeng, and Yu, 2008) or reinforcement learning (Feltovich, 2005), *how can one choose or compare different learning algorithms in light of the heterogeneity of various forms of parameterized intelligence?* This is the second kind of issue facing agent-based economic modelling, and is slightly different and more advanced than the one discussed in Section 5.1. To make this clear, we shall use Charness and Levin (2009) as an illustration.

Charness and Levin (2009) further pursued the issue between intelligence and learning behaviour. Their finding shows that the failure to perform Bayesian updating can be a cause of the winner's curse, and the ability to perform Bayesian updating is dependent upon the agents' cognitive capacity. In this case, obviously, two learning algorithms are involved, and they are assumed to be associated with different types of cognitive loading.

Intuitively, intelligence can affect the way in which agents learn from the environment's feedback, because different learning algorithms, as decision making, have different degrees of complexity. In computational learning theory, there is even a formal treatment on the complexity of learning machine (Kearns, 1990; Hutter, 2000). Although these complexity measures are not necessarily computable, it is conceivable that some learning algorithms may be more complex than others. For example, Bayesian learning can generally be more complex than reinforcement learning.

Due to the general negligence of intelligence effects in ACE, there is also no effort being made to consider a mixture of various learning algorithms, which can reasonably reflect the empirical lessons drawn from market or game experiments, and to explore its consequence. To the best of our knowledge, Chan *et al.* (1999) is probably the only study of this kind. In a context of agent-based artificial stock markets, they consider three different types of agents, namely, momentum traders (chartists), empirical Bayesian traders and K-nearest-neighbour (KNN) traders.

The efficient markets hypothesis implies that there are no profitable strategies, and hence learning, regardless of its formalism, does not matter. As a result, the three types of traders, momentum traders, empirical Bayesian and KNN traders, should behave equally well, at least in the long run. However, when the market is not efficient, and learning may matter, it is expected that smarter agents can take advantage of dumber agents. In their experiments, Chan *et al.* (1999) found that momentum traders, who never learn, performed the worst during the transition period when the market is not efficient. Furthermore, the empirical Bayesian traders were also outperformed by the KNN traders. Although the two types of traders started learning at the same time and competed against each other to discover the true price, the KNN traders were evidently able to exploit predictability more quickly than the empirical Bayesian traders.

Like Feltovich (2005), Chan *et al*. (1999) did not make any explicit reference to agents' heterogeneity in intelligence; therefore, the way in which they introduced the three types of agents is not empirically driven. One of the next steps of ACE should be to attempt to empirically ground the mixture of different learning algorithms in either field data or experimental data so as to ensure the empirical relevance of the then-established agent-based models of heterogeneously intelligent agents.

6. Modularity in Agent-based Computational Economics

6.1 *Modularity: Legacy of Herbert Simon*

As we mentioned at the very beginning of this paper, the integrated framework presented here is inspired by Herbert Simon, who conducted a number of pioneering interdisciplinary studies in economics, psychology, computer science, artificial intelligence and complex systems. Among many of his great works, the one to which he devoted almost his entire academic life is *modularity* and its relationship with complex systems. He started this work and continued it to the end of his life (Callebaut and Rasskin-Gutman, 2005). Herbert Simon viewed *hierarchy* as a general principle of complex structures (Simon, 1965). Hierarchy, he argued, emerges almost inevitably through a wide variety of evolutionary processes, for the simple reason that hierarchical structures are *stable*. To demonstrate the importance of a *hierarchical structure* or *modular structure* in production, Simon offered his well-known story about a competition between Hora and Tempus, two imaginary watchmakers. In this story, Hora prospered because he used the modular structure in his design of watches, whereas Tempus failed to prosper because his design was not modular. Therefore, the story is mainly about a lesson: the advantage of using a *modular design* in production.

Modularity is becoming more important today because of the increased complexity of modern technology. Using the computer industry as an example, Baldwin and Clark (2000) show that the industry has experienced previously unimaginable levels of innovation and growth because it embraced the concept of modularity. Kamrani (2002) also asserts that embracing the principle of modular design can enable organizations to respond rapidly to market needs and allow the changes to take place in a cost-effective manner.

Nevertheless, the idea of modularity is not restricted to economics, and it has drawn no less attention from psychologists (Fodor, 1983, 2000) and neuroscientists (Churchland and Sejnowski, 1992; Brocas and Carrillo, 2008a). The recent progress in neuroscience has allowed us to identify a number of brain modules at various levels of granularity. In addition, various hypotheses regarding *the modularity of mind* also exist, such as the famous *massive modularity hypothesis* (Williams, 1966; Sawkins, 1976). The recent literature has indicated that a trend integrating economics, psychology and neural science has emerged as a new interdisciplinary research subject, and an easy way to see the drive of this collaboration is from the familiar *dual system conjecture*.

6.2 *Dual System Conjecture*

The dual system conjecture generally refers to the hypothesis that human thinking and decision making are governed by two different but interacting systems. This conjecture has been increasingly recognized as being influential in psychology (Kahneman, 2003), neural science (McClure *et al.*, 2004) and economics. The two systems are an *affective system* and a *deliberative system* (Loewenstein and O'Donoghue, 2005) or a *reflexive system* and a *reflective system* (Lieberman, 2003). The affective system is considered to be myopic, activated by environmental stimuli, and primarily driven by affective states. The deliberative system is generally described as being goal oriented and forward looking. The former is associated with the areas of the brain that we have labelled the ventral striatum (nucleus accumbens, ventral caudate and ventral putamen), the right striatum, neostriatum and amygdala, among others, whereas the latter is associated the areas of the brain that we have the labelled ventromedial and dorsolateral prefrontal and anterior cingulate, among others.

The dual system of the brain has become the neuroeconomic area which economic theorists take the most seriously. This has also helped with the formation of the new field known as *neuroeconomic theory*. A number of dual-process models have been proposed in economics with applications to *intertemporal choice* (Loewenstein and O'Donoghue, 2005; Fudenberg and Levine, 2006; Brocas and Carrillo, 2008a), *risk preferences* (Loewenstein and O'Donoghue, 2005) and *social preferences* (Loewenstein and O'Donoghue, 2005). All these models view economic behaviour as being determined by the interaction between two different systems.

The application of the dual system conjecture to learning is just the beginning. Earlier, we have mentioned the cognitive loading between different learning algorithms, such as reinforcement learning versus Bayesian learning (Section 5.2). This issue has been recently discussed in experimental economics (Charness and Levin, 2005), and now also in neuroeconomics (Bossaerts *et al.*, 2008).

6.3 *Software Agents with Neurocognitive Dual System*

Although agents with dual systems have been considered a new research direction in neuroeconomic theory (Brocas and Carrillo, 2008a, 2008b), software agents or autonomous agents in agent-based modelling mostly largely follow a single system. However, the dual system interpretation exists for many agent-based economic models. Consider the fundamentalist–chartist model as an example, where the fundamentalist's and chartist's behaviour can be differentiated by the associated neural systems, say, assuming the former is associated with a deliberative system whereas the latter is associated with the affective system.

Another example is the *individual learning* versus *social learning*. These two learning schemes have been frequently applied to model the learning behaviour in experiments and their fitness to the experimental data are different (Hanaki, 2005). Agent-based simulation has also showed that their emergent patterns were different. For example, in the context of an artificial stock market, Yeh and Chen

(2001) show that agents using individual learning behave differently from agents using social learning in market efficiency, price dynamics and trading volume. If individual learning can be associated with, say, the deliberative system, and social learning can be connected to the affective system, then the dual system can also be applied to agent-based modelling. This issue opens the future to collaboration between agent-based economics and neuroeconomics.

6.4 *From Modular Mind/Brain to Modular Preference*

At present, modularity is still not a part of agent-based economic modelling. This absence is a little disappointing because ACE is regarded as a complement to mainstream economics in terms of articulating the mechanism of evolution and automatic discovery. One way to make progress is to enable autonomous agents to discover the modular structure of their surroundings, and hence they can adapt by using modules. This is almost equivalent to causing their 'brain' or 'mind' to be designed in a modular way as well.

The only available work in agent-based economic modelling which incorporates the idea of modularity is the *agent-based models of innovation* initiated by Chen and Chie (2004). They proposed a *modular economy* whose demand side and supply side both have a *decomposable* structure.[8] Although the decomposability of the supply side, i.e. production, has already received intensive treatment in the literature, the demand side has not. Inspired by the study of *neurocognitive modularity*, Chen and Chie (2004) assume that *the preference of consumers can be decomposable*.[9] In this way, the demand side of the modular economy corresponds to a market composed of a set of consumers with *modular preference*.

In the modular economy, the assumption of modular preference is made in the form of a dual relationship with the assumption of modular production. Nevertheless, whether in reality the two can have a nice mapping, e.g. a one-to-one relation, is an issue related to the distinction between *structural modularity* and *functional modularity*. Although in the literature this distinction has been well noticed and discussed, 'recent progress in developmental genetics has led to remarkable insights into the molecular mechanisms of morphogenesis, but has at the same time blurred the clear distinction between structure and function' (Callebaut and Rasskin-Gutman 2005, p. 10).

The modular economy considered by Chen and Chie (2004) does not distinguish two kinds of modularity, and they are assumed to be identical. One may argue that the notion of modularity that is suitable for preference is structural, i.e. *what it is*, whereas the one that is suitable for production is process, i.e. *what is does*. However, this understanding may be partial. Using the LISP parse-tree representation, Chen and Chie (2004) have actually integrated the two kinds of modularity. Therefore, consider drinking coffee with sugar as an example. Coffee and sugar are modules for both production and consumption. Nevertheless, for the former, producers *add* sugar to coffee to deliver the final product, whereas for the latter, the consumers drink the mixture by knowing the existence of both components or by 'seeing' the development of the product.

Chen and Chie (2007) tested the idea of augmented GP (augmented with automatic automatically defined terminals) in a modular economy. Chen and Chie (2007) considered an economy with two oligopolistic firms. Although both of these firms are autonomous, they are designed differently. One firm is designed with simple GP, whereas the other firm is designed with augmented GP. These two different designs match the two watchmakers considered by Simon (1965). The modular preferences of consumers not only define the search space for firms, but a search space with different hierarchies. Although it is easier to meet consumers' needs with very low-end products, the resultant profits are negligible. To gain higher profits, firms have to satisfy consumers up to higher hierarchies. However, consumers become more and more heterogeneous when their preferences are compared at higher and higher hierarchies, which calls for a greater diversity of products.[10] It can then be shown that the firm using modular design performs better than the firm not using modular design, as Simon predicted.

7. Concluding Remarks

The literature on complexity in economics has been superbly surveyed and collected in Rosser (2004), which includes emergent complexity in agent-based economic models. Although this work has also included a few agent-based economic models with autonomous agents, emergent complexity is mostly discussed at the macro level. In this paper, we argue that the advantage of using autonomous agents is that it enables us to explore the possible emergent complexity at the micro level. In addition, the use of autonomous agents also bridges the gap between agent-based economic simulation and human-subject economic experiments. We particularly emphasize some recent augmentations of autonomous agents, inspired by experimental economics and neuroeconomics, by incorporating the *intelligence heterogeneity of human intelligence* and the *modularity of brain, mind and preference*. This work leads to the development of neurocognitive software agents, and starts with a 'molecular' foundation of aggregate dynamics. More work could be done along these lines in the future. For example, personality, social preference and culture can be included. Hence, the emergent complexity in economics and psychology can be firmly connected, and, hopefully, this is one of the aims further pursued by the economics profession.

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Notes

1. ACE is only part of the more broad field known as *computational economics*. For example, it appears only in two out of the eighteen chapters in Kendric, Mercado, and Amman (2006).
2. Having said that, we must notice that behavioural finance does study the market outcomes emerging from cognitive biases (Hirshleifer, 2001; Barberis and Thaler, 2003; Stracca, 2004). Nonetheless, most behavioural financial models are still confined to the representative agent assumptions.
3. It has been shown that various versions of cellular automata are capable of universal computation. For example, see Kari (2005). However, the computational-theoretic foundation of agent-based economic models has not been rigorously addressed in the literature. Few exceptions are Velupillai (2000, 2010a, 2010b).
4. To some extent, this way of rule selection is similar to that of finding the best performing random number generators (algorithms).
5. For a survey on this class of agent-based financial models and their emergent complexities, see Hommes (2006).
6. Probably one of the most controversial issues is whether individual differences in cognitive ability have a high heritability. See Rushton and Jensen (2005) for the recent development. Of course, our survey from human-subject experiments to computational-agent modelling would not involve this issue.
7. The idea of using population size as a proxy variable for working memory is first proposed in Casari (2004), who literally treated the population size used in the genetic algorithm equivalent to the number of chunks that human can process at a time. According to the famous '7 ± 2' rule proposed by Miller (Miller, 1956), the capacity lies between five to nine. Casari (2004) then set the population size of genetic algorithms to 6, 'which implies that decision makers have a hardwired limitation in processing information at six strategies at a time. (Casari, 2004, p. 261)' This is the probably the earliest article which connects the *population size* used in evolutionary computation to *working memory capacity* in cognitive psychology. However, in Casari (2004), agents are still treated as homogeneous.
8. Although this assumption is easier for the purpose of modelling, it is too much stronger than the assumption of only *near decomposability* as what Herbert Simon has developed (Simon, 1965, 2002).
9. Whether one can build preference modules upon the brain/mind modules is of course an issue deserving further attention.
10. If the consumers' preferences are randomly generated, then it is easy to see this property through the combinatoric mathematics. On the other hand, in the parlance of economics, moving along the hierarchical preferences means travelling through different regimes, from a primitive manufacturing economy to a quality service economy, from the mass production of homogeneous goods to the limited production of massive quantities of heterogeneous customized products.

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