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動態空間交互模型的理論與實證分析

The Theoretical and Empirical Analysis of the Dynamic Spatial Interaction Model

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中文摘要

本研究主要理論上推衍以最大熵概念為基礎的動態空間交互模型，並以電腦模擬探討此模型之特性。分析結果顯示以最大熵目標所推導的最適靜態區位流量估計式為基礎的區位選擇機率估計式恰符合數學 logit 模型的型式。由此靜態的區位選擇機率式所推導的動態機率估計式為基礎的動態空間交互模型有許多重要的特性：（一）此動態模型所解釋的成長過程對於參數值非常敏感。不同的參數值可導致完全不同結構的衍化過程。模擬結果顯示，同一動態模型可產生穩定與振動兩種不同的區域成長過程。同時在穩定的成長型式裏，參數與起始條件的不同會導致"不隨機"與"隨機"兩種不同的衍生結果。（二）此動態空間交互模型可模擬都市成長現象中最明顯卻又缺乏理論說明的法則：「Zipf's Law」。模擬結果發現衍化時間越長，都市分佈越趨近於大小不均（與實證結果相近），同時區位間利益的差異越大，衍化速度越快。區域中城市分佈的結構會收斂的因素主要是來自於變動的區位利益（聚集經濟）。

關鍵字：區位利益，衍化過程，聚集經濟

Keywords: location attractivity, Zipf's law evolution process, agglomeration effect.

The Theoretical and Empirical Analysis of the Dynamic Spatial Interaction Model

Abstract

The purpose of this paper is to theoretically derive a dynamic spatial interaction model based on the entropy theory and use this derived growth process to explain the mysterious Zipf's law. Empirical findings show that (1) the purposed dynamic process possibly generates both stable and unstable patterns according to the value of the parameters. (2) In the stable evolution, the model possibly generates both deterministic and stochastic growth processes. (3) Both deterministic and stochastic growth processes could converge to Zipf's pattern. (4) Evidence from cities in Taiwan shows the diminishing estimated intercept and slope as the proposed model predicted. Size distribution in Taiwan converges to Zipf's pattern.

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1. Introduction

It is well known that the size distribution of cities is surprisingly well described by Zipf's law across countries with various economic structures and histories. Its robust empirical evidence and significant regularity in economics make Zipf's law the minimal criterion for any urban growth model. However, there is a lack of plausible theoretical models to explain this empirically robust distribution.

The expression of Zipf's law can be visualized by taking a cross sectional data on city size and city rank. Then draw a graph with the log of rank along the y-axis, and the log of the population along the x-axis. The resulting graph will mostly show a straight line with slope very close to −1, based on the regression. The linear relation of log of size versus log of rank is explained as the famous Zipf's law. This amazing result is shown in various data sets: most modern countries (Rosen and Resnick [1980]), India in 1911 (Zipf [1949]), U.S. history (Dobkins and Ioannides [1998], Krugman [1996], Zipf [1949]) and China in the mid-nineteenth century (Rozman [1990]). Empirical exercises of different countries and periods show the general explanation power of Zipf's law. There have been quite a few important efforts to explain or resolve the puzzle of rank-size rule: such as economics models (Losch [1954], Hoover [1954], Beckman

[1958]), Krugman's spatial model (Krugman [1999]), and Simon's random-growth model (Simon [1955]). The puzzle still remains even though these efforts do offer different ways to analyze the possible theoretical foundation.

Gabaix (1999) proposed Gibrat's law to explain Zipf's law. He finds that homogeneous growth processes will lead the distribution to converge into a Zipf pattern. Homogeneity of growth processes refers to the common mean and common variance of city growth rate. In spite of the driving forces of the growth of cities, or the economic structures of the countries; as long as they satisfy Gibrat's law, Zipf distribution will appear. According to Gibrat's law, both mean and variance of growth rate are independent of the size of the city. Randomly growing cities with the same expected growth rate and the same variance will converge to a Zipf pattern.

Gabaix's work proposes a general and neat interpretation to explain that puzzling regularity - Zipf's law. Gabaix uses Eeaton and Eckstein's data to show that the variance of the growth rate does not seem different across sizes. Eaton and Eckstein (1997) find there is no correlation between the initial size and the growth rate for both Japanese and French cities. These empirical results show some evidences for Gabaix's finding. However, the interaction behavior among cities, that is the essential driving force of agglomeration, in the region is not expressed in Gabaix's work.

The purpose of this paper is try to explain Zipf's law by a growth process oriented from a spatial interaction model, which is theoretically derived from the concept of entropy in physics. Furthermore, there are a few questions about Zipf's law that we wish to understand more from this paper: What does Zipf's law indicate in cities' distribution? What does the slope of the curve is close to −1 means? Is it possible to change this "law" in terns of the timing and the slope?

The major concept of entropy is to derive the maximum uncertainty estimator given the limited information. This feature has been greatly applied in urban and regional modelling for commuting pattern and location choice probability in transport and location models. These urban and regional modelling mostly focus on the static solution from entropy, such as probability distribution and the implied spatial interaction model. Nijkamp and Reggiani (1991) have derived the dynamic process of the location choice probability distribution from entropy, nevertheless, the evolution property and the size distribution have not been investigated. Due to the legitimate explaining power of the static probability estimator from entropy in regional modelling, it is quite essential to investigate the properties of the followed dynamic process and the converged distribution. Due to the surprising regularity of Zipf's law in empirical size distribution of various countries and at different times, it is greatly motivated to exam the features of the evolution process from entropy and the possible relation between this evolution process and the mysterious Zipf's law.

Section 2 is the theoretical background of the proposed dynamic logit model. Section 3 investigates the property of the proposed model through simulations, and examines the size distribution of cities in Taiwan. Section 4 presents the conclusion.

2. Residential location and the spatial interaction model

2.1 Entropy in a spatial interaction model

The application of entropy-maximizing methods to the trip distribution has been discussed in Wilson (1967). The major concept is to maximize the "uncertainty" in terms of possible assignment subject to all prior information with respect to the trip distribution. The entropy theory applied in this topic aims to derive the most probable trip or migrant distribution given additivity conditions and transport cost budget constraint. The solved optimal trip estimator is:

$$T_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij}) \tag{1}$$

where A_i and B_i are balancing factors, and $\exp(-\beta c_{ij})$ is the distance friction function. Parameter, β, in the distance friction function is the marginal possible states (the objective function in entropy problem) per unit of transport cost. And the parameter, c_{ij}, represents the general transport cost between location i and j. The function of this optimal flow appears corresponding to the idea of gravity theory. Please see appendix for the details of the deriving process or variable explanation.

The probability of transport or migrant from location i to j derived from the gravity type migrant flow (equation (1)) is:

$$P_{ij} = \frac{T_{ij}}{\overline{O}_i} = A_i B_j \overline{D}_j \exp(-\beta c_{ij}) = \frac{B_j \overline{D}_j \exp(-\beta c_{ij})}{\sum_j B_j \overline{D}_j \exp(-\beta c_{ij})} = \frac{W_j \exp(-\beta c_{ij})}{\sum_j W_j \exp(-\beta c_{ij})} \quad (2)$$

2.2 Qualitative choice model

Traditional location theory assumes households maximize their utility subject to budget constraints in residential location decisions. Assuming V_{ij} as the systematic household utility and ε_{ji} as the error term, the household utility function is as follows:

$$U_{ij} = V_{ij} + \varepsilon_{ji}. \quad (3)$$

Under the consideration of household utility maximization, and given the distribution for the unsystematic part of utility (ε_{il})[1], the probability that the household will migrant from city i to city j is

$$P_{ij} = \Pr ob(U_{ij} > U_{il}, for \quad all \quad l, \quad l \neq j) = \frac{e^{V_{ij}}}{\sum_l e^{V_{il}}}, \quad (4)$$

[1] Assume that each ε_{il} is distributed independently, identically in accordance with the extreme value distribution.

This is the multinomial logit model. The utility function plays the role of location advantage. The larger the observed utility a household could achieve at city j, the more attractive that city j is to the household; consequently, the higher the probability that the household would choose to migrant to city j. The probability that the household would migrant from city i to city j (equation (2)) is the relative location advantage.

2.3 Dynamic process of the discrete choice model

A simplified probability model with time variable from equation (2) is:

$$P_{j,t} = \frac{\exp(V_{j,t})}{\sum_{l} \exp(V_{l,t})} \tag{5}$$

where $\sum_{i=1}^{n} P_{i,t} = 1$. This is in a multinomial logit form based on the assumption that a household chooses alternative j to achieve the maximized observed utility V_j. The negative term, $-\beta c_{ij}$ (in equation (2)), represents the major concern that location choice is based on in this simplified model. It indicates the reduction of the possible number of states due to city i's location in the region. The shorter the distance between city i and other cities, the higher the location accessibility at city I; consequently, the higher the selection advantage at city i. In an extended model, the location endowment other than location differences will also be included in this location advantage term. The discrete dynamic logit model derived by Nijkamp and Reggiani (1991) is as follows:

$$P_{j,t+1} = (\dot{V}_j + 1)P_{j,t} - \dot{V}_j P_{j,t}^{2} - P_{j,t}\sum_{j \neq l} \dot{V}_l P_{l,t} \qquad (6)$$

The first two terms at the right-hand side is the logistic growth of choice probability P_j. The third term is the interaction effects within region. This dynamic spatial interaction process from entropy expresses that the change of choice probability for city j not only is influenced by its current choice probability in a decreasing rate, other cities' choice probabilities also play competitive roles in city j's growth.

The variable V_j is the observed utility or location benefit in city j. This systematic location benefit is assumed to consist of two parts according to the time variable: (1) Geographical advantage, a location advantage caused by known geographical endowment and benefit, is fixed through time. It is the source of the deterministic force in the dynamic process. (2) Agglomeration advantage, another kind of location advantage caused by external effect from population and employment gathering together, is varied through time. It is historical dependent and the source of possible stochastic force in the growth of cities.

$$V_{i,t} = \Psi_i + h(y_{i,t}) \qquad (7)$$

3. The long runs location pattern of the spatial interaction model

3.1 The model

The discrete dynamic logit model is simulated. We assume a region with n cities. Each city grows due to immigrants or industries from outside region. Assuming there is no inter-cities immigrants. The growth process is based on the discrete dynamic location choice probability as in equation (6). We simulate the spatial interaction model to exam the property of the evolution process by varying variables and the initial condition: number of city (n), length of time path (t), change of utility (a), and the initial value of location choice probability P_j.

Households choose residential location where utility is maximized, and the industries choose location where their profits are maximized. Both utility and profit in corresponding location reflect the location advantage to the decision makers. The location advantages are the major concern of decision-makers in their location decisions of this model. Under the assumption that the location advantage is decomposed into two parts: fixed and time varying parts. They are corresponding to determined and stochastic forces of growth processes. The determined location advantage named geographical advantage affects the growth of city through the initial location choice probability that reflects the relative geographical advantage. Furthermore, the time varying location advantage influences the growth of city through the change of utility in the model. The change of utility through time is the change of the time-dependent advantage (agglomeration benefit).

In the simple case, assuming the change of utility as a constant α_j :

$$P_{j,t+1} = (\alpha_j + 1)P_{j,t} - \alpha_j P_{j,t}^{\ 2} - P_{j,t}\sum_{j \neq l}\alpha_l P_{l,t} \tag{8}$$

If change of the utility α_j equals zero, the choice probability will be fixed through time and converge to the real size proportion in the long run. The growth process reaches steady state when the choice probability converges to the real size proportion.

3.2 Simulation

The evolution process based on the dynamic spatial interaction modelpossibly leads to two different kinds of dynamic patterns: stable and unstable processes depending on the parameter values.

(1) Stable process

Assume there are 50 cities in the region, and all has uniform initial city zise. The change of the utility is assumed as a constant 'a'. It is generated from the random number within a range (0, 0.04). The simulated time path for cities in the region is presented in Fig. 1.1 and Fig. 1.2. The dynamic probabilities and city sizes are converging to a stable trajectory.

(2) Oscillating process

Another experiment based on the same assumption and initial condition as in the previous experiment. The range of the random number generated as the change of utility is changed to (0, 2.7). The simulated time path for all 50 cities is presented in Fig. 2.1 and Fig.2.2. The dynamic probabilities and city sizes are oscillating with the unstable trajectories. These two experiments show that the same dynamic interaction rule would lead to two essentially different evolution processes due to the value of parameter (scale of time varying location advantage).

(3) Features in stable evolution

Assume the same numbers of cities, (n=50), evolution time, t (100), and the scale of time varying location advantage, a (0.04), are all the same as in experiment (1). The simulated distribution of city size and rank is presented in Fig. 3.1. The Zipf plot that shows the distribution of log size versus log rank is presented in Fig. 3.2. We run the regression of Zipf's law.

$$\ln(Rank) = A - B \ln(Size),$$

The result is

$$\ln(Rank) = 6.17 - 0.67 \ln(Size),$$
$$(0.77 \quad 0.57)$$

where the 95% confidence interval of estimated slope is in parentheses, and the R^2 is 0.787. The estimated slope in Zipf plot is different from 1 which Zipf's law would predict. The experiment results are in Table 1.1 and Table 1.2. The small value of the standard deviations of both estimated intercept and slope implies that a negative slope Zipf plot could always be generated from a dynamic logit model. Also, given the same conditions and randomly generated change of utility parameters, different growth processes have close estimated value of both intercepts and slopes in the Zipf plot.

(3.1) Evolution time and region size

The simulations in this section are based on the same number of cities (n=50), and the scale of time-varying location advantage (a=0.04). The only difference is the evolution time path (t). The regression result is in Table 2.1. The simulation result of the region of 100 cities (n=100) is in Table 2.2. The corresponding evolution graphs and the Zipf plots are in Fig. 4 and Fig. 5.

The absolute value of the estimated slope in the Zipf relation is getting smaller after a longer evolution time. The longer the time the more divergence of the size of cities in the region. This is due to the cumulated effect of the location advantage. A longer evolution time reduces the scale of the slope in the Zipf relation. At a certain time during evolution, the absolute value of the slope will be close to 1. In Table 2.1, the number of cities is 50, and the estimated slope is close to one at t=67; in Table 2.2, the number of cities is 100, and the estimated slope is close to one at t=75. A larger region (more number of cities) does not change the features that city sizes get less homogeneous in longer evolution time. On the contrary, larger number of cities in the region reduces the speed of the interaction process.

(3.2) The scale of the time-varying location advantage

The change of the time-varying location advantage is assumed to be a constant, α_i in equation (8), for each city through time. Table 3 lists simulation results given different values of parameter α_i. The larger the value of parameter α_i, the smaller the absolute value of the slope. This implies that the more significant the difference of each city's change of utility that affect migrants' choices, the more divergent the city sizes within the region.

(3.3) Test of lock-in effect

In this experiment, we change the value of parameter α_i of city 3 into three times the original scale at time t=50, and exam whether the final choice probability distribution (t=100) would be changed. The correlation coefficient of the final distribution with and without the change of the parameter is 0.86. The city which dominates the region is still be the dominant city even if city 3 has relatively higher time varying location advantage than the original dominate city in the middle of the evolution. This result implies the possible "lock-in" property of the dynamic process. This property is one of the essential features of the self-organization system.

(3.4) The average and variance of the growth rate across sizes

The condition of Gibrat's law is examined given initial arbitrary probability distribution. Fig. 6 shows the plot of growth rate versus normalized population size. The mean growth rates at the first period is clearly independent of city sizes; the mean growth rates at period 100 also does not show significant relation with the city sizes.This feature has empirical evidence in Eeaton and Eckstein (1997). The mean and variance of the average growth rate is in Table 4. The variance of growth rate across sizes is the same. The average growth rate across sizes does not seem the same across cities.[2] However, the differences between average growth rate across cities are within 0.0391.

(3.5) Determinism versus chance

To distinguish the time-dependent property in the location advantage V_j as in equation (7) allows both deterministic and stochastic features into the growth process. The geographical advantage is determined by the given location benefit, which is fixed cross time. City with higher geographical advantage has selection advantage. Agglomeration advantage depends on current size of population and employment; it is changed through time and is historical independent.

[2] An F-test evaluates the equality of the average growth rate of N cities show significant differences across N cities in both initial distributions. The F-statistic is F=1056 given initial uniform

Allowing only known geographical advantage in the location advantage (utility or profit) without time-varying location advantage will make the region growth pattern becomes deterministic. The dominant city will always be the one with highest geographical advantage. Including the time-varying location advantage but assuming constant value (constant change of agglomeration and other time-dependent advantage through time) will also lead to a deterministic long tern pattern. The long run distribution is based on the initial known location advantage and known effect from agglomeration.

Relaxing the assumption of the constant change of utility into a time-varying variable will include the stochastic features into the dynamic process. Assume the agglomeration advantage is bounded. The simulation results show possible multi-dominate cities in the steady state; these dominant cities do not necessarily endow the highest geographical advantage or the largest time-varying location advantage. The historical dependent force dominates the known geographical advantage in this case. This implies the special feature of the model that a deterministic rule may lead to a stochastic long-term pattern. And experiment results find that stochastic growth process also possible generates Zipf pattern in the steady state.

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distribution, and F=1493 given initial arbitrary distribution.

3.3 Evidence on the size distribution of Cities in Taiwan

We collect data on the population of 209 to 216 cities in Taiwan for the years: 1971, 1974, 1977, 1980, 1983, 1986, 1989, 1992, 1995, and 1998.[3] The criterion for selection is population of at least 20,000 inhabitants. The regression result for all 10 years is in Table 5. Data of cities in Taiwan shows that the absolute value of the estimated slope is decreasing and converge to 1 through time. The adjusted R^2 is 0.96 in 1971 and increases through time. The size distribution of cities in Taiwan tends to converge to the Zipf's law. Similar to the result of previous simulation (Table 2), both estimated intercept and slope have diminishing absolute values cross time. This indicates that the urban system in Taiwan converges to a less homogeneous city size distribution. This may due to the cumulated effect of location advantage including both fixed geographical and time-varying location advantages..

[3] The data are from Statistics Annals by Ministry of Interior.

4. Conclusions

In this paper, we exam the property and long run distribution pattern of a growth process derived from the entropy concept. The proposed model possibly generates both deterministic and stochastic growth processes. Both deterministic and stochastic processes could reach Zipf pattern in the long run.

Zipf's law indicates certain degree of the combination of different size of cities. The decreasing absolute value of the slope as time pass by from both empirical data and simulation result indicates that cities grow from a more homogeneous states into a more heterogeneous distribution. In the evolution process, Zipf's law shows that the region will not evolve beyond certain degree of "heterogeneous distribution". The absolute value of the slope will not decrease infinitely. The converged level is at a certain distribution which corresponds to the slope equals -1. The converged state is at the balance point of two contradicting forces: positive and negative agglomeration effects in cities. The timing of the converging time mainly depends on the following conditions: the initial location differences and endowments which affect decision makers' location advantage, and the change of the location advantage through time. The change of the location advantage essentially indicates the change of the net agglomeration effect (positive and negative agglomeration effects) in cities. A change of the interaction effect and the structure of both positive and negative agglomeration effect may change the converging distribution (slope).

Some major findings about the properties of the growth process of this model are as follows. (1) The purposed dynamic process possible generates both stable and unstable patterns according to the value of the parameters. (2) In the stable evolution process, the purposed model possibly generates both deterministic and stochastic growth processes. (3) The dynamic logit choice model possibly generates Zipf's pattern. (4) The longer the evolution time; the less homogeneous of cities in the region; the smaller the absolute value of the slope in Zipf's plot. This is due to the cumulated effect of location advantages. (5) The number of cities in the region affects the speed to reach Zipf's pattern. The larger the size of region (number of cities), the slower the evolution process. (6) The larger the change of utility through times the faster the speed of the evolution process. (7) Evidence from cities in Taiwan shows the diminishing estimated intercept and slope as the proposed model predicted. Size distribution in Taiwan converges to Zipf's pattern.

Simulation findings correspond to the findings from previous studies that for most modern countries with different economic and social structures, the distribution of city sizes tends to follow Zipf's law. Although there exist countries or urban systems which differ from a Zipf pattern, but they show patterns converge to Zipf distribution. Given the assumption of the dynamic process of location choice probability and the randomly generated geographical advantage, an urban system with certain parameter value will evolve to a Zipf pattern in the long run. The timing of the Zipf pattern achieved depends on the number of cities and the relative location advantage. The location advantage includes both fixed and time-dependent advantages. This helps us to explain why most countries with different properties could converge to the same long-run Zipf's pattern. The influence from the change of relative location advantage (or related parameters) on the timing of steady state for Zipf distribution is an important question for future research in policy implication.

APPENDIX: Entropy Theory in Spatial Interaction and the Dynamic Logit Model

Let T_{ij} be the number of trips (or migrants) and c_{ij} is the travel cost between zones i and j; let O_i be the total outflows from zone i, and D_j be the total inflows to zone j. The entropy $w(T_{ij})$ measures the uncertainty of assignments of individual units to an origin-destination matrix. Maximizing logarithm of $w(T_{ij})$ subject to the additivity conditions (2), (3) and transport cost budget constraint (4) derive the most probable arrangement of spatial distribution of trips in the system.

$$w(T_{ij}) = \frac{T!}{\prod_i \prod_j T_{ij}!} \tag{1}$$

$$\sum_j T_{ij} = O_i \tag{2}$$

$$\sum_i T_{ij} = D_j \tag{3}$$

The travel budget C is expressed as follows:

$$\sum_i \sum_j c_{ij} T_{ij} = C \tag{4}$$

The following consistency condition should also hold:

$$\sum_i \sum_j T_{ij} = \sum_i O_i = \sum_j D_j = T \tag{5}$$

The optimal flow T_{ij} is derived:

$$T_{ij} = A_i B_j O_i D_j \exp(-\beta c_{ij}) \tag{6}$$

where $A_i = \{\sum_j B_j D_j \exp(-\beta c_{ij})\}^{-1}$

and $B_j = \{\sum_i A_i O_i \exp(-\beta c_{ij})\}^{-1}$ (7)

The term $-\beta c_{ij}$ represents the reduction in total numbers of possible states induced from transport cost between i and j, and the term $\exp(-\beta c_{ij})$ is the distance friction function. A_i and B_j are balancing factors. The function of this optimal flow appears corresponding to the idea of gravity theory,

The derived gravity type migrant flow from i to j (Equ. (6)) gives the probability of a destination choice from i to j as the following:

$$P_{ij} = \frac{T_{ij}}{\overline{O_i}} = A_i B_j \overline{D_j} \exp(-\beta c_{ij}) = \frac{B_j \overline{D_j} \exp(-\beta c_{ij})}{\sum_j B_j \overline{D_j} \exp(-\beta c_{ij})} = \frac{W_j \exp(-\beta c_{ij})}{\sum_j W_j \exp(-\beta c_{ij})} \qquad (8)$$

where $W_j = B_j \overline{D_j}$ is the weight. Consider possible time varying probability, adding the time variable into equation (8):

$$P_{ij,t} = \frac{W_{j,t} \exp(-\beta c_{ij,t})}{\sum_j W_{j,t} \exp(-\beta c_{ij,t})} \qquad (9)$$

where $c_{ij,t}$ represents the distance between i and j at time t.

This equation is transformed into a simple form by omitting the symbol of the origin i and assuming weight $W_{j,t} = 1$, and $-\beta c_{ij,t} = u_{j,t}$.

$$P_{j,t} = \frac{\exp(u_{j,t})}{\sum_l \exp(u_{l,t})} \qquad (10)$$

where $u_{j,t}$ could be interpreted as a choice factor, which is the utility achieved by choosing alternative j. The above probability is the formula of multinomial logit models in discrete choice models, which assume a household chooses alternative j to achieve the maximized utility u_j.

The evolution of the dynamic multinomial logit model is expressed by the change of probability $P_{j,t}$ with respect to time t:

$$\frac{\mathrm{d}P_{j,t}}{\mathrm{d}t} = \dot{P}_{j,t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\exp(\mathrm{u}_{j,t})}{\sum_{\mathrm{n}}\exp(\mathrm{u}_{\mathrm{n,t}})}\right] \tag{11}$$

$$\dot{P}_j = \dot{u}_j P_j(1-P_j) - P_j\sum_{n\neq j}\dot{u}_n P_n \tag{12}$$

where the symbol t is omitted for the sake of simplicity. The term $\dot{u}_j$ represents the change of utility through time; it is assumed to be a constant α_j. Expression (12) is a system of Lotka-Volterra type. The first term at the right-hand side is the logistic growth of population P_j, and the second term is the interaction effects among population. Equation (12) is approximate by discrete time and derive:

$$P_{j,t+1} = (\alpha_j + 1)P_{j,t} - \alpha_{jj}P_{j,t}^{\;2} - P_{j,t}\sum_{j\neq l}\alpha_l P_{l,t} \tag{13}$$

where $\alpha_j = \dot{u}_j$

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Table 1.1

Simulation result of 'rank-size' regression (n=50) *

$\ln (Rank) = A - B \ln (Size)$

	A	B
	10.097	1.393
	10.637	1.483
	10.387	1.441
	9.952	1.362
	9.681	1.311
	10.947	1.548
	9.879	1.351
	9.174	1.211
	10.377	1.439
	11.815	1.712
	9.970	1.364
	9.896	1.343
Mean	10.234	1.413
Standard deviation	0.678	0.122

* City size (n=50), evolution time (t=50), scale of location advantage (a=0.04)

Table 1.2

Simulation result of 'rank-size' regression (n=100) *

ln (*Rank*) = A – B ln (*Size*)

	A	B
	6.221	0.679
	5.943	0.634
	6.192	0.679
	6.143	0.679
	6.636	0.761
	6.867	0.800
	5.946	0.633
	5.945	0.646
	6.535	0.757
	6.467	0.735
	5.899	0.639
Mean	6.295	0.702
Standard deviation	0.330	0.059

* City size (n=50), evolution time (t=100), scale of location advantage (a=0.04)

Table 2.1

Simulation result of 'rank-size' regression (n=50)

ln (*Rank*) = A – B ln (*Size*)

Time	A (Estimated constant)	B (Estimated slope)
50	3.62	1.28
60	8.69	1.13
65	8.36	1.07
67	7.96	0.998
70	7.73	0.96
75	7.54	0.93
80	7.13	0.85
100	3.09	0.70
200	2.54	0.34
300	2.28	0.26

Notes: Scale of location advantage (a=0.04)

Table 2.2

Simulation result of 'rank-size' regression (n=100)

ln (*Rank*) = A – B ln (*Size*)

Time	A (Estimated constant)	B (Estimated slope)
50	9.91	1.41
60	8.56	1.12
65	8.72	1.16
70	8.10	1.04
75	7.97	1.01
80	7.44	0.90
100	6.53	0.73
200	4.61	0.36
300	3.88	0.24

Notes: Scale of location advantage (a=0.04)

Table 3
Change of scale of location advantage (a)*

a	A	B
0.004	68.723	12.415
0.04	9.762	1.324
0.4	2.765	0.115
0.9	2.318	0.030

* Number of cities (n=50), evolution time (t=50)

Table 4

Means and variances of the average and variance of growth rates

	Average growth rate	Variance of growth rate
Mean	-0.0042	8.8277e-006
Variance	0.000132	1.1357e-039
Minimum	-0.0242	8.8277e-006
Maximum	0.0149	8.8277e-006
Observations	50	50

Table 5*

'Rank-size' regression of cities in Taiwan

$$\ln(Rank) = A - B \ln(Size)$$

Year	Observation	A	B	Adj-R^2
1971	216	21.138	1.478	0.96
1974	216	19.491	1.412	0.97
1977	216	18.932	1.355	0.97
1980	216	18.430	1.304	0.98
1983	216	18.055	1.265	0.98
1986	213	17.748	1.234	0.99
1989	210	17.351	1.196	0.99
1992	207	16.946	1.154	0.99
1995	207	17.060	1.163	0.99
1998	209	16.822	1.140	0.99

* Source: Statistics Annals by Ministry of Interior

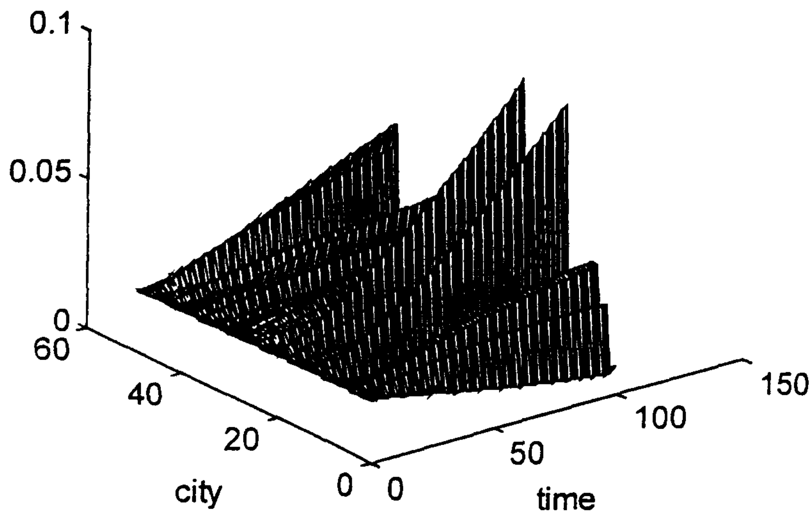

Fig. 1.1 The dynamic probability path of all cities in the region
(n=50, t=100, a=0.04)

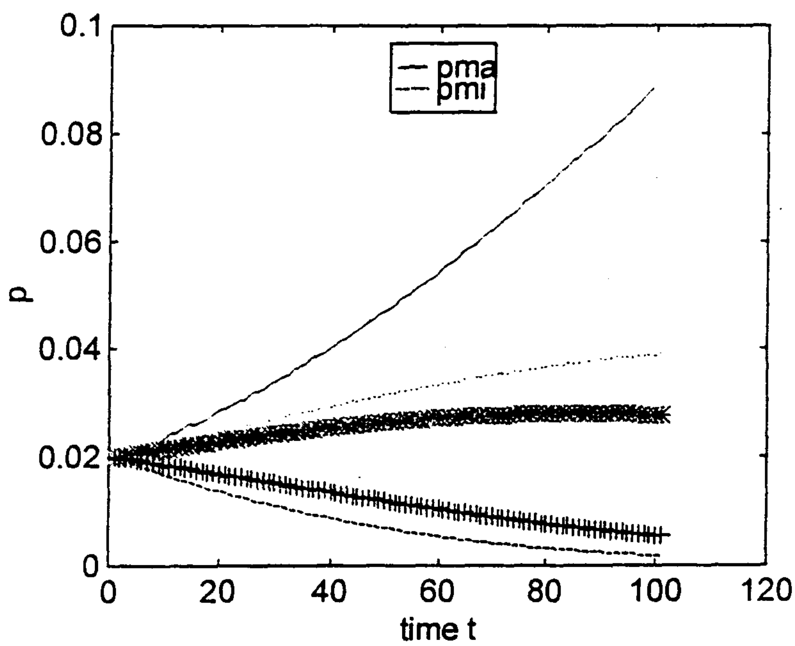

Fig. 1.2 The dynamic probability path of four cities in the region
(Including the cities with highest and lowest choice probability)

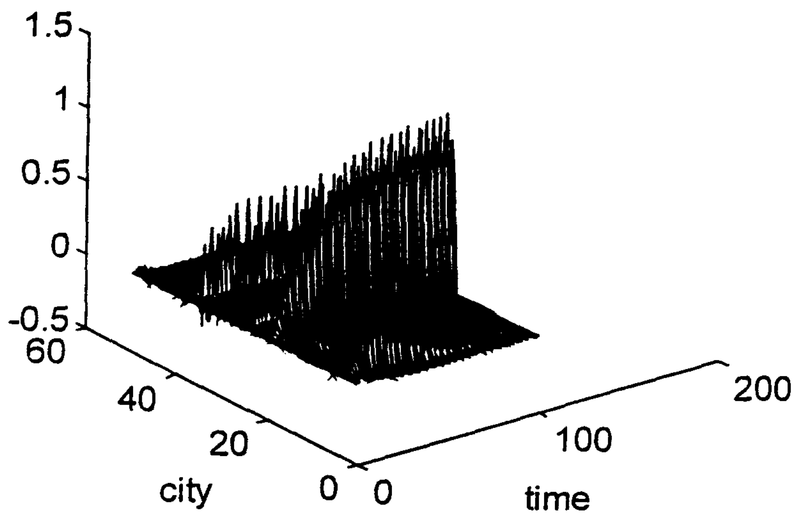

Fig. 2.1 The dynamic probability path of all cities in the region
(n=50, t=100, a=2.7)

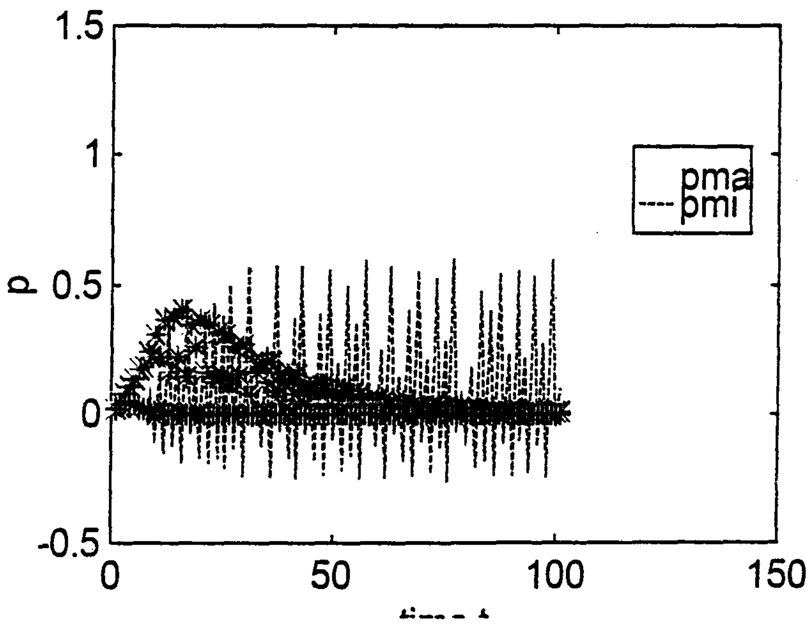

Fig. 2.2 The dynamic probability path of four cities in the region
(Including the cities with highest and lowest choice probability)

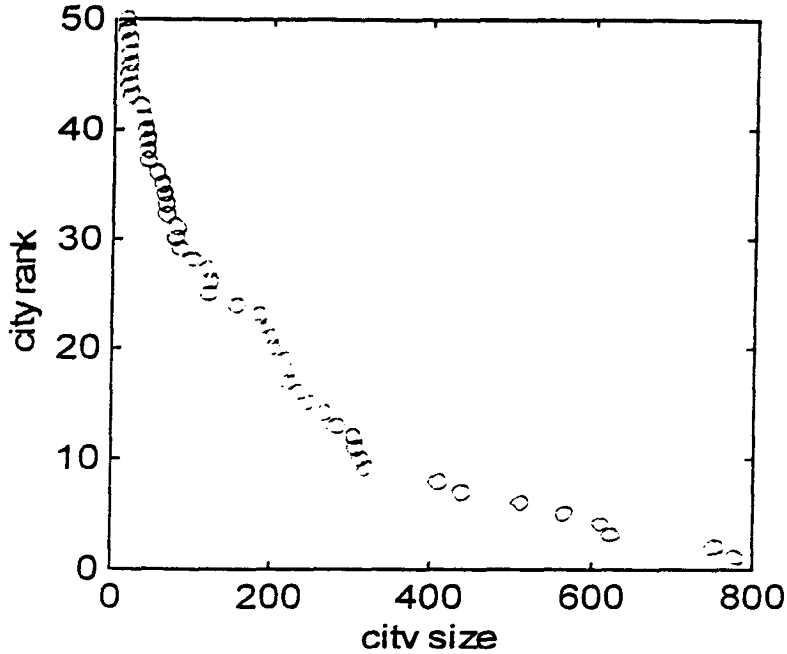

Fig. 3.1 City size versus rank. (n=50, t=100, a=0.04)

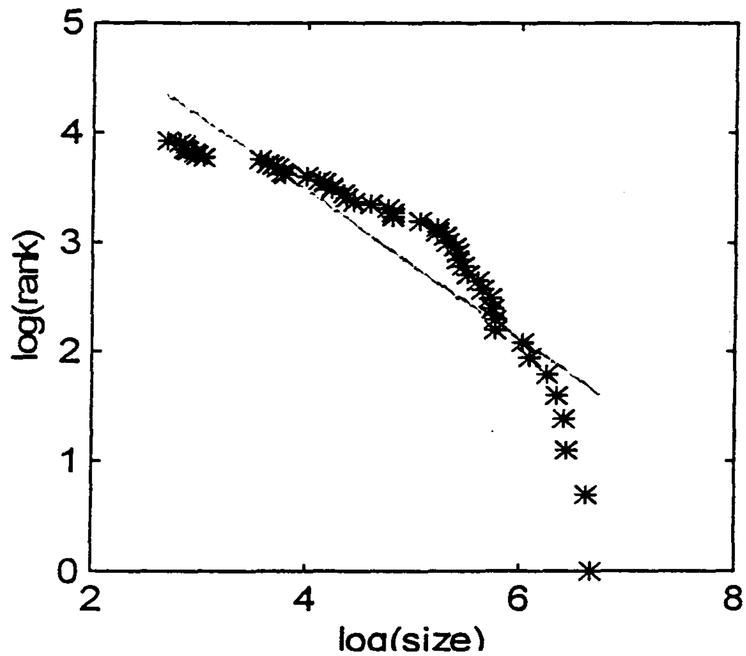

Fig. 3.2 Log size versus log rank

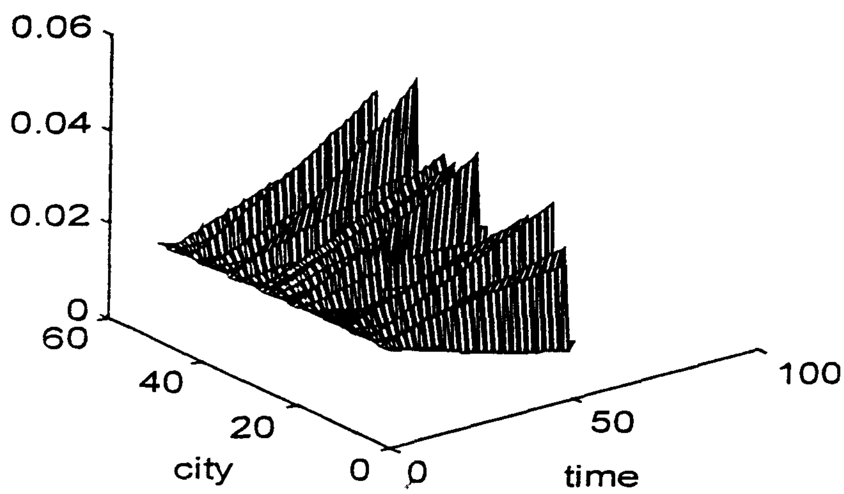

Fig. 4.1 The dynamic probability path of all cities in the region
(n=50, t=50, a=0.04)

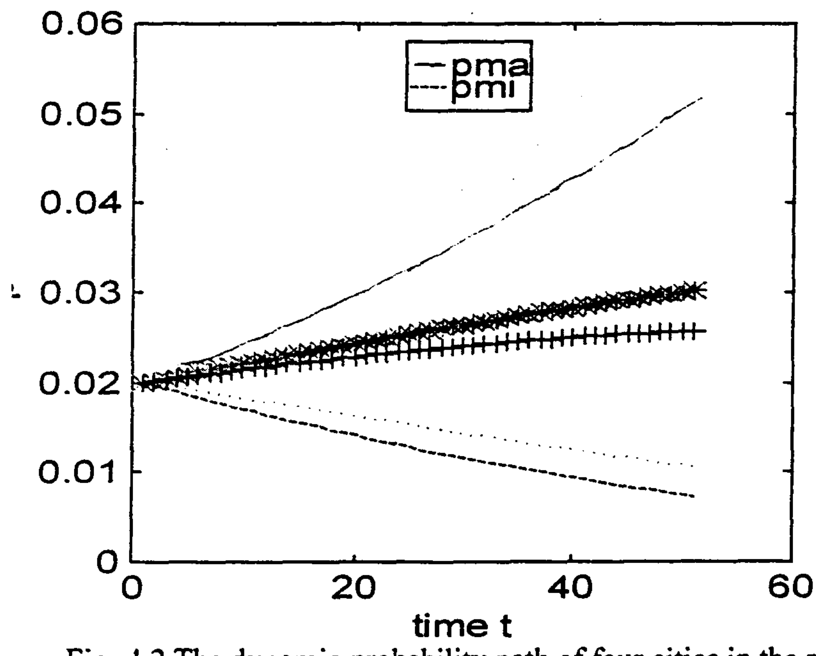

Fig. 4.2 The dynamic probability path of four cities in the region
(Including the city with highest and lowest choice probability)

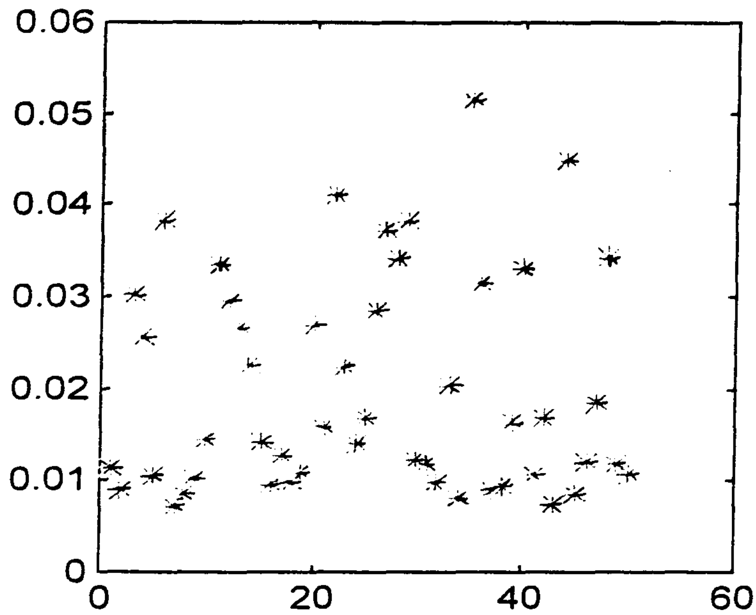

Fig. 4.3 Cities versus choice probability at t=50

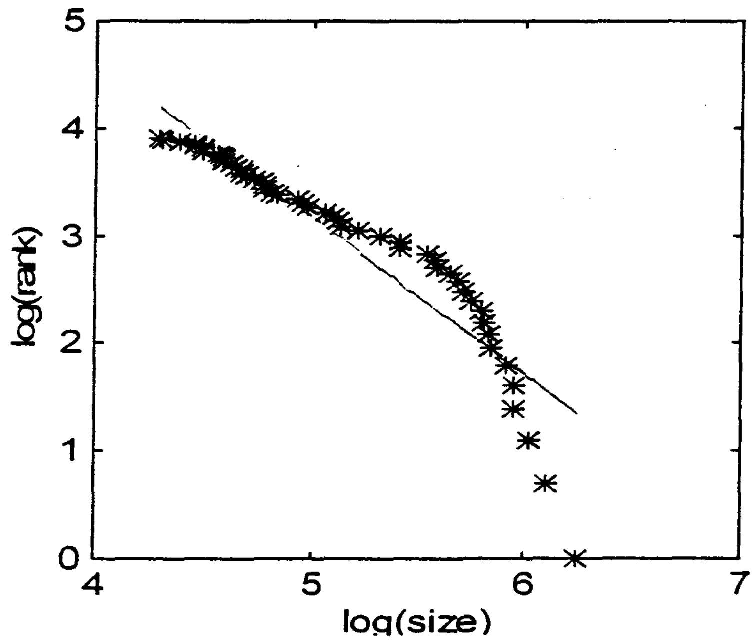

Fig. 4.4 log size versus log rank

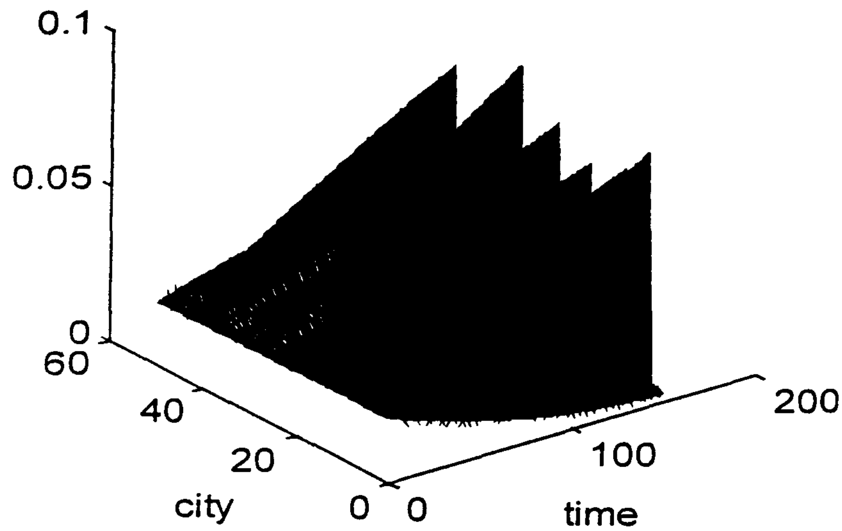

Fig. 5.1 The dynamic probability path of all cities in the region
(n=50, t=150, a=0.04)

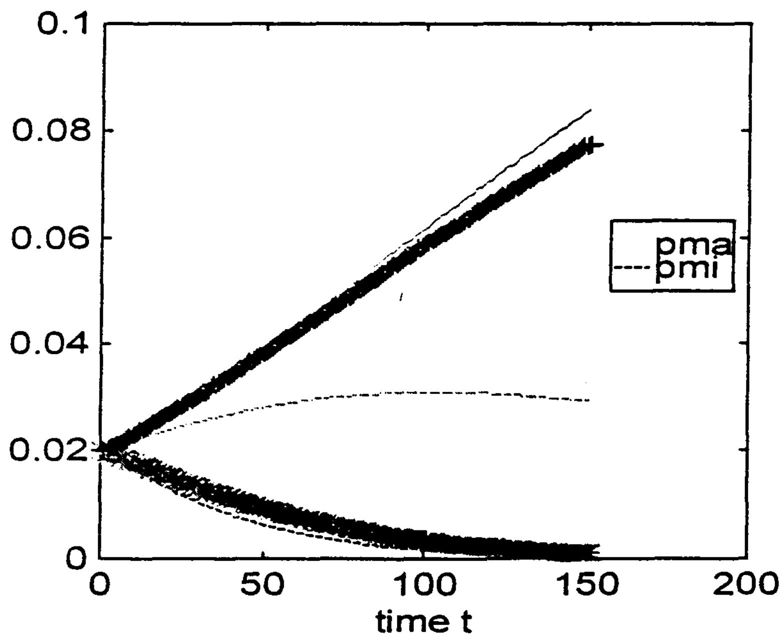

Fig. 5.2 The dynamic probability path of some cities in the region
(Including the cities with highest and lowest choice probability)

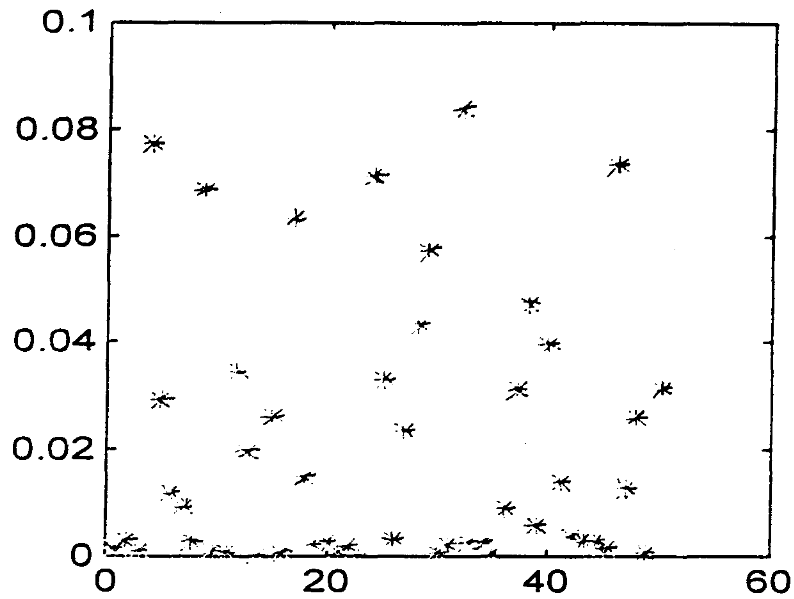

Fig. 5.3 Cities versus choice probability at t=150

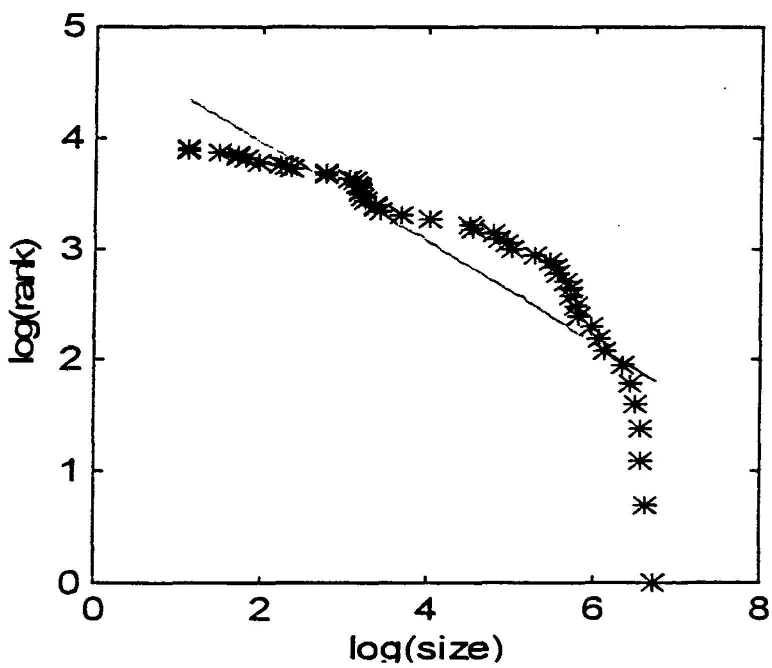

Fig. 5.4 Log size versus log rank

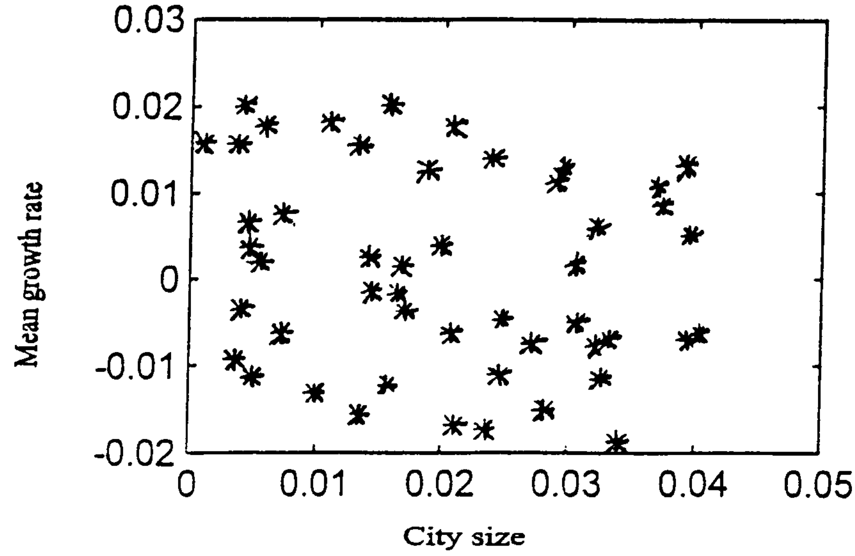

Fig. 6.1 Mean growth rates versus city sizes at t=1

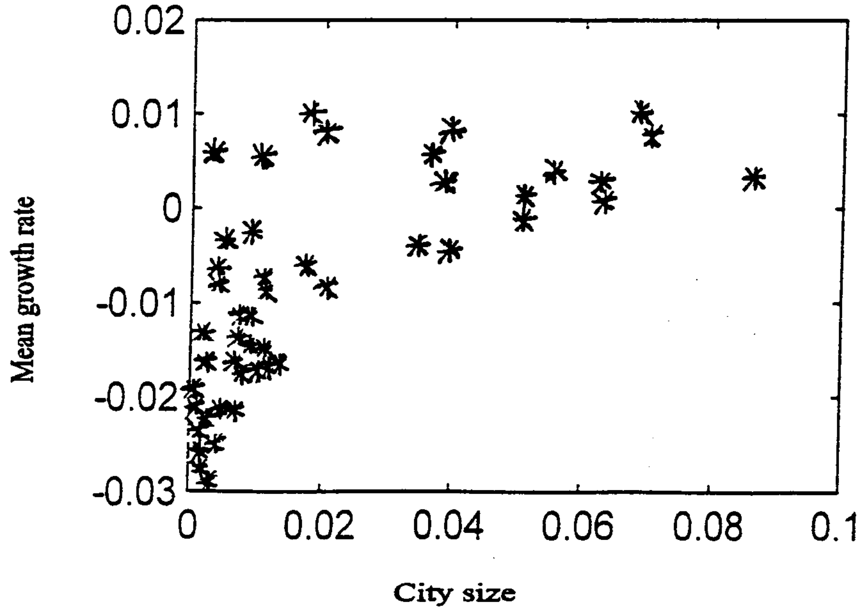

Fig. 6.2 Mean growth rates versus cities size at t =100