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具相關誤差之線性迴歸模型的剖面曲線監測
研究成果報告(精簡版)

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中文摘要： 剖面曲線監測（profile monitoring）在品質管制的領域
中，是一相對新且漸為受到重視的方法；其應用於在給定的
時間內，以一剖面曲線（profile）描繪該一製程。本研究計
畫主要討論的剖面曲線為誤差具自我相關移動平均過程
（autoregressive moving-average process）的複迴歸模
型，將提出相關統計方法在此類剖面曲線的製程下，藉以發
現失控樣本的診斷設計。研究以電腦模擬方式檢驗所提出的
方法；同時，藉由一個實際的資料分析陳示此方法。

中文關鍵詞： 誤差相關；Hotelling 's T2 統計量；統計製程管制。

英文摘要：

英文關鍵詞：

Monitoring Profile Based on a Linear Regression Model with Correlated Errors

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1. Introduction

Profile monitoring is the use of control charts for cases in which the quality of a process or product can be characterized by a functional relationship between a response variable and one or more explanatory variables. Due to advances in technology, such monitoring is becoming more common to obtain profiles at each time period, especially when a series of data points forms a curve representing the quality state of a process. These cases appear to be increasingly common in practical applications. Indeed, it is crucial in Phase I of a control chart scheme to determine which of the data points are similar to each other and which are outliers in some way. This ensures that the Phase II application will be adequate for real-time monitoring. Recent research has focused on how to determine those outlying profiles in Phase I applications. For a detailed overview of profile monitoring, examples of its application, and a review of the literature, see Woodall *et al*. (2004), Woodall (2007), and references therein.

The majority of works on profile monitoring have focused on situations where the profiles are modeled parametrically using a simple linear regression. For example, see Kang and Albin (2000), Kim *et al*. (2003), Mahmoud and Woodall (2004), Wang and Tsung (2005), Gupta *et al*. (2006), Mahmoud *et al*. (2007), Zou *et*

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al. (2008), and Jensen *et al*. (2008).   These methods often fit the profile with separate linear regression models and monitor the coefficients of the fitted regression model to determine outlying profiles, thus reducing the profiles to a smaller set of values that simplifies the monitoring scheme.   Jensen and Birch (2009) extend the use of non-linear mixed models to monitor the nonlinear profiles in order to account for the correlation structure.   Noorossana *et al*. (2009) specifically focus on Phase I monitoring of multivariate multiple linear regression profiles.   Mahmoud *et al*. (2007) and Kazemzadeh *et al.* (2008) propose Phase I methods for profiles represented by multiple and polynomial regression models, respectively.

All the above-mentioned studies on the monitoring of simple or multiple linear profiles assume that the observations within each profile are uncorrelated.   Zou *et al*. (2007) propose a monitoring scheme for a general linear profile using a multivariate exponentially weighted moving average approach.   Noorossana *et al*. (2008) specifically study the case when profiles are not independent from each other over time, by considering a simple linear regression with a first-order autoregressive model (AR(1)) for errors.   Soleimani *et al*. (2009) look at a simple linear profile and assume that there is an AR(1) model between observations in each profile.   However, a situation often exists where an AR(1) model is not adequate to depict the correlation structure among errors in practice.   The general within-profile correlation model has been considered by Qiu *et al*. (2010), and more explanatory variables in the regression model may be required to improve the model fitting.   This project considers a multiple linear regression with random errors following an autoregressive moving-average (ARMA) model, which can be regarded as a special parametric form of Qiu *et al*. (2010).

2. Linear regression model with ARMA errors

Consider the regression model:

$$y_t = x_t^{'}\beta + \varepsilon_t, \qquad t=1, 2, \cdots, T, \qquad\qquad (1)$$

where y_t is the observed variable at time t, $x_t^{'}$ is an $1 \times k$ vector of explanatory variables, and β is a $k \times 1$ vector of unknown parameters. Here, the random error ε_t follows an ARMA process, which can be expressed as:

$$\Phi(B)\varepsilon_t = \Theta(B)v_t, \qquad v_t \sim WN(0,\sigma^2), \qquad\qquad (2)$$

where $\Phi(B) = 1 + \phi_1 B + \phi_2 B^2 + \cdots + \phi_p B^p$ and $\Theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \cdots + \theta_q B^q$ are respectively the pth-order and qth-order lag operator polynomials and B denotes the lag operator. Harvey (1990, Section 7.2) provides the estimation of parameters for models (1) and (2) using the generalized least squares (GLS) approach under the frequency domain. The maximum likelihood (ML) estimation for this can be seen in Harvey and Phillips (1979).

Let $y = (y_1,..., y_T)$ and $\varepsilon = (\varepsilon_1,...,\varepsilon_T)$ be $T \times 1$ vectors, and $X = (x_1, x_2..., x_T)'$ be a $T \times k$ matrix. If $V = Var(\varepsilon)$ is a positive definite (p.d.) matrix, then there exists a p.d. lower triangular matrix, L, and a p.d. diagonal matrix F, such that

$$V^{-1} = L^{'}F^{-1}L.$$

We pre-multiply model (1) in matrix form by L and define y^*, X^*, and ε^* as Ly, LX, and $L\varepsilon$, respectively. This leads to the following heteroscedastic regression for models (1) and (2):

$$y^* = X^*\beta + \varepsilon^*, \qquad Var(\varepsilon^*) = F,$$

or

$$y_t^* = x_t^{*'}\beta + \varepsilon_t^*, \qquad Var(\varepsilon_t^*) = f_t, \qquad t=1, 2, ...,T.$$

Under the normality condition, the asymptotic distribution of the ML estimate (as well

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as the GLS estimate) of β, denoted by $\hat{\beta} = (\sum_t f_t^{-1} x_t^* x_t^{*'})^{-1} \sum_t f_t^{-1} x_t^* y_t^{*'}$, is a normal

distribution with mean β and variance-covariance matrix $(X^{*'} F^{-1} X^*)^{-1}$ (e.g.

Brockwell and Davis, 2009).

Direct use of GLS requires finding the inverse of the covariance matrix for the

ε_t's, which can be achieved more easily using the Kalman filter (Durbin and

Koopman, 2001). Many time-series models used in econometrics are special cases

of the class of linear state space models developed by engineers to describe physical

systems. The Kalman filter, an efficient recursive method for computing optimal

linear forecasts in such models, can be exploited to compute the exact Gaussian

likelihood function. The computation for the estimates is easily implemented by

converting models (1) and (2) into the state space form and applying the Kalman filter

recursive approach. The linear state-space model postulates that an observed time

series is a linear function of a (generally unobserved) state vector and the law of

motion for the state vector is a first-order vector autoregression.

Let α_t denote the values taken at time t by a vector of s state variables, and A

and b are $s \times s$ and $s \times 1$ matrices of constants, respectively. We assume that y_t

is generated by:

$$y_t = b' \alpha_t + u_t \qquad\qquad (3)$$
$$\alpha_t = A \alpha_{t-1} + v_t, \qquad\qquad (4)$$

where the scalar u_t and the vector v_t are white noise processes with a zero mean,

and are independent of each other, and (4) has initial value α_0. We denote

$\sigma^2 = E(u_t^2)$ and $\Sigma = E(v_t v_t')$. Equation (3) is sometimes called the "measurement"

equation while (4) is called the "transition" equation. The assumption that the

autoregression is first-order is not restrictive, since higher-order systems can be

handled by adding additional state variables (see Harvey (1989) for details).

The first d non-stationary elements are taken to have a diffuse prior as discussed

in Harvey (1989). A set of *T-k* standardized generalized residuals:

$$e_t = (y_t^* - x_t^* \hat{\beta}) / f_t^{1/2}, \qquad t = k+1, \ldots, T,$$

is produced as a by-product of the recursive equations. When ϕ and θ are known, these residuals are independently and normally distributed with mean zero and constant variance, σ_*^2, which can be estimated by $\hat{\sigma}^2 = \sum_t e_t^2 /(T-k)$.

3. Hotelling's T^2 test

This section discusses the diagnostic scheme for the profile models (1) and (2) by applying Hotelling's T^2 ststistic. Let $X_1, \ldots, X_n$ be a random sample from a *p*-dimensional normal population with mean μ and positive-definite variance-covariance matrix Σ. Consider the testing problem $H_0 : \mu = \mu_0$ versus $H_a : \mu \neq \mu_0$, where μ_0 is a known vector. To deal with this problem, Hotelling's T^2 test is used, which is defined as

$$T^2 = n(\bar{x} - \mu_0)^T S^{-1}(\bar{x} - \mu_0), \qquad (5)$$

where $\bar{x}$ is the sample mean and *S* is the sample covariance matrix. If H_0 is true, then result (5) is distributed as $\frac{(n-1)p}{n-p} F_{p,n-p}$, where $F_{u,v}$ denotes the *F*-distribution with degrees of freedom *u* and *v* (see, for example, Iwashita (1997). Several different formulations of the T^2 statistic have been proposed to monitor the profile coefficients resulting from different kinds of profile models in the literature (e.g. Kang and Albin 2000; Williams, Woodall, and Birch 2007; Vaghefi et al. 2009; and references therein).

3.1 T^2 for model coefficients

Many monitoring problems can be stated as the problem of detecting a change in the

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parameters of the model of interest. These problems result from the increasing complexity of most technological processes, for which details of this issue can be found in Basseville and Nikiforov (1993). To monitor the departure of coefficients, $\delta = (\beta_0, \beta_1, ..., \beta_k, \phi_1, ..., \phi_p, \theta_1, ... \theta_q)$, from the profile models (1) and (2) applied to m datasets, the analogous Hotelling's T^2 test of (5) is thus:

$$T_{1i}^2 = (\hat{\delta}_i - \overline{\delta})^T \Delta^{-1} (\hat{\delta}_i - \overline{\delta}), \qquad i = 1, 2, \dots, m, \qquad (6)$$

where $\hat{\delta}_i$ denotes the estimate of δ for the ith dataset, $\overline{\delta}$ entails the averages of all $\hat{\delta}_i$'s , and $\Delta = \sum_{i=1}^{m} (\hat{\delta}_i - \overline{\delta})(\hat{\delta}_i - \overline{\delta})^T / (m-1)$. Section 2 discusses the asymptotic normality and related proporties of $\hat{\delta}_i$. The estimate $\hat{\delta}_i$ is unbiased, but it does not have a closed form for variance. The Kalman Filter approach can easily give $\hat{\delta}$, $Var(\hat{\delta})$, and $\hat{\sigma}^2$. This paper carries out the estimation for these, which are computed by using the arima function in R (http://www.r-project.org/).

The result (6) follows an *F*-distribution as the test statistic (5), but with different degrees of freedom according to the number of parameters, *r=k+p+q+1*, in the model and sample size. A similar discussion can be found in Kazemzadeh *et al.* (2008). The 100(1-α) percentile of the *F* distribution is used to construct an upper control limit (UCL) for (6) represented by $\frac{(T-1)r}{T-r} F_{\alpha, r, T-r}$ for the Phase I control chart. Soleimani *et al.* (2009) suggest that the upper control limit is $\chi^2_{\alpha, r-1}$, which is the 100(1-α) percentile of the χ^2 distribution with *r* degrees of freedom. We suggest using the former, because it provides better results in terms of ARL values than those based on the latter, as we will see in the subsequent section for a simulation study under the in-control case. Here, $\frac{r(T+1)(T-1)}{T(T-r)} F_{\alpha, r, T-r}$ is used as UCL for the Phase II scheme, which follows Chenouri, Steiner, and Variyath (2009).

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3.2 T^2 based on the residuals

This subsection uses the residuals of profile models (1) and (2) to monitor the possible deviation due to σ^2 , which is suggested by Soleimani *et al*. (2009). If e_i denotes the $T \times 1$ residual vector for the *i*th dataset and $\hat{\sigma}_i^{\,2}$ is the corresponding estimate of σ^2 for dataset i , then we check the stability of the variance, σ^2 , in the profile using the following test statistic:

$$T_{2i}^2 = (e_i - 0)'\Sigma_e^{-1}(e_i - 0), \qquad i=1,\,2,\ldots,\,m, \tag{7}$$

where $\Sigma_e = \overline{\sigma}^2 I$, $\overline{\sigma}^2$ is the average of all $\hat{\sigma}_i^{\,2}$'s, and I is the identity matrix. Here, T_2^2 approximately follows an χ^2 distribution with T-1 degrees of freedom. The UCL for (7) is $\chi_{\alpha, T-1}^2$, which is the 100(1- α) percentile of the χ^2 distribution with T-1 degrees of freedom.

4. Simulation study

This section conducts a simulation study for models (1) and (2) to verify the performance of both T_1^2 and T_2^2 statistics based on the average run length (ARL) criterion. There are many possible forms for models (1) and (2) depending on the number of explanatory variables and the values of *p* and *q* for the ARMA model. To simplify the simulation study, we consider the following model:

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \varepsilon_t, \qquad t = 1,\,2,\ldots,T, \tag{8}$$

$$\varepsilon_t = \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \phi_3 \varepsilon_{t-3} + v_t, \tag{9}$$

where $v_t \sim WN(0, \sigma^2)$. Model (9) is a third-order autoregressive model for the error terms, which corresponds to the real data illustration in section 5.

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Given that $\beta_0 = \beta_2 = 1$, $\phi_2 = 0.1$, and $\phi_3 = -0.1$ in models (8) and (9), we focus on evaluating the impact of changes in β_1, ϕ_1, and σ on the monitoring scheme. Both x_{1t} and x_{2t} are independently generated from a uniform distribution between values 1 and 10. There are 50000 replicates used for Phase II diagnostic monitoring, while 20000 replicates are carried out for the Phase I control chart scheme. For the latter, both values of β_1 and σ are assigned to be 1, while the values of ϕ_1 vary between -0.6 and 0.6 in order to prevent a non-stationary series from occurring in the data generating process. This is used to construct the in-control profile. The sample size, T, is 150 and 300.

Tables 1, 2, and 3 present the values of ARL, where the varying parameter and its ranges are given in the top row of each table. The baseline case is printed in bold type for each row. The overall in-control ARL can be calculated by $1/ARL_{overall} = 1 - (1 - \alpha_1)(1 - \alpha_2)$, where α_1 and α_2 denote the probability of committing false alarms for T_1^2 and T_2^2, respectively. Given $\alpha_1 = \alpha_2 = 0.0027$, the combination of T_1^2 and T_2^2 control chart schemes (in (6) and (7), respectively) is considered to yield an overall in-control ARL of approximately 185.

Table 1 presents the values of ARL when the true value of β_1 shifts from 1.0 to 1.1, given the same value of ϕ_1 in each row. The first column denotes the in-control case, in which the values indicate the number of false alarms occurring in 50,000 replications. Almost all of those values printed in bold are less than or close to 185. The ARL values show that the test statistics are sensible to a change in β_1. It is noted that the results are more sensible to a change when the sample size becomes larger, which can be seen by comparing the columns of $\beta_1 = 1.02$, 1.04, and 1.06 between T=150 and T=300. It also shows that the sensitivity of T^2 is related to the values of ϕ_1. The values of out-of-control ARL become larger as the value of

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ϕ_1 varies from -0.6 to 0.6, no matter what the sample size is. Here, we only reach a conclusion that the detection of the change in β_1 may depend on the values of ϕ_1. The pattern for how this differs may depend on more factors, such as the complexity of error function (2) and/or simultaneous shifts in different parameters. To verify this, more simulations should be expected.

Table 2 considers the change of ϕ_1 in the profile model. We reach similar conclusions about T^2 statistics as with those of Table 1. Again, the ARL value is more sensible to the change for T=150 than that for T=300. However, there lacks a clear pattern with regards to the change of ARL values when comparing the sensitivity due to the left or right deviation from the true value of ϕ_1. If we look at each row, ARL values are larger for shifts from the left side of the true value than those from the right side for ϕ_1 = -0.6, -0.4, and -0.2. This phenomenon is different for positive values of ϕ_1, in which ARL values seem more similar on both sides of the true value.

Table 3 examines the deviation of σ from the profile model. Comparing the ARL values row by row yields a similar pattern no matter what the value of ϕ_1 is. The effect of the sample size appears the same for T=150 and T=300 as shown in Tables 1 and 2. It is noted that both T_1^2 and T_2^2 are able to identify the change in the value of σ, resulting in a significant drop in the ARL values when σ shifts from 1 to 1.1. Nevertheless, T_2^2 is more sensitive to the change than T_1^2 in terms of ARL. While T_1^2 is a very useful statistic for the change in β_1 and ϕ_1 in Tables 1 and 2, respectively, T_2^2 is not so sensitive to the deviation in β_1 and ϕ_1.

Given α =0.0027, Tables 4 and 5 show the accurate detection rate based on T_1^2 when the values β_1 and ϕ_1 vary in the same way as the previous tables, respectively. The accurate detection rate is calculated by 1/(out-of-control ARL) given the probability of committing a type I error is 0.0027. The values printed in

the bold type are then the estimated false alarm rate.   Table 6 presents the accurate

detection rate based on T_2^2 when the values of σ change from 1 to 1.1.   All these

three tables confirm that the proposed statistical approach is sensitive to the change in

parameters of concern.   It is also noted that the powers of the tests increase as the

sample size is larger.

To examine the high correlation problem for an AR(3) errors in models (8)

and (9), a new parameter for $\phi_2 = -0.1$ is given to avoid the nonstationarity.   Table

7 presents the values of out-of-control ARL with a sample size of 150, where the

varying parameter and its ranges are similar to the previous tables, except for the

change of ϕ_1.   The performance of using T_1^2 and T_2^2 together on detecting the

shift of any one parameter β_1, ϕ_1, or σ with high correlation errors in the data is

similar to those with mild or weak correlation ones.

5. Conclusions

This project proposes approaches to monitoring the profile of the linear regression

model with ARMA errors.   The first test statistic diagnoses the shift in the

coefficients of the linear regression model and the coefficients of the correlated error

model.   The second test monitors the variance of random errors.   The simulation

study shows the successful performance in terms of ARL.   A real dataset exemplifies

the proposed procedure, and graphical analyses enhance the results.   This research

considers both the Phase I control scheme and Phase II monitoring application, in

which the proposed T^2 statistics are quite successful in detecting the abnormal cases.

Several issues remain for futher investigation.

According to the simulation, there exists a complex situation on the

performance of the proposed test statistics, which depends on the different values of

ϕ_1.   We only confirm that this will lead to different conclusions as shown in Tables 1 and 2, which are unable to identify the pattern.   This will become more difficult when the error function is more dynamic - for example, the ARMA model with different orders in functions $\Phi(B)$ and $\Theta(B)$.   Furthermore, only one single parameter is allowed to change in the simulation study shown in Tables 1-3.   If several parameters simultaneously deviate from that of the candidate profile model, then we expect that the approach will be able to identify the out-of-control situation, but this may require further examinations in order to find out exactly how it happens.

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Table 1. Simulated ARL values when β_1 varies from 1 to 1.10 (in-control ARL=185).

		$\beta_1=1$	1.02	1.04	1.06	1.08	1.10
$T=150$	$\phi_1=$-0.6	**142.045**	19.231	1.588	1.007	1.000	1.000
	-0.4	**157.233**	29.851	2.249	1.044	1.000	1.000
	-0.2	**176.056**	46.685	3.618	1.194	1.004	1.000
	0.0	**174.216**	66.225	7.198	1.663	1.053	1.001
	0.2	**166.667**	79.872	13.221	2.685	1.257	1.022
	0.4	**187.970**	107.527	25.253	5.247	1.882	1.164
	0.6	**168.350**	125.313	40.783	10.156	3.181	1.573
$T=300$	$\phi_1=$-0.6	**175.439**	4.224	1.011	1.000	1.000	1.000
	-0.4	**188.679**	6.973	1.068	1.000	1.000	1.000
	-0.2	**176.678**	11.743	1.249	1.001	1.000	1.000
	0.0	**186.567**	20.309	1.750	1.016	1.000	1.000
	0.2	**185.874**	34.916	2.962	1.116	1.001	1.000
	0.4	**193.050**	54.171	5.503	1.468	1.031	1.000
	0.6	**180.505**	68.213	9.750	2.190	1.152	1.010

Table 2. Simulated ARL values under the varying values of ϕ_1 (in-control ARL=185).

		$\phi_1=0$	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6
T=150	ϕ_1=-0.6	1.000	1.001	1.024	1.283	3.089	19.216	**145.773**
	-0.4	1.149	1.968	6.539	43.365	**177.936**	112.867	16.706
	-0.2	10.301	60.976	**157.729**	108.225	20.938	3.544	1.305
T=300	ϕ_1=-0.6	1.000	1.000	1.000	1.003	1.282	7.504	**138.122**
	-0.4	1.000	1.056	2.058	17.094	**177.305**	39.777	2.302
	-0.2	2.802	27.337	**187.266**	44.326	3.294	1.111	1.001
		$\phi_1=0$	0.1	0.2	0.3	0.4	0.5	0.6
T=150	ϕ_1=0.0	**174.216**	78.125	12.994	2.935	1.317	1.032	1.001
	0.2	13.203	76.453	**148.810**	77.760	12.713	2.621	1.206
	0.4	1.260	2.460	9.651	60.168	**164.474**	79.365	10.231
	0.6	1.000	1.006	1.094	1.709	5.572	38.580	**162.338**
T=300	ϕ_1=0.0	**186.567**	33.201	3.043	1.142	1.002	1.000	1.000
	0.2	3.128	32.489	**168.919**	34.626	2.903	1.097	1.001
	0.4	1.001	1.094	2.511	24.606	**190.840**	31.726	2.169
	0.6	1.000	1.000	1.000	1.023	1.706	14.510	**171.233**

Table 3. Simulated ARL values under the shifts of σ from 1 to 1.5 (in-control ARL=185).

		$\sigma =1$	1.1	1.2	1.3	1.4	1.5
T=150	ϕ_1=-0.6	**142.045**	5.677	1.415	1.034	1.001	1.000
	-0.4	**157.233**	5.578	1.399	1.034	1.001	1.000
	-0.2	**176.056**	5.648	1.410	1.033	1.002	1.000
	0.0	**174.216**	5.752	1.411	1.033	1.002	1.000
	0.2	**166.667**	5.633	1.407	1.032	1.001	1.000
	0.4	**187.970**	5.476	1.399	1.032	1.001	1.000
	0.6	**168.350**	5.725	1.412	1.033	1.001	1.000
T=300	ϕ_1=-0.6	**175.439**	2.619	1.040	1.000	1.000	1.000
	-0.4	**188.679**	2.571	1.040	1.000	1.000	1.000
	-0.2	**176.678**	2.588	1.040	1.000	1.000	1.000
	0.0	**186.567**	2.614	1.041	1.000	1.000	1.000
	0.2	**185.874**	2.605	1.040	1.000	1.000	1.000
	0.4	**193.050**	2.582	1.041	1.000	1.000	1.000
	0.6	**180.505**	2.618	1.041	1.000	1.000	1.000

Table 4. False alarm rate and accurate detection rate based on T_1^2 when β_1 varies from 1 to 1.10.

		$\beta_1=1$	1.02	1.04	1.06	1.08	1.10
T=150	ϕ_1=-0.6	**0.003**	0.049	0.628	0.993	1.000	1.000
	-0.4	**0.002**	0.030	0.442	0.958	1.000	1.000
	-0.2	**0.002**	0.018	0.273	0.837	0.996	1.000
	0.0	**0.002**	0.012	0.136	0.599	0.950	0.999
	0.2	**0.002**	0.008	0.072	0.370	0.795	0.979
	0.4	**0.002**	0.006	0.035	0.187	0.529	0.858
	0.6	**0.002**	0.004	0.021	0.095	0.312	0.634
T=300	ϕ_1=-0.6	**0.003**	0.234	0.989	1.000	1.000	1.000
	-0.4	**0.002**	0.140	0.936	1.000	1.000	1.000
	-0.2	**0.002**	0.082	0.800	0.999	1.000	1.000
	0.0	**0.002**	0.046	0.570	0.984	1.000	1.000
	0.2	**0.002**	0.025	0.336	0.895	0.999	1.000
	0.4	**0.002**	0.015	0.179	0.680	0.970	1.000
	0.6	**0.002**	0.011	0.100	0.455	0.868	0.990

Table 5. False alarm rate and accurate detection rate based on T_1^2 under the varying values of ϕ_1.

		$\phi_1=0$	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6
$T=150$	$\phi_1=-0.6$	1.000	0.999	0.977	0.779	0.321	0.048	**0.003**
	-0.4	0.870	0.506	0.149	0.019	**0.002**	0.005	0.056
	-0.2	0.094	0.012	**0.002**	0.005	0.044	0.279	0.766
$T=300$	$\phi_1=-0.6$	1.000	1.000	1.000	0.997	0.780	0.129	**0.004**
	-0.4	1.000	0.947	0.484	0.055	**0.002**	0.021	0.432
	-0.2	0.355	0.034	**0.002**	0.019	0.301	0.900	0.999
		$\phi_1=0$	0.1	0.2	0.3	0.4	0.5	0.6
$T=150$	$\phi_1=0.0$	**0.002**	0.009	0.073	0.338	0.758	0.969	0.999
	0.2	0.072	0.009	**0.002**	0.009	0.075	0.379	0.828
	0.4	0.793	0.404	0.099	0.012	**0.002**	0.008	0.093
	0.6	1.000	0.994	0.914	0.583	0.176	0.022	**0.002**
$T=300$	$\phi_1=0.0$	**0.002**	0.027	0.326	0.875	0.998	1.000	1.000
	0.2	0.317	0.027	**0.002**	0.025	0.342	0.911	0.999
	0.4	0.999	0.914	0.396	0.037	**0.002**	0.028	0.459
	0.6	1.000	1.000	1.000	0.978	0.585	0.066	**0.002**

Table 6. False alarm rate and accurate detection rate based on T_2^2 under the shifts of σ from 1 to 1.5.

		$\sigma=1$	1.1	1.2	1.3	1.4	1.5
$T=150$	$\phi_1=-0.6$	**0.004**	0.171	0.703	0.967	0.999	1.000
	-0.4	**0.004**	0.176	0.711	0.966	0.999	1.000
	-0.2	**0.004**	0.174	0.706	0.967	0.998	1.000
	0.0	**0.004**	0.170	0.705	0.967	0.998	1.000
	0.2	**0.004**	0.174	0.707	0.968	0.999	1.000
	0.4	**0.004**	0.179	0.711	0.968	0.999	1.000
	0.6	**0.004**	0.171	0.705	0.967	0.998	1.000
$T=300$	$\phi_1=-0.6$	**0.003**	0.377	0.961	1.000	1.000	1.000
	-0.4	**0.003**	0.386	0.961	1.000	1.000	1.000
	-0.2	**0.003**	0.383	0.961	1.000	1.000	1.000
	0.0	**0.003**	0.379	0.960	1.000	1.000	1.000
	0.2	**0.003**	0.380	0.961	1.000	1.000	1.000
	0.4	**0.003**	0.384	0.960	1.000	1.000	1.000
	0.6	**0.003**	0.378	0.960	1.000	1.000	1.000

Table 7. Simulated ARL values for high correlation in an AR(3) errors (in-control ARL=185).

	Change in β_1					
	$\beta_1=1$	1.02	1.04	1.06	1.08	1.10
$\phi_1=-0.9$	**126.263**	7.634	1.066	1.000	1.000	1.000
-0.8	**142.857**	9.137	1.101	1.000	1.000	1.000
0.8	**178.571**	126.263	32.489	7.515	2.501	1.341
0.9	**179.211**	130.890	35.638	8.628	2.818	1.445

	Change in ϕ_1					
	$\phi_1=-0.4$	-0.5	-0.6	-0.7	-0.8	-0.9
$\phi_1=-0.9$	1.000	1.000	1.024	1.503	7.777	**121.065**
-0.8	1.011	1.187	2.653	17.953	**152.439**	87.260
	$\phi_1=0.4$	0.5	0.6	0.7	0.8	0.9
0.8	1.034	1.394	4.101	32.425	**174.216**	55.127
0.9	1.000	1.010	1.198	2.873	23.191	**161.290**

	Change in σ					
	$\sigma=1$	1.1	1.2	1.3	1.4	1.5
$\phi_1=-0.9$	**126.263**	5.547	1.397	1.034	1.001	1.000
-0.8	**142.857**	5.534	1.396	1.033	1.001	1.000
0.8	**178.571**	5.679	1.409	1.033	1.001	1.000
0.9	**179.211**	5.644	1.409	1.032	1.002	1.000

國科會補助計畫衍生研發成果推廣資料表

國科會補助計畫	計畫名稱: 具相關誤差之線性迴歸模型的剖面曲線監測
	計畫主持人: 鄭宗記
	計畫編號: 100-2118-M-004-002-　　　　學門領域: 品質管制

<div align="center">

無研發成果推廣資料

</div>

100 年度專題研究計畫研究成果彙整表

計畫主持人：鄭宗記　　計畫編號：100-2118-M-004-002-

計畫名稱：具相關誤差之線性迴歸模型的剖面曲線監測

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	1	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	2	2	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等,請以文字敘述填列。)	無。

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與(閱聽)人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以 100 字為限） 　　□實驗失敗 　　□因故實驗中斷 　　□其他原因 　說明：
2. 研究成果在學術期刊發表或申請專利等情形： 論文：□已發表 ■未發表之文稿 □撰寫中 □無 專利：□已獲得 □申請中 ■無 技轉：□已技轉 □洽談中 ■無 其他：（以 100 字為限）
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限） Profile monitoring is a relatively new technique and is becoming popular in the area of quality control, because it is used when the process is characterized by the relationship between a response variable and a set of explanatory variables (i.e., profile) at each time period. This project considers the situation where profiles are modeled parametrically using a multiple linear regression model with random errors following an autoregressive moving-average process. Diagnostic schemes to find out-of-control samples are developed for this purpose. This project conducts a simulation study to examine the performance of the proposed approach based on the average run length criterion. A real example is implemented to illustrate the results, after considering both Phase I and Phase II schemes.