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以免疫理論探討壽險公司生存風險避險之最適產品組合策略

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I. 中文摘要

死亡率的下降會導致年金商品的負債大幅增加，因此人類平均餘命的日益延長對於販售年金商品的人壽保險公司將造成重大的威脅。從資產負債管理的觀點，壽險和年金商品的負債對於死亡率變動是呈反向變化的。當死亡率下降時，壽險商品的負債就會下降，但是年金商品的負債反而會上升。因此，壽險商品可以用來作為年金商品在面對未來死亡率不確定變動的動態避險工具，也就是所謂的自然避險(Natural Hedging)。在本研究案中，我們探討如何利用壽險商品來作為規避年金商品在死亡率降低（即長壽風險）的最適避險策略。透過免疫理論的概念，本研究求出壽險商品與年金商品在規避死亡率降低風險時的最適商品組合比例。此外，我們利用美國壽險商品與年金商品的實際死亡率經驗資料以及目前在美國廣為販售的壽險與年金商品，以本研究所建議之理論模型進行避險策略分析，我們發現利用本研究提出的免疫理論模型所計算的最適商品組合策略，可以有效地提供人壽保險公司規避因死亡率降低所造成的長壽風險。

關鍵詞 ：生存風險、資產負債管理、自然避險、免疫理論、年金保險、壽險、
最適產品組合比例

I. Abstract

The increase in the population longevity rate presents a critical threat to the insurance companies because it substantially increases the liability for the annuity. From an asset liability management perspective, life insurance and annuity are sensitive in opposing directions to the changes in the mortality rate. Therefore, life insurance can serve as a dynamic hedge vehicle against unexpected mortality risk of annuity (the so-called natural hedging). In this paper, we investigate the optimal strategy for hedging longevity risk of annuity by using life insurance products. We propose an immunization model to calculate the optimal level of product mix between annuity and life insurance to against the longevity risks. Using the mortality experience and insurance products in the United State, we demonstrate that our proposed model is useful in obtaining the optimal level of product mix between annuity and life insurance to effectively reduce longevity risks for life insurance company.

Key Words: longevity risk, asset liability management, natural hedging, immunization model, annuity and life insurance products, optimal level of product mix.

II. Introduction

As increasing shares of the defined contribution pension plans and self-directed retirement accounts, the amount of annuity sold from life insurance companies is growing substantially. In term of annuity written premium, the annuity market in the United States is approximately two billion dollars per year, while the United Kingdom annuity market is approximately six billion pounds per year. On the other hand, the longevity improvement presents a critical threat to the government as well as private insurance companies since it increases the payout period as well as the liability cost for providing annuity. More importantly, recent research articles, such as Willets (2004), also point out that the historical longevity improvements do not occur in a smooth upward fashion, but rather exhibit certain cyclical patterns, with some special cohorts having better improvement rate than the others (the so called cohort effect). In addition, recent medical discoveries might increase the lifespan far beyond currently projected mortality table used by the insurance company. All above issues have increased the difficulty for insurance actuaries to price annuity correctly. However, for a life insurance company, the inaccurate mortality assumptions of annuity pricing may in terms presents other major risks, such as under-pricing risk, incorrect payout estimation risk, under-reserve risk, and cash flow/duration mismatch risk, etc…. Thus, hedging longevity risk definitely plays an increasing important role for the operation of life insurance companies.

From a theoretical perspective, life insurance and annuity are sensitive in opposing directions to the changes in the mortality rate. If the future mortality of a cohort improves relative to current expectations, the life insurers would gain because they can pay the death benefit later than initially expected, whereas annuity insurer would suffer a loss because they have to pay annuity benefit longer than initially expected. Therefore, from asset-liability management perspective, life insurance can serve as a dynamic hedge vehicle against unexpected mortality risk of annuity (the so-called natural hedging). However, in real practices, annuity is sold to the old, whereas life insurance is purchased by the young. Thus, the durations of these two products can not be easily matched in order to properly hedge each other. However, relatively few academic papers have extended the line of research on the natural hedging issue.

Many prior studies have investigated mortality risk and pricing issues for various annuity products. These research include Friedman and Warshawsky (1990) and Mitchell, Poterba, Warshawsky and Brown (2001). Brown, Mitchell and Poterba (2000) further explore the mortality risk and inflation risk of various annuity products in the United State and other nations. They find that the present value of annuity payouts falls below the cost of these products by 10 to 20 percent for a randomly selected person. In addition, other studies focus on the impact of mortality changes on life insurance and annuity separately, or investigate the combination of life and pure endowment contracts. Marceau and Gaillardetz (1999) calculated the reserves in a stochastic mortality and interest rate environment for a portfolio of term life insurance contract and pure endowment policies. Milevsky and Promislow (2001) proposed a model for pricing on future mortality and suggest that both mortality and interest risk can be hedged, and the option to annuitize can be priced by locating a replicating portfolio involving insurance, annuity and default-free bonds. More recently, Milevsky and Promislow (2002) investigate the impact of unexpected improvement in longevity on the profitability of annuity. Their numerical results indicate that annuity profitability is quite sensitive to changes in mortality rates and that insurer can generate a hedging effect by invest the asset to earn the spread cause by mortality improvement. In addition, many financial vehicles, such as mortality derivatives and survival bond, have been suggested to reduce or hedge the mortality risk of annuity. Blake and Burrows (2001) suggest that issuing survival bonds would help pension fund to insure against the mortality risk. Charupat and Milevsky (2001) report that the existence of a tax arbitrage opportunity could involve a mortality swap because the investor take on mortality risk by acquiring an immediate life annuity and then can swap it back by purchasing life insurance. As for the natural hedging issue, Lin and Cox (2003) proposed a pricing model for mortality swap and provide empirical evidence to support that insurers who utilize natural hedging can charge low risk premium than other insurers. However, there is no discussion in the literature so far related to the products strategy for the natural hedging. This paper intends to fill up this gap.

In this paper, we investigate the optimal strategy for hedging longevity risk of annuity by using life insurance products. We propose an immunization model to calculate the optimal level of product mix between annuity and life insurance to against the longevity risks. Using the mortality experience and insurance products in the United State, we demonstrate that our proposed model is useful in obtaining the optimal level of product mix between annuity and life insurance products to effectively reduce longevity risks for insurance company.

II. Model

In this paper, we try to calculate the optimal level of product mix between annuity and life insurance against longevity risks. First, we derive the mortality curve by using the mortality experience in the United State. Milevsky and Promislow (2001) provide some parametric specification for the hazard-mortality rate function and demonstrate how this approach can be applied to annuity pricing. Following most literature, we assume the mortality curve is an exponential function of parameters a, b, and c as follow:

$$M(t) = ae^{bt^2+ct^3} \quad \text{……………………………………………………………}(1)$$

If we transfer the mortality curve by a log function, we can get

$$\ln M(t) = \ln a + bt^2 + ct \quad \text{……………………………………………………….}(2)$$

From equation (2), we can observe that the change in each parameter has different meaning. The change of parameter $\ln a$ implies the change in intercept of mortality curve. For parameter b and c, the change in parameters stand for the change in convexity and the change in slope of mortality curve respectively.

For simplicity, we first assume that an insurance company only sells two products: annuity and life insurance. Thus, the value of the liability of the insurance company is as follow:

$$V = \alpha V_{life} + (1-\alpha)V_{annuity} \quad \text{…………………………………………………………….(3)}$$

where V is the liability value of product mix of annuity and life insurance,

V_{life} is the liability value of life insurance,

$V_{annutiy}$ is the liability value of annuity,

α is the weight of product mix of life insurance.

If insurance company does not like to take any risk on the changes in mortality rate, then the best strategy is to keep $\frac{\partial V}{\partial a}=0$, $\frac{\partial V}{\partial b}=0$, and $\frac{\partial V}{\partial c}=0$. Thus, we can obtain optimal level for α_a^*, α_b^*, and α_c^*, respectively. Assume K value, (K=$\frac{\alpha}{1-\alpha}$), is the ratio of product mix between life and annuity products. Thus, the K ratio implies that if the insurance company sells one unit of annuity product then it has to sell K unit of life insurance product in order to achieve the effect of immunization against longevity risks. However, in reality the insurance company will not sell only two products: life insurance and annuity. To cope with this problem, we can further revise the portfolio of product mix model by adding more product classifications.

Let's assume that an insurance company sells a portfolio of product mix between different types (such as term, endowment or whole life) of annuity and life insurance products. Then, the liability of the portfolio of product mix is as follow:

$$V_p = \sum_{i=1}^{m} w_i V_i^{life} + \sum_{j=1}^{n} w_j V_j^{annuity} \qquad \text{………………………………………………(4)}$$

where V_p is the liability value of the portfolio of product mix of annuity and life insurance,

V_i^{life} is the liability value of the i th type of life insurance product,

$V_j^{annuity}$ is the liability value of the j th type of annuity product,

w_i is the weight of product mix of the i th type of life insurance with m types of products,

w_j is the weight of product mix of the j th type of annuity with n types of product,

If the insurance company does not like to take any risk on the changes in mortality rate, then the best strategy is to keep $\frac{\partial V_p}{\partial a}=0$, $\frac{\partial V_p}{\partial b}=0$, $\frac{\partial V_p}{\partial c}=0$, and $\sum_{j=1}^{m} w_i + \sum_{j=1}^{n} w_j = 1$. Separately, it may not be difficult for risk managers to cope with each risk, such as $\frac{\partial V_p}{\partial a}=0$. However, immunization strategies may conflict with each other and/or may not even be completely compatible. To integrate the immunization strategies against mortality rate risk as a whole, we further propose an integrated model by using the goal-programming algorithm as follows:

$$
\begin{aligned}
&Min\ \ d \\
&s.t \\
&W_i(\frac{\partial V_i^{life}}{\partial a}) + W_j(\frac{\partial V_j^{annuity}}{\partial a}) - d \le 0, \\
&W_i(\frac{\partial V_i^{life}}{\partial b}) + W_j(\frac{\partial V_j^{annuity}}{\partial b}) - d \le 0, \\
&W_i(\frac{\partial V_i^{life}}{\partial c}) + W_j(\frac{\partial V_j^{annuity}}{\partial c}) - d \le 0, \\
&\sum W_i + \sum W_j = 1, \quad W_i \ge 0, \ \ W_j \ge 0,
\end{aligned}
\qquad \text{..(5)}
$$

where d is the maximum value of the change for each parameter a, b and c in mortality rate,

V_p is the liability value of the portfolio of product mix of annuity and life insurance,

V_i^{life} is the liability value of the i th type of life insurance product,

$V_j^{annuity}$ is the liability value of the j th type of annuity product,

w_i is the weight of product mix of the i th type of life insurance with m types of products,

w_j is the weight of product mix of the j th type of annuity with n types of product,

Thus, integrated model can help to generate a product-mix allocation ratio for various products of life insurance to achieve an effective level of immunization against longevity risks. To explore more detail information, we further examines the impact of the changes of underlined factors guiding the process of the mortality rates and the changes in the product mix to the mortality risk in different situations by using numerical analysis with the mortality experience and insurance products in the real practice.

III. The Numerical Analysis

To demonstrate the application of our model, we construct some numerical analysis for insurance company in this section. Using the proposed algorithms in this paper, we can further calculate the optimal product mix ratio between life and annuity products, i.e K ratio. The K ratio implies that if the insurance company sells one unit of annuity product, then it has to sell K unit of life insurance product to achieve the effect of immunization for the change in the mortality rate.

The Assumptions of Numerical Analysis

Assume the insurance company sells two products, with one life insurance and the other annuity product. By the coverage period, the products can be further classified into three different types of products: ten-year term, twenty-year term and whole-life insurance. For simplicity, we also assume there is only one customer who buys the insurance products at the age of twenty-five (Male or Female). The pricing interest rate of insurance product is 4 percent. For life insurance products, the pay-out benefit is US$ 1,000,000. For annuity products, the pay-out benefit is US$ 10,000 per year and will be paid after thirty years from the issue date. For each kind of product, the insurance premium can be paid either by single premium (SP) or by level premium (LP).

Hedging Strategy for Two Products

For illustration purpose, we used the product mix of whole life insurance and pure life annuity to explain the implementation of our proposed models. If the insurance company consider the risk on the parameter change in mortality rate separately, then the best strategy is to keep $\frac{\partial V_p}{\partial a} = 0$, $\frac{\partial V_p}{\partial b} = 0$ or $\frac{\partial V_p}{\partial c} = 0$. Table 1 reports the results of estimated K_a, K_b, K_c values for the product mix of whole-life insurance and pure life annuity for hedging per unit change in parameters a, b, and c, respectively.

Table 1 K value for Whole-life Insurance VS. Pure Life Annuity

Factor \ Product Mix	Male	Female
K_a	0.04	0.04
K_b	0.02	0.20
K_c	0.04	0.08

From Table 1, to against the longevity risk, if the insurance company sells one unit of life annuity to a man, it need to sell 0.04 unit of whole life insurance to hedge the per unit change in parameter a. By the same token, the insurance company has to sell 0.02 and 0.04 unit of whole life insurances to hedge per unit change in parameter b and parameter c respectively to achieve the effect of natural hedging. On the other hand, if the insurance sells one unit of pure life annuity to woman, it has to sell more units of whole life insurance to against the mortality risk. For selling each unit of pure life annuity, the insurance company need to sell 0.04 unit of whole life insurance to hedge per unit change in parameter a, 0.20 unit to hedge per unit change in parameter b, and 0.08 unit to hedge per unit change in parameter c to achieve the effect of natural hedging.

Hedging Strategy for two products by using Goal Programming Model

However, the immunization strategies in previous section may conflict with each other and/or may not even be completely compatible. To integrate the immunization strategies against mortality rate risk as a whole, we may further use the proposed goal-programming algorithm in Equation 5. We analyze the ratio of product mix between life insurance and annuity products (K Value) under different product settings. From table 2, if the insurance company sells one unit of whole life annuity paid with single premium to a man, it has to sell 0.08 unit of whole life insurance to hedge the longevity risk considering the changes of all the parameters as a whole. In addition, we also observe that the K value of life insurance product with longer coverage period is smaller than life insurance with shorter coverage period. It implies, compared with short-term annuity product, long-term annuity product takes more units of life insurance product to achieve the effect of natural hedging. The results of K values for female are similar to that of male. However, insurance company has to sell much more units of life insurance product to female than to male in order to hedge the longevity risk because the life expectancy of female is longer than male. We found that K value of female is twice as high as that of male in the numerical analysis. Due to the difference of life expectancy between male and female, life insurance company has to take different immunization strategy when selling the annuity product.

Table 2 K Values for Different Product Mix for Male – single premium

	10-year term life	20-year term life	Whole life
20-year term Annuity	0.43	0.18	0.02
30-year term Annuity	0.96	0.39	0.04
Whole life annuity	1.92	0.79	0.08

In summary, we find that compared with short-term annuity, long-term annuity needs more units of life insurance product to achieve the effect of “natural hedging”. For the same annuity product, the effect of natural hedging for the long-term life insurance product is better than that of short-term life insurance product. For the same annuity product, women need more units of life insurance product than men to achieve the effect of natural hedging.

Hedging Strategy for portfolio of products by using Goal Programming Model

In real practice, the insurance company usually will not sell only two products. Therefore, we implement “goal programming” to calculate the optimal weight of each product in the portfolio setting when the life insurance company sells more than two products. Assume there are six products in the portfolio: ten-year term life, twenty-year term life, whole life, twenty-year annuity, thirty-year annuity, and whole life annuity. All these products can be paid either by single premium or level premium. Table 3 and Table 15 reports results of product mix by using goal-programming model proposed in this paper for simple premium and level premium. From Table 3, we find if the insurance company sells the portfolio paid with single premium to men, it need to allocate 25.03% in 10-year term life, 6.30% in whole life, 52.29% in thirty-year annuity, and 16.38% in life annuity of this portfolio to achieve the effect of natural hedging. By the same token, if the insurance company sells the portfolio paid with single premium to women, it need to allocate 65.29% in 10-year term life, 34.71% in twenty-year annuity of this portfolio to achieve the effect of natural hedging. Thus, using the idea of goal programming, insurance company can make the optimal strategy in allocating the product portfolio effectively to hedge longevity risk.

Table 3 Optimal Product-mix for using Goal-programming Model (simple premium) Portfolio A

	TL_SP_10	TL_SP_20	WL_SP	AN_SP_20	AN_SP_30	AN_SP_W
Male	25.03%	0.00%	6.30%	0.00%	52.29%	16.38%
Female	65.29%	0.00%	0.00%	34.71%	0.00%	0.00%

IV. Conclusion

This paper investigates the optimal strategy for hedging longevity risk of annuity by using life insurance products. We propose immunization models to calculate the optimal level of product mix between annuity and life insurance to against longevity risks. Furthermore, we analyze the impacts of the changes of underlined factors guiding the process of the mortality rates and calculate the optimal product mix to hedge the mortality risk in different situations by using numerical analysis. If the insurance company considers the change of each parameter separately, our paper provides the approach to calculate individual K ratio for each product mix to achieve the effect of immunization for controlling longevity risk. Furthermore, by using the goal-programming algorithm proposed by this paper, we can further obtain a optimal ratio of product mix by taking the changes of all the parameters into consideration simultaneously. The results of the numerical analysis show that our proposed models can help the life insurance company to make better strategy in obtaining the product mix for hedging the longevity risk. We demonstrate that our proposed models can serve as a useful tool in obtaining the optimal level of product mix between annuity and life insurance product to effectively reduce longevity risks. However, due to a lack of suitable data for actual market price for various insurance products, we are unable to analyze the actual natural hedge cost in the real practice for insurance companies. This issue is very important in the insurance literature and certainly deserves more investigation for future research.

References

ACLI, 1999, *Life Insurance Fact Book* (Washington, DC: American Council of Life Insurance).

ACLI, 2003, *Life Insurance Fact Book* (Washington, DC: American Council of Life Insurance).

Benjamin, B., and A.S. Soliman, 1993, *Mortality on the Move*. Institute of Actuaries, Oxford.

Blake, D., and W. Burrows, 2001, Survivor bonds: Helping to hedge mortality risk, *Journal of Risk and Insurance*, 68(2): 339-348.

Charupat, N., and M. A. Milevsky, 2001, Mortality swaps and tax arbitrage in the Canadian insurance and annuity markets, *Journal of Risk and Insurance*, 68(2): 277-303.

Coppola, M., Di Lorenzo, and M. Sibilo, 2000, Risk sources in a life annuity portfolio: decomposition and measurement tools, *Journal of Actuarial Practice*, 8: 43-61.

Frees. E.W., J. Carriere, and E. Valdez, 1996, Annuity Valuation with Dependent Mortality, *Journal of Risk and Insurance*, 63(2): 229-261.

Lee, R.D., and L. Carter, 1992, Modeling and forecasting the time series of US mortality, *Journal of American Statistical Association*, 87: 659-671.

Lin, Yijia, and Samuel H. Cox, 2005, Securitization of mortality risks in life annuities, *Journal of Risk and Insurance,* 72(2): 227-252

Marceau, Etienne, and Patrice Gaillardetz, 1999, On life insurance reserve in a stochastic mortality and interest rates environment, *Insurance, Mathematics & Economics*, 25(3): 261-280.

McDonald, A.S., 1997, ed.,. *The Second Actuarial Study of Mortality in Europe* (Oxford: Groupe Consultatif des Associations d’Actuaires des Pays des Communautés Européennes).

McDonald, A.S., A.J.C. Cairns, P.L. Gwilt, and K.A. Miller, 1998, An international comparison of recent trends in mortality, *British Actuarial Journal*, 4: 3-141.

Milevsky, M.A., and S.D. Promislow, 2001, Mortality derivatives and the option to annuitize, *Insurance: Mathematics and Economics*, 29:299-318.

Mitchell, O.S., J.M. Poterba, M.J. Warshawsky, and J.R. Brown, 2001, New Evidence on the Money’s Worth of Individual Annuities, in J.R. Brown, O.S. Mitchell, J.M. Poterba, and M.J. Warshawsky, eds., *The Role of Annuity Markets in Financing Retirement* (Boston: MIT Press), pp.71-152.

Olivieri, A., 2001, Uncertainty in mortality projections: an actuarial perspective, *Insurance: Mathematics and Economic*, 29: 231-245.

Renshaw, A.E., and S. Haberman, 2003a, Lee-Carter mortality forecasting with age specific enhancement, *Insurance: Mathematics and Economics* 33: 255-272.

Renshaw, A.E., and S. Haberman, 2003b, Lee-Carter mortality forecasting: a parallel generalized linear modeling approach for England and Wales mortality projections, *Applied Statistics* 52: 255-272.

Willets, R., 2004, *The Cohort Effect: Insights and Explanations*, presented to the Faculty of Actuaries, March 15, 2004, and to the Institute of Actuaries, April 26, 2004. Printed by Bell and Bain Ltd., Glasgow.