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# Minimum winning versus oversized coalitions in public finance: the role of uncertainty

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**Abstract** This paper extends Persson et al.'s (J Polit Econ 108:1121–1161, 2000) simple legislature in the context of public finance with certainty to uncertainty. In our uncertain world, oversized coalitions (OSCs) as well as minimum winning coalitions (MWCs) may arise in equilibrium, and the agenda setter's proposed policy may fail to receive a majority support. This is in marked contrast to the certain world, in which only MWCs can arise in equilibrium and the agenda setter's proposal never fails to pass. When OSCs arise, we show that both public good provision and redistribution are likely to achieve their first-best solution, even if the legislature is simple.

## 1 Introduction

Baron and Ferejohn (1989, hereafter BF) have developed a so-called "majoritarian bargaining" model, which extends the two-player bargaining game invented by Stahl (1972) and Rubinstein (1982) to an $n$-player bargaining setting under majority rule: a "pie" can be and will be split among players as long as a majority of the players agree.

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The BF model has been applied widely and has become a workhorse for the analysis of a variety of political issues.[1]

Using the BF model as a backbone, Persson et al. (2000, hereafter PRT) devise a model of public finance, in which there exists an agency problem between voters and their political agents (legislators), and the voters can discipline their agents only by removing them from posts through elections.[2] The implications of the PRT model critically rely on the two sharp predictions of the BF model:[3]

(1) *Minimal size of coalitions*—only a minimal majority of players receive a positive allocation of benefits from the bargaining.
(2) *No rejection*—the agenda setter's motion always receives a majority support and never fails to pass.

Putting these two predictions together gives rise to the so-called "minimum winning coalition" (MWC).

When the MWC applies, the equilibrium must always allocate zero to the players who are excluded from the winning coalition. This is because giving positive allocations to the excluded players would only force the otherwise MWC to give up benefits unnecessarily. In addition, when the MWC applies, a player who receives a positive allocation of benefits from a proposal will cast a yea vote for the proposal. This is because casting a nay vote against the proposal will never make the player better off.

An agenda setter in the legislature will seek majority support in the cheapest way. As a consequence of the MWC, a lower "reservation utility" is actually beneficial to a player, since it raises the likelihood of being included in the winning coalition and getting at least the allocation of reservation utility. Realizing this, voters in each district will have an incentive to set their reservation utility lower than voters in other districts, so their political agents can be included in the MWC. This underbidding of reservation utilities causes a race to the bottom and results in a corner solution in equilibrium: all voters, except for those in the agenda setter's district, will choose the lowest reservation utility to discipline their political agents. This corner solution of delegation between voters and their political agents is extensively exploited by PRT as a stepping stone toward deriving their main results.

The sharp predictions of the MWC are powerful in analysis. However, are MWCs typically observed in the political arena of the real world? The answer is a resounding "no". According to Druckman and Thies (2002), there have been 80 oversized coalitions (OSCs) and only 74 MWCs in European parliamentary democracies since World War II. Furthermore, the empirical evidence shows that divisions on legislative roll calls are seldom near 50–50 (see, for example, Uslaner 1975; Lutz and Williams 1976). Last but not least, the hypothesis that the agenda setter's motion always receives a majority support and never fails to pass is clearly invalid in the real world.

[1] See Persson and Tabellini (2000) for applications and references.

[2] The seminal work on "elections as a discipline device" includes Barro (1973) and Ferejohn (1986).

[3] These two predictions hold under the so-called "closed rule," which is the setting assumed by PRT. The closed rule does not permit amendments on the floor and requires that a motion be voted on immediately. Sinclair (1995) documents that some types of restrictive closed rules have been employed with increasing frequency in the US Congress and accounted for 66% of the bills in the 102nd Congress.

This paper considers a simple modification of the PRT model such that (i) OSCs as well as MWCs may arise in equilibrium, and (ii) the agenda setter's motion may fail to receive a majority support. Our main thrust is to introduce uncertainty into legislative bargaining.

The MWC hypothesis was first put forth by Riker (1962), who forcefully argues that, to divide the benefits of controlling the executive, parties joining a government will include just enough parties to ensure majority support. However, Riker does not rule out the possibility of OSCs and, in fact, he himself provides a celebrated answer to the creation of OSCs:

> "Since the members of a winning coalition may be uncertain about whether or not it is winning, they may in their uncertainty create a coalition larger than the actual minimum winning size." (p. 48)

The creation of OSCs is meant to mitigate or to avoid the uncertainty of losing the political battle according to this answer. OSCs arise in our model because of uncertainty, too. We flesh out Riker's argument in terms of the tradeoff between the share of the "pie" offered to coalition members and the probability of winning.

PRT (p. 1137) conclude from their analysis of the so-called "simple legislature":

> "In our model, only the voters from one of three regions can secure redistribution toward their region, whereas the other voters get nothing. Voters of the non-agenda-setting regions cannot discipline their representatives to ask for more equitable redistribution because they compete with each other to be included in the [MWC] majority."

How will the strategic delegation between voters and their political agents be altered when legislative bargaining leads to OSCs rather than MWCs in the simple legislature? We show that, instead of underbidding each other and racing to the bottom, voters in the non-agenda-setting regions may set a higher reservation utility to discipline their representatives and ask for some redistribution.

PRT note that the simple legislature displays three fundamental political failures: "underprovision of public goods, wasteful allocation of tax revenues, and redistribution toward a powerful minority" (p. 1129). PRT explore the possibility of rectifying these political failures by the introduction of institutional features such as the separation of power exhibited in the US presidential-congressional system or the legislative cohesion exhibited in the European parliamentary system. We show that when OSCs arise, both public good provision and redistribution are likely to achieve their first-best solution even if the legislature is simple.

## 2 Model

Our model is essentially the same as the PRT model, except for a key departure: a legislator's discount factor is private information, unknown to other players. Legislators are assumed to have a common discount factor in the BF model. BF offer two interpretations for the discount factor: (i) the political imperative derived from legislators' reelection concerns to distribute benefits sooner rather than later, and (ii) the probability that a legislator will be reelected in the next election. Following either of the interpretation, it seems natural to allow for heterogeneous discount factors among legislators. In this paper, we consider a simplest possible such heterogeneity: a legislator's discount factor is either high or low. The private information assumption may be justified by, for example, a legislator knowing much better about the winning chance of her own reelection than other players. As will be seen, this slight modification of the PRT model may result in equilibrium outcomes that are quite different from those found in PRT. To facilitate the comparison with the PRT model, we use the same notation as PRT whenever possible.

### 2.1 A model of public finance

Consider an economy, in which there are three districts (or regions). Each district has a continuum of homogeneous voters with unit mass, and voters in each district are represented by exactly one legislator in the legislature. The game lasts two periods. Preferences of a voter in district $i$ over the two periods are given by

$$U^i(q_1) + U^i(q_2)$$

where $q_t$ is a vector of policies in period $t = 1, 2$. $U^i$ is the utility function per period and specifically,

$$U^i(q_t) = 1 - \tau_t + r_t^i + H(g_t)$$

where $\tau_t \leq 1$ is a common tax rate across districts, $r_t^i$ is a transfer payment to voters in district $i$, and $g_t$ is the supply of public goods evaluated by all voters with $H(0) = 0$, $H_g > 0$, $H_{gg} < 0$, and $H_g(0) > 1$. Note that we do not discount the voter utility derived from the second period. This setting is a simplification without loss of generality because, as will be seen, $U^i(q_2) = 0$ in equilibrium.

The policy vector $q_t$ is defined as

$$q_t = [\tau_t, g_t, \{r_t^i\}, \{s_t^l\}]$$

where $s_t^l$ denotes the rent captured by legislator $l$ in period $t$, and all components of $q_t$ are constrained to be nonnegative. It is assumed that the policy vector in each period $t$ must be the budget balance, that is,

$$3\tau_t = r_t + s_t + g_t \tag{1}$$

where $r_t = \sum_i r_t^i$ and $s_t = \sum_l s_t^l$.

As emphasized by PRT, the above formulation of public finance is quite general in the sense that it incorporates three conflicts of interest between players: "policy makers may abuse their power in office and capture public funds for their own benefit at the voters' expense; different groups of voters disagree on the allocation of tax revenues; and the political representatives, each pursuing their own career and personal interests, disagree over the distribution of current and future rents." (p. 1123)

## 2.2 A simple legislature

Preferences of an incumbent legislator $l$ are given by

$$s_1^l + \rho s_2^l D^l$$

where $0 \leq \rho \leq 1$ is a discount factor, and $D^l = 1$, if the legislator still holds office in period 2 and $D^l = 0$ otherwise. Our key departure from the PRT model lies in the design of the legislator's discount factor $\rho$. First, we allow for heterogeneous discount factors across legislators and consider a simplest possible such heterogeneity: a legislator's discount factor is either high ($\rho = \bar{\rho}$) or low ($\rho = \underline{\rho} < \bar{\rho}$). The former is called a type H, while the latter is called a type L. It is assumed that $\bar{\rho} = \delta$ and $\underline{\rho} = 0$. That is, the type L does not care about the rent in the second period. This is a simplifying assumption without loss, as far as our main results are concerned. Second and more importantly, we assume that whether a legislator is a type H or L is private information, unknown to other players. All that other players know is that either type is equally likely a priori. This extends the PRT model to a world with uncertainty.

Legislative bargaining in both periods follows the style of the BF model. At the end of the first period, each district holds a separate election under the plurality rule, in which the incumbent legislator runs against opponents who are not inherently different in any attributes. The sequence of events is as follows:

1. Nature randomly selects an agenda setter $a$ among the three legislators.
2. Voters formulate their reelection strategies, which are publicly known.
3. Legislator $a$ proposes a policy $q_1$.
4. The proposed policy is voted on immediately in the legislature. If a majority support the proposal, it is implemented. If not, a default policy is implemented, with $\tau_1 = s_1^l = \sigma > 0$ and $g_1 = r_1^i = 0$.
5. All voters observe the events that occurred previously. Elections are held at the end of period 1.
6. Legislative bargaining in period 2 repeats the events described in 1, 3 and 4, with a proposed policy $q_2$ and a default policy $\tau_2 = s_2^l = \sigma > 0$ and $g_2 = r_2^i = 0$.

It is assumed as in PRT that voters from the same district coordinate their strategies, but voters across districts do not cooperate. In event 2, voters in all districts simultaneously and independently set their "reservation utilities" $b^i$ in a utility-maximizing fashion with

$$D^l = 1 \quad \text{if and only if } U^i(q_1) \geq b^i, i = l.$$

That is, voters will reelect their incumbent legislator if and only if the policy $q_1$ does not bring about a utility lower than $b^i$.

PRT note that the simple legislature described above can illustrate "three fundamental political failures: underprovision of public goods, wasteful allocation of tax revenues, and redistribution toward a powerful minority." (p. 1129) The focus of our paper is on how these three fundamental political failures might be modified in the presence of uncertainty when OSCs rather than MWCs arise.

## 3 Analysis

In this section, we analyze the (subgame perfect) equilibrium of the game. Given the voters' reelection strategies $\{b^i\}$, we first derive the legislators' policy proposals.

### 3.1 Bargaining in the legislature

We solve the legislative bargaining game backward.

*Second period*

Legislators always appropriate maximum rents once in office in the second period. This is due to the fact that the second period is the last period of the game and so there is no reelection at the end of the period. As a result, legislators have no incentive to behave well since voters can no longer discipline their political agents through reelection. This implies that any legislator who is selected to become the agenda setter will propose a policy $q_2$ that satisfies

$$\tau_2 = 1; \quad g_2 = r_2 = 0.$$

Let $a$ be the agenda setter and let $m, n \neq a$ denote the legislators representing the other two districts. If the agenda setter $a$ seeks legislator $m$ (resp. $n$) as her coalitional partner, $s_2^m = \sigma$ and $s_2^n = 0$ (resp. $s_2^m = 0$ and $s_2^n = \sigma$). Note that a MWC will always form in the second period. This is simply because the creation of an oversized coalition would require $s_2^m = \sigma$ and $s_2^n = \sigma$, which means that $s_2^a = 3 - 2\sigma$, an amount strictly smaller than $s_2^a = 3 - \sigma$ (the agenda setter's payoff from forming a MWC). Thus, in equilibrium, $(s_2^a, s_2^m, s_2^n)$ will be either $(3 - \sigma, \sigma, 0)$ or $(3 - \sigma, 0, \sigma)$.

On the basis of the equilibrium $(s_2^a, s_2^m, s_2^n)$, the continuation value of the game for any legislator at the start of the second period (before nature has selected the agenda setter) is

$$\delta \left[ \frac{1}{3}(3 - \sigma) + \frac{2}{3} \cdot \frac{1}{2}\sigma \right] = \delta$$

if she is a type H (since $\bar{\rho} = \delta$). Put in words: each legislator has a $\frac{1}{3}$ chance to become the agenda setter $a$ and obtain the payoff of $3 - \sigma$ in the second period; once becoming the agenda setter, she seeks either $m$ or $n$ as her coalitional partner with equal probability in the formation of a MWC.[4] On the other hand, the continuation value is 0 if she is a type L (since $\underline{\rho} = 0$).

[4] Since $s_2^l = \sigma$ for $l = m, n$ if the default policy is implemented, the agenda setter is indifferent over which of the other two legislators to include in the formulation of a MWC. There are many possible ways of breaking the tie of this indifference. We assume that the agenda setter will break the tie symmetrically so that legislators $m$ and $n$ are equally likely to be included in the MWC coalition. This symmetry assumption seems to be a neutral benchmark since it does not favor either of the two otherwise identical legislators *a priori*. For an exploration of asymmetric tie-breaking rules in legislative bargaining, see Norman (2002).

*First period*

Before going into the analysis, we make three remarks. First, the information revealed in the first period with regard to the legislators' types will not exert any impact on a legislator's continuation value in the second period. Put differently, there is no room for signaling in our model. This greatly simplifies the analysis and enables us to focus on the role of uncertainty.[5]

Second, when the MWC applies, a legislator who receives a positive allocation of benefits from a proposal will always cast a yea vote for the proposal. It is thus unnecessary to make a distinction between the coalition ex ante (i.e., those legislators who receive positive allocations of benefits from a proposal) and the coalition ex post (i.e., those legislators who cast yea votes for the proposal). However, as will be seen, the coalition ex ante may differ from the coalition ex post in our context. Unless specified otherwise, the so-called "coalition" in this paper will always mean the coalition in the ex ante sense for convenience.

Third, the agenda setter $a$ (who may differ from the agenda setter in the second period) is indifferent toward $m$ and $n$ if she wants to form a MWC. We assume, as in the second period, that the agenda setter will break the tie symmetrically in such an indifferent situation. For ease of exposition, however, we will only report the case where legislator $m$ becomes the coalitional partner of the agenda setter $a$ ex post whenever a MWC is formed.

The agenda setter $a$ may or may not want to seek reappointment at the end of the first period. Clearly, a type L will not seek reappointment since any benefit in the subsequent period has no value to her. She has no incentive to behave well in the first period, but still has to pay $\sigma$ to legislator $m$ to win a majority support in the legislature. Thus, the proposed policy $q_1$ will be

$$\tau = 1; \quad g = r = 0; \quad (s^a, s^m, s^n) = (3 - \sigma, \sigma, 0).$$

where from now on, we let $\tau = \tau_1$, $g = g_1$, $r = r_1$, and $(s^a, s^m, s^n) = (s_1^a, s_1^m, s_1^n)$ for simplicity of notation.

On the other hand, the agenda setter $a$ if she is a type H may want to seek reappointment. Suppose that she does. To have the support of legislator $m$ for the proposed policy, agenda setter $a$'s offer will satisfy either

$$s^m + \delta = \sigma \tag{2a}$$

or

$$s^m = \sigma. \tag{2b}$$

The right-hand side of (2) is the payoff to legislator $m$ if the agenda setter's proposal fails to pass and hence the default policy is implemented. Since the default policy gives $g = r^i = 0$ with $\tau > 0$, it is clear that legislator $m$ will lose reelection if the default policy is implemented. The offer $s^m$ in (2) would leave $m$ indifferent between

[5] For an analysis of signaling in legislative bargaining, see Tsai and Yang (2009).

this fail-to-pass outcome and the alternative outcome that the agenda setter's proposal receives a majority support and then $m$ gets reappointment. From the analysis of legislative bargaining in the second period, we know that the continuation value of reappointment equals $\delta$ if $m$ is a type H, while it equals 0 if $m$ is a type L. This leads to two different equalities (2a) and (2b). A similar result and interpretation applies to $s^n$ for legislator $n$, that is, either

$$s^n + \delta = \sigma \tag{3a}$$

or

$$s^n = \sigma. \tag{3b}$$

On the basis of (2)–(3), there are three relevant proposals for the H-type agenda setter $a$ to consider if she seeks reappointment:[6]

($\alpha$) offering $(s^m, s^n) = (\sigma, 0)$ to buy a sure vote from legislator $m$ (this is a sure vote because legislator $m$ will cast a yea vote for the agenda setter's proposal, regardless of whether she is a type H or L);

($\beta$) offering $(s^m, s^n) = (\sigma - \delta, \sigma - \delta)$ to buy two risky votes from legislators $m$ and $n$ (this is risky because there will be two nay votes against the agenda setter's proposal if both legislators $m$ and $n$ turn out to be of the L type; the probability of failing to pass the proposal equals $\frac{1}{4}$ in this case because this is the probability that both legislators $m$ and $n$ are of the L type);

($\gamma$) offering $(s^m, s^n) = (\sigma - \delta, 0)$ to buy a risky vote from legislator $m$ (this is risky because legislator $m$ will cast a yea vote for the agenda setter's proposal if she is a type H, but legislator $m$ will cast a nay vote against the proposal if she is a type L; the probability of failing to pass the proposal equals $\frac{1}{2}$ in this case because both types are equally likely *a priori*).

From the budget constraint (1), we have

$$s = 3\tau - r - g. \tag{4}$$

Utilizing (2)–(4), the H-type agenda setter $a$'s overall (two-period) expected payoffs from offering proposals $\alpha$, $\beta$ and $\gamma$ equal, respectively

$$\begin{aligned}
v_\alpha &\equiv s - \sigma + \delta = 3\tau - r - g - \sigma + \delta;\\
v_\beta &\equiv \frac{3}{4}[s - 2(\sigma - \delta) + \delta] + \frac{1}{4}\sigma = \frac{3}{4}(3\tau - r - g - 2\sigma + 3\delta) + \frac{1}{4}\sigma;\\
v_\gamma &\equiv \frac{1}{2}[s - (\sigma - \delta) + \delta] + \frac{1}{2}\sigma = \frac{1}{2}(3\tau - r - g - \sigma + 2\delta) + \frac{1}{2}\sigma
\end{aligned}$$

**Lemma 1** *It is never optimal for a H-type agenda setter to offer* $(s^m, s^n) = (\sigma - \delta, 0)$ *if she seeks reappointment.*

[6] Proposals other than these three are strictly dominated since they give the agenda setter a lower rent, but do not increase the probability of winning a majority support.

*Proof* We will show that offering $(s^m, s^n) = (\sigma - \delta, 0)$ is strictly dominated by offering $(s^m, s^n) = (\sigma, 0)$. Suppose that this is not true, then there must exist an $(\tau, g, r)$ such that $v_\gamma \geq v_\alpha$, which implies that $r \geq 3\tau - g - 2\sigma$. On the other hand, since the agenda setter seeks reappointment, we must have $v_\gamma \geq 3 - \sigma$, where the right-hand side of the inequality represents the agenda setter's payoff from not seeking reappointment. This inequality implies that $r \leq 3\tau - 6 - g + 2\sigma + 2\delta$. Note that $3\tau - 6 - g + 2\sigma + 2\delta < 3\tau - g - 2\sigma$. Thus, there does not exist an $(\tau, g, r)$ that satisfies $v_\gamma \geq v_\alpha$, which is a contradiction. □

Because of Lemma 1, we can confine our attention to the following three proposals for $(s^a, s^m, s^n)$: $(s-\sigma, \sigma, 0)$ (a MWC that seeks reelection), $(s-2\sigma+2\delta, \sigma-\delta, \sigma-\delta)$ (an OSC that seeks reelection), and $(3-\sigma, \sigma, 0)$ (a MWC that does not seek reelection).

It is most interesting to observe that the agenda setter may propose an OSC in the legislature, that is, $(s^a, s^m, s^n) = (s - 2\sigma + 2\delta, \sigma - \delta, \sigma - \delta)$ may hold. As we will show later, OSCs can indeed arise in equilibrium in our model. What is the motivation for players to form an OSC? Riker (1962, p. 48) provides a celebrated answer:

> "Since the members of a winning coalition may be uncertain about whether or not it is winning, they may in their uncertainty create a coalition larger than the actual minimum winning size."

According to Riker, the creation of OSCs is meant to mitigate or avoid uncertainty as to whether or not a coalition is winning. This argument fits our result well if the agenda setter's proposals are confined to whether to buy a risky vote from legislator $m$ (proposal $\gamma$) or to buy two risky votes from both legislators $m$ and $n$ (proposal $\beta$). The probability of failing to have a majority support equals $\frac{1}{2}$ for the minimal-size proposal $\gamma$, but it will be reduced to $\frac{1}{4}$ for the OSC proposal $\beta$. Thus, the creation of an OSC could mitigate the uncertainty of a coalition with regard to "whether or not it is winning" as suggested by Riker. Observe that there is a tradeoff between the share offered to coalition members ($2(\sigma - \delta)$ vs. $(\sigma - \delta)$) and the probability of winning ($\frac{3}{4}$ vs. $\frac{1}{2}$).

A similar tradeoff applies to proposals $\alpha$ and $\beta$. Depending on the magnitude of $\delta$, it can be cheaper in the ex ante sense for the agenda setter to buy two risky votes from legislators $m$ and $n$ than to buy a sure vote from legislator $m$. The cost paid for this ex ante "cheaper" OSC is that rejection may occur *ex post* in equilibrium: there is the $\frac{1}{4}$ chance that both legislators $m$ and $n$ would turn out to be of the L type so the OSC proposal $\beta$ would receive two nay votes and fail to pass. By contrast, rejection would not arise if the more expensive MWC proposal $\alpha$ were offered instead.

It is arguable that the tradeoff between the share of the "pie" offered to coalition members and the probability of winning may be the key idea underlying Riker's answer to the formation of oversized coalitions. More generally, Riker's argument seems to suggest that complete information is a sufficient condition for the formation of MWCs. This is logically equivalent to our finding that incomplete information is a necessary condition for the formation of OSCs.

Which proposal for $(s^a, s^m, s^n)$ will prevail in equilibrium? To answer the question, we need to address voters' strategies first.

### 3.2 Voters' reelection strategies

A L-type agenda setter will not seek reelection and hence it is impossible for voters to discipline her. The focus of our analysis is on a H-type agenda setter.

How will voters set their "reservation utilities" $b^i$ to discipline their political agents? The answer critically depends on voters' expectations regarding whether a MWC or an OSC will form in the legislature. Utilizing the power of the MWC, PRT show that $r^m = r^n = 0$ in equilibrium. This outcome arises because agenda setter $a$ seeks a cheaper district partner who has a lower $s^l + r^i$ $(i = l)$ in the formulation of a MWC and, realizing this, voters in districts $m$ and $n$ will underbid each other so that their political agents can be included in the MWC. This underbidding between voters in districts $m$ and $n$ results in a corner solution, $r^m = r^n = 0$, in equilibrium.

The result $r^m = r^n = 0$ implies that $r^a = r$, which means that all the transfers are distributed to voters in district $a$. Voters in district $a$ in essence become the "residual claimant" on the resources of the economy, which implies that they will ask their political agent to set $\tau = 1$. Substituting in $\tau = 1, U^a(q_1) = 1-\tau+r+H(g) = r+H(g)$. This utility function indicates that voters in district $a$ trade off redistribution toward their district and public good provision *one for one*. As a result, it will be true that $g = \hat{g}$ in equilibrium, where $\hat{g}$ is the public good level with $H_g(\hat{g}) = 1$.

The reasoning and results become quite different if voters anticipate that OSCs will form in the legislature. When an OSC rather than a MWC is formed, both legislators $m$ and $n$ will be included in the majority. This implies that there is no need for voters in districts $m$ and $n$ to underbid each other in order for their political agents to be included in the majority. As a consequence, the outcome shown in PRT (i.e. $r^m = r^n = 0$) may no longer hold in equilibrium. Indeed, since voters in districts $m$ and $n$ no longer underbid each other, the division of $r$ between the three districts is basically indeterminate. Despite indeterminacy in nature, it is arguable in this scenario that a natural focal point for the division of $r$ is that $r^a = r^m = r^n = \frac{r}{3}$. This is because voters in any district will have neither an advantage nor a disadvantage in delegation relative to voters in the other two districts whenever OSCs arise in the legislature. The equal division of $r$ implies that no voters are the "residual claimant" on the resources of the economy and, therefore, one would expect that $\tau = 1$ and $g = \hat{g}$ may no longer hold in equilibrium. This is indeed true, as we demonstrate below.

From the equality $v_\beta = 3 - \sigma$, we obtain $r(\delta) = 3\delta - g - \frac{1}{3}\sigma + 3\tau - 4$. $r(\delta)$ is the maximal $r$ that makes the agenda setter $a$ merely indifferent between forming an OSC that seeks reelection and a MWC that does not seeking reelection. Given $\tau$ and $g$, it is clear that voters in district $a$ will set their reservation utility so that it satisfies $r = r(\delta)$ if they anticipate that an OSC that seeks reelection will form in the legislature. With the equal division of $r$, voters across districts are symmetric so that $U^i(q_1) = 1 - \tau + \frac{r}{3} + H(g)$ for $i = a$, $m$ and $n$. Substituting $r = r(\delta)$ in this utility function yields

$$U^i(q_1) = -\frac{1}{3} + \delta - \frac{1}{3}g - \frac{1}{9}\sigma + H(g).$$

It is clear from the above derived utility function that: (i) all voters are indifferent with respect to the choice of the tax rate $\tau$ since it does not appear in the utility function, and (ii) the choice of the public good $g$ by all voters will give rise to $g = g^*$ with $H_g(g^*) = \frac{1}{3}$, which is the first-best solution of public good provision in the economy.

Since taxes are nondistortionary in our model, the taxes that are used to finance $r$ by voters in a district will all be returned to the voters in that district if $r^a = r^m = r^n = \frac{r}{3}$. This explains why all voters are indifferent with respect to the choice of $\tau$ when $r$ is equally divided between the three districts. If taxes were even slightly distortionary, then we would have $r = 0$. When redistribution is all toward a minority, voters from the minority trade off redistribution to their own district with public good provision *one for one*. This leads to $H_g(\hat{g}) = 1$ in equilibrium. By contrast, with $r^a = r^m = r^n = \frac{r}{3}$, all voters trade off redistribution to their own district with public good provision *one-third for one*. This explains why the public good provision in this case will satisfy $g = g^*$ with $H_g(g^*) = \frac{1}{3}$ in equilibrium.

PRT note that the simple legislature displays three fundamental political failures: "underprovision of public goods, wasteful allocation of tax revenues, and redistribution toward a powerful minority" (p. 1129). PRT explore the possibility of rectifying these political failures through the introduction of institutional features such as the separation of power exhibited in the US presidential-congressional system or the legislative cohesion exhibited in the European parliamentary system. We show here that if OSCs that seek reelection arise in equilibrium, both public good provision and redistribution may achieve their first-best solution even if the legislature is simple.

In light of the arguments above, the key question becomes: can OSCs arise in equilibrium in the legislature? Within our model, we provide a positive answer to the question in the following section.

## 4 MWCs versus OSCs

To facilitate our analysis, we impose the following two assumptions:

**Assumption 1** $\delta \geq \max\{\hat{g}, \frac{1}{3}g^* + \frac{1}{9}\sigma + \frac{1}{3}\}$.

This is a technical assumption that ensures that $r$ will not become negative in equilibrium.

**Assumption 2** $r^a = r^m = r^n = \frac{r}{3}$ and $\tau = 1$ in equilibrium if an OSC that seeks reelection is formed in the legislature.

As we have argued, $r^a = r^m = r^n = \frac{r}{3}$ is a natural focal point for the division of $r$ whenever OSCs are formed in the legislature. This in turn implies that $g = g^*$ in equilibrium. As to $\tau = 1$ in equilibrium, this is a convenient assumption since all voters are indifferent with respect to the choice of $\tau$ when $r^a = r^m = r^n = \frac{r}{3}$.

Under the above two assumptions, we show that OSCs as well as MWCs can arise in equilibrium in our model. Specifically, we provide two sufficient conditions, showing that OSCs will arise under one condition (i.e. $3 + \sigma \leq 5\delta + 4\hat{g} - 3g^*$), whereas MWCs will arise under the other condition (i.e. $3 + \sigma \geq 6\delta$).

Before proceeding, we note two things. First, $\tau = 1$ holds in equilibrium. This is due to PRT's argument when a MWC is formed, or due to Assumption 2 when an OSC is formed. Second, because $g$ may equal $\hat{g}$ or $g^*$ in equilibrium, for ease of exposition, we use the notations $v_\alpha(g)$ and $v_\beta(g)$ instead of $v_\alpha$ and $v_\beta$ from now on.

## 4.1 $3 + \sigma \geq 6\delta$

This is the regime associated with Fig. 1, which is characterized by the feature that the point of the intersection between the straight line representing $v_\alpha(\hat{g})$ and the straight line representing $v_\beta(\hat{g})$ is located at or below the horizontal line $3 - \sigma$.[7] Voters will choose the maximal $r$ that leaves the agenda setter merely indifferent between seeking and not seeking reelection. This maximal $r$ is given by $r(\delta)$ in Fig. 1. Voters in district $a$ will not ask for less since giving rents to the legislators is costly to the voters. Moreover, suppose that voters in district $a$ ask for an $r$ that is smaller than $r(\delta)$. Then, voters in other districts could ask for more since asking for more need not violate the legislators' reelection incentives. Since $v_\alpha(\hat{g}) \geq v_\beta(\hat{g}) > v_\beta(g^*)$ $(g^* > \hat{g})$ and $v_\alpha(\hat{g}) \geq 3 - \sigma$ at $r = r(\delta)$, voters anticipate that a MWC that seeks reelection will form in the legislature ($v_\alpha(\hat{g}) > v_\beta(g^*)$ and $v_\alpha(\hat{g}) \geq 3 - \sigma$) and, furthermore, it is not profitable for the agenda setter to deviate and form an OSC instead ($v_\alpha(\hat{g}) \geq v_\beta(\hat{g})$).

As we have argued, voters in district $a$ would choose $r(\delta)$ if they wish to reelect the H-type agenda setter. $r(\delta)$ is implicitly determined by the equality $v_\alpha(\hat{g}) = 3 - \sigma$, which leads to $r(\delta) = \delta - \hat{g}$. Substituting it in (4) with $\tau = 1$ and $g = \hat{g}$ yields $s = 3 - \delta$. Thus, we have the following $\{s^l\}$ in equilibrium: a type H makes a MWC proposal that seeks reelection, i.e., $(s^a, s^m, s^n) = (3 - \sigma - \delta, \sigma, 0)$, while a type L makes a MWC proposal that does not seek reelection, i.e., $(s^a, s^m, s^n) = (3 - \sigma, \sigma, 0)$.

The equality $U^a(q_1) = b^a$ must hold in equilibrium. This is because $U^a(q_1) < b^a$ will cause a replacement in reelection, while $U^a(q_1) > b^a$ will be costly and unnecessary to voters. Thus, if the agenda setter is a type H, $b^a = r^a + H(\hat{g}) = \delta - \hat{g} + H(\hat{g})$ ($\delta \geq \hat{g}$ by Assumption 1), while $b^m = b^n = H(\hat{g})$ since $r^m = r^n = 0$. All legislators will be reelected. If the agenda setter is a type L, voters will not reelect her and all legislators will be replaced since $\tau = 1$ and $g = r = 0$.

To sum up, we obtain:

**Proposition 1** *Let* $3 + \sigma \geq 6\delta$.

1. *If the agenda setter is a type H, the equilibrium outcome gives:*
   $\tau = 1$;
   $g = \hat{g}$;
   $(s^a, s^m, s^n) = (3 - \sigma - \delta, \sigma, 0)$ *(a MWC that seeks reelection);*
   $r^a = \delta - \hat{g}$, $r^m = r^n = 0$ *(redistribution all toward district a)*;
   *all legislators are reelected.*
2. *If the agenda setter is a type L, the equilibrium outcome gives:*
   $\tau = 1$;
   $g = 0$;
   $(s^a, s^m, s^n) = (3 - \sigma, \sigma, 0)$ *(a MWC that does not seek reelection);*
   $r^a = r^m = r^n = 0$;
   *all legislators are replaced.*

[7] First, note that $\frac{\partial v_\alpha(g)}{\partial r} = -1$ and $\frac{\partial v_\beta(g)}{\partial r} = -\frac{3}{4}$. The point of the unique intersection between the straight line representing $v_\alpha(\hat{g})$ and the straight line representing $v_\beta(\hat{g})$ is implicitly determined by $v_\alpha(\hat{g}) = v_\beta(\hat{g})$, which gives the corresponding $r = 3 - \hat{g} + \sigma - 6\delta + \delta$. Substituting in this $r$ yields $v_\alpha(\hat{g}) = v_\beta(\hat{g}) = -2\sigma + 6\delta$. Comparing this height of the intersection with $3 - \sigma$ gives rise to $3 + \sigma \geq 6\delta$ if the location of the intersection between $v_\alpha(\hat{g})$ and $v_\beta(\hat{g})$ is at or below the horizontal line $3 - \sigma$.

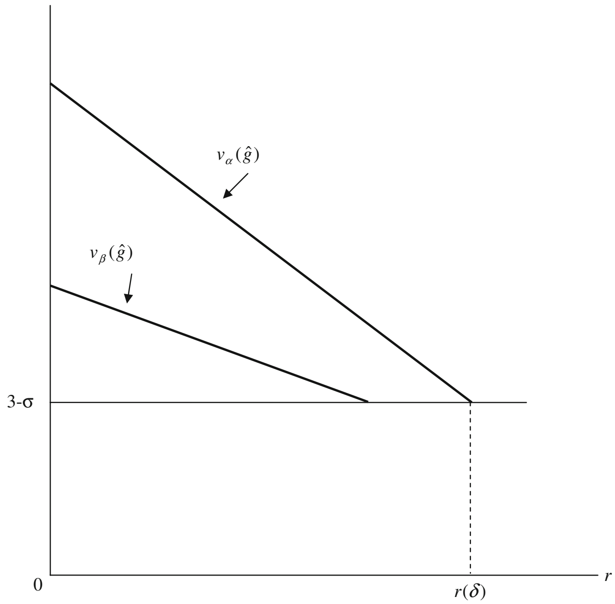


**Fig. 1** Intersection of $v_\alpha(\hat{g})$ and $v_\beta(\hat{g})$ is below $3 - \sigma$

Although a simplified version, our Proposition 1.1 shows three fundamental political failures as PRT's Proposition 1: (i) an underprovision of public goods ($H_g(\hat{g}) = 1$ instead of $H_g(g^*) = \frac{1}{3}$); (ii) a wasteful allocation of tax revenues ($s > 0$ instead of $s = 0$); and (iii) a redistribution toward a powerful minority ($r^a = r$). A main difference is that while an equilibrium always entails a MWC that seeks reelection in PRT's Proposition 1, an equilibrium may entail a MWC that does not seek reelection in our Proposition 1.2. Regardless of whether or not reelection is sought, the two sharp predictions of the MWC in legislative bargaining (i.e., the minimal size of coalitions and no rejection) remain robust in this regime. This is true even though uncertainty exists.

### 4.2 $3 + \sigma \leq 5\delta + 4\hat{g} - 3g^*$

This is the regime associated with Fig. 2, which is characterized by the feature that $v_\beta(\delta; g^*) \geq v_\alpha(\delta; \hat{g})$ at $r = 0$.[8] Again, voters will choose the maximal $r$ that leaves the agenda setter merely indifferent between seeking and not seeking reelection. This maximal $r$ is given by $r(\delta)$ in Fig. 2. Since $v_\beta(g^*) \geq v_\alpha(\hat{g}) > v_\alpha(g^*)$ ($g^* > \hat{g}$) and $v_\beta(g^*) \geq 3 - \sigma$ at $r = r(\delta)$, voters anticipate that an OSC that seeks reelection will form in the legislature ($v_\beta(g^*) \geq v_\alpha(\hat{g})$ and $v_\beta(g^*) \geq 3 - \sigma$) and, furthermore, it is not profitable for the agenda setter to deviate and form a MWC instead ($v_\beta(g^*) > v_\alpha(g^*)$).

[8] Substituting $r = 0$ in $v_\beta(g^*) \geq v_\alpha(\hat{g})$ gives the condition that defines the regime.

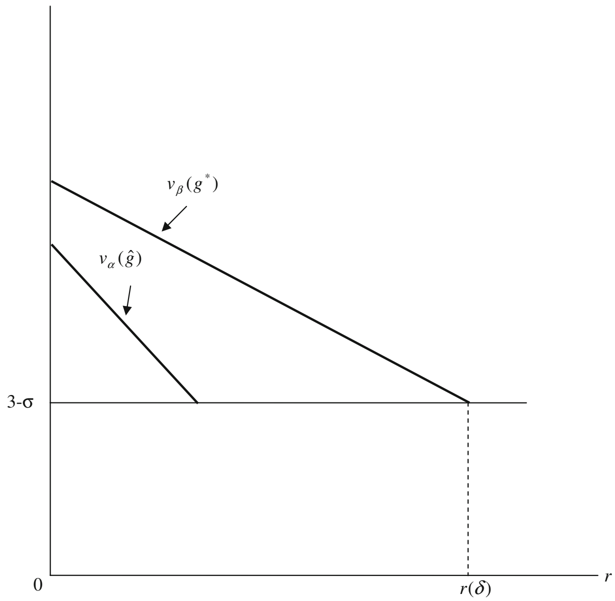


**Fig. 2** $v_\beta(g^*) \geq v_\alpha(\hat{g})$ at $r = 0$

Within this regime, voters in district $a$ would choose $r = r(\delta)$ if they want to reelect the H-type agenda setter. $r(\delta)$ is implicitly determined by the equality $v_\beta(g^*) = 3 - \sigma$, which leads to $r(\delta) = 3\delta - g^* - \frac{1}{3}\sigma - 1$. Substituting it in (4) with $\tau = 1$ and $g = g^*$ yields $s = 4 + \frac{1}{3}\sigma - 3\delta$. Thus, we have the following $\{s^l\}$ in equilibrium: a type H makes an OSC proposal that seeks reelection, i.e., $(s^a, s^m, s^n) = (4 - \delta - \frac{5}{3}\sigma, \sigma - \delta, \sigma - \delta)$, while a type L makes a MWC proposal that does not seek reelection, i.e., $(s^a, s^m, s^n) = (3 - \sigma, \sigma, 0)$.

Given $r^m = r^n = \frac{r}{3}$, if voters in the agenda-setting district deviate from this equilibrium by requesting $r^a > \frac{r}{3}$, the voters will definitely be worse-off with $r = r(\delta)$. This is because demanding $r^a + r^m + r^n > r(\delta)$ implies that the H-type agenda setter will not seek reelection any longer. Demanding $r^a < \frac{r}{3}$ with $r = r(\delta)$ is not optimal either because these voters will only give up rents to voters in other districts. Thus, if the agenda setter is a type H, $b^a = \frac{r(\delta)}{3} + H(g^*) = \delta - \frac{1}{3}g^* - \frac{1}{9}\sigma - \frac{1}{3} + H(g^*)$ ($\delta \geq \frac{1}{3}g^* + \frac{1}{9}\sigma + \frac{1}{3}$ by Assumption 1). Note that the probability of failing to pass a proposal equals $\frac{1}{4}$ when an OSC is formed. The analysis is analogous for voters in districts $m$ and $n$ because their utility functions are exactly the same.

To sum up, we obtain:

**Proposition 2** *Let* $3 + \sigma \leq 5\delta + 4\hat{g} - 3g^*$.

1. *If the agenda setter is a type H, the equilibrium outcome gives:*
   $\tau = 1$;
   $g = g^*$;
   $(s^a, s^m, s^n) = (4 - \delta - \frac{5}{3}\sigma, \sigma - \delta, \sigma - \delta)$ *(an OSC that seeks reelection);*

*$r^a = r^m = r^n = \delta - \frac{1}{3}g^* - \frac{1}{9}\sigma - \frac{1}{3}$ (equal redistribution among the three districts); all legislators are reelected with probability $\frac{3}{4}$.*

2. *If the agenda setter is a type L, the equilibrium outcome is the same as that of Proposition* 1−2.

Both predictions of the MWC are refuted in this regime: coalitions that seek reelection will always be OSCs rather than MWCs, and the proposals made by the agenda setter may fail to pass. PRT (p. 1137) conclude from their analysis of the simple legislature:

> "In our model, only the voters from one of three regions can secure redistribution toward their region, whereas the other voters get nothing. Voters of the non-agenda-setting regions cannot discipline their representatives to ask for more equitable redistribution because they compete with each other to be included in the [MWC] majority."

This conclusion is applicable to our model if the relevant regime is associated with Fig. 1. However, if the relevant regime is associated with Fig. 2, the opposite result may occur. Our Proposition 2 characterizes an equilibrium, in which redistribution is equally distributed, and voters in non-agenda-setting districts can now discipline their representatives to ask for some redistribution.

Weingast (1979) and others have pioneered a line of the literature known as "universalism," which attempts to theoretically explain the stylized empirical observation that legislators under majority rule often distribute benefits with broad majorities rather than in line with the prediction of the MWC. Persson and Tabellini (2000) criticize this strand of the literature and point out that the literature "has weak micropolitical underpinnings" since "it is hard to model as the outcome of an extensive form game" (p. 190). OSCs that result from our model are equilibrium outcomes of an extensive form game.[9]

## 4.3 Summary

From the above results, one can see that given $\delta$, a higher $\sigma$ tends to be associated with the formation of a MWC in the legislature (Proposition 1), while a lower $\sigma$ tends to be associated with the formation of an OSC (Proposition 2). Since $\sigma$ is the default policy, it can be viewed as an outside option when the proposed policy fails to pass. A larger $\sigma$ implies that the price to buy each yea vote is higher, which leads to the disadvantage of the formation of an OSC. On the other hand, given $\sigma$, one can see that OSCs are likely to arise if $\delta$ is large (i.e., $5\delta \geq 3 + \sigma - 4\hat{g} + 3g^*$), whereas MWCs are likely to arise if $\delta$ is small (i.e., $6\delta \leq 3 + \sigma$). This result is also intuitive. When $\delta$ is large, it

[9] OSCs can also arise in equilibrium in Diermeier and Merlo (2000) and Baron and Diermeier (2001), in which political agents care about the policy chosen as well as the rents obtained. The key to their result lies in the agenda setter being able to extract rents from other players through offering a compromised policy. This mechanism is obviously different from ours, since, like the PRT model, legislators in our model only care about rents obtained, not resulting policies. Note also that a proposal never fails to pass in the Baron-Diermeier-Merlo model, whereas it may fail to pass in our model.

will be cheaper in the ex ante sense for the agenda setter to offer the (relatively small) $2(\sigma - \delta)$ share of the "pie" to buy two risky votes from legislators $m$ and $n$ than to offer the (relatively large) $\sigma$ share of the "pie" to buy one sure vote from legislator $m$. On the other hand, when $\delta$ is small, the opposite will be true.

## 5 Conclusion

Since Riker (1962), uncertainty has been recognized as a plausible reason for the creation of OSCs. In this paper, we incorporate uncertainty into the PRT model and flesh out Riker's argument in terms of the tradeoff between the share of the "pie" offered to coalition members and the probability of winning. The formulation of OSCs in our model embodies the intuitive idea that a sure vote (a MWC) may be too expensive to buy relative to two risky votes (an OSC).

In the presence of uncertainty, we show that OSCs as well as MWCs can arise, and that the agenda setter's proposed policy may fail to receive a majority support. These two results are in marked contrast to the certain world, in which only MWCs can arise in equilibrium and the agenda setter's proposal never fails to pass. When OSCs arise, equilibrium outcomes may differ substantially from those in PRT. In particular, we show that: (i) instead of underbidding each other and racing to the bottom as the MWC applies, voters in the non-agenda-setting districts may set a higher reservation utility to discipline their representatives and ask for some redistribution, and (ii) both public good provision and redistribution are likely to achieve their first-best solution even if the legislature is simple.

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