# Pollution, Factor Ownerships, and Emission Taxes 

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#### Abstract

This paper employs Cournot's (1838) model of complementary goods to analyze the optimal emission taxation under joint and independent ownership with pollution. When the marginal damage is small (large), an emission taxation is unnecessary (necessary), because the quantity (environmental) distortion is more serious than the environmental (quantity) distortion. This finding has never been presented in the literature. In contrast to Cournot (1838), a striking result is that independent ownership may be welfare superior to joint ownership when the marginal damage of externality is large in the absence of governmental intervention.


Keywords Emission taxation • Joint ownership • Independent ownership • Pollution

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## Introduction

One of the well-known imperfect competitive market models is Cournot's (1838, Chapter 9) model of complementary goods, which has been given renewed attention in the analysis of key issues such as product compatibility and networks in recent years (for

[^0]example, Economides and Salop 1992; Carter and Wright 1994; Brueckner 2004). Within this model, a homogenous final product (say brass) is produced by two monopolized factors (say zinc and copper) and each factor firm buys another factor from the rival firm in order to produce the final product and sells its product to a final goods market. ${ }^{1}$ This market structure is defined as independent ownership (or independentlyowned monopolists). If these two factors are owned by a single monopolist, then the market structure is defined as joint ownership (or integrated monopolist).

Cournot (1838) shows that the product price under joint ownership is lower than that under independent ownership. This implies that joint ownership is welfare superior to independent ownership. An interesting question naturally arises: Does this standard result remain unchanged when the mutual relations of production generate some pollution? This paper will answer this question by way of a partial equilibrium analysis.

In the fusion process of zinc and copper, some greenhouse gas $\left(\mathrm{CO}_{2}\right)$ will be produced. ${ }^{2}$ To solve production externalities, one popular policy is to enact an emission tax on the final goods production. Buchanan (1969) graphically demonstrates that when externalities exist under imperfect competition, the emission tax may not be appropriate to solve this problem. Baumol and Oates (1975), Baumol (1972), Barnett (1980), Goel and Hsieh (1997), and Martin (1986) also discuss this issue.

There are two distortions under imperfect competition with externalities (pollution). First, negative externality results in over-production when compared to the social optimal level. Second, imperfect competition results in under-production. An emission tax reduces the first distortion, while it enlarges the second one. Therefore, one must be careful to use an emission tax under an imperfect market structure to solve the problem of externalities.

To achieve the social optimal production, an emission taxation (subsidy) should be different in those two ownership structures. Moreover, if the externality distortion is high enough to offset the quantity distortion, then the welfare superiority of joint ownership over independent ownership may not be valid under some situations. The rest of this paper is organized as follows. The second section presents the model, and the last section offers the conclusion.

## The Model

## Joint Ownership (Integrated Monopolist)

Suppose that two factors of production (zinc and copper) are owned by a single jointly-owned monopolist and that the linear inverse demand function ${ }^{3}$ of the final product (brass) is:

$$
\begin{equation*}
p=\alpha-\beta q, \tag{1}
\end{equation*}
$$

[^1]where $\alpha$ and $\beta$ are constants. It is assumed that $m_{1}\left(0<m_{1}<1\right)$ unit of zinc and $1-m_{1}$ unit of copper yield one unit of brass. ${ }^{4}$ Following Cournot's assumption, both factors have no other use than that of being jointly fused together as an alloy. For simplicity, the production cost of brass is assumed to be zero.

The (brass) producing process results in some pollution to the environment. The marginal damage ( $M D$, in terms of money) of pollution, although it is ignored by the producer, is defined as follows.

$$
\begin{equation*}
M D=k \cdot q \tag{2}
\end{equation*}
$$

where $k>0$ is a constant. ${ }^{5}$ The total revenue (TR) and marginal revenue $(M R)$ of the integrated monopolist are defined as follows, respectively,

$$
\begin{equation*}
T R=(\alpha-\beta q) q, M R=\alpha-2 \beta q \tag{3}
\end{equation*}
$$

Suppose the government enacts an emission tax $t_{J}$ on per unit brass production for the integrated monopolists, where the subscript $J$ represents joint ownership. Solving for $M R=M C=t_{J}$ yields:

$$
\begin{equation*}
q_{J}=\frac{\alpha-t_{J}}{2 \beta} \tag{4}
\end{equation*}
$$

The equilibrium market price and joint profit are respectively:

$$
\begin{equation*}
p_{J}=\frac{\alpha+t_{J}}{2}, \pi_{J}=\frac{\left(\alpha-t_{J}\right)^{2}}{4 \beta} \tag{5}
\end{equation*}
$$

The consumer surplus (CS) is:

$$
\begin{equation*}
C S_{J}=\int_{0}^{q_{J}} p d q-p_{J} \cdot q_{J}=\frac{\left(\alpha-t_{J}\right)^{2}}{8 \beta} \tag{6}
\end{equation*}
$$

The social welfare is then:

$$
\begin{equation*}
W_{J}=C S_{J}+\pi_{J}+t_{J} \cdot q_{J}-\int_{0}^{q_{J}} M D d q=\frac{\left(\alpha-t_{J}\right)\left(3 \beta \alpha+t_{J} \beta-k \alpha+k t_{J}\right)}{8 \beta^{2}} \tag{7}
\end{equation*}
$$

[^2]

Fig. 1 Joint ownership market equilibrium and optimal emission tax
Solving $\partial W_{J} / \partial t_{J}=0$ yields an optimal taxation rate: ${ }^{6}$

$$
\begin{equation*}
t_{J}^{*}=\frac{\alpha(k-\beta)}{\beta+k} . \tag{8}
\end{equation*}
$$

There are two effects of the emission tax: The first effect is a social gain in that the firm produces less externalities (the area $\Delta$ acd in Fig. 1), and the second one is a social loss (excess burden) resulting from the taxation (the area $\Delta a b c$ in Fig. 1). ${ }^{7}$ Since these two effects are in opposite directions, the sign of the total effect depends on the relative strength of marginal damage and excess burden. ${ }^{8}$

Independent Ownership (Independently-owned Monopolists)
If both factors (zinc and copper) are independently owned by two firms (the zinc firm and the copper firm), then each firm must buy a necessary factor from the rival firm. The game structure is that both firms play Nash against each other to set up the factor price. The brass price is determined by the summation of zinc price and copper price. Since there is only one brass market, Eqs. 1 and 2 are thus applied. Once the brass price is determined, the equilibrium quantity and damage are automatically determined. The profits and welfares are also determined simultaneously. In other words, summing up the zinc and copper prices with their relative proportions yields $p_{I}=m_{1} p_{I 1}+\left(1-m_{1}\right) p_{I 2}$, where subscript I represents independent ownership on

[^3]the production of brass under independent ownership of inputs, and subscript 1 represent zinc, and 2 for copper. Substituting $p=p_{I}$ into (1) thus yields equilibrium quantity of brass $q_{I}=\left(\alpha-m_{1} p_{I 1}-p_{I 2}+m_{1} p_{I 2}\right) / \beta$.

If the government now enacts an emission $\operatorname{tax} t_{I}$, then the profit functions of these two raw material firms are:

$$
\begin{equation*}
\pi_{I 1}=m_{1}\left(p_{I 1}-t_{I}\right) \cdot q_{I}, \pi_{I 2}=\left(1-m_{1}\right)\left(p_{I 2}-t_{I}\right) \cdot q_{I} \tag{9}
\end{equation*}
$$

Solving for $\partial \pi_{I 1} / \partial p_{I 1}=0$ and $\partial \pi_{I 2} / \partial p_{I 2}=0$ yields:

$$
\begin{equation*}
p_{I 1}=\frac{\alpha-t_{I}+3 m_{1} t_{I}}{3 m_{1}}, p_{I 2}=\frac{\alpha+2 t_{I}-3 m_{1} t_{I}}{3\left(1-m_{1}\right)} . \tag{10}
\end{equation*}
$$

The equilibrium market price and quantity are then:

$$
\begin{equation*}
p_{I}=\frac{2}{3} \alpha+\frac{1}{3} t_{I}, q_{I}=\frac{\alpha-t_{I}}{3 \beta} . \tag{11}
\end{equation*}
$$

Note that $p_{I}$ and $q_{I}$ are independent to the factor proportion ( $m_{1}, 1-m_{1}$ ) of the final product. ${ }^{9}$ This is because the Leontief production type makes them share profit equally. The equilibrium profits are:

$$
\begin{equation*}
\pi_{I 1}=\pi_{I 2}=\frac{\left(\alpha-t_{I}\right)^{2}}{9 \beta} \tag{12}
\end{equation*}
$$

The consumer surplus is:

$$
\begin{equation*}
C S_{I}=\int_{0}^{q_{I}} p_{I} d q-p_{I} \cdot q_{I}=\frac{\left(\alpha-t_{I}\right)^{2}}{18 \beta} . \tag{13}
\end{equation*}
$$

The social welfare is:

$$
\begin{equation*}
W_{I}=C S_{I}+\pi_{I 1}+\pi_{I 2}+t_{I} q_{I}-\int_{0}^{q_{I}} M D d q=\frac{\left(\alpha-t_{I}\right)\left(5 \beta \alpha+t_{I} \beta-k \alpha+k t_{I}\right)}{18 \beta^{2}} . \tag{14}
\end{equation*}
$$

Solving $\partial W_{I} / \partial t_{I}=0$ yields:

$$
\begin{equation*}
t_{I}^{*}=\frac{\alpha(k-2 \beta)}{(\beta+k)} \tag{15}
\end{equation*}
$$

Subtracting (8) from (15) yields:

$$
\begin{equation*}
t_{I}^{*}-t_{J}^{*}=-\frac{\alpha \beta}{(\beta+k)} 0 \tag{16}
\end{equation*}
$$

This result is due to the fact that the quantity distortion (under-production) is more serious under independent ownership. Therefore, the environmental correction policy should be milder.

[^4]Comparison of Optimal Environmental Policies Between Joint and Independent Ownerships

With $t_{I}^{*}$, the market price should be $p^{o}$ and market quantity should be $q^{o} .{ }^{10}$ Therefore, $C S_{I}=C S^{o}$ and $W_{I}=W^{o}$. The results of $t_{J}^{*}$ and $t_{I}^{*}$ can be stated as the following proposition.

Proposition One A government's optimal externality control policy must satisfy the following rules: (A) When $k<\beta$, then there is a subsidy under both joint and independent ownerships; (B) When $\beta<k<2 \beta$, then there is taxation (subsidy) under joint (independent) ownership; (C) When $k>2 \beta$, then there is a taxation under both joint and independent ownerships; (D) When $k=\beta$, there is no intervention (subsidy) under joint (independent) ownership; (E) When $k=2 \beta$, there is no intervention (taxation) under independent (joint) ownership.

Figure 2 is depicted to illuminate the graphical presentation of Proposition One. To provide more economic intuition for Proposition One, we see that the proposition hinges on the relative sizes of $k$ (the marginal damage of the externality) and $\beta$ and $2 \beta$ (the slopes of demand curve and marginal revenue curve, respectively). The former represents environmental distortion, while the latter monopoly distortion.

The most important fact is that when $k=\beta(k=2 \beta)$ in the joint (independent) case, the government needs not do anything, because the monopoly distortion and environmental distortion cancel out each other. More specifically, when these two opposite distortions are the same at $k=\beta$ under joint ownership case, their effects cancel out each other and thus no intervention is necessary. Under the independent ownership case, the monopoly distortion is reinforced due to the double marginalization and thus the case of the cancelation happens at $k=2 \beta$.

Further note that the price under independent ownership is higher than that under joint ownership ( $p_{I}>p_{J}$ ), while the quantity under independent ownership is smaller than under joint ownership $\left(q_{I}<q_{J}\right)$. The lower quantity also means less damage to the environment. When the marginal damage $(k)$ is relatively small, the monopoly distortion is more serious. In order to encourage more production, the government must use a subsidy to achieve this goal. On the other hand, when $k$ is large, then one should go back to the traditional externality literature whereby a tax is necessary. The policy implication is that the government may investigate the value of all parameters and market structures before enacting an emission tax.

Without governmental intervention, the above results reduce to Cournot's (1838) findings with externalities. To see this, plugging $t_{J}=0$ and $t_{I}=0$ into (7) and (14) yields:

$$
\begin{equation*}
W_{J}-W_{I}=\frac{\alpha^{2}(7 \beta-5 k)}{72 \beta^{2}}<(\geq) 0, \text { ifk }>(\leq) \frac{7}{5} \beta \tag{17}
\end{equation*}
$$

[^5]

Regions for an optimal pollution control policy
$A$ : Subsidy under joint and independent ownerships
$B$ : Taxation (subsidy) under joint (independent) ownership
$C$ : Taxation under joint and independent ownership
$k=\beta$ : No intervention (subsidy) under joint (independent) ownership
$k=2 \beta:$ No intervention (taxation) under independent (joint) ownership
Fig. 2 Optimal environmental policies with joint and independent ownerships

Cournot (1838, Chapter 9) implicitly shows that joint ownership is welfare superior to independent ownership without externalities. ${ }^{11}$ Equation 17 indicates that Cournot's claim will be overturned when $k \frac{7}{5} \beta$, leading to the following proposition.

Proposition Two When the externality effect is very strong ( $k>\frac{7}{5} \beta$ ), independent ownership is welfare superior to joint ownership in the absence of governmental intervention.

This result is obviously in sharp contrast to the standard one without externalities.

## Conclusion

This paper analyzes the optimal emission taxation under both joint and independent ownerships of factors with production externalities. Since the quantity distortion and environmental distortion are in opposite directions, the government may let the market be free from intervention under some damage and demand situations. When the marginal damage is small, an emission subsidy is necessary, because the quantity distortion (under-production) is more serious. When the marginal damage is large, the optimal intervention is to enact an emission tax, because the environmental distortion (over-production) is more serious. This finding has never been presented in the literature.

Without any externalities, Cournot (1838) implicitly shows that joint ownership is welfare superior to independent ownership. Nevertheless, the current paper shows that the welfare superiority of joint ownership will be reversed when a serious

[^6]externality (large marginal damage) is taken into consideration. Using a partial equilibrium analysis, this paper ignores the (emission) tax collection or subsidy distribution. If the model is based on a general equilibrium analysis, then the optimal taxation or subsidy may be different from the current framework. Under negative externalities which are directly linked to the quantity produced, any scenario where integration among firms creates a higher quantity produced than that without integration ${ }^{12}$ is also expected to have the same welfare implication as in this paper. These extensions will be left for future research.

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[^7]
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[^1]:    ${ }^{1}$ The final product's price is solely determined on the supply side. In fact, Cournot (1838) does not define the final market types. In our viewpoint, he implies that there are many buyers for the final product and thus all of them are price takers.
    ${ }^{2}$ The casting of zinc actually produces some zinc oxide fumes which may cause serious diseases.
    ${ }^{3}$ Goel and Hsieh (1997) show that convexity or concavity of the demand function may affect the welfare effect of an emission tax when solving the externality problem. To achieve closed-form solutions and without loss of generality, this paper assumes a linear inverse demand function.

[^2]:    ${ }^{4}$ As Fisher (1898, p.128) points out, "His [Cournot's] study here, unlike the rest of his work, is confined to a special case; namely, that where the component articles enter in perfectly definite proportions into the joint article". However, the essence of Cournot's model is "mutual relations of producers" and the striking result is that these two firms share profit equally as long as the proportion is $0<m_{1}<1$. The economic intuition behind the mutual relations of producers is that both firms are indispensable for each other, instead of the fixed proportion of production. In reality, brass is produced with fixed proportion of zinc and copper. The more zinc, the more yellow color and the more hardness for the alloy. For example, it is called red brass when containing $15 \%$ of zinc and is called yellow brass when containing $35 \%$ of zinc. This is similar to the fact that the 18 k gold contains a fixed proportion ( $75 \%$ ) of gold. The key point of Cournot's model is that both firms not only compete with each other, but also must cooperate together. In this sense, the fixed proportion in production may be considered as a normal case, instead of a special case.
    ${ }^{5}$ The total damage is thus quadratic to the quantity produced.

[^3]:    ${ }^{6}$ Note that there are two industries (the polluted industry and the damaged industry) in Buchanan's (1969) model. Therefore, even though the polluted industry has been optimally taxed, the damaged industry will still be polluted and thus only the second-best goal can be reached. However, our model is a partial equilibrium analysis under one industry with one policy instrument. Thus, $t_{J}^{*}$ indeed reaches the first-best solution. Plugging $t_{J^{2}}^{*}$ into $p_{J}, q_{J}, C S_{J}$, and $W_{J}$ yields $p_{J}^{*}=p^{o}=\frac{\alpha k}{\beta+k}, q_{J}^{*}=q^{o}=\frac{\alpha}{\beta+k}, C S_{J}^{*}=C S^{o}=\frac{1 \beta \alpha^{2}}{2(\beta+k)^{2}}$, and $W_{J}^{*}=W^{o}=\frac{\alpha^{2}}{2(\beta+k)}$, where the superscript " $o$ " represents "optimum".
    ${ }^{7}$ Note that the social loss is due to the further reduction in output that would result in an already output constrained monopoly market. We owe this point to the Referee.
    ${ }^{8}$ An emission tax can be charged on output or emissions (see Carlton and Loury 1986 and Kohn 1986). In this paper the emission is monotonic to output quantity $(M D=k q)$, and it is thus not necessary to clarify whether the pollution is charged on output or emissions.

[^4]:    ${ }^{9}$ Note also $p_{I}>p_{J}$ is without taxation. This is because the independent firms ignore the price markup externality to each other, while the integrated monopolist internalizes this externality (See Economides and Salop (1992), page 106).

[^5]:    ${ }^{10}$ Note that the demand function and marginal damage function are identical for both joint and independent ownerships. Thus, the social optimal quantity and price should be identical in both types of ownerships. Therefore, the optimal tax rate may not equal the marginal damage. At $q_{0}$, the marginal damage is $\alpha k /(\beta+k)$, which is obviously greater than $t_{I}$ and $t_{J}^{*}$.

[^6]:    ${ }^{11}$ In fact, one can plug $k=0$ into (17) to have $W_{J}-W_{I}>0$.

[^7]:    ${ }^{12}$ For example, a vertical integration in successive monopolies will produce a higher quantity of the final product due to the elimination of double marginalization.

