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Environmental resilience and economic growth: Command economy's optimization and environmental Kuznets curve[☆]

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ABSTRACT

This paper constructs a two-sector environmental growth model with explicit mathematical derivation and economic intuition in a social planning economy. Through the optimal allocation of man-made capital between the production sector and the environmental sector, this paper shows that the trade-off between economic growth and environmental protection exists only when an economy deviates from its steady state. We also provide short-run transitions for both the whole economic system and individual control and state variables. In addition, technological progress in the production sector benefits economic growth rate while the improvement of technology in the environmental sector has only level effects on economic variables. This paper ends with a link between the theory and a hot empirical issue — the environmental Kuznets curve (EKC) hypothesis.

1. Introduction

In the early 1970s, the theoretical application of economic growth models to environmental issues was initiated by some representative studies — Anderson (1972), Keeler et al. (1972) and Forster (1973), for example. The summary of their results generally indicated that the speed of capital accumulation should be controlled due to the unavoidable pollution in the process of production, and due to the representative agent's disutility from a contaminated environment. It was also shown that in a regulated economy, compared to an unregulated one, the optimal steady-state levels of consumption and capital stock are lower because of the need to divert resources to pollution abatement. In both types of economy however, with decreasing returns to capital, the long-run economic growth rate is exogenously determined by non-reproducible factors, such as labor growth and technological progress as proposed by Solow (1956), Swan (1956), Cass (1965) and Koopmans (1965), for example.

The development of endogenous growth models after the late 1980s improves the theoretical settings in analyzing the environmental issue under the framework of dynamic optimization. These environment-based endogenous growth models point in two opposing directions. On the one hand, the agent experiences disutility from the emissions of pollutants, which are inevitably generated by the accumulation of physical capital (e.g. Huang and Cai (1994) and Shieh et al. (2001)). On the other hand, environmental quality and natural resources are considered in an agent's utility-maximization problem, so it is beneficial to upgrade environmental quality and reproduce natural resources through the allocation of economic resources (e.g. Tahvonen and Kuuluvainen (1991) and Bovenberg and Smulders (1995)). In general, the above-mentioned models focused mostly on the analysis of steady-state equilibrium and hardly discussed the off-steady-state evolution of endogenous variables.

A recent environmental growth model constructed by Aznar-Márquez and Ruiz-Tamarit (2005) emphasizes the importance of short-run dynamics. Their model provides a linkage between final-good and natural resource sectors in which the former sector requires the exploitation from the latter one. In a decentralized economy, they analyzed both short-run transitional dynamics and steady-state properties of endogenous variables and concluded that under some circumstances, sustainable long-run growth is possible.

This paper utilizes the two-sector framework adopted by Aznar-Márquez and Ruiz-Tamarit (2005) under a command economy. The model developed in this paper requires physical and human capital inputs to improve the speed of environmental resilience. The exploitation of natural resources, on the other hand, needs not be an input in the production process but can directly provide a positive but diminishing marginal utility to the representative agent. Based on these settings, the model in this paper cannot only analyze the economy in both short and long run, but also map the pattern of the environmental Kuznets Curve (EKC) in a closed command economy.

This paper is arranged as follows. Section 2 describes the technology structures in the final-good and the environmental sectors and defines specific preferences for the representative agent. With the

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establishment of the dynamic optimization problem, Section 3 discusses how an intertemporal utility maximizing solution chosen by a social planner generates the long-run steady-state equilibrium of the system. The analysis of short-run dynamics is sketched in the form of phase diagrams in Section 4. The effects of shocks, especially technological progress from both sectors, are derived by comparative statics in Section 5. The concluding Section 6 connects the theoretical results to the EKC hypothesis, an unproven but plausible proposition broadly motivating the interests of economists and environmentalists.

2. An innovative environmental growth model

By incorporating the concept of renewable resources and environmental quality, and allowing an interaction between the economic and ecological system, we shed a different light on the relationship between economic growth and environmental resilience. To upgrade our natural environment, it requires not only the existing stock of natural resources and environmental renewability, but also the expenditure of economic resources, especially physical capital and human capital. By defining these two types of capital in a broad sense as 'man-made capital', it exhibits the property of constant returns in the production process (Rebelo, 1991; Lucas, 1988). This simple but intuitive setting in the production technology makes it possible to distribute the proportion of man-made capital across final-good and environmental sectors.

2.1. Technology structure of the two sectors

First, we consider an AK technology in the final goods production with reproducible man-made capital:

$$Y(t) = A[1-u(t)]H(t); \quad u(t) \in [0,1] \tag{1}$$

where Y and H represent the aggregate levels of final output and man-made capital, respectively. $(1-u)$ is the share of man-made capital used in final goods production (the remaining portion of man-made capital, uH, is diverted to the environmental sector) and A is the exogenous productivity level in this sector.

Unlike the output sector that benefits people's physical life by producing final goods and services, the environmental sector provides renewable resources and upgradable environmental quality that can be consumed by all humans. In the environmental sector, its renewal requires joint factor inputs as:

$$E(t) = B[N(t)]^{\eta}[u(t)H(t)]^{1-\eta}; \quad \eta \in (0,1) \text{ and } \eta > (1-\eta). \tag{2}$$

The assumption that $\eta \in (0,1)$ ensures the Inada conditions hold, under which natural capital N (defined by Bovenberg and Smulders (1995)) and man-made capital provide positive but diminishing marginal renewal of the environment. The environmental production structure E (gross renewal) reveals three properties. First, to upgrade the environment, the economy requires not only its self-resilience capability, N, but also the input of man-made capital, uH, from the production sector. Second, the importance of natural capital is assumed to be greater than man-made capital because of the belief that natural environment itself is a giant base of reviving environmental quality. Therefore, the share of natural capital η is greater than that of man-made capital $(1-\eta)$ in the environmental sector. Lastly, B represents the exogenous parameter of technology level in renewing the environment.

The decision maker, at every point of time, decides the optimal proportion of man-made capital u to ameliorate environmental quality, and $(1-u)$ to produce final goods and services. The decision maker also controls the agent's consumption of final output, C, and consumption of environmental resources, R. The evolutions of man-made capital and natural capital are therefore as follows:

$$\dot{H}(t) = A[1-u(t)]H(t) - C(t) \tag{3}$$

$$\dot{N}(t) = B[N(t)]^{\eta}[u(t)H(t)]^{1-\eta} - R(t). \tag{4}$$

The time derivative is shown by a dot on any variable. For simplicity, a zero rate of depreciation for both types of capital is assumed.[1] The evolution of man-made capital is a stereotyped setting with total output minus the flow of final-good consumption. The evolution of natural capital, on the other hand, is considered by different intuition. Instead of pollution that was generally used as a deduction of the flow of environmental renewal (Bovenberg and Smulders (1996), for instance), our model directly assumes that the major source of environmental degradation is from human utility-maximizing behavior. For example, increasing demand for petroleum and electricity leads to rapid depletion of natural resources and more emissions of greenhouse gasses, which hurt the environment. As the traditional consumption, C, does in the final-good sector, the environmental consumption, R, provides positive but decreasing marginal utility to the agent. R therefore acts as an offsetting flow of environmental renewal, E.

2.2. Agent's preferences

According to the concepts of traditional final-good consumption, C, and environmental consumption, R, the utility function of a representative agent can be assumed as:

$$U[C(t),R(t)] = \frac{C(t)^{1-\theta}-1}{1-\theta} + \frac{R(t)^{1-\theta}-1}{1-\theta}; \quad \theta \in (0,1) \tag{5}$$

where θ denotes the inverse of constant intertemporal elasticity of substitution for both final-good and environmental consumption. Lower θ implies that C and R can provide higher contemporaneous marginal utility and have better exchangeability in terms of consumption today and tomorrow. Higher intertemporal elasticity of substitution, by intuition, can benefit an agent's saving strategy and boost the economic growth rate through a higher real rate of return.

The utility function (5) consists of two additive and separable constant-elasticity-of-intertemporal substitution (CEIS) functions because the two arguments, C and R, are assumed to be two independent sources of consumption. This leads to the result that $U_{CR} = U_{RC} = 0$.[2]

3. Social planner's command economy

3.1. Pareto optimization

A benevolent social planner in an autarchic economy maximizes an agent's infinite lifetime utility defined by Eq. (5) (discounted at every point of time), subject to the resource constraints (3) and (4). The social planner not only controls the household's consumption of final output and environmental renewal, but also determines the proportion of aggregate man-made capital allocated in the two sectors. A dynamic optimization problem is therefore formed as follows:

$$\max_{C(t),R(t),u(t)} \int_0^{\infty} exp(-\rho t)\left[\frac{C(t)^{1-\theta}-1}{1-\theta} + \frac{R(t)^{1-\theta}-1}{1-\theta}\right]dt$$

[1] The adoption of positive and different rates of depreciation for man-made capital and natural capital will not affect the main results of the model.

[2] The setting of the utility function here is not original. Forster (1973) and Selden and Song (1995), for instance, assume that pollution (P) and consumption (C) are additively separable in an agent's utility function: $U_1(C) + U_2(P)$.

subject to

$$\dot{H}(t) = A[1-u(t)]H(t)-C(t) \qquad H(0) = H_0 \text{ given}$$
$$\dot{N}(t) = B[N(t)]^{\eta}[u(t)H(t)]^{1-\eta}-R(t) \qquad N(0) = N_0 \text{ given}$$

where a constant ρ represents the discount rate of time preference. After defining $\nu(t)$ and $\mu(t)$ as the shadow prices of man-made capital and natural capital, respectively, a current value Hamiltonian function $\mathbb{H}$ is then presented:

$$\mathbb{H} = \left[\frac{C(t)^{1-\theta}-1}{1-\theta} + \frac{R(t)^{1-\theta}-1}{1-\theta}\right] + \nu(t)\{A[1-u(t)]H(t)-C(t)\} + \mu(t)\left\{B[N(t)]^{\eta}[u(t)H(t)]^{1-\eta}-R(t)\right\}. \tag{6}$$

The first order conditions that solve the above Hamiltonian function are $\frac{\partial\mathbb{H}}{\partial C} = \frac{\partial\mathbb{H}}{\partial R} = \frac{\partial\mathbb{H}}{\partial u} = 0$, $\frac{\partial\mathbb{H}}{\partial H} = \rho\nu-\dot{\nu}$ and $\frac{\partial\mathbb{H}}{\partial N} = \rho\mu-\dot{\mu}$. They are sequentially listed as follows:

$$C^{-\theta} = \nu \tag{7}$$

$$R^{-\theta} = \mu \tag{8}$$

$$\nu A = \mu(1-\eta)B\left(\frac{uH}{N}\right)^{-\eta} \tag{9}$$

$$\nu A(1-u) + \mu(1-\eta)B\left(\frac{uH}{N}\right)^{-\eta}u = \rho\nu-\dot{\nu} \tag{10}$$

$$\mu\eta B\left(\frac{uH}{N}\right)^{1-\eta} = \rho\mu-\dot{\mu}, \tag{11}$$

and finally, the following transversality conditions ensure that long-run non-explosive solutions exist:

$$\lim_{t\to\infty} \exp(-\rho t)\nu(t)H(t) = \lim_{t\to\infty} \exp(-\rho t)\mu(t)N(t) = 0. \tag{12}$$

Eqs. (7) and (8) reveal that the marginal utility of the consumption of the final product and environmental renewal equal the shadow prices of man-made capital and natural capital, ν and μ, respectively. This means that along the optimal growth path, one additional unit of expenditure of certain capital has to be compensated by the marginal utility of the output generated by that capital. In addition, Eq. (9) implies that the marginal revenue products (MRPs) of man-made capital in the two sectors should be equal at all times.

By combining (7), (8) and (9), we can show that, at any point in time, the decision maker's optimal solution satisfies the condition in which the marginal rate of substitution between R and C (MRS_{RC}) equals their relative price, $\frac{\mu}{\nu}$. Appendix A defines P as the relative price of environmental renewal to the final output (the price of final output is normalized to unity) and proves that $P \equiv \frac{\mu}{\nu}$. Appendix A further shows that this relative price equals the marginal rate of technological substitution of man-made capital between the two sectors ($MRTS_{EY}$). The following equation provides a summary of this Pareto optimization concept:

$$MRS_{RC} = \left(\frac{C}{R}\right)^{\theta} = P \equiv \frac{\mu}{\nu} = \frac{A}{B(1-\eta)}\left(\frac{uH}{N}\right)^{\eta} = MRTS_{EY}. \tag{13}$$

3.2. Growth rates of variables

We now derive the growth rates of the two shadow prices ν and μ by combining (9), (10) and (11) as follows:[3]

$$-\gamma_{\nu} = A-\rho \tag{14}$$

$$-\gamma_{\mu} = \eta B\left(\frac{uH}{N}\right)^{1-\eta} - \rho, \tag{15}$$

where a lower discount rate, ρ, ensures that the above growth rates of shadow prices are always negative along the accumulation paths of man-made and natural capital.

Next, by taking logarithms for Eqs. (7) and (8) and differentiating them with respect to time, together with (14) and (15), the growth rates of final-good consumption and environmental consumption are:

$$\gamma_C = \frac{1}{\theta}(A-\rho) \tag{16}$$

$$\gamma_R = \frac{1}{\theta}\left[\eta B\left(\frac{uH}{N}\right)^{1-\eta} - \rho\right]. \tag{17}$$

It is not surprising to have a constant growth rate of final-good consumption in that an *AK* model has inherently guaranteed perpetual growth in an economy as long as $\rho<A$. In contrast, the growth rate of environmental consumption depends on the interaction and dynamic paths among u, H and N.

The growth rates of ν and μ also give us the growth rate of the relative price of environmental renewal, P:

$$\gamma_P = \gamma_{\mu} - \gamma_{\nu} = A - \eta B\left(\frac{uH}{N}\right)^{1-\eta}. \tag{18}$$

Substituting Eq. (13) to the right-hand side of (18) further yields the following convergent price function:

$$\gamma_P = A - \eta B\left[\frac{A}{B(1-\eta)}\right]^{\frac{\eta-1}{\eta}} P(t)^{\frac{1-\eta}{\eta}}. \tag{19}$$

The above equation satisfies the stability condition that $\frac{\partial\gamma_P}{\partial P} < 0$ such that there exists a non-explosive and unique steady-state equilibrium, $P^* = \left[\frac{A}{B(1-\eta)}\right]\left(\frac{A}{\eta B}\right)^{\frac{\eta}{1-\eta}}$, at which $\gamma_P = 0$.

The existence of P^* further shows that, based on Eq. (13), MRS_{RC} and $MRTS_{EY}$ are both time-invariant in the steady state. In Section 4, constant steady-state ratios of $\frac{N}{H}$, $\frac{C}{H}$ and $\frac{R}{N}$ will be derived, so the steady-state growth rates of C, R, H and N are identical and constant as follows:[4]

$$\gamma_C = \gamma_R = \gamma_N = \gamma_H = \gamma^{**} = \frac{1}{\theta}(A-\rho). \tag{20}$$

Therefore, this model guarantees that a command economy will experience sustainable growth in the steady state under social planner's optimization.

Most existing environmental growth models are able to derive a constant economic growth rate (Huang and Cai (1994) and Bovenberg and Smulders (1995), for example), but the analysis of short-run transitional paths of variables towards steady state was sometimes too complicated to be discussed. With simplified settings in our

[3] The growth rate in any variable x is defined as $\gamma_x = \frac{\dot{x}}{x}$.

[4] In fact, both final output Y and the environmental renewal E also share the same growth rate γ^* in the steady state.

model, the next section sketches several phase diagrams to show that the discussion of the off-steady-state dynamics of variables in this system is possible.

4. Transitional dynamics towards steady state

4.1. Steady-state equilibrium and phase diagrams of the system

The very first Eqs. (3) and (4) are now re-written in the form of the off-steady-state growth rates of man-made capital and environmental capital, respectively as follows:

$$\gamma_H = A(1-u)-\chi \quad (21)$$

$$\gamma_N = Z-\phi \quad (22)$$

where $\chi = \frac{C}{H}$ and $\phi = \frac{R}{N}$ represent the ratio of control variables (C and R) to state variables (H and N) in each sector. $Z = B\left(\frac{uH}{N}\right)^{1-\eta}$ stands for the average environmental renewal, $\frac{E}{N}$, and represents a transitional combination of the two components, uH and N. Z increases when uH grows relatively faster than N. Therefore, observing the magnitude of Z is a way of identifying an economy's current condition. For example, a small value of Z indicates that an economy has a clean environmental sector with abundant natural capital so little man-made capital is needed in this sector.

The logarithmic transformation of Eq. (13) is differentiated with respect to time as:

$$\gamma_P = \eta(\gamma_u + \gamma_H - \gamma_N). \quad (23)$$

Substituting (21) and (22) into an equation that equalizes (18) and (23) yields the growth rate of the share of man-made capital u:

$$\gamma_u = \chi - A(1-u) - \phi + \frac{A}{\eta}. \quad (24)$$

Let $\omega = \frac{N}{H}$ represent the natural–man-made capital ratio. From Eqs. (16), (17), (21), and (22), the off-steady-state growth rates of χ, ϕ, and ω can be derived respectively as:

$$\gamma_\chi = \gamma_C - \gamma_H = \frac{1}{\theta}(A-\rho) - A(1-u) + \chi \quad (25)$$

$$\gamma_\phi = \gamma_R - \gamma_N = \frac{1}{\theta}(\eta Z-\rho) - Z + \phi \quad (26)$$

$$\gamma_\omega = \gamma_N - \gamma_H = \chi - A(1-u) - \phi + Z. \quad (27)$$

Along the balanced growth path, the steady-state equilibrium (with a star on the superscript of the variable) of u, Z, χ, ϕ and ω can be solved by setting Eqs. (24) to (27) to zero, together with $\frac{\chi^*}{\omega^*} = \phi^*(P^*)^{\frac{1}{\theta}}$ implied by Eq. (13), as follows:

$$u^* = \frac{A - \frac{1}{\theta}(A-\rho)}{A+\Omega} \quad (28)$$

$$Z^* = \frac{A}{\eta} \quad (29)$$

$$\chi^* = \Omega\left[\frac{A-\frac{1}{\theta}(A-\rho)}{A+\Omega}\right] \quad (30)$$

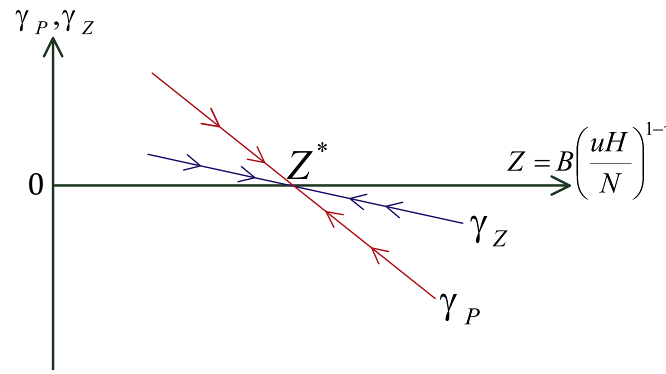

Fig. 1. Transitional dynamics of the relative price P and the average product of environmental renewal Z.

$$\phi^* = A\left(\frac{\rho}{A\theta} + \frac{1}{\eta} - \frac{1}{\theta}\right) \quad (31)$$

$$\omega^* = \left[\frac{A-\frac{1}{\theta}(A-\rho)}{A+\Omega}\right]\left(\frac{A}{\eta B}\right)^{\frac{-1}{1-\eta}}; \text{ where } \Omega = \phi^*(P^*)^{\frac{1}{\theta}}\left(\frac{A}{\eta B}\right)^{\frac{-\eta}{1-\eta}}. \quad (32)$$

A phase diagram depicts the paths of the above variables that converge to their respective steady-state equilibrium. For example, we can combine the growth rates of P and Z from Eqs. (24), (27) and (29) into the following deviation form:

$$\gamma_P = \eta(\gamma_u - \gamma_\omega) = -\eta(Z - Z^*) \quad (33)$$

$$\gamma_Z = (1-\eta)(\gamma_u - \gamma_\omega) = -(1-\eta)(Z - Z^*), \quad (34)$$

and their phase diagram is sketched in Fig. 1.

Fig. 1 shows that, when the natural environment in an economy is at a pure stage, the value of Z is far lower than its steady-state equilibrium. Under this developing stage, an economy's focus is to accumulate man-made capital to stimulate final goods production and consumption. In the meantime, however, the degradation of environmental quality is a trade-off. The (relative) price of environmental renewal gradually increases with a declining growth rate at this stage. The explanatory intuition is that an agent's marginal utility of environmental consumption rises due to the depletion of the environment.

On the contrary, in an over-developed economy with a large amount of man-made capital stock and low level of natural capital, the average product of environmental renewal exceeds its stationary equilibrium and will be driven back towards Z^*. At this stage, environmental renewal becomes a profitable good with a soaring price level that comes from high marginal utility of environmental consumption. Therefore, people pursue a cleaner environment by shifting economic resources from the accumulation of man-made capital back to natural capital and consequently,[5] Z and P will gradually fall back to their equilibrium points.

Given that the share of natural capital is greater than man-made capital in renewing the environment ($\eta > 1-\eta$), the dynamic path of P is therefore steeper than that of Z in Fig. 1. This suggests that P has a higher speed of convergence than Z when the average product of the environment deviates from its equilibrium.

The model construction in this system can form a more complicated phase diagram for the four variables: u, Z, ϕ and χ all together. By substituting Eqs. (28) to (31) into (24)–(26), we can transform the growth rates of u, χ and ϕ to the following deviation form:

$$\gamma_u = A(u-u^*) - (\phi-\phi^*) + (\chi-\chi^*) \quad (35)$$

[5] This proposition can be confirmed by observing the transition of u in later Fig. 2.

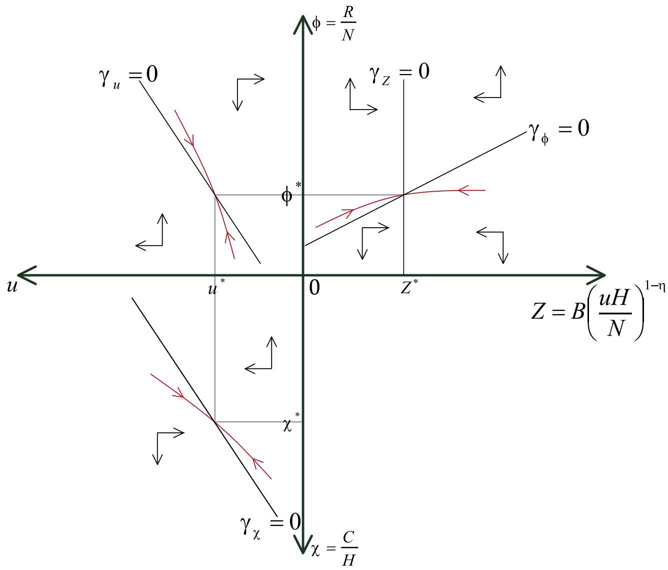

Fig. 2. Transitional dynamics of u, Z, ϕ and χ.

$$\gamma_\chi = A(u - u^*) + (\chi - \chi^*) \tag{36}$$

$$\gamma_\phi = \left(\frac{\eta - \theta}{\theta}\right)(Z - Z^*) + (\phi - \phi^*). \tag{37}$$

Combining the above three growth rates with Eq. (34) and assuming $\theta > \eta$ yield the phase diagram in Fig. 2. It shows the transitional dynamics of Z, u, χ, and ϕ with mutual interaction.

There are two assumptions behind the interpretation of Fig. 2. The first one is that the inverse of the intertemporal elasticity of substitution, θ, must be greater than the share of natural capital η to ensure that a positive and unique steady-state equilibrium exists in this system.[6] Second, since this figure shows a saddle equilibrium in the $Z - \phi$ space, the initial values of Z, ϕ and u have to be on their stable arms to prevent them from explosive dynamic paths.

With the two assumptions, if the starting values of $Z(0)$, $\phi(0)$, and $u(0)$ are below their steady-state levels, they will all converge upwards to their respective equilibrium. This again suggests that, when an economy starts with abundant natural capital, its main focus is to accumulate man-made capital to boost the production of final goods and services. Although C and H both grow steadily over time, χ is declining because of the faster accumulation of man-made capital. Meanwhile, increasing environmental consumption and the exploitation of natural resources both push up the level of $\phi = \frac{R}{N}$. However, a benevolent social planner devotes increasing level of uH to the environmental sector to balance the development in both sectors.

On the contrary, when Z, ϕ, and u are above their steady-state levels, an economy is over-developed with high stock of man-made capital from which high level of uH is allocated on rescuing the environment. In this over-developed economy, the decision maker's focuses are to recover the natural environment and to reduce the level of environmental consumption. In the meantime, the proportion of uH flows back to the production sector through the reduction in u. This implies that the improvement of the environment is not entirely at the expense of economic development in an optimized economy.

[6] For example, ϕ^* in Eq. (31) may be negative if $\eta > \theta$.

4.2. Phase diagrams of individual variables

In addition to Z, the natural–man-made capital ratio, $\omega = \frac{N}{H}$, is also a key indicator that represents the current status of an economy. For example, higher initial level of $\omega(0)$ indicates that the amount of man-made capital is far less than natural capital (a virgin economy). On the contrary, an over-developed economy with high rate of man-made capital accumulation and deteriorated environmental quality bears low level of $\omega(0)$.

The transitional dynamics in Fig. 2 can further be decomposed according to the evolution of individual variables, C, R, N, H, and u, against ω. Taking the two types of consumption in this model for the first instances, final-good consumption itself exhibits a constant growth rate as shown in Eq. (16) while the growth rate of environmental consumption (17) can be re-written as $\gamma_R = \frac{1}{\theta}\left[\eta B\left(\frac{u}{\omega}\right)^{1-\eta} - \rho\right]$ and converges gradually to the steady-state growth rate $\gamma^* = \gamma_C = \frac{1}{\theta}(A - \rho)$. Therefore, Fig. 3 depicts a comparison of the short-run evolution between C and R in two different economic stages.

Fig. 3 is consistent with our previous discussion that people sacrifice environmental quality and accumulate man-made capital in the under-developed stage. Compared to the consumption of final goods and services that has a constant growth rate over time, the growth rate of environmental consumption increases steadily from a negative value towards the positive equilibrium when an economy starts to develop. An opposite case occurs in the over-developed stage where the growth rate of environmental consumption is too high and declines towards γ^* over time.

A similar pattern is observed in the growth rates of natural capital and man-made capital. Their growth-rate deviation forms can be derived from Eqs. (17) and (37), (16) and (36), respectively as:

$$\gamma_N = \gamma_R - \gamma_\phi = \gamma^* + (Z - Z^*) - (\phi - \phi^*) \tag{38}$$

$$\gamma_H = \gamma_C - \gamma_\chi = \gamma^* - A(u - u^*) - (\chi - \chi^*). \tag{39}$$

Fig. 4 points out two opposite dynamic paths between natural and man-made capital. In an under-developed economy, natural capital accumulation exhibits a negative but increasing growth rate, while man-made capital accumulation has a high growth rate but it decreases over time. The mathematical explanation behind this was consistently proposed in the dynamic paths of the whole system in Fig. 2. The evolution of these two types of capital in an over-developed economy is an opposite story as has been discussed in Fig. 2.

Finally, Eq. (35) and the discussion in Fig. 2 creates the dynamic path of the proportional variable, u, shown in Fig. 5. In an under-developed economy, the growth rate of u is positive as the social planner diverts gradually accumulated man-made capital to the environmental sector to ease the depletion of the environment. In contrast, a negative growth rate of u is observed when an over-developed economy needs to recover the environment. In such an over-developed economy, the stock of man-made capital will be shifted back to the production sector to maintain appropriate levels of final output and consumption.

5. Technological progress in two sectors — comparative statics

How the short-run transitions and long-run equilibrium of variables can be affected by the exogenous rate of technological progress is always of policy makers' interest. This problem is analyzed by performing comparative statics of the main variables, Z, ϕ, and u, with respect to the changes in the two technology parameters A and B in the two sectors.

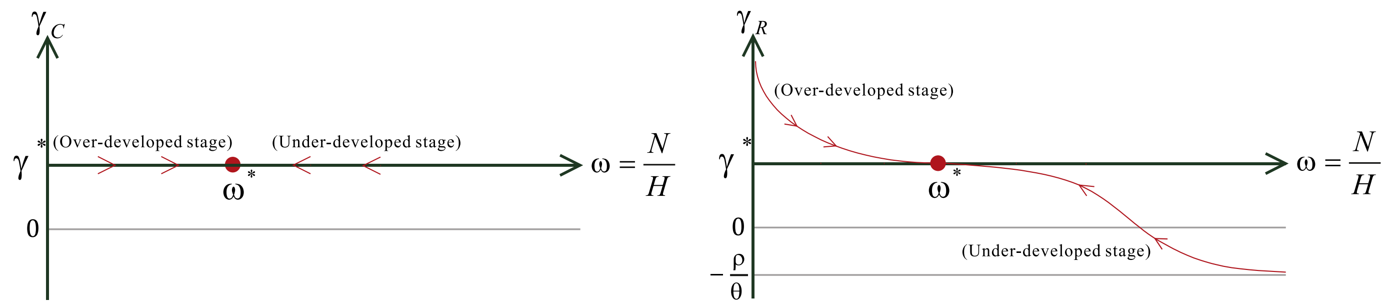

Fig. 3. Short-run transitions of C and R in the process of two stages.

We first consider the impact of technological change in the final-good sector when the productivity parameter A increases permanently. From Eqs. (29), (31) and (28), the following partial derivatives show:

$$\frac{\partial Z^*}{\partial A} > 0, \ \frac{\partial \phi^*}{\partial A} > 0, \text{ and } \frac{\partial u^*}{\partial A} \gtreqless 0. \tag{40}$$

Since Z^* is positively affected by A, the growth rates of Z and ϕ are therefore shifted respectively rightwards and upwards as shown in Fig. 6.

In Fig. 6, the improvement of the technology in the final-good sector results in a permanent increase of the steady-state positions of Z^* and ϕ^*. The intuition is that the improvement in the technology of producing final goods and services benefits the accumulation of man-made capital through increasing marginal product. Therefore, the transitional dynamics of Z and ϕ jumps up to the new path towards new steady-state levels in which the stock of natural capital is relatively lower than environmental consumption and environmental expenditure. In addition, Eq. (20) shows that the steady-state growth rate γ^* rises accordingly with the increase of A. The overall effect is summarized by the following proposition:

Proposition 1. *In a command economy, the improvement of technology in the production sector benefits the economic growth rate and results in a lower steady-state natural–man-made capital ratio if u^* is non-decreasing. Moreover, the steady-state relative price P^* goes to a higher equilibrium level, as $\frac{\partial P^*}{\partial A} > 0$ consistently shows.*

The uncertain effect on the steady-state share of man-made capital spent on the environmental sector, u^*, results from two offsetting forces. On the one hand, technological improvement in the production sector increases the marginal product of man-made capital. Hence, it is more profitable to divert man-made capital back to its sector. On the other hand, the growth rate of the average product of environmental renewal γ_Z is shifting positively because of the introduction of new technology in the production sector. This may attract higher spending of man-made capital from the output sector to renewing the environment. As a result, the impact on u^* and γ_u depends on the magnitudes of these two competing forces.

Now we consider another productivity parameter that represents the technology level in the environmental sector, B. Taking the same partial differentiation for the following variables yields:

$$\frac{\partial Z^*}{\partial B} = 0, \frac{\partial \phi^*}{\partial B} = 0, \text{ and } \frac{\partial u^*}{\partial B} > 0. \tag{41}$$

The above partial derivatives suggest that the technological progress in the environmental sector shifts γ_u rightwards with a lower level of u^* but leaves the position of γ_Z and γ_ϕ unchanged. These effects are shown in Fig. 7.

The technological improvement in the environmental sector neither changes the steady-state values of Z^* and ϕ^* nor the long-run growth rate of the economy. This is because the engine of economic growth is mainly driven by the accumulation of man-made capital, which is independent from the renewal structure in the environmental sector. However, higher productivity level in environmental renewal leads to the application of more resources to this sector so that the steady-state share of man-made capital spent on the environmental sector increases with higher B. In addition, the productivity improvement in the environmental sector also reduces the steady-state level of relative price P^*, as can be shown by $\frac{\partial P^*}{\partial B} < 0$. In fact, given that P^* drops, we can see from Eq. (13) that the level of environmental consumption increases relative to that of traditional consumption. We also know that the ratio $\frac{R}{N}$ does not change, meaning that higher environmental consumption is accompanied by higher level of environmental capital accumulated after better technology is introduced in this sector. We then summarize the above discussions with the following proposition:

Proposition 2. *Technology progress in the environmental sector does not change the growth rate of final output, but it increases the expenditure of man-made capital spent on this sector, and cheapens the steady-state value of environmental renewal. This in turn decreases the ratio of traditional consumption to environmental consumption.*

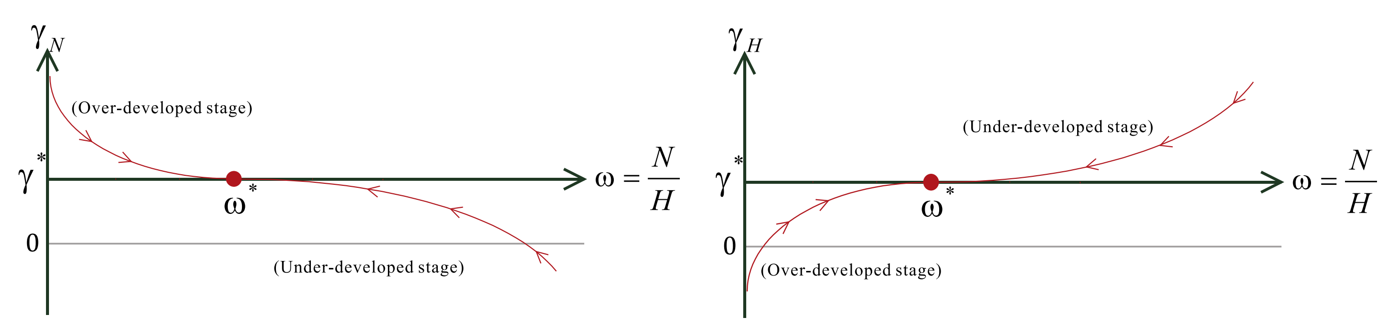

Fig. 4. Short-run transitions of N and H in the process of two stages.

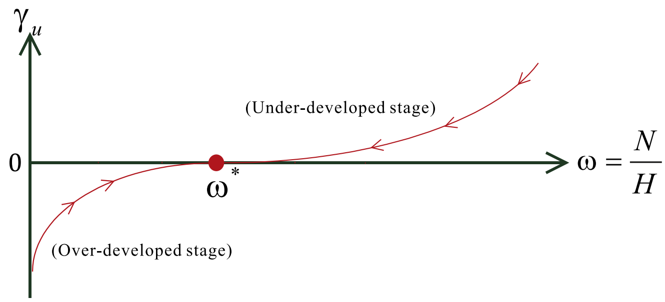

Fig. 5. Short-run transition of u in the process of two stages.

6. Conclusion and future empirical issues

This paper constructs a two-sector environmental growth model in a closed economy that allows an agent to distribute resources between the production sector and the environmental sector. In this system, a decision maker cares not only about the consumption of material goods but also about the consumption of natural environment. After solving the social planner's optimization problem, this model shows that the optimum guarantees a positive and identical steady-state growth rate for all endogenous variables except for the proportion of man-made capital in the environmental sector. Therefore, the conflict between economic growth and the preservation of the natural environment does not exist in the steady state under the social planner's optimization.

The value-added of this paper also includes the short-run transitional dynamics of key endogenous variables. In an economy's developing stage, a high level of the natural–man-made capital ratio is observed so the priority is to accumulate man-made capital for fostering economic development. In the meantime, though, this economy consumes an increasing level of natural environment such that natural capital exhibits a low or even negative rate of accumulation. The sacrifice of the environment in the process of economic development gradually increases people's willingness to protect their surroundings. In order to compensate for the deteriorated environmental quality, people devote an increasing proportion of man-made capital to renewing the environment. Based on this balanced mechanism, the growth in final-good production and environmental upgrading will both be achieved. Conversely, when this command economy starts with a high level of man-made capital and a scarce amount of natural capital stock, all the variables discussed above will converge oppositely to restore the whole system towards the eventual balanced growth path.

The improvement of technology in the production sector not only benefits the marginal product of man-made capital in its own sector but also boosts the long-run growth rate of major variables, including those in the environmental sector. However, the progress in environmental technology only increases the steady-state share of man-made capital spent on this sector, but has no influence on the long-run economic growth rate.

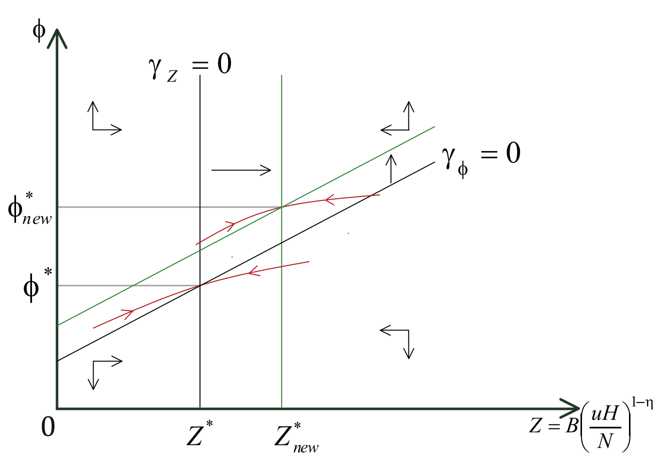

Fig. 6. The effect of permanent technological progress in the production sector.

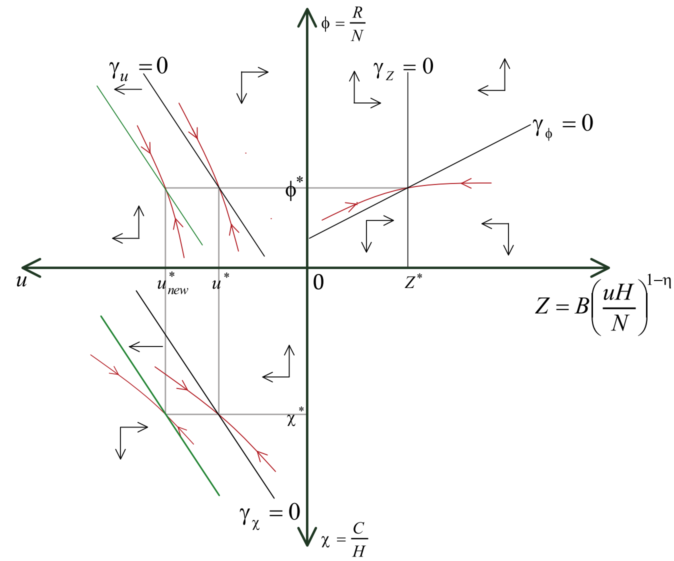

Fig. 7. The effect of permanent technological progress in the environmental sector. Note: from equation (30), there is also uncertain effect on X^* after B increases. For simplicity, X^* is assumed to be unchanged in this diagram and only the curve $\gamma_x =$ shifts leftwards.

The theoretical model in this paper also provides a link to an unproven empirical hypothesis called the environmental Kuznets curve (EKC). It suggests that the degree of environmental deterioration rises in the early stages of economic development, and only after an economy reaches a certain wealth level will environmental quality be recovered. Such an inverted U-shaped curve of environmental degradation against a general index, income per capita, has been empirically investigated since the early 1990s (Grossman and Krueger, 1991, for instance).

From Fig. 4, our model can similarly depict an EKC-shaped path of environmental degradation through the transformation of the

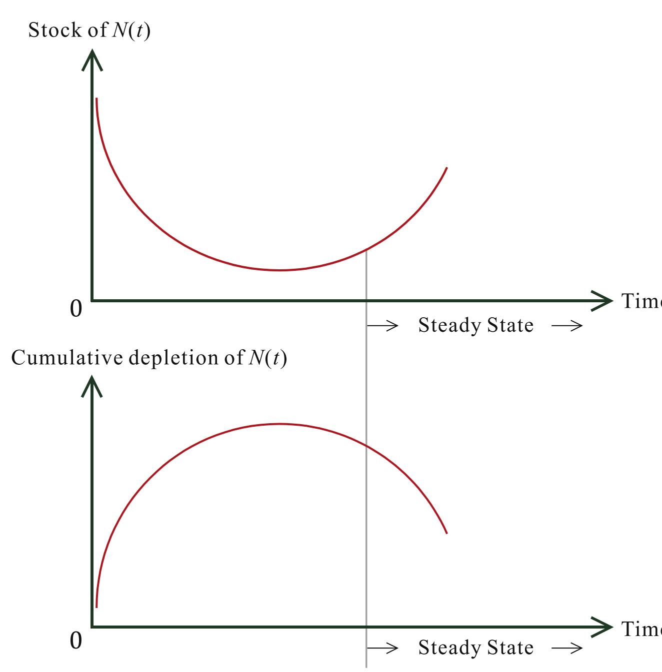

Fig. 8. Linking the theoretical model to the environmental Kuznets curve hypothesis.

evolution of N. This bell-shaped cumulative depletion of N is shown in Fig. 8.

It has already been concluded that people's material wealth (final-good consumption) steadily increases across time from developing to developed stage. Therefore, if the depletion of natural capital shown in Fig. 8 follows a bell-shaped trend against time, it should exhibit a similar EKC pattern against the time-dependent growing income per capita.

The theory proposed in this paper holds an optimistic point of view towards the coexistence of economic growth and environmental protection in the long run. As long as an economy is becoming wealthier, the deterioration of the environment will be eased. Therefore, in a social planning economy, the EKC phenomenon seems to exist as the economy is moving towards its steady state.

Appendix A. Profit-maximization in relation to capital's shadow prices of two sectors

Define P as the relative price of environmental renewal to the final output (the price of final output is normalized to unity), and assume that there is a representative producer using man-made capital with the rental cost R_H in producing the two goods, the profit-maximization in both sector can therefore be constructed as follows:

$$\max_{[1-u(t)]H(t)} \pi_Y = A[1-u(t)]H(t) - R_H[1-u(t)]H(t) \tag{A.1}$$

$$\max_{u(t)H(t)} \pi_E = P \cdot B[N(t)]^{\eta}[u(t)H(t)]^{1-\eta} - R_H[u(t)H(t)], \tag{A.2}$$

where π_Y and π_E represent two profit functions in the output and environmental sector, respectively. The producer decides profit-maximizing $(1-u)$ and u for producing final goods and environmental renewal, respectively. The first order condition for Eqs. (A.1) and (A.2) gives the following equation with the linkage of R_H:

$$A = R_H = P \cdot (1-\eta) B \left(\frac{uH}{N}\right)^{-\eta}. \tag{A.3}$$

After reorganizing Eq. (A.3), we have:

$$P = \frac{A}{B(1-\eta)} \left(\frac{uH}{N}\right)^{\eta}. \tag{A.4}$$

Comparing (A.4) to the Eq. (13) in Section 3, we can prove that the relative price P is equivalent to the ratio of shadow prices of natural capital to man-made capital, that is $P = \frac{\mu}{\nu}$. Therefore, the shadow prices of both types of capital not only represent their own value but also provide a way of calculating the price of the environmental renewal given the price index in the output sector.

References

Anderson, K.P., 1972. Optimal growth when the stock of resources is finite and depletable. Journal of Economic Theory 4 (2), 256–267.

Aznar-Márquez, J., Ruiz-Tamarit, J.R., 2005. Renewable natural resources and endogenous growth. Macroeconomic Dynamics 9, 170–197.

Bovenberg, A.L., Smulders, S., 1995. Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. Journal of Public Economics 57, 369–391.

Bovenberg, A.L., Smulders, S., 1996. Transitional impacts of environmental policy in an endogenous growth model. International Economic Review 37 (4), 861–893.

Cass, D., 1965. Optimum growth in an aggregative model of capital accumulation. The Review of Economic Studies 32, 233–240.

Forster, B.A., 1973. Optimal capital accumulation in a polluted environment. Southern Economic Journal 39, 544–547.

Grossman, G.M., Krueger, A.B., 1991. Environmental impacts of a North American Free Trade Agreement. NBER Working Paper 3914. National Bureau of Economic Research (NBER), Cambridge.

Huang, C.-H., Cai, D., 1994. Constant-returns endogenous growth with pollution control. Environmental and Resource Economics 4, 383–400.

Keeler, E., Spence, M., Zeckhauser, R., 1972. The optimal control of pollution. Journal of Economic Theory 4, 19–34.

Koopmans, T.C., 1965. On the Concept of Optimal Economic Growth. The Economic Approach to Development Planning, North-Holland, Amsterdam.

Lucas, R.E., 1988. On the mechanics of economic development. Journal of Monetary Economics 22, 3–42.

Rebelo, S., 1991. Long-run policy analysis and long-run growth. Journal of Political Economy 99 (3), 500–521.

Selden, T.M., Song, D., 1995. Neoclassical growth, the J curve for abatement, and the inverted U curve for pollution. Journal of Environmental Economics and Management 29 (2), 162–168.

Shieh, J.-Y., Lai, C.-C., Chen, J.-H., 2001. A comment on Huang and Cai's constant-returns endogenous growth with pollution control. Environmental and Resource Economics 20, 165–172.

Solow, R.M., 1956. A contribution to the theory of economic growth. Quarterly Journal of Economics 70, 65–94.

Swan, T., 1956. Economic growth and capital accumulation. The Economic Record 32, 344–361.

Tahvonen, O., Kuuluvainen, J., 1991. Optimal growth with renewable resources and pollution. European Economic Review 35, 650–661.