Causality in quantiles and dynamic stock return–volume relations

Chia-Chang Chuang [a], Chung-Ming Kuan [b,*], Hsin-Yi Lin [c]

[a] Department of International Business, National Taipei College of Business, Taiwan
[b] Department of Finance, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan
[c] Department of Economics, National Chengchi University, Taiwan

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ABSTRACT

This paper investigates the causal relations between stock return and volume based on quantile regressions. We first define Granger non-causality in all quantiles and propose testing non-causality by a sup-Wald test. Such a test is consistent against any deviation from non-causality in distribution, as opposed to the existing tests that check only non-causality in certain moment. This test is readily extended to test non-causality in different quantile ranges. In the empirical studies of three major stock market indices, we find that the causal effects of volume on return are usually heterogeneous across quantiles and those of return on volume are more stable. In particular, the quantile causal effects of volume on return exhibit a spectrum of (symmetric) *V*-shape relations so that the dispersion of return distribution increases with lagged volume. This is an alternative evidence that volume has a positive effect on return volatility. Moreover, the inclusion of the squares of lagged returns in the model may weaken the quantile causal effects of volume on return but does not affect the causality per se.

1. Introduction

The relationship between financial asset return and trading volume, henceforth the return–volume relation, is important for understanding operational efficiency and information dynamics in asset markets. Models related to this topic include, e.g., the sequential information arrival model (Copeland, 1976; Jennings et al., 1981; Jennings and Barry, 1983) and mixture of distributions model (Clark, 1973; Epps and Epps, 1976; Tauchen and Pitts, 1983). There are also equilibrium models that emphasize the information content of volume, e.g., Harris and Raviv (1993); Blume et al. (1994), Wang (1994) and Suominen (2001). For instance, Blume et al., 1994) stress that volume carries information that is not contained in price statistics and hence is useful for interpreting the price (return) behavior. On the empirical side, there have been numerous studies on contemporaneous return–volume relation since Granger and Morgenstern (1963) and Ying (1966); see Gallant et al. (1992) and also Karpoff (1987) for a review. Yet, as far as prediction and risk management are concerned, the dynamic (causal) relation between return and volume is more informative.

Causal relations between variables are typically examined by testing Granger non-causality. While Granger non-causality is defined in terms of conditional distribution, it is more common to test non-causality in conditional mean based on a linear model (Granger, 1969, 1980). Granger et al. (1986) and Cheung and Ng (1996) consider testing non-causality in conditional variance, whereas Hiemstra and Jones (1994) derive a test for nonlinear causal relations. These tests have been widely used in the literature (e.g., Fujihara and Mougoué, 1997; Silvapulle and Choi, 1999; Chen et al., 2001; Ciner, 2002; Lee and Rui, 2002). A serious limitation of this approach is that non-causality in mean (or in variance) need not carry over to other distribution characteristics or different parts of the distribution. Diks and Panchenko (2005) also give examples that the test of Hiemstra and Jones (1994) may not test Granger non-causality. These motivate us to consider characterizing and testing causality differently.

This paper investigates causal relations from the perspective of conditional quantiles. We first define Granger non-causality in a given quantile range and non-causality in all quantiles. The quantile causal effects are then estimated by means of quantile regressions (Koenker and Bassett, 1978; Koenker, 2005). The hypothesis of non-causality in all quantiles is tested by the sup-Wald test of Koenker and Machado (1999). This test checks significance of the entire parameter process in quantile regression models and hence is consistent against any deviation from non-causality in distribution, as opposed to the conventional tests of non-causality in a moment and the tests of Lee and Yang (2006) and Hong et al. (forthcoming). The test of Koenker and Machado (1999) is easily

* Corresponding author. Tel.: +886 2 3366 9541; fax: +886 2 2366 0764.
  E-mail addresses: jjjuang@webmail.ntcb.edu.tw (C.-C. Chuang), ckuan@ntu.edu.tw (C.-M. Kuan), linh@nccu.edu.tw (H.-Y. Lin).

extended to evaluate non-causality in different quantile ranges and enables us to identify the quantile range for which causality is relevant. Our approach thus provides a detailed description of the causal relations between return and volume.

In the empirical study we examine the causal relations between return and (log) volume in three stock market indices: New York Stock Exchange (NYSE), Standard & Poor 500 (S&P 500), and Financial Times-Stock Exchange 100 (FTSE 100). Despite that the conventional test may suggest no causality in mean, there are strong evidences of causality in quantiles in these indices. For NYSE and S&P 500, we find two-way Granger causality in quantiles between return and volumes; for FTSE 100, only volume Granger causes return in quantiles. In particular, the causal effects of volume on return are heterogeneous across quantiles, in the sense that they possess opposite signs at lower and upper quantiles and are stronger at more extreme quantiles. On the other hand, the causal effects of return on volume, if exist, are mainly negative and remain stable across quantiles.

With log volume on the vertical axis and return on the horizontal axis, the quantile causal effects of volume on return exhibit a spectrum of symmetric *V*-shape relations for NYSE and S&P 500.While many existing results (e.g., Karpoff, 1987) find a simple *V*-shape relation based on a least-squares regression of absolute return on volume, our *V*-shape results are very different. First, what we find are dynamic rather than contemporaneous relations. Second, these relations hold across quantiles rather than at the mean only. Moreover, the identified *V* spectrum suggests that distribution dispersion increases with lagged volume. This constitutes an alternative evidence that volume has a positive effect on return volatility and is compatible with the empirical finding based on conditional variance models (e.g., Lamoureux and Lastrapes, 1990; Gallant et al., 1992).

It is interesting to note that the quantile causal relations we find are quite robust to different sample periods and different model specifications. Indeed, the inclusion of the squares of lagged returns in the model may weaken the quantile causal effects of volume on return but does not affect the causality per se. Thus, lagged volumes carry information that is not contained in lagged returns and their squares, as argued by Blume et al. (1994). Our results also confirm that non-causality in mean bears no implication on non-causality in distribution (quantiles). A conventional test may find no causality in mean because the positive and negative quantile causal effects cancel out each other in least-squares estimation, as demonstrated in our study. It is therefore vulnerable to draw a conclusion on causality solely based on a test of non-causality in mean.

This paper is organized as follows. We introduce the notion of Granger (non-)causality in quantiles in Section 2 and discuss the sup-Wald test of non-causality in quantiles in Section 3. The empirical results of different causal models are presented in Section 4. Section 5 concludes the paper.

2. Causality in mean and quantiles

Following Granger (1969, 1980), we say that the random variable *x* does not Granger cause the random variable *y* if

$$F_{y_t}(\eta|(\mathscr{Y}, \mathscr{X})_{t-1}) = F_{y_t}(\eta|\mathscr{Y}_{t-1}), \quad \forall \eta \in \mathbb{R}, \tag{1}$$

holds almost surely (a.s.), where $F_{y_t}(\cdot|\mathscr{F})$ is the conditional distribution of y_t, and $(\mathscr{Y}, \mathscr{X})_{t-1}$ is the information set generated by y_i and x_i up to time $t-1$. That is, Granger non-causality requires that the past information of *x* does not alter the conditional distribution of y_t. The variable *x* is said to Granger cause *y* when (1) fails to hold. In what follows, Granger non-causality defined by (1) will be referred to as Granger non-causality in distribution.

As estimating and testing conditional distributions are practically cumbersome, it is more common to test a necessary condition of (1), namely,

$$\mathbb{E}[y_t|(\mathscr{Y}, \mathscr{X})_{t-1}] = \mathbb{E}[y_t|\mathscr{Y}_{t-1}], \quad a.s. \tag{2}$$

where $\mathbb{E}(y_t|\mathscr{F})$ is the mean of $F_{y_t}(\cdot|\mathscr{F})$. We say that *x* does not Granger cause *y* in mean if (2) holds; otherwise, *x* Granger causes *y* in mean. Similarly, we may define non-causality in variance (Granger et al., 1986; Cheung and Ng, 1996) and non-causality in other moments. Hong et al. (forthcoming) consider "non-causality in risk," a special case of (1) in which η is the negative of a VaR (Value at Risk). Note that these notions of non-causality are necessary for, but not equivalent to, Granger non-causality in distribution.

The hypothesis (2) is usually tested by evaluating a linear model of $\mathbb{E}[y_t|(\mathscr{Y}, \mathscr{X})_{t-1}]$:

$$\alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=1}^{q} \beta_j x_{t-j},$$

which depends on the past information of $y_{t-1}, \ldots, y_{t-p}$ and $x_{t-1}, \ldots, x_{t-q}$. Testing (2) now amounts to testing the null hypothesis that $\beta_j = 0, j = 1, \ldots, q$, in the postulated model; that is, whether any lagged *x* has a significant impact on the conditional mean of y_t. [1] Rejecting this null hypothesis suggests that *x* Granger causes *y*. Yet, failing to reject the null is compatible with non-causality in mean but says nothing about causality in other moments or other distribution characteristics.

Given that a distribution is completely determined by its quantiles, Granger non-causality in distribution can also be expressed in terms of conditional quantiles. Letting $Q_{y_t}(\tau|\mathscr{F})$ denote the τth quantile of $F_{y_t}(\cdot|\mathscr{F})$, (1) is equivalent to

$$Q_{y_t}(\tau|(\mathscr{Y}, \mathscr{X})_{t-1}) = Q_{y_t}(\tau|\mathscr{Y}_{t-1}), \quad \forall \tau \in (0, 1), \quad a.s. \tag{3}$$

We say that *x* does not Granger cause *y* in all quantiles if (3) holds. We may also define Granger non-causality in the quantile range $[a, b] \subset (0, 1)$ as

$$Q_{y_t}(\tau|(\mathscr{Y}, \mathscr{X})_{t-1}) = Q_{y_t}(\tau|\mathscr{Y}_{t-1}), \quad \forall \tau \in [a, b], \quad a.s. \tag{4}$$

Note that Lee and Yang (2006) considered only non-causality in a particular quantile, i.e., the equality in (3) holds for a given τ.

3. Testing non-causality in quantiles

This paper proposes to verify causal relations by testing (3), rather than testing non-causality in a moment (mean or variance) or non-causality in a given quantile. To this end, we postulate a model for $Q_{y_t}(\tau|(\mathscr{Y}, \mathscr{X})_{t-1})$ and estimate this model by the quantile regression method of Koenker and Bassett (1978); see Koenker (2005) for a comprehensive study of quantile regression.

Letting $\mathbf{y}_{t-1,p} = [y_{t-1}, \ldots, y_{t-p}]'$, $\mathbf{x}_{t-1,q} = [x_{t-1}, \ldots, x_{t-q}]'$, and $\mathbf{z}_{t-1} = [1, \mathbf{y}'_{t-1,p}, \mathbf{x}'_{t-1,q}]'$, we assume that the following model is correctly specified for the τth conditional quantile function:

$$Q_{y_t}(\tau|\mathbf{z}_{t-1}) = a(\tau) + \mathbf{y}'_{t-1,p}\boldsymbol{\alpha}(\tau) + \mathbf{x}'_{t-1,q}\boldsymbol{\beta}(\tau) = \mathbf{z}'_{t-1}\boldsymbol{\theta}(\tau),$$

where $\boldsymbol{\theta}(\tau) = [a(\tau), \boldsymbol{\alpha}(\tau)', \boldsymbol{\beta}(\tau)']'$ is the k-dimensional parameter vector with $k = 1 + p + q$. Note that the τth conditional quantile of the error $e_{t\tau} = y_t - \mathbf{z}'_{t-1}\boldsymbol{\theta}(\tau)$ is zero, a consequence of correct model specification. For a given τ, the parameter vector $\boldsymbol{\theta}(\tau)$ is estimated by minimizing asymmetrically weighted absolute deviations:

$$\min_{\boldsymbol{\theta}} \sum_{t=1}^{T} (\tau - \mathbf{1}_{\{y_t < \mathbf{z}'_{t-1}\boldsymbol{\theta}\}}) |y_t - \mathbf{z}'_{t-1}\boldsymbol{\theta}|,$$

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[1] Clearly, this approach would be valid provided that the postulated linear model is correctly specified for the conditional mean function.

where $\mathbf{1}_A$ is the indicator function of the event A. The solution to this problem, denoted as $\hat{\boldsymbol{\theta}}_T(\tau)$, can be computed using a linear programming algorithm.

In what follows, let $\xrightarrow{D}$ denote convergence in distribution, $\Rightarrow$ weak convergence (of associated probability measures), and $\|\cdot\|$ the Euclidean norm. Under suitable regularity conditions, $\hat{\boldsymbol{\theta}}_T(\tau)$ is consistent and asymptotically normally distributed such that

$$\sqrt{T}\left[\hat{\boldsymbol{\theta}}_T(\tau) - \boldsymbol{\theta}(\tau)\right] \xrightarrow{D} [\tau(1-\tau)]^{1/2}\boldsymbol{\Omega}(\tau)^{1/2}\mathcal{N}(\mathbf{0}, \mathbf{I}_k),$$

where $\boldsymbol{\Omega}(\tau) = \mathbf{D}(\tau)^{-1}\mathbf{M}_{zz}\mathbf{D}(\tau)^{-1}$, $\mathbf{M}_{zz} := \lim_{T\to\infty} T^{-1}\sum_{t=1}^T \mathbf{z}_{t-1}\mathbf{z}'_{t-1}$, and

$$\mathbf{D}(\tau) := \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^T f_{t-1}(F_{t-1}^{-1}(\tau))\mathbf{z}_{t-1}\mathbf{z}'_{t-1},$$

with F_{t-1} and f_{t-1} being, respectively, the distribution and density functions of y_t conditional on $\mathscr{Z}_{t-1}$, the information set generated by $\mathbf{z}_{t-1}, \mathbf{z}_{t-2}, \ldots$; see Koenker (2005) and Koenker and Xiao (2006).[2]

Given a linear model for conditional quantiles, testing (3) amounts to testing

$$H_0 : \boldsymbol{\beta}(\tau) = \mathbf{0}, \quad \forall \tau \in (0,1). \tag{5}$$

To this end, we must check significance of the entire parameter process $\boldsymbol{\beta}(\cdot)$. Letting $\boldsymbol{\Psi}$ be a $q \times k$ selection matrix such that $\boldsymbol{\Psi}\boldsymbol{\theta}(\tau) = \boldsymbol{\beta}(\tau)$, we have

$$\sqrt{T}[\hat{\boldsymbol{\beta}}_T(\tau) - \boldsymbol{\beta}(\tau)] = \sqrt{T}\boldsymbol{\Psi}[\hat{\boldsymbol{\theta}}_T(\tau) - \boldsymbol{\theta}(\tau)]$$
$$\xrightarrow{D} [\tau(1-\tau)]^{1/2}[\boldsymbol{\Psi}\boldsymbol{\Omega}(\tau)\boldsymbol{\Psi}']^{1/2}\mathcal{N}(\mathbf{0}, \mathbf{I}_q). \tag{6}$$

For a given τ, the Wald statistic of $\boldsymbol{\beta}(\tau) = \mathbf{0}$ is

$$\mathscr{W}_T(\tau) := T\hat{\boldsymbol{\beta}}_T(\tau)'(\boldsymbol{\Psi}\hat{\boldsymbol{\Omega}}(\tau)\boldsymbol{\Psi}')^{-1}\hat{\boldsymbol{\beta}}_T(\tau)/[\tau(1-\tau)],$$

where $\hat{\boldsymbol{\Omega}}(\tau)$ is a consistent estimator of $\boldsymbol{\Omega}(\tau)$. In the special case that $f_t(\cdot) = f(\cdot)$, the unconditional density of y_t, $\boldsymbol{\Omega}(\tau) = f(F^{-1}(\tau))^{-2}\mathbf{M}_{zz}^{-1}$, and the Wald statistic becomes

$$\mathscr{W}_T(\tau) = T\hat{\boldsymbol{\beta}}_T(\tau)'(\boldsymbol{\Psi}\widehat{\mathbf{M}}_{zz}^{-1}\boldsymbol{\Psi}')^{-1}\hat{\boldsymbol{\beta}}_T(\tau)\hat{f}^2/[\tau(1-\tau)],$$

where $\widehat{\mathbf{M}}_{zz} = T^{-1}\sum_{t=1}^T \mathbf{z}_{t-1}\mathbf{z}'_{t-1}$, and $\hat{f}$ is a consistent estimator of f. To test (5), Koenker and Machado (1999) suggest using a sup-Wald test, i.e., the supremum of $\mathscr{W}_T(\tau)$.

Note that $\mathbf{B}_q(\tau)$, a vector of q independent Brownian bridges, equals $[\tau(1-\tau)]^{1/2}\mathcal{N}(\mathbf{0}, \mathbf{I}_q)$ in distribution. Thus, (6) can be expressed as

$$\sqrt{T}[\hat{\boldsymbol{\beta}}_T(\tau) - \boldsymbol{\beta}(\tau)] \xrightarrow{D} [\boldsymbol{\Psi}\boldsymbol{\Omega}(\tau)\boldsymbol{\Psi}']^{1/2}\mathbf{B}_q(\tau). \tag{7}$$

Under suitable conditions, (7) holds uniformly on a closed interval $\mathscr{T} \subset (0,1)$, so that under the null hypothesis (5),

$$\mathscr{W}_T(\tau) \Rightarrow \left\|\frac{\mathbf{B}_q(\tau)}{\sqrt{\tau(1-\tau)}}\right\|^2, \quad \tau \in \mathscr{T},$$

where the weak limit is the sum of squares of q independent Bessel processes.[3] This immediately leads to the following result:

$$\sup_{\tau \in \mathscr{T}} \mathscr{W}_T(\tau) \xrightarrow{D} \sup_{\tau \in \mathscr{T}} \left\|\frac{\mathbf{B}_q(\tau)}{\sqrt{\tau(1-\tau)}}\right\|^2. \tag{8}$$

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[2] Note that when $\mathbb{E}[\tau - \mathbf{1}_{\{e_{t\tau}<0\}}|\mathscr{Z}_{t-1}] = 0, \mathbf{z}_{t-1}[\tau - \mathbf{1}_{\{e_{t\tau}<0\}}]$ is a martingale difference sequence and hence obeys a central limit theorem:

$$T^{-1/2}\sum_{t=1}^T \mathbf{z}_{t-1}[\tau - \mathbf{1}_{\{e_{t\tau}<0\}}] \xrightarrow{D} [\tau(1-\tau)]^{1/2}\mathbf{M}_{zz}^{1/2}\mathcal{N}(\mathbf{0}, \mathbf{I}_k).$$

The asymptotic normality of $\hat{\boldsymbol{\theta}}_T(\tau)$ and the asymptotic covariance matrix $\boldsymbol{\Omega}(\tau)$ readily follow from the Bahadur representation and this result. For some regularity conditions ensuring $\mathbb{E}[\tau - \mathbf{1}_{\{e_{t\tau}<0\}}|\mathscr{Z}_{t-1}] = 0$, see Koenker and Xiao (2006).

[3] Note that $\|\mathbf{B}_q(\tau)/\sqrt{\tau(1-\tau)}\|$ tends to infinity when $\tau \to 0$ or 1 (Andrews, 1993). Thus, $\mathscr{W}_T(\tau), \tau \in \mathscr{T}$, would not have a well defined limit unless $\mathscr{T}$ is a closed interval in $(0,1)$.

Table 1
The critical values of the sup-Wald test on $[0.05, 0.95]$.

	$q=1$	$q=2$	$q=3$
1%	13.01	16.30	19.21
5%	9.84	12.77	15.28
10%	8.19	11.05	13.49

Note: q is the dimension of the parameter vector being tested.

Table 2
Summary statistics for stock returns r_t and volume v_t.

	NYSE		S&P 500		FTSE 100	
	r_t	v_t	r_t	v_t	r_t	v_t
Mean	0.03	769.43	0.04	863.18	0.02	778.10
Standard deviation	0.89	541.27	1.02	761.37	1.02	662.77
Median	0.05	609.31	0.05	494.88	0.04	426.80
Skewness	−0.23	0.53	−0.09	0.61	−0.11	0.94
Kurtosis	4.15	−0.96	3.74	−1.07	3.11	−0.07
Minimum	−6.79	31.64	−7.25	2.08	−5.89	26.36
Maximum	5.18	2767.75	5.61	3345.21	5.90	4461.01

Note: Volumes here are traded share volumes times 10^{-6}.

In practice, we may set $\mathscr{T} = [\epsilon, 1-\epsilon]$ for some small ϵ in $(0, 0.5)$ and choose n points ($\epsilon = \tau_1 < \cdots < \tau_n = 1 - \epsilon$). The sup-Wald test for (5) is computed as

$$\text{sup} - \mathscr{W}_T = \sup_{i=1,\ldots,n} \mathscr{W}_T(\tau_i).$$

When n is large, the right-hand side of (8) with $\mathscr{T} = [\epsilon, 1-\epsilon]$ ought to be a good approximation to the null limit of sup-$\mathscr{W}_T$. See Koenker and Machado (1999) for some simulation results on the finite-sample performance of this test. Similarly, we may test the null:

$$H_0 : \boldsymbol{\beta}(\tau) = \mathbf{0}, \quad \forall \tau \in [a, b] \tag{9}$$

by the supremum of $\mathscr{W}_T(\tau_i)$ with $a = \tau_1 < \cdots < \tau_n = b$. It is clear that the limit in (8) carries over to $\mathscr{T} = [a, b]$. The results of the sup-Wald test on various $[a, b]$ may be used to identify the quantile range from which causality arises. For example, if the null hypothesis (5) is rejected but (9) is not rejected for some interval $[a, b]$, one may infer that causality mainly arises from the quantiles outside $[a, b]$.

Remark. The linear model considered here is convenient for model estimation and hypothesis testing. Yet, our approach to testing causality, the sup-Wald test in particular, would be valid provided that the linear model is correctly specified for conditional quantile functions.

To determine the critical values for the sup-Wald test, we note that, for $s = \tau/(1-\tau)$, the one-dimensional Bessel process $B(\tau)/\sqrt{\tau(1-\tau)}$ and the normalized, one-dimensional Brownian motion $W(s)/\sqrt{s}$ are equal in distribution. It follows that

$$\mathbb{P}\left\{\sup_{\tau \in [a,b]} \left\|\frac{\mathbf{B}_q(\tau)}{\sqrt{\tau(1-\tau)}}\right\|^2 < c\right\} = \mathbb{P}\left\{\sup_{s \in [1, s_2/s_1]} \left\|\frac{\mathbf{W}_q(s)}{\sqrt{s}}\right\|^2 < c\right\},$$

with $s_1 = a/(1-a), s_2 = b/(1-b)$, and $\mathbf{W}_q$ a vector of q independent Brownian motions. That is, the critical values c are determined by the sum of squared normalized Brownian motions. The critical values for some q and s_2/s_1 have been tabulated in DeLong (1981) and Andrews (1993); other critical values can be easily computed via simulations. The simulated critical values of the sup-Wald test (with $q = 1, 2, 3$) on $[0.05, 0.95]$ are summarized in Table 1.[4]

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[4] Our simulation approximates the standard Brownian motion using a Gaussian random walk with 3000 i.i.d. $\mathcal{N}(0,1)$ innovations; the number of replications is 20,000.
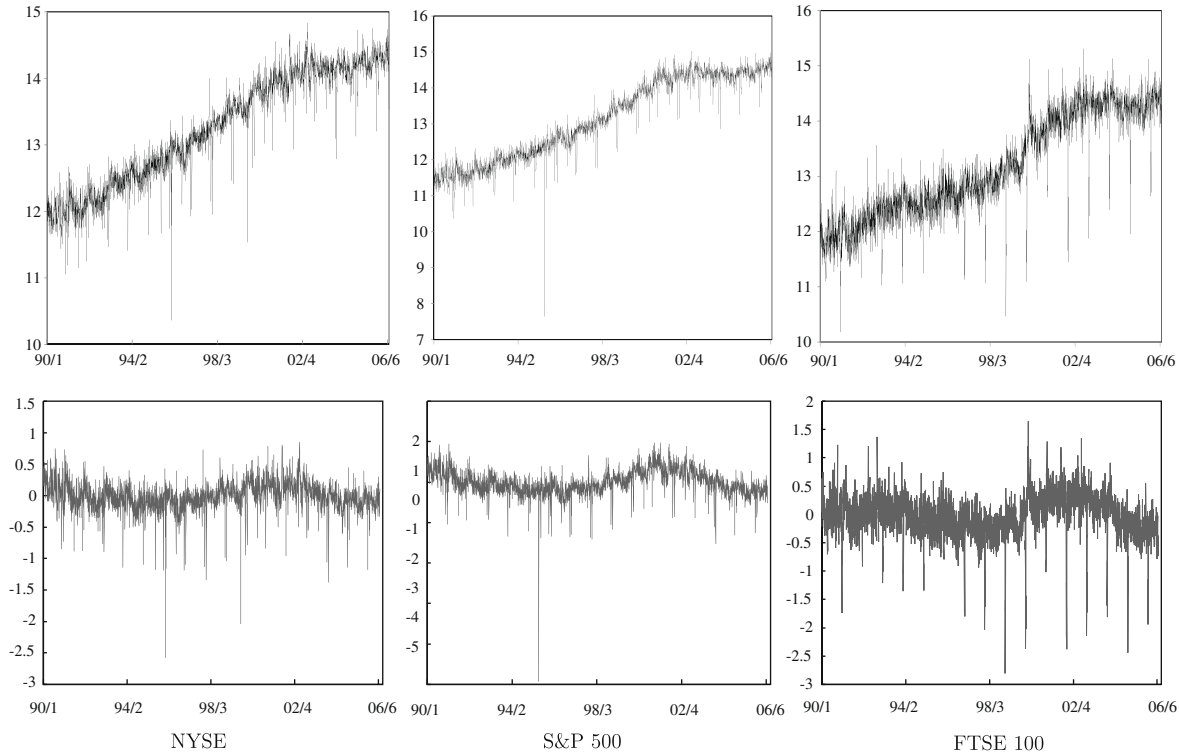
Fig. 1. The series of log volume (upper panel) and detrended residuals (lower panel).

4. Empirical study

Our empirical study of return–volume relations focuses on three stock market indices: NYSE, S&P 500 and FTSE 100. The daily data from the beginning of 1990 (Jan. 2 or Jan. 4) to June 30, 2006 are taken from *Datastream* database, and there are 4135, 4161 and 4166 observations for NYSE, S&P 500 and FTSE 100, respectively. As will be shown in Section 4.4, our results are quite robust to different sample periods.

Returns are calculated as $r_t = 100 \times (\ln(p_t) - \ln(p_{t-1}))$, where p_t is index at time t; volumes v_t are the traded share volumes of these indices. Their summary statistics are collected in Table 2. It can be seen that the mean and median returns are all close to zero and their standard deviations are close to one. Also, the return series behave similarly to what we usually observe in the literature: they fluctuate around their respective mean levels and exhibit volatility clustering and excess kurtosis. For each volume, the mean and median are quite different, and its kurtosis coefficient is small.

There are pronounced trending patterns in the volume series. Following Gallant et al. (1992), we consider log volume series and remove their trends by regressing $\ln v_t$ on a constant, t/T and $(t/T)^2$; see also Chen et al. (2001) and Lee and Rui (2002). To conserve space, we plot only the log volume series and their detrended residuals in Fig. 1. It can be seen that there is no trend in these residual series. Our subsequent analysis of return–volume relations is thus based on r_t and $\ln v_t$ while controlling the time trend effects.[5]

4.1. Causal effects of volume on return: model without r_{t-j}^2

We first consider the following model for return and estimate this model using the least-squares (LS) and quantile regression (QR) methods:

$$r_t = a(\tau) + b(\tau)\frac{t}{T} + c(\tau)\left(\frac{t}{T}\right)^2 + \sum_{j=1}^{q} \alpha_j(\tau) r_{t-j}$$
$$+ \sum_{j=1}^{q} \beta_j(\tau) \ln v_{t-j} + e_t, \tag{10}$$

where T is the sample size and $q \geqslant 1$; this model will be referred to as a lag-it q model. In the light of Fig. 1, we include t/T and $(t/T)^2$ as regressors in the model so as to control the trending effect in $\ln v_t$. We do not report the results of the model with detrended $\ln v_t$ (i.e., the residuals of regressing $\ln v_t$ on t/T and $(t/T)^2$ as regressors because, as far as causality is concerned, all regressors should be in the information set so that the model involves no future information.[6] Although we may specify different models for the conditional mean and quantile functions, we estimate the same model (10) in our study so that the LS and QR estimates can be compared directly.

We apply the sup-Wald test to determine an appropriate lag order q^*. If the null of $\beta_q(\tau) = 0$ for τ in $[0.05, 0.95]$ is not rejected for the lag-q model but the null of $\beta_{q-1}(\tau) = 0$ for τ in $[0.05, 0.95]$ is rejected for the lag-$(q-1)$ model, we infer that $\ln v_{t-q}$ does not Granger cause r_t in quantiles but $\ln v_{t-q+1}$ does. The desired lag order is then set as $q^* = q - 1$. For simplicity, we do not consider the model that includes r_{t-j} and $\ln v_{t-j}$ with different lag orders. For NYSE, the sup-Wald test of $\beta_4(\tau)$ in the lag-4 model is 11.813 and that of $\beta_3(\tau)$ in the lag-3 model is 18.261. The latter is significant at 1% level, but the former is not; see the critical values in Table 1 (under $q = 1$). For S&P 500, the sup-Wald test of $\beta_3(\tau)$ in the lag-3 model is 12.421 which is insignificant at 1% level, but that of $\beta_2(\tau)$ in the lag-2 model is 25.227 which is significant. For FTSE 100, the sup-Wald test of $\beta_3(\tau)$ in the lag-3 model is 7.7 which is insignificant even at 10% level, and that of $\beta_2(\tau)$ in the lag-2 model is 13.567 which is significant at 1% level. Thus, we set $q^* = 3$ for NYSE and $q^* = 2$ for S&P 500 and FTSE 100.[7] For each lag-q^* model (10), 91

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[6] Nonetheless, we find that the QR estimates of (10) are very close to those of the model with lagged r_t and lagged detrended $\ln v_t$ as regressors.

[7] At 5% level, we find $q^* = 5$ for NYSE, $q^* = 5$ for S&P 500, and $q^* = 2$ for FTSE 100. To ease our illustration, we choose 1% level and deal with simpler models.
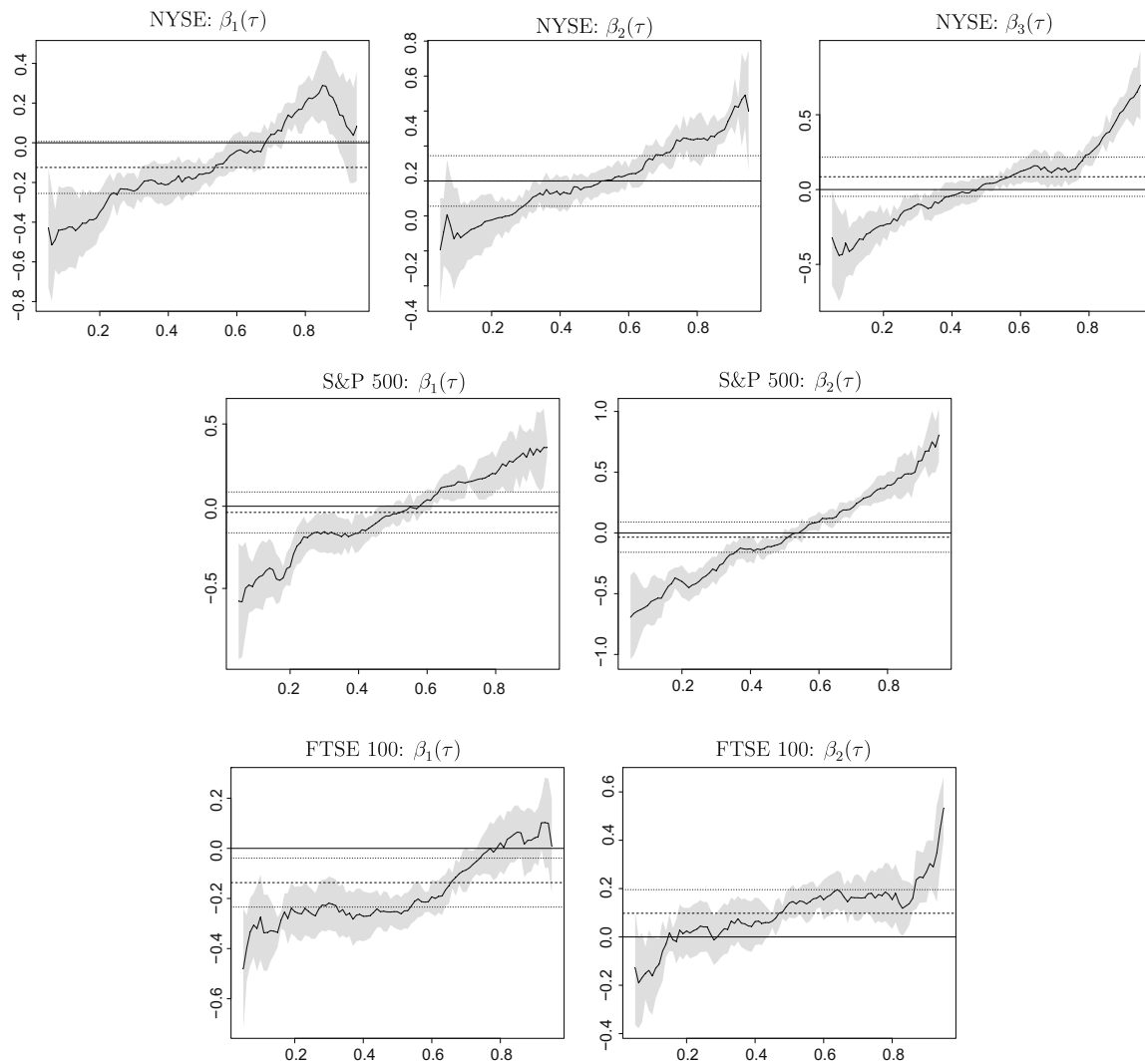
Fig. 2. QR and LS estimates of the causal effects of log volume on return: model without r_{t-j}^2.

quantile regressions (with $\tau = 0.05, 0.06, \ldots, 0.95$) are estimated using the R program (version 2.4.0) with the "quantreg" package (version 4.01) written by Koenker.[8]

In Fig. 2, we plot against τ the QR estimates of $\beta_j(\tau)$ (solid line) and their 95% confidence intervals (in shaded area), together with the LS estimate (dashed line) and its 95% confidence interval (dotted lines). It can be seen that, for NYSE and S&P 500, the LS estimates of β_j, the mean causal effects of log volumes, are all negative but insignificantly different from zero. This suggests no causality in mean in these two series. Yet, the QR estimates of $\beta_j(\tau)$ vary with quantiles and exhibit an interesting pattern. First, the QR estimates are negative at lower quantiles and positive at upper quantiles. Second, the magnitude of these estimates increases as τ moves toward 0 and 1. Third, these estimates are, in general, significant at tail quantiles.[9] Thus, lagged log volume exerts opposite and heterogeneous quantile causal effects on the two sides of the return distribution, and such effects are stronger at more extreme quantiles.

The estimation results for FTSE 100 are quite different. The LS estimate of β_1 is significantly negative at 5% level, but that of $\beta_2(\tau)$ is insignificant. This shows that there is causality in mean in FTSE 100. The QR estimates of $\beta_j(\tau)$ are also heterogeneous across τ. The QR estimates of $\beta_1(\tau)$ are significantly negative at lower quantiles but insignificant at upper quantiles, and the QR estimates of $\beta_2(\tau)$ are significantly positive at most upper quantiles.[10]

To be sure, we apply the sup-Wald test to check joint significance of all coefficients of lagged log volumes. The null hypothesis for NYSE is $\beta_1(\tau) = \beta_2(\tau) = \beta_3(\tau) = 0$ on [0.05, 0.95], and the null for S&P 500 and FTSE 100 is $\beta_1(\tau) = \beta_2(\tau) = 0$ on [0.05, 0.95]. As shown in Table 3, these statistics overwhelmingly reject the null of non-causality at 1% level, suggesting causality in quantiles in these indices. We also test $\beta_i(\tau) = 0$ on the ranges of quantiles at which the estimates of $\beta_i(\tau)$ are found insignificant individually. As shown in Table 3, none of these null hypotheses can be rejected at 5% level. Thus, we conclude that, for NYSE and S&P 500, the quantile causal effects are mainly due to the tail quantiles outside the interquartile range (except that of $\ln v_{t-1}$ for NYSE). Our results are in contrast with many existing findings of non-causality that

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[8] These programs are available from the CRAN website: http://cran.r-project.org/.

[9] For NYSE, we obtain insignificant QR estimates of $\beta_1(\tau)$ for τ in [0.53, 0.79] and [0.87, 0.95], insignificant estimates of $\beta_2(\tau)$ for τ in [0.30, 0.72], and insignificant estimates of $\beta_3(\tau)$ for τ in [0.26, 0.62] and [0.68, 0.76]. For S&P 500, there are insignificant QR estimates of $\beta_1(\tau)$ for τ in [0.24, 0.64] and insignificant estimates of $\beta_2(\tau)$ for τ in [0.34, 0.65].

[10] For FTSE 100, the quantiles range of insignificant QR estimates are [0.67, 0.95] for $\beta_1(\tau)$ and [0.08, 0.49] and [0.83, 0.85] for $\beta_2(\tau)$.

Table 3
The sup-Wald tests of non-causality in different quantile ranges.

Index	$\beta_i(\tau)=0, i=1,2,3$	$\beta_1(\tau)=0$		$\beta_2(\tau)=0$	$\beta_3(\tau)=0$	
NYSE	[0.05,0.95]	[0.53,0.79]	[0.87,0.95]	[0.3,0.72]	[0.26,0.62]	[0.68,0.76]
	79.12**	3.48	3.11	2.65	2.86	2.26
	$\beta_i(\tau)=0, i=1,2$	$\beta_1(\tau)=0$	$\beta_2(\tau)=0$			
S&P 500	[0.05,0.95]	[0.24,0.64]	[0.34,0.65]			
	134.27**	5.80	5.71			
FTSE 100	[0.05,0.95]	[0.67,0.95]	[0.08,0.49]	[0.83,0.85]		
	27.53**	2.18	2.98	2.78		

Note: Each interval in the square bracket is the quantile range on which the null hypothesis holds; the entry below each interval is the sup-Wald statistic. The critical values for the tests on [0.05,0.95] are in Table 1; the other critical values are obtained by simulations.
** Denotes significance at 1% level.

are based on a test for linear causality in mean (e.g., Kocagil and Shachmurove, 1998; Chen et al., 2001; Lee and Rui, 2002).

Following Buchinsky (1998), we test whether the pairwise causal effects at the τth and $(1-\tau)$th quantiles are symmetric about the median, i.e., $\beta_i(\tau)+\beta_i(1-\tau)=2\beta_i(0.5)$ with $i=1,2,3$ for NYSE and $i=1,2$ for both S&P 500 and FTSE 100. This amounts to checking whether

$$\hat{\delta}_{i,T}(\tau)=\hat{\beta}_{i,T}(\tau)+\hat{\beta}_{i,T}(1-\tau)-2\hat{\beta}_{i,T}(0.5)$$

is sufficiently close to zero. To this end, we conduct a $\chi^2(1)$ test based on the square of the normalized $\hat{\delta}_T(\tau)$ for the τ pairs: $(0.05,0.95),(0.1,0.9),\ldots,(0.45,0.55)$, where the standard error of $\hat{\delta}_T(\tau)$ is computed via design matrix bootstrap. We may also conduct a joint test to check if $\hat{\delta}_{i,T}(\tau), i=1,\ldots,k$, are close to zero. For NYSE, it is a $\chi^2(3)$ test; for S&P 500 and FTSE 100, it is a $\chi^2(2)$ test. The testing results of all indices are summarized in Table 4.

Table 4 shows that, for NYSE and S&P 500, the null of symmetric causal effects can not be rejected at 5% for all τ pairs we considered. This is so for both individual test and joint test. For FTSE 100, these effects are not symmetric for some middle τ pairs of $\beta_1(\tau)$. These symmetry results are somewhat different from those of Hutson et al. (2008). The symmetry of these quantile causal effects helps to explain why the conventional methods, such as correlation coefficient and LS estimation, usually yield an insignificant estimate of the causal effect of volume, as the positive and negative effects at corresponding upper and lower quantiles tend to cancel out each other in "averaging."

The estimation and testing results for NYSE and S&P 500 lead to a vivid pattern of quantile causal effects. By putting lagged log volume on the vertical axis and return on the horizontal axis, the quantile causal effects of log volume on return exhibit a spectrum of symmetric V-shape relations, in which the V's at more extreme quantiles have wider opening. Thus, an increase in lagged log vol-

ume results in a larger return in either sign, and such effect is stronger for returns with larger magnitude. This dynamic V-shape pattern complements the findings of Karpoff (1987); Gallant et al. (1992) and Blume et al. (1994). These V-shape relations also imply that the dispersion of return increases with lagged volume, so that return volatility depends positively on lagged volume, analogous to the results in the context of conditional variance, e.g., Lamoureux and Lastrapes (1990); Gallant et al. (1992); Moosa and Al-Loughani (1995); Kocagil and Shachmurove (1998); Chen et al. (2001) and Xu et al. (2006).

4.2. Causal effects of volume on return: model with r_{t-j}^2

From the preceding subsection we find that the dispersion (volatility) of return changes with lagged log volume. To see if the quantile causal effects of log volume are robust, we take squared return as a proxy for return volatility and consider an extension of (10) which includes lagged r_{t-j}^2 as additional regressors:

$$r_t = a(\tau)+b(\tau)\frac{t}{T}+c(\tau)\left(\frac{t}{T}\right)^2+\sum_{j=1}^{q}\alpha_j(\tau)r_{t-j}+\sum_{j=1}^{q}\beta_j(\tau)\ln v_{t-j}$$
$$+\sum_{j=1}^{q}\gamma_j(\tau)r_{t-j}^2+e_t. \tag{11}$$

This allows us to examine whether log volume still Granger causes return in the presence of r_{t-j}^2. The model (11) carries the flavor of an ARCH-in-mean model and is also able to capture some nonlinearity in lagged return.

We again apply the sup-Wald test to determine an appropriate lag order q^*. For NYSE, the sup-Wald test of $\beta_3(\tau)$ in the lag-3 model is 12.402 which is significant at 1% level and that of $\beta_2(\tau)$ in the lag-2 model is 21.158 which is significant. For S&P 500, the sup-Wald test of $\beta_3(\tau)$ in the lag-3 model is 11.362, and that of $\beta_2(\tau)$

Table 4
Testing symmetry of quantile causal effects: Models without r_{t-j}^2.

τ pair $(\tau,1-\tau)$	NYSE				S&P 500			FTSE 100		
	β_1	β_2	β_3	Joint	β_1	β_2	Joint	β_1	β_2	Joint
0.05	0.027	0.139	2.692	4.325	0.335	0.760	0.724	0.042	0.447	0.874
0.10	0.023	0.699	0.056	1.695	0.000	0.384	0.608	3.615	1.477	3.857
0.15	1.665	0.029	0.001	3.134	0.020	0.452	0.566	4.253*	1.119	4.597
0.20	2.160	0.338	0.658	3.967	0.329	0.612	0.629	8.339**	1.082	10.236**
0.25	2.931	0.598	1.432	5.259	0.349	0.013	0.740	7.150**	0.883	8.749**
0.30	1.484	0.495	0.234	3.120	0.790	0.009	1.487	6.224*	1.434	7.040*
0.35	0.929	1.162	0.002	4.301	0.120	0.601	1.571	1.815	0.106	2.567
0.40	1.025	0.008	0.006	1.463	0.234	0.867	0.871	0.697	1.311	1.280
0.45	0.327	0.135	0.526	0.873	0.062	0.095	0.100	1.296	3.508	3.408

Note: Each entry is a test statistic for the hypothesis that the quantile causal effects are symmetric about the median. The corresponding critical values are 6.63 and 3.84 for $\chi^2(1)$, 9.21 and 5.99 for $\chi^2(2)$, and 11.34 and 7.81 for $\chi^2(3)$.
* Denotes significance at 5%.
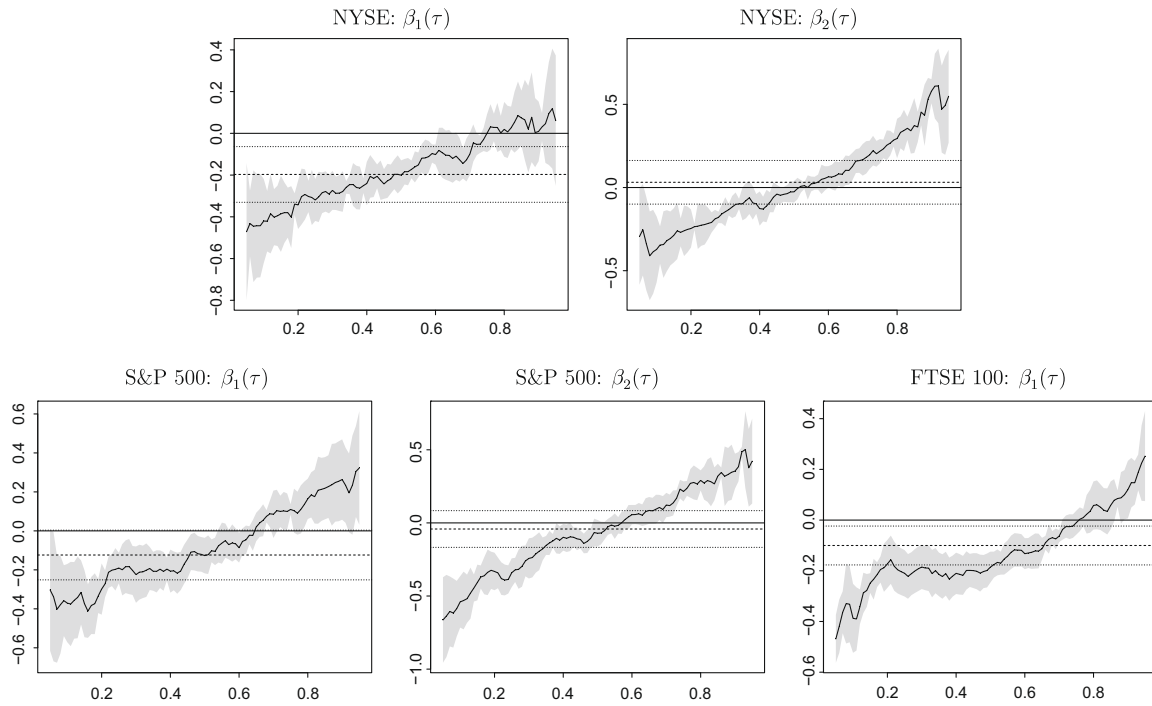** Denotes significance at 1%.

Fig. 3. QR and LS estimates of the causal effects of log volume on return: model with r_{t-j}^2.

in the lag-2 model is 20.554; the latter is significant at 1% level but the former is not. Therefore, we estimate (11) with $q^* = 2$ for both NYSE and S&P 500. For FTSE 100, $q^* = 1$ because the sup-Wald test of $\beta_2(\tau)$ in the lag-2 model is 5.27 which is insignificant at 10% level and that of $\beta_1(\tau)$ in the lag-1 model is 20.513 which is significant at 1% level. Thus, the desired lag order q^* may be affected when r_{t-j}^2 are included in the model. For each lag-q^* model, we also estimate 91 quantile regressions. The resulting QR and LS estimates and their 95% confidence intervals are plotted in Fig. 3.

For NYSE and S&P 500, we observe that the LS estimates of mean causal effect are insignificantly different from zero, except that the estimate of β_1 for NYSE is significantly negative at 5% level (but still insignificant at 1% level). Thus, one may still conclude that there is no causality in mean in NYSE and S&P 500. On the other hand, their quantile causal effects are similar to those in Fig. 2. For NYSE, the QR estimates of $\beta_1(\tau)$ at upper quantiles are mostly insignificant, and those of $\beta_2(\tau)$ are negative (positive) at lower (upper) quantiles and significant at tail quantiles. For S&P 500, the QR estimates for each $\beta_j(\tau)$ also have opposite signs at two sides of the return distribution and are significant at tail quantiles.[11] Moreover, we find that the magnitude of the QR estimates at tail quantiles are weaker than the corresponding estimates in Fig. 2. For FTSE 100, the LS estimate of β_1 is significantly negative, and the QR estimates are significantly negative at middle and lower quantiles and significantly positive at right tail quantiles (except for τ in [0.66, 0.89]). These are somewhat similar to the results of FTSE 100 in Fig. 2.

The sup-Wald test of non-causality again significantly rejects the null of $\beta_i(\tau) = 0$ on [0.05, 0.95] for these indices. The results of the symmetry test are collected in Table 5, which are similar to those in Table 4. For NYSE and S&P 500, the quantile causal effects are symmetric about the median for all τ pairs. For FTSE

100, the causal effects are symmetric, except for some pairs of middle quantiles. To summarize, the presence of r_{t-j}^2 in the model may reduce the strength of quantile causal effects of log volume but does not affect causality in quantiles per se. For S&P 500, these quantile causal effects exhibit a spectrum of "smaller", symmetric V-shape relations. This is also the pattern of the quantile causal effects of $\ln v_{t-2}$ on the return of NYSE.

We also examine the effects of r_{t-j}^2 on r_t i.e., the estimates of $\gamma_j(\tau)$ in (11). These estimates are plotted in Fig. 4. It is quite interesting to see that the heterogeneity of the estimated $\gamma_j(\tau)$ is somewhat similar to that of the estimated $\beta_j(\tau)$. In particular, the quantile causal effects increase with τ. The estimated $\gamma_2(\tau)$ for NYSE and S&P 500 and the estimated $\gamma_1(\tau)$ for FTSE 100 have opposite signs at the two side of the return distribution. Putting r_{t-j}^2 on the vertical axis and r_t on the horizontal axis, we would also obtain a spectrum of V-shape relations based on these estimates. This result, together with the quantile causal effects in Figs. 2 and 3, confirms that both $\ln v_{t-j}$ and r_{t-j}^2 are able to account for distribution dispersion (volatility) in a similar manner.

Table 5
Testing symmetry of quantile causal effects: Models with r_{t-j}^2.

τ pair $(\tau, 1-\tau)$	NYSE			S&P 500			FTSE 100
	β_1	β_2	joint	β_1	β_2	joint	β_1
0.05	0.005	1.732	2.003	1.256	0.105	1.409	0.629
0.10	0.003	2.520	2.885	0.612	0.061	0.748	0.660
0.15	0.259	1.093	2.576	0.395	0.123	1.299	3.390
0.20	0.426	0.811	2.422	0.751	0.489	2.921	8.367[**]
0.25	0.617	0.341	1.584	1.703	0.042	3.221	3.969[*]
0.30	0.063	0.801	1.519	1.357	0.031	2.408	2.820
0.35	0.217	0.554	1.322	0.513	0.581	2.569	0.349
0.40	0.953	0.044	0.962	0.202	1.374	1.544	0.157
0.45	0.017	0.481	0.696	0.745	0.000	1.274	0.270

Note: Each entry is a test statistic for the hypothesis that the quantile causal effects are symmetric about the median. The corresponding critical values are 6.63 and 3.84 for $\chi^2(1)$ and 9.21 and 5.99 for $\chi^2(2)$.
[*] Denotes significance at 5%.
[**] Denotes significance at 1%.

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[11] For NYSE, the QR estimates of $\beta_1(\tau)$ are insignificant for τ in [0.55, 0.95], and those of $\beta_2(\tau)$ are insignificant for τ in [0.31, 0.67]. For S&P 500, the QR estimates of $\beta_1(\tau)$ are insignificant at [0.22, 0.29] and [0.44, 0.79], and those of $\beta_2(\tau)$ are insignificant at [0.34, 0.72].
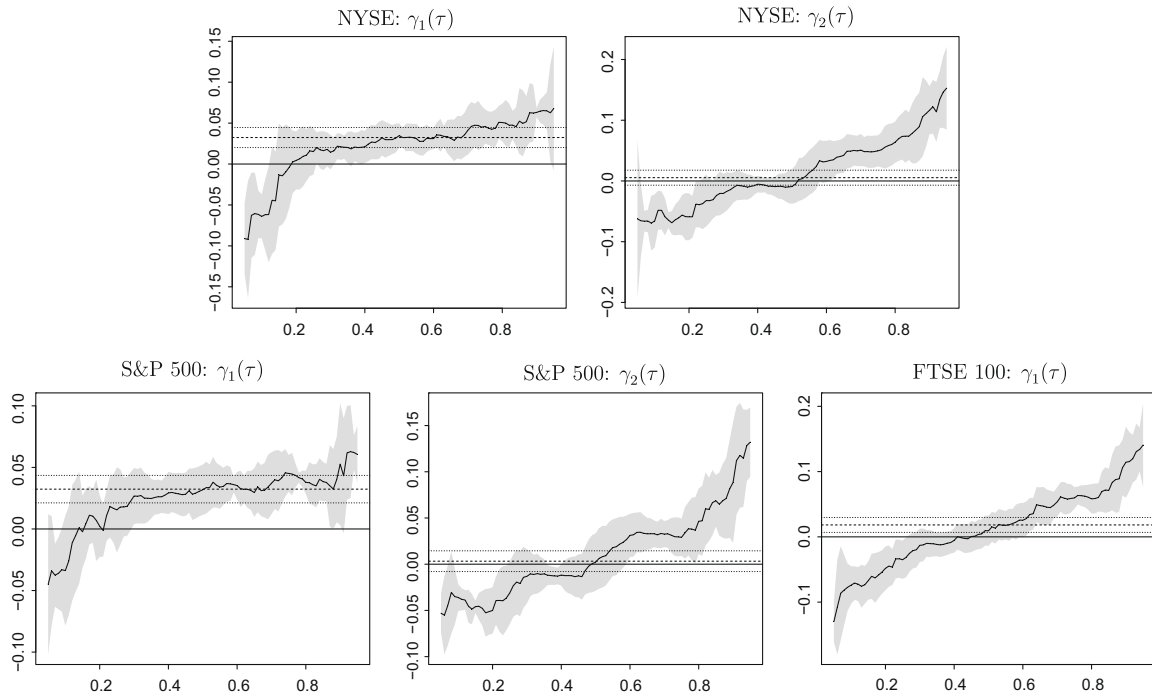
Fig. 4. QR and LS estimates of the causal effects of r_{t-j}^2 on r_t.

4.3. Causal effects of return on volume

To see if there is two-way causality between return and log volume, we now consider the following models for $\ln v_t$:

$$\ln v_t = a^\dagger(\tau) + b^\dagger(\tau)\frac{t}{T} + c^\dagger(\tau)\left(\frac{t}{T}\right)^2 + \sum_{j=1}^{q}\alpha_j^\dagger(\tau)\ln v_{t-j}$$
$$+ \sum_{j=1}^{q}\beta_j^\dagger(\tau)r_{t-j} + e_t;$$

$$\ln v_t = a^\dagger(\tau) + b^\dagger(\tau)\frac{t}{T} + c^\dagger(\tau)\left(\frac{t}{T}\right)^2 + \sum_{j=1}^{q}\alpha_j^\dagger(\tau)\ln v_{t-j}$$
$$+ \sum_{j=1}^{q}\beta_j^\dagger(\tau)r_{t-j} + \sum_{j=1}^{q}\gamma_j^\dagger(\tau)r_{t-j}^2 + e_t.$$

The first model is common in empirical studies; the second one extends the first by including r_{t-j}^2 as regressors and is compatible with the model (11).

As in the preceding subsections, we first determine an appropriate lag order q^* by the sup-Wald test. For NYSE, the sup-Wald test of $\beta_2^\dagger(\tau)$ in the lag-2 model without r_{t-j}^2 is 11.138 which is insignificant at 1% level and that of $\beta_1^\dagger(\tau)$ in the lag-1 model without r_{t-j}^2 is 16.298 which is significant. For S&P 500, the sup-Wald test of $\beta_3^\dagger(\tau)$ in the lag-3 model without r_{t-j}^2 is 10.107 and that of $\beta_2^\dagger(\tau)$ in the lag-2 model is 16.636. The latter is significant at 1% level and the former is not. Therefore, for models without r_{t-j}^2, we set $q^* = 1$ for NYSE and $q^* = 2$ for S&P 500. We also find that, for models with r_{t-j}^2, $q^* = 1$ for NYSE and S&P 500.[12] On the other hand, the lagged return does not Granger cause log volume in FTSE 100 because the sup-Wald test of $\beta_1^\dagger(\tau)$ in the lag-1 model without and with r_{t-j}^2 yields 6.161 and 7.767 which are insignificant even at 10% level.

We now focus on NYSE and S&P 500 and summarize the LS and QR estimates of $\beta_j^\dagger(\tau)$ in the models without and with r_{t-j}^2

in Figs. 5 and 6. We observe that the LS estimates in these models are all significantly negative. The QR estimates are also significantly negative at most quantiles and stay within the confidence interval of the corresponding LS estimate. Thus, the causal effects of return on log volume are relatively stable (homogeneous) across quantiles for NYSE and S&P 500. These results are consistent with the existing findings, such as Moosa and Al-Loughani (1995); Silvapulle and Choi (1999); Chen et al. (2001) and Lee and Rui (2002). We also note that the quantile causal effects of r_{t-1}^2 on log volume are all significantly positive.

To summarize, we find from these subsections that there are two-way quantile causal relations between return and log volume for NYSE and S&P 500 but only one-way causality in quantiles from log volume to return for FTSE 100. The quantile causal effects of a lagged log volume on return still exhibit symmetric V shapes in general, yet the causal effects of a lagged return on log volume are negative. Thus, lagged log volume carries important information that is not contained in the past returns and past squared returns. Similarly, lagged return also carries information that is not contained in the past volumes and past squared returns. This is in line with Blume et al. (1994).

4.4. Robustness check

As our sample extends a fairly long time (17 years), we check the robustness of our results by evaluating the causal relations between return and volume in different sub-samples. Specifically, we conduct the same causality analysis on two sample periods: January 1995–June 2006 and January 2000–June 2006. The choice of these sub-samples is arbitrary.

We briefly summarize our estimation results here; the detailed statistics and estimation results are available upon request. At 1% level, the sup-Wald test suggests $q^* = 1$ in the model (10) for each index in each sub-sample considered, except for S&P 500 in 1995–2006 the sup-Wald test of $\beta_1(\tau) = 0$ in the lag-1 model is 9.69 which is almost significant at 5% level. To ease our comparison, we estimate all models with $q^* = 1$. The resulting estimates of

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[12] For NYSE, the sup-Wald test of $\beta_2(\tau)$ in the lag-2 model is 6.808 and that of $\beta_1(\tau)$ in the lag-1 model is 13.512. For S&P 500, the sup-Wald test of $\beta_2(\tau)$ in the lag-2 model is 12.003 and that of $\beta_1(\tau)$ in the lag-1 model is 20.564.
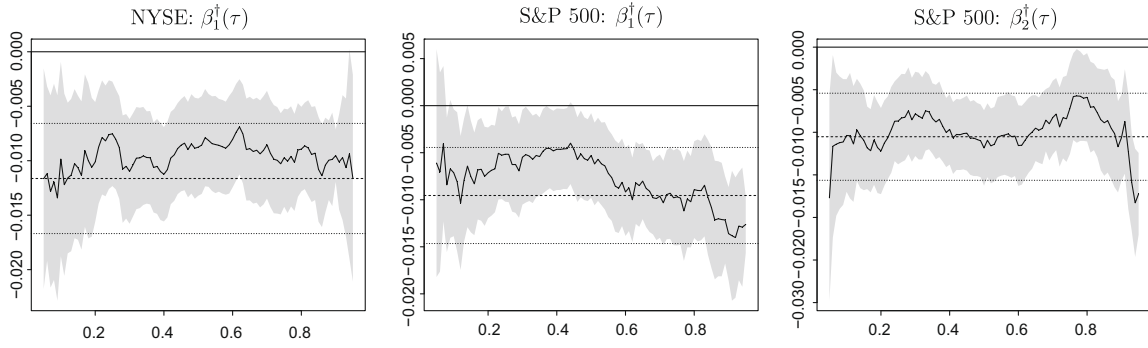
Fig. 5. QR and LS estimates of the causal effects of return on log volume: model without r_{t-j}^2.
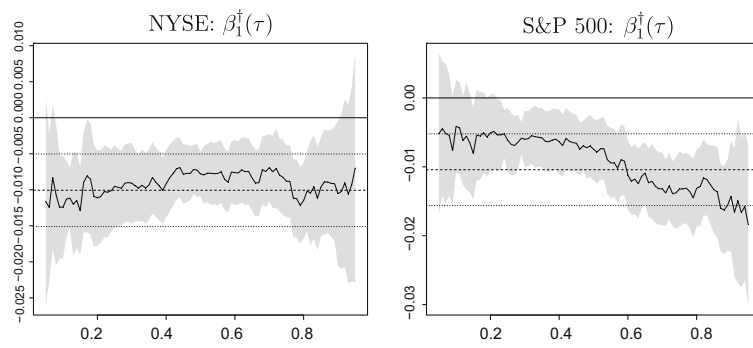

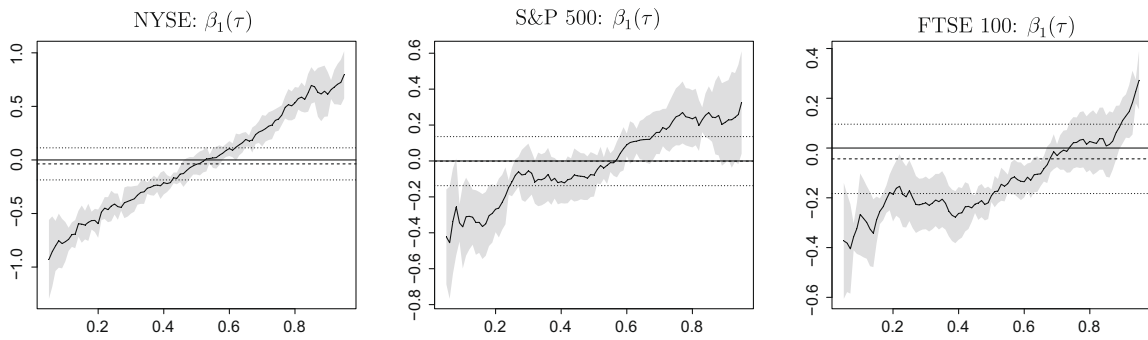**Fig. 6.** QR and LS estimates of the causal effects of return on log volume: model with r_{t-j}^2.

Fig. 7. QR and LS estimates of the causal effects of log volume on return: 1995–2006, model without r_{t-j}^2.
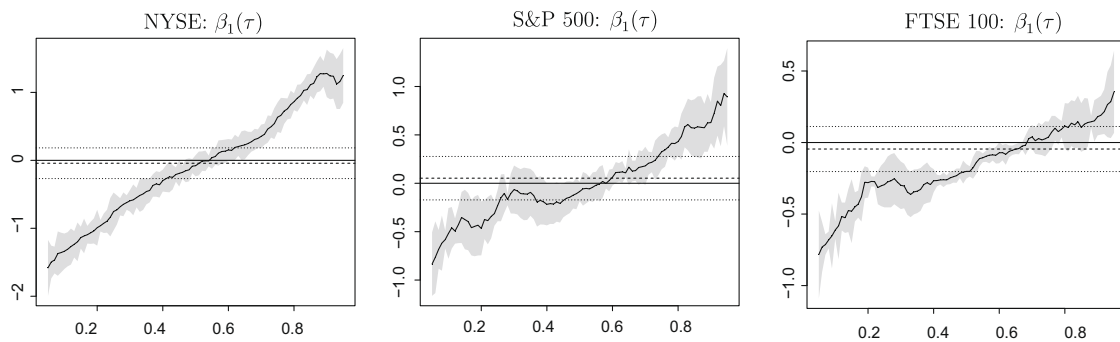

Fig. 8. QR and LS estimates of the causal effects of log volume on return: 2000–2006, model without r_{t-j}^2.

$\beta_1(\tau)$ based on the sample of 1995–2006 are plotted in Fig. 7, and those based on the sample of 2000–20006 are plotted in Fig. 8. It is readily seen that the LS estimates in these plots are all insignificant at 5% level while the quantile causal patterns of these estimates are qualitatively similar to those in Fig. 2. Indeed, we observe V-shape causal relations for NYSE and S&P 500 in these sub-samples. Estimating the model (11) with r_{t-j}^2 also yields similar results. To conserve space, the plots of these parameter estimates are not presented here.

5. Concluding remarks

In this paper we estimate quantile causal effects and test Granger non-causality in different quantile ranges based on the quantile regressions of return (log volume). We find that there are quantile causal relations between return and log volume. More importantly, our results indicate that the causal relations may be far more complicated than what can be described using least-squares regression. Indeed, the causal effects may be heterogeneous across quantiles and that the causal effects at tail quantiles may be much different from those at middle quantiles and at the mean. Thus, the conclusion on non-causality based solely on a conventional test on the mean relation may be misleading.

The empirical results of causality in quantiles, however, can not be explained by existing equilibrium models (e.g., Campbell et al., 1993). These models typically yield implications on the conditional mean but say little about the behaviors of conditional quantiles. Therefore, different models are needed to account for the quantile causal patterns found in this paper. It is also interesting to note that quantile causal relations provide detailed information about distribution dispersion and hence can complement conventional volatility measures, such as conditional variance. How to incorporate such information to improve on the evaluation of volatility and related assets (e.g., VIX option) is an interesting topic and currently being investigated.

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