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Social networks, social interaction and macroeconomic dynamics: How much could Ernst Ising help DSGE?

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ABSTRACT

In this paper, two different versions of the agent-based DSGE (dynamic stochastic general equilibrium) model are studied in comparison. The first version is the mesoscopic modeling of market sentiment using the Brock–Hommes adaptive belief system (ABS). The second version is the microscopic modeling of market sentiment by applying the Ising model to different social networks. The issue is to examine whether the distribution of market sentiment generated by the ABS machine can emerge from some kinds of local mimetic interactions, and hence the macroeconomic behavior of the two versions will be essentially the same. Our simulation results show that it is rather hard to have the equivalence of these two versions in the Kolmogorov–Smirnov sense. Hence, directly incorporating social networks and social interactions into microscopic modeling has its own values and may not be replaced or simplified by the mesoscopic counterpart.

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1. Motivation

The New Keynesian DSGE (dynamic stochastic and general equilibrium) models have been widely used by central banks for policy analysis; however, despite this dominant position in macroeconomics, the DSGE model has received intensive criticisms during and after financial crises (Colander et al., 2008; Colander, 2010; Solow, 2010; Velupillai, 2011). Partially motivated by these criticisms, some modified or extended versions are, therefore, proposed. In actual fact, one recent form of progress

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associate with the DSGE model is that it has been endowed with the three most criticized missing elements, namely, bounded rationality, heterogeneity and interactions (Orphanides and Williams, 2007; Branch and McGough, 2009; Milani, 2009; Chen and Kulthanavit, 2010). More specifically, one major implementation has been to introduce the adaptive belief system (Brock and Hommes, 1997, 1998) to the DSGE model and attempts to formulate the DSGE model in the agent-based fashion (Bask, 2007; De Grauwe, 2010a,b; Lengnick and Wohltmann, 2010; Assenza et al., 2011).

The adaptive belief system (ABS) is basically a stochastically discrete choice model. Unlike the normal discrete choice model which is to be applied to each individual, the ABS machine is used to describe the evolution of the mesoscopic structure of individuals. For example, in a two-type agent-based model (Chen et al., 2012), it is used to model the fractions of fundamentalists and chartists or the factions of optimists and pessimists.

However, in using the agent-based model, one can actually go down one level further and model the social interaction at the individual level with an explicitly embedded social network. If we do so, very naturally, we will encounter the consistency issue, i.e., whether the mesoscopic structure as described by the ABS machine can be generated bottom up. This inquiry is related to [Kampouridis et al. \(2011, in press\)](#), who use genetic programming and self-organizing maps to generate a mesoscopic structure of traders (fractions of different types of traders) bottom up, and is even more closely related to [Chen et al. \(2010\)](#), who use agent-based financial market simulation to show that the well-known *elasticity puzzle* is mainly a result of the micro-macro inconsistency.

In this paper, we study the possible effects of the social network and social learning on the *distribution of market sentiment* (optimism vs. pessimism), also called *fraction distribution* (Chen et al., 2012), within the context of a stylized New Keynesian DSGE model.¹ We want to examine whether the direct application of the stochastic choice model or the ABS machine at the mesoscopic level can emerge bottom up through the *mimetic effects* operated in some familiar classes of network topologies. In addition, since we are getting to each of the individuals and their interaction with neighbors, we need to first describe the interaction model among them. It is at this point that we introduce the famous Ising model, invented by the physicist Ernst Ising in his PhD thesis in 1924, as our model for interacting agents. In sum, we study an “agent-based version” of the DSGE model bottom up by allowing for individuals’ mimetic effects, through Ising’s model, under different network topologies. We then statistically examine how well these bottom-up settings can fit the mesoscopic structure generated by the direct high-level modeling using the discrete choice model or the ABS machine.

A highlight of our results is briefly given here. In general, we fail to find a good bottom-up match to the ABS-generated fraction distribution. This indicates two possibilities: first, we have not sampled enough of the network topologies, or, second, the use of the ABS machine directly at the aggregate level may not be a good approximation to any macroeconomic model which takes the social network into explicit account. While the first possibility is certainly an issue left for further study, it is the second possibility which motivates a theoretical inquiry into the appropriateness of the use of the discrete choice model at the mesoscopic level when individual interactions are completely governed by the embedded social networks.

The rest of the paper is organized as follows. [Section 2](#) gives an introduction to the DSGE models, including the *standard* New Keynesian version as well as the *agent-based extension* of it proposed by De Grauwe, when the homogeneous rational expectations in the former are replaced by the heterogeneous boundedly rational expectations using Brock–Hommes’ adaptive belief systems. [Section 3](#) introduces the social networks employed in this study, mainly the fully-connected network, regular network, small-world network and scale-free network. [Section 4](#) gives a brief review of the Ising model and its incorporation into our agent-based version of the DSGE model. Simulations of both agent-based versions of the DSGE models and the comparative analysis of the simulation results are given in [Section 5](#), followed by the concluding remarks in [Section 6](#).

¹ The fraction distribution gives the distribution of the portfolio of market beliefs. In the statistical-physical macroeconomic model (Aoki and Yoshikawa, 2006), it is a pivotal variable because various types of aggregate behavior will be fundamentally determined by and fed back to it. When there are only two types of agents, say, optimists and pessimists, the fraction distribution is simply the distribution of the share of the optimists in the market.

2. Agent-based DSGE model with the ABS machine

This section serves as a brief introduction to the DSGE model. There are two DSGE models to be considered here, the standard one (Section 2.1) and its agent-based extension (Section 2.2). There is another agent-based version augmented with social networks, which will be presented in a later section (Section 4). The introduction here is very brief, and readers who are not familiar with the basic DSGE model are referred to DeJong and Dave (2007) for more details.

2.1. The standard DSGE model

Given the purpose specified above, we only consider a simple standard New Keynesian version of the DSGE model. This is done by presenting a standard aggregate demand and aggregate supply model augmented with a Taylor rule. More specifically, we follow De Grauwe's two-type agent-based version of the DSGE model (De Grauwe, 2010a,b). This economic system has the following structure:

$$y_t = a_1 E_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - E_t \pi_{t+1}) + \varepsilon_t, \quad (1)$$

$$\pi_t = b_1 E_t \pi_{t+1} + (1 - b_1) \pi_{t-1} + b_2 y_t + \eta_t, \quad (2)$$

$$r_t = c_1 (\pi_t - \pi_t^*) + c_2 y_t + c_3 r_{t-1} + u_t. \quad (3)$$

Eq. (1) is referred to as the standard *aggregate demand* that describes the demand side of the economy. It is derived from the Euler equation which is the result of the dynamic utility maximization of a representative household with the market-clearing assumption applied to the goods market. The notations in Eq. (1) have the following meaning: y_t denotes the output gap in period t , r_t is the nominal interest rate and π_t is the rate of inflation. Here, we add a lagged output gap y_{t-1} in the aggregate demand equation to describe *habit formation* (Fuhrer, 2000).

Eq. (2) is a *New Keynesian Phillips curve* that represents the supply side in the economic system. Under the assumption of nominal price rigidity and monopolistic competition, the New Keynesian Phillips curve can be derived from the profit maximization of a representative final-goods producer and the profit maximization of intermediate-goods producers which are composed of a number of heterogeneous households. To reflect the price rigidity, the intermediate-goods producers can adjust their price through the Calvo pricing rule (Calvo, 1983). By combining the first-order conditions of the final-goods producer, the intermediate-goods producer and the Calvo pricing rule, we can obtain the New Keynesian Phillips curve (Eq. 2).

Eq. (3) represents the *Taylor rule* commonly used for describing the behavior of the central bank in the standard New Keynesian DSGE model (Taylor, 1993). The central bank reacts to deviations of inflation and output from targets. In Eq. (3), π^* refers to the inflation target of the central bank. In addition, the lagged interest rate in Eq. (3) represents the smoothing behavior.

Finally, ε_t , η_t , and u_t are all white noise added to aggregate demand, aggregate supply and the interest rate. Given these stochastic elements, E_t , in Eqs. (1) and (2), is the expectations operator, denoting people's expectations of the GDP gap and the inflation rate.

The reduced form of the New Keynesian DSGE model is found by substituting Eq. (3) into (1) and then by rewiring in the matrix notation. This yields:

$$\begin{aligned} \begin{bmatrix} 1 & -b_2 \\ -a_2 c_1 & -a_2 c_2 \end{bmatrix} \times \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} &= \begin{bmatrix} 1 - b_1 & 0 \\ -a_2 & a_1 \end{bmatrix} \times \begin{bmatrix} E_t \pi_{t+1} \\ E_t y_{t+1} \end{bmatrix} + \begin{bmatrix} 1 - b_1 & 0 \\ 0 & 1 - a_1 \end{bmatrix} \times \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ a_2 c_3 \end{bmatrix} \times r_{t-1} + \begin{bmatrix} \eta_t \\ a_2 u_t + \varepsilon_t \end{bmatrix}, \end{aligned} \quad (4)$$

or, in a compact matrix notation,

$$AZ_t = BE_t Z_{t+1} + CZ_{t-1} + br_{t-1} + V_t, \quad (5)$$

where all notations above correspond one-to-one to the matrices in Eq. (4) in a order from left to right. The solution to (5) in terms of Z_t is:

$$Z_t = A^{-1}[BE_t Z_{t+1} + CZ_{t-1} + br_{t-1} + V_t]. \quad (6)$$

The solution exists only if matrix A is non-singular. In other words, matrix A has to satisfy $(1 - a_2 c_2) \times a_2 b_2 c_1 \neq 0$. By obtaining the inflation rate (π_t) and output gap (y_t) through Eq. (6) and substituting them into Eq. (3), the interest rate (r_t) can be determined accordingly.

2.2. The DSGE model augmented with the ABS machine

After describing the stylized New Keynesian DSGE model, we come to a variant initiated by De Grauwe, which differs from the standard DSGE model in terms of the formation of expectations and its heterogeneity. In the standard New Keynesian DSGE model, the representative agent always has rational expectations. De Grauwe relaxed this stringent assumption and started the agent-based version of the DSGE model by replacing the homogeneous rational expectations of output with the heterogeneous boundedly rational counterparts.

This is done by using a *two-type agent-based model*, which distinguishes agents into optimists and pessimists, each with different expectations. Basically, the optimistic agents systematically bias upward the output expectations and the pessimistic agents systematically bias downward the output expectations. Specifically, the optimists' rule is defined by $E_{o,t} y_{t+1} = g$ and the pessimists' rule is defined by $E_{p,t} y_{t+1} = -g$, where $g > 0$ denotes the degree of bias in the expectations of the output gap.

Being an optimist or a pessimist has been formulated as a stochastic discrete-choice (binary-choice) problem in De Grauwe's DSGE model. In this formulation, the agent's current choice is mainly determined by the rewards or utilities which he experienced when choosing different alternatives. Let us assume that these experienced rewards or utilities have been constantly updated with time t , and let $V_{o,t}$ and $V_{p,t}$ be the experienced utilities of being an *optimist* and *pessimist*, respectively, updated at time t . In other words, $V_{o,t}$ is the temporal realized utility gained or experienced from being an optimist, and $V_{p,t}$ is that gained from being a pessimist. De Grauwe further equates these utilities to their forecast errors, specifically, the weighted sum square errors, as shown in Eqs. (7) and (8). Each past forecast error contributes to the negative utility with a sequence of weights, ρ_k .

$$V_{o,t} = - \sum_{k=1}^{\infty} \rho_k (y_{t-k} - E_{o,t-k-1} y_{t-k})^2, \quad (7)$$

$$V_{p,t} = - \sum_{k=1}^{\infty} \rho_k (y_{t-k} - E_{p,t-k-1} y_{t-k})^2. \quad (8)$$

As usual, ρ_k declines with k . For example, $\rho_k = \rho^k$ ($0 < \rho < 1$).

Despite his use of an agent-based model, De Grauwe does not model each individual's learning behavior or expectation formation process. Instead, by following the Brock–Hommes' adaptive belief systems (Brock and Hommes, 1997, 1998), one of the most prominent agent-based economic models, De Grauwe applies the above two equations directly to the mesoscopic structure of the economy and uses them to determine the fractions of the optimists ($\alpha_{o,t}$) and the pessimists ($\alpha_{p,t}$) of the whole economy at time t through the familiar *logit distribution* or the *Gibbs–Boltzmann distribution* (Luce, 1959; Blume et al., 1993), as characterized by Eqs. (9) and (10).

$$\alpha_{o,t} = \frac{\exp(\lambda V_{o,t})}{\exp(\lambda V_{o,t}) + \exp(\lambda V_{p,t})}, \quad (9)$$

$$\alpha_{p,t} = 1 - \alpha_{o,t} = \frac{\exp(\lambda V_{p,t})}{\exp(\lambda V_{o,t}) + \exp(\lambda V_{p,t})}. \quad (10)$$

Parameter λ in the two above equations is known as the *intensity of choice*, because it basically measures the extent to which agents are sensitive to additional advantages from the superior choice relative to the inferior one.

$$\alpha_{p,t} \rightarrow \begin{cases} 1/2, & \text{if } \lambda \rightarrow 0. \\ 1, & \text{if } \lambda \rightarrow \infty \text{ and } V_{o,t} > V_{p,t}. \\ 0, & \text{if } \lambda \rightarrow \infty \text{ and } V_{o,t} < V_{p,t}. \end{cases} \quad (11)$$

Given the fractions of the optimists and pessimists, $\alpha_{p,t}$ and $\alpha_{o,t}$, the expected output gap of the whole economy in period $t+1$ can then be regarded as the weighted average of the forecast held by the two groups of agents, weighted by their fractions, as shown in Eq. (12).

$$E_t = \alpha_{o,t} E_{o,t} y_{t+1} + \alpha_{p,t} E_{p,t} y_{t+1}. \quad (12)$$

In sum, De Grauwe's DSGE model can be read as the first step in preparing the standard top-down equation-based DSGE model into its agent-based variants. In this initial attempt, the mesoscopic structure, as represented by Eqs. (9), (10) and (12), is added to the standard DSGE model. Furthermore, as we shall see in this article, the work can be further extended into the micro-structure by articulating an interaction scheme among agents, and part of the interaction scheme may be explicitly based on the assumed network topologies to which we now turn.

3. Social networks

3.1. Network thinking in economics

While it is only very recently that the social network has drawn the attention of macroeconomists, network thinking has a long history in economics. The idea of providing a network representation of the whole economy started with Quesnay's *Tableau Economique* in 1758, which depicted the circular flow of funds in an economy as a network. Quesnay's work later on inspired the celebrated input–output analysis founded by Wassily Leontief (1905–1999) in the 1950s (Leontief, 1951), which was further generalized into the social accounting matrices by Richard Stone (1913–1991) (Stone, 1961) in the 1960s. This series of developments forms the backbone of computable general equilibrium analysis, a kind of applied micro-founded macroeconomic model, pioneered by Herbert Scarf in the early 1970s (Scarf, 1973).

These earlier “network representations” of economic activities enable us to see the interconnections and interdependence of various economic participants. This “visualization” helps us to address the fundamental issue in macroeconomics, i.e., how disruption propagates itself from one participant (sector) to others through the network. Nowadays, a vast amount of research had been carried out. For example, network analysis is not only applied to examine the transmission of information regarding job opportunities, trade relationships, how diseases spread, how people vote and which languages they speak, but is also used in empirical works, such as the World Trade Web, the Internet, ecological networks and co-authorship networks.

3.2. Graphs and social networks

So far, the most powerful mathematical treatment of social networks is mathematical graph theory. To recap, a graph (G) or a network $G(V, E)$ is defined by a set of vertices (nodes) V and a set of edges (links) E . In many social applications like ours, each node corresponds to a single agent, and $V = \{1, \dots, N\}$ denotes the set of all the agents considered in the economy. The number N is then the cardinality of the set V or the size of the network. The set E can be represented as an $N \times N$ binary matrix. $E = \{b_{ij} : i, j \in V\}$ denotes the pairwise connections existing among the agents; normally, $b_{ij} = 1$ if such a connection exists between i and j , and zero if there is no such connection. In addition, since self-connection has little application value, we normally assume that $b_{ii} = 0$. The network is *undirected* if the matrix E is symmetric, i.e., $b_{ij} = b_{ji}$. All the social network topology considered in this paper is *undirected*.

Following the current practice in the social network analysis, we consider a number of frequently used network topologies in the literature. These network topologies are at most served for the purpose of thought experimentations; it would be hard to take any of it literally from an empirical viewpoint. Presumably, the best use of them is simply to help us have a rough idea or a picture of the effect of social networks on the macroeconomic performance. With this scope in mind, we have considered the following network topologies: the fully-connected network, the circle and the regular network, the small-world network, and the scale-free network. Each of them will be briefly described as follows. All these networks are also depicted in Fig. 1.

3.2.1. Fully-connected network

The fully-connected network has the feature that agents are completely connected with each other. In other words, each agent has $(N - 1)$ links: $b_{ij} = 1$, for all $i \neq j$. An example of the fully-connected network is given in Fig. 1 (first row, left).

3.2.2. Circle and regular networks

In the fully-connected network, all interactions are global; however, in many realistic settings, interactions are rather local and are confined to the geographical constraints. There are a number of spatial networks, such as *cellular automata*, that may be a better representation of these constraints. We, however, consider an alternative with similar virtues but much less computationally demanding, which is known as a *regular network*. In a regular network, all agents are distributed and placed like a ring (Fig. 1, first right and second left) and each agent is connected with his k neighbors both on the left and the right; k is a constant. A special case called the circle appears when the interaction is extremely limited and $k = 1$ (Fig. 1, first right). In addition to this extreme case, a regular network with $k = 2$ is also considered (Fig. 1, second left).

3.2.3. Small-world and random networks

The regular network focuses only on local interactions. It captures a kind of clustering activity, but does not allow for interactions crossing clusters. Nevertheless, inter-cluster interactions are important in reality. Sociologist Mark Granovetter first noticed its significance in the labor market and proposed the so-called *weak-tie connection* (Granovetter, 1973). A network which allows for both local and bridging interaction was first proposed by Watts and Strogatz (1998) and is known as the small-world network.

The small-world network combines the ideas of random networks and regular networks. These two kinds of networks can be interestingly compared by the two essential characterizations of network topologies, namely, the *clustering coefficient* and the *average distance*. The clustering coefficient is a formal measurement of the extent to which that *friends of mine are also friends to each other*. The average distance, defined as the average length of the shortest path between two nodes, is used to measure the average distance between two nodes, which corresponds to the degree of separation in a social network. Watts and Strogatz (1998) show that regular networks tend to have a larger clustering coefficient and also a larger diameter; random networks of the equivalent size tend to have a smaller diameter and also smaller clustering coefficient.

To have a network with a large clustering coefficient but also a small average distance, Watts and Strogatz proposed a network generation algorithm as follows. Firstly, it generates a regular network with N nodes, each with $2k$ neighbors. Secondly, a rewiring probability, p , is applied to each link of each agent. If rewiring takes place, then that link will be disconnected and rewired to a randomly selected agent. By fine-tuning the probability parameter, p , a spectrum of networks, known as the small-world network, which has the random network and the regular network as two extremes, can be generated. In fact, when $p = 0$, we have the regular network and, when $p = 1$, we have the random network (Fig. 1, first on the fifth row). In this study, small-world networks with p equal to 0.1, 0.3, 0.5, 0.7, 0.9 are employed and they are exemplified in Fig. 1 (right on the second, the two on the third and fourth). Small-world networks, as compared to other random graphs with the same number of nodes and edges, are characterized by clustering coefficients significantly larger than expected and average shortest-path length smaller than expected.

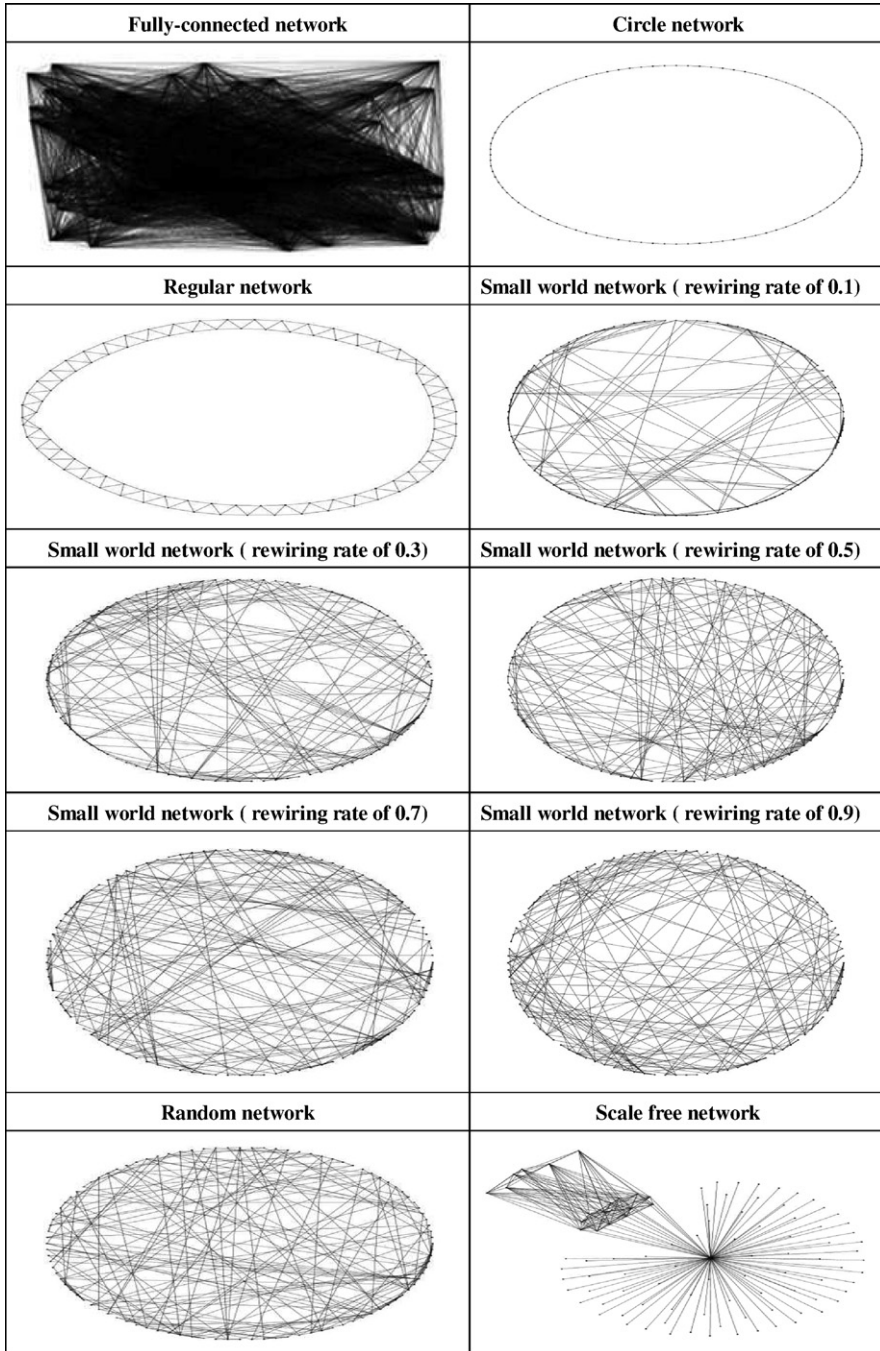


Fig. 1. Network topologies. The 10 network topologies shown here are based on the parameter values given in Table 2.

3.2.4. Scale-free network

Now, we have networks with a mixture of local and bridging connections, each up to a different degree. In addition, due to the randomness introduced by the rewiring parameter, nodes (agents) can have different numbers of connections, a property which is not shared by the regular networks. This phenomenon, known as the *degree distribution* (see Section 3.3.1), corresponds well with what we experience in real social settings: some agents have many more connections than others. However, the way the degree distribution is introduced by the rewiring parameter is basically *random* rather than by a certain social mechanism. Therefore, one cannot directly control the degree distribution in a manner that mimics the empirical degree distribution observed in real social contexts, such as the *power law distribution*.²

The power law distribution of the degree has been found in many social contexts, such as the citation network of scientific publications (Redner, 1998), the World Wide Web and the Internet (Albert et al., 1999; Faloutsos et al., 1999), telephone call and e-mail graphs (Aiello et al., 2002; Ebel et al., 2002), and in the network of human sexual contacts (Liljeros et al., 2001). A class of networks which is strongly motivated by this observed prevalent power law distribution is known as the scale-free network. It is called a scale-free network because it is able to feature a power law distribution of degree, and the power law distribution is scale-free.

The *scale-free network* was first proposed by Barabási and Albert (1999), and hence is also known as the *BA model* (the Barabási-Albert model). The BA model is based on two mechanisms: (1) networks grow incrementally, by the adding of new vertices, and (2) new vertices attach *preferentially* to vertices that are already well connected. Let us assume that initially the network is composed of m_0 vertices, and that each is connected to m other vertices ($m < m_0$). Then, at each point in time, a number of new vertices, m_T are added to the network, each of which is again connected to m vertices of the net by the *preferential linking*. This idea of *preferential attachment* is similar to the classical “rich get richer” model originally proposed by Simon (1955).

It is implemented as follows. At time T , each of the new m_T vertices is randomly connected to a node $i \in V_T$ according to the following distribution:

$$\pi_i = \frac{k_i}{\sum_{j \in V_T} k_j}, \quad i \in V_T, \quad (17)$$

where $V_T = \{1, 2, \dots, \sum_{t=0}^{T-1} m_t\}$. That is, the probability of becoming attached to a node of degree k is proportional to k , π_k , and nodes with high degrees attract new connections with a high probability. An example of the scale-free network is shown in Fig. 1 (right, the last row).

This completes our brief description of the networks included in this study. Of course, this is not an exhaustive list, and one can always add others, but as a pioneering study, we believe that the four

² A power law distribution is a density function which is proportional to a power function, i.e.,

$$y = f(x) = \text{Prob}(X = x) \sim x^{-\gamma}, \quad (13)$$

where X is a random variable. A nice feature of the power distribution is that it is *scale free*. A random variable X is called *scale free* or said to have a scale-free distribution if

$$f(bx) = g(b)f(x). \quad (14)$$

Intuitively, the shape of the distribution in an interval $[x_1, x_2]$ is the same as that of $[bx_1, bx_2]$ except for a multiplicative constant. The definition above obviously applies to the power-law distribution since

$$f(bx) = (bx)^{-\gamma} = b^{-\gamma}x^{-\gamma}. \quad (15)$$

The power law distribution has gained a quite significant popularity these days in sciences. It has been often cited by scientists, while, sometimes, in different names. For example, when the exponent γ is equal to 1, it is also known as *Zipf's law*, in memory of Harvard linguistics professor George Zipf (1902–1950). Alternatively, it has also been cited as *Pareto's Law* when what interests us is the tail distribution of Eq. (13), i.e.,

$$\text{Prob}(X \geq x) \sim x^{-\beta}, \quad (16)$$

where $\beta = \gamma - 1$.

which we choose are rich enough to cover many network behaviors ranging from global connections to local connections and further to overarching connections, and from even degree distributions to uneven power distributions. Later on, our agent-based DSGE model will be embedded within these different network settings, and their effects on macroeconomic behavior will be simulated and studied.

3.3. Characterizations of network topologies

To facilitate the later simulation study, it would be useful to characterize the chosen network topologies by a few key variables, and then examine the effects of these variables on the resultant macroeconomic behavior. Based on what we have discussed throughout this section and also the literature on social network analysis, we restrict our attention to the following two major characterizations, basically, the two averages, *average degree* and *average clustering coefficient*. They shall be briefly described as follows.

3.3.1. Degree distribution and average degree

The degree of a specific vertex is the number of links emanating from that vertex. A degree distribution $f(k)$ gives the probability of a randomly chosen vertex which has exactly k links. The power law distribution discussed earlier, which has the form $f(k) = k^{-\gamma}$, is one example of the degree distribution.³ When the network has a finite size, $f(k)$ can also be read as a histogram that gives the percentage of the agents who have exactly k links. The average degree is the mean associated with distribution $f(k)$. When V is finite, it is simply

$$\bar{k} = \frac{\sum_{i=1}^N k_i}{N}, \quad (18)$$

where N is the size of the network (the total number of agents in the network) and k_i is the number of the degrees of agent i .

3.3.2. Average clustering coefficient

The clustering coefficient measures the tightness of the local connection. Specifically, we are asking: if agent j is connected to i , and l is also connected to i , is j also connected to l ? Let ϑ_i be the set of neighbors of agent i ,

$$\vartheta_i = \{j : b_{ij} = 1, j \in V\}. \quad (19)$$

Then the *clustering coefficient* of agent i , C_i , is defined as follows:

$$C_i = \frac{\#\{(h, j) : b_{hj} = 1, h, j \in \vartheta_i, h < j\}}{\#\{j : j \in \vartheta_i\}}. \quad (20)$$

The definition of the *average clustering coefficient* is thus straightforward.

$$\bar{C} = \frac{1}{n} \sum_{i=1}^n C_i. \quad (21)$$

By Eq. (20), if agent i 's neighborhood is fully connected then the clustering coefficient C_i is 1; otherwise, if they are poorly connected, then C_i is closer to zero. Hence, the average clustering coefficient \bar{C} gives a general picture of how well agents are locally connected.

3.3.3. A summary table

Table 1 gives the average degree and the average clustering coefficient of various network topologies used in this study.

³ See footnote 2.

Table 1

Basic characterizations of the 10 network topologies.

Social network	Characterization	
	Average degree	Average clustering coefficient
Fully-connected	99	1.0000
Circle	2	0.0000
Regular	4	0.5000
SW01	4	0.2540
SW03	4	0.0979
SW05	4	0.0027
SW07	4	0.2650
SW09	4	0.2700
Random	4	0.0360
Scale free	4.52	0.1470

SW01 (03,05,07,09) refers to the small-world network with a rewiring rate of 0.1 (0.3,0.5,0.7,0.9), respectively. The average degree and the average cluster coefficient is derived from the parameter values specified in Table 2.

4. Social interactions

4.1. The Ising model

For describing the social interaction behavior among agents, we use the Ising model as our interaction model. The Ising model originated from the dissertation of Ernst Ising (1900–1998). Ising studied a linear chain of magnetic moments, which are only able to take two positions or states, either up or down, and which are coupled by interactions between nearest neighbors. The model is strikingly successfully in the search for the transition between the ferromagnetic and the paramagnetic states. In addition to physics, the model is also used in biology and the social sciences. In economics, it was first used in Follmer (1974), and has been used to model opinion dynamics (Orlean, 1995), financial markets (Iori, 1999; Iori and Dave, 2002; Sornett and Zhou, 2006) and tax evasion (Zaklan et al., 2009).

In this paper, the Ising model is composed of a finite number of agents who are arranged in a specific network structure as we introduced in Section 3. The Ising model characterizes each individual's decision as a *stochastic discrete choice*. In our case, each agent's choice of being an optimist or a pessimist is influenced by his neighbors (magnetic field) in a stochastic manner. Specifically, agent i 's probability of being an optimist or a pessimist in period t , " $i(t) = o$ " or " $i(t) = p$ ", is stochastically determined by his interactions with his neighbors in previous periods. For implementation, we further assume that this local interaction can be summed up by a few statistics, for example, the number of his optimistic neighbors and the number of his pessimistic neighbors in period $t - 1$. Then his choice in time t is assumed to be influenced by these local statistics. A simple formalization of this stochastic discrete choice is given by Eq. (22).

$$\text{prob}(i(t) = o) = \text{prob}(Z_{i,t} = 1) = \frac{1}{1 + \exp(-2\lambda m_{i,t})}. \quad (22)$$

Here, we introduce a dummy variable $Z_{i,t}$ in correspondence to the event that agent i in period t is an optimist, " $i(t) = o$ ". In other words,

$$Z_{i,t} = \begin{cases} 1, & \text{if } i(t) = o. \\ -1, & \text{if } i(t) = p. \end{cases} \quad (23)$$

In Eq. (22), the decision to be optimistic ($Z_{i,t} = 1$) or pessimistic ($Z_{i,t} = -1$) only involves one local statistic, i.e., $m_{i,t}$. In line with the Ising model, this can typically be the weighted average of the

local optimistic forces and pessimistic forces (or simply the *local market sentiment*), i.e., the market sentiment that agent i can experience from his neighbors (magnetic field). More concretely,

$$m_{i,t} = \sum_{j=1, j \in \vartheta_i}^N w_{ij} Z_{j,t-1}. \quad (24)$$

Recall that ϑ_i is the set of all j , $b_{ij} = 1$, i.e., the set of agent i 's neighbors. w_{ij} is the weight that represents the interaction strength between i and j or j 's influential power on i . In a very simple setting, we consider the uniform weight, i.e.,

$$w_{ij} = \frac{1}{\#\{j : j \in \vartheta_i\}}. \quad (25)$$

In addition to the local statistic $m_{i,t}$ (the forces in the magnetic field), the other variable appearing in Eq. (22) is λ , the same parameter which we use in Eq. (9) to denote the intensity of choice. The reason why we keep this same notation here is because we can rewrite Eq. (22) in a form very similar to Eq. (9) as follows:

$$\text{prob}(i(t) = o) = \text{prob}(Z_{i,t} = 1) = \frac{\exp(\lambda m_i)}{\exp(\lambda m_{i,t}) + \exp(-\lambda m_{i,t})}. \quad (26)$$

Hence, the λ appearing here can also be interpreted as the intensity of choice, which works in the same way as λ works in the ABS machine.

$$\text{prob}(Z_{i,t} = 1) \rightarrow \begin{cases} 1/2, & \text{if } \lambda \rightarrow 0. \\ 1, & \text{if } \lambda \rightarrow \infty \text{ and } m_{i,t} > 0. \\ 0, & \text{if } \lambda \rightarrow \infty \text{ and } m_{i,t} < 0. \end{cases} \quad (27)$$

4.2. The ABS machine and the Ising model

Despite this similarity, there is also one essential difference between Eqs. (9) and (22). What the former one does is to give a mesoscopic distribution or market sentiment directly from the top based on the forecasting performance of the two groups of agents (optimists vs. pessimists), while the latter does not give such a distribution directly, but enables the market sentiment to emerge through local interaction with a simple mimetic (herding) mechanism. Therefore, in the former, the decision can be made, individually and independently, based on the publicly available information on the forecasting performance of various rules; there is no need for social interaction, not to mention social networks.

However, for the latter case, either such information is not publicly available or else the agent's reasoning and decision-making is more influenced by what the herd did or will do rather than how they performed in the past, since what the herd did or will do depends on the agent's perception of the herd, which in turn depends on his own network and neighbors and can be quite heterogeneous among agents. It is this difference bringing in the neglected social networks by which the mimetic effect is operated.

In sum, Eq. (9) is more inclined toward individual learning and decision making, which involves little interaction among agents, whereas Eq. (22) is more inclined toward social learning and mimetic decision making, which involves substantial local interactions. The purpose of this paper is to see, when the two, referred to as the ABS machine and the Ising model, are equivalent. If the equivalence between the two can be demonstrated in a very relaxing environment, then in terms of model simplicity, one can replace the latter with the former as a good approximation of the reality. On the other hand, if the equivalence between the two can be shown only in very restricted settings, then to have a good harness of the economy local interaction and social networks remain indispensable elements. This is the fundamental pursuit of this paper, which, in our understanding, has not been formally addressed before.

Since in the models built in the spirit of statistical physics the macroeconomic behavior will be determined by the mesoscopic structure or, more generally, the Chapman–Kolmogorov equation, also known as the Master Equation (Aoki and Yoshikawa, 2006), we shall restrict our equivalence checking

Table 2

Parameters setting of the two-type agent-based DSGE model.

Parameter	Value	Description
π^*	0	The central bank's inflation target
a_1	0.5	Aggregate demand sensitivity to the expected output, Eq. (1)
a_2	-0.2	Aggregate demand sensitivity to the real interest rate, Eq. (1)
b_1	0.5	Price sensitivity to the expected inflation, Eq. (2)
b_2	0.05	Price sensitivity to the aggregate demand, Eq. (2)
c_1	1.5	Interest sensitivity to the inflation gap, Eq. (3)
c_2	0.5	Interest sensitivity to the aggregate demand, Eq. (3)
c_3	0.5	Interest adjustment smoothness, Eq. (3)
$\varepsilon_t, \eta_t, u_t$	0.005	Standard deviation of shocks, Eqs. (1)–(3)
ρ	0.5	Coefficient of geometrically declining factor, Eqs. (7) and (8)
g	0.01	Optimists' forecasts of the GDP gap
λ	100, 500 1000, 5000 10,000, 50,000	Intensity of choice
k	1, 2	Radius of neighbors of the regular networks
m_0	20	Initial nodes of the scale-free networks
p	0.1, 0.3, 0.5 0.7, 0.9, 1	Cutting (rewiring) probability of the small-world networks
N	100	Network size (number of individuals)

at the mesoscopic level, i.e., to market sentiment, instead of to the behavior of all macroeconomic variables. Hence, so as to not overload the presentation, our analysis of the simulation results, as shown in the next section, will also be restricted to market sentiment.

5. Simulation settings and simulation results

5.1. Simulation settings

We run the De Grauwe's version of the DSGE model using his calibrated parameters. We then replace his stochastic discrete choice model (the ABS machine) with the Ising model and re-run this DSGE model with different embedded network topologies. In addition, different values of the intensity of choice are simulated for both De Grauwe's DSGE model and the Ising model. The values of the parameters in these two kinds of agent-based DSGE models, the one with the ABS machine and the other with the Ising model, can be found in Table 2. The three blocks of Table 2 give the values of the *structural parameters* (parameters related to the structural equations of the DSGE model), the values of the *behavioral parameters* (parameters related to the agents' utility or error function, expectations and learning) and the values of the *network parameters* (parameters associated with different network topologies), respectively. The last row in Table 2 denoted by N is the number of agents in the economy, also referred to as the network size. It is set to 100 throughout all simulations. The entire setting of the third block (values of the network parameters) is, of course, only applicable to the Ising model.

For both models, each setting is run 100 times, each with 300 iterations. Therefore, for De Grauwe's DSGE model, what we have is 100 observations of the mesoscopic structure, i.e., the market sentiment in terms of the fraction of optimists in the economy, $\{\alpha_{o,t}\}_{t=1}^{300}$, in each of the 300 periods. This allows us to use the empirical distribution to reasonably approximate the theoretical distribution of market sentiment. We can then further examine whether this fraction distribution generated by the ABS machine can be simulated *bottom up* by testing whether the two samples are from the same distribution using the Kolmogorov–Smirnov test. To be assured that our comparison is made based on the

Table 3The p -values of the Kolmogorov–Smirnov statistic.

λ /networks	Fully connected	Circle	Regular	SW01	SW03
100	3.276E–17	2.206E–08	9.122E–09	2.168E–10	4.261E–13
500	5.774E–37	1.220E–05	9.122E–09	8.083E–11	2.206E–08
1000	1.317E–38	2.849E–06	3.695E–09	3.695E–09	1.466E–09
5000	1.551E–45	4.705E–02	2.206E–08	2.849E–06	2.446E–05
10,000	1.551E–45	3.215E–04	8.083E–11	1.466E–09	9.122E–09
50,000	1.402E–13	1.335E–06	3.695E–09	1.466E–09	2.752E–07
λ /networks	SW05	SW07	SW09	Random	Scale free
100	9.122E–09	1.466E–09	2.168E–10	3.696E–12	1.466E–09
500	4.261E–13	3.695E–09	2.951E–11	2.951E–11	2.446E–05
1000	5.223E–08	1.335E–06	1.466E–09	7.174E–28	8.216E–03
5000	5.223E–08	1.335E–06	1.785E–03	2.849E–06	3.031E–03
10,000	3.031E–03	5.956E–06	1.220E–05	2.752E–07	5.223E–08
50,000	3.215E–04	2.168E–10	2.752E–07	9.122E–09	1.220E–05

limit distributions of the two, our statistical analysis of market sentiment is based only on the last observation, i.e., period 300.⁴

Below in Section 5.2.1 we first apply the Kolmogorov–Smirnov statistic to test the null hypothesis that the distributions of market sentiment generated by the ABS machine and the Ising model are the same. In Section 5.2.2, we then have a deeper look at the geometric shapes of these distributions to see how similar or dissimilar they are.

5.2. Simulation results

5.2.1. Quantitative analysis: the Kolmogorov–Smirnov statistic

According to Table 3, which presents the p -values of the Kolmogorov–Smirnov statistic, the null hypothesis that the market-sentiment distributions from the ABS machine and the Ising model are identical is rejected by all cases of various combinations of network topologies and intensities of choice. In other words, none of the simulated market-sentiment distributions of the different social network topologies under the Ising model can match those generated by the ABS machine.

However, if we look at the results closely, some cases can mimic the target distribution, the one derived by the ABS machine, better than other cases. To see these differences, we highlight some numbers in Table 3 with different colors. The topology whose generated market-sentiment distribution can mimic the target distribution most closely, i.e., the one with the highest p -value in each row, will be in blue ink. In addition, in the same manner, we also use red ink for the second best and the third best.

A look at Table 3 shows that these colored numbers are distributed intensively on a few settings and they are by no means random, which tends to suggest some possible underlying effects of network topologies. For example, among the 10 chosen network topologies, regardless of the intensity of choice, the behavior of the *circle network* is always in the top three (always colored). The *scale-free network* has a similar performance except in the case $\lambda=10,000$. On the other hand, the fully-connected network and the random network are the ones which remain uncolored over all settings.

The *small-world network*, which is somewhere between the regular network and the random network, has a more complex behavior. In most of the settings, it does not mimic the target distribution well. A few occasionally colored cases are scattered here and there and lack the kind of consistency to draw our attention. Probably the only interesting case is the

⁴ This number is much larger than what is probably required. In fact in most simulations the distribution of market sentiment already converges before period 100.

Table 4

Shapes of the fraction distribution.

Networks/ λ	100	500	1000	5000	10,000	50,000
Target	C-0.5	Bell	Bell	U	U	0-or-1
Fully	U	C-1	C-1	C-1	C-1	C-1
Circle	Bell	M	M	U	U	C-1
Regular	Bell	M	M	U	U	C-1
SW01	Bell	M	M	U	C-1	C-1
SW03	Bell	M	M	U	C-1	C-1
SW05	Bell	M	M	M	U	0-or-1
SW07	Bell	M	M	U	C-1	U
SW09	Bell	M	M	U	C-1	C-1
Random	Bell	M	U	U	C-1	C-1
Scale free	Bell	M	M	M	M	C-1

The target distribution is the one generated by the ABS machine (the De Grauwe model). “C-x” refers to the distribution with a concentration on x (that normally happens at 0, 0.5 or 1), “U” refers to the U-shaped distribution, “Bell” refers to the bell-shaped distribution, and “M” refers to the M-shaped distribution (the distribution with two modes).

small-world network with a rewiring probability of 0.5. From Table 3, this network is capable of mimicking the target distribution most successfully when the intensity of choice is high.

5.2.2. Qualitative analysis by distribution types

To have a better idea of how well or how poorly the distribution of the market sentiment can be mimicked bottom up, a qualitative analysis exclusively based on the type of the simulated distribution is provided to accompany the above quantitative analysis. To do so, we first roughly categorize the observed simulated distribution into the following four common types. They are the U-shaped, M-shaped, Bell-shaped, and degenerated (or almost degenerated) distributions (see Figs. A.2–A.7). The last type refers to the distribution of the market sentiment that almost concentrates on one single value x . In our simulations, these degenerated values can occur at 0, 0.5, and 1.

Of course, by this rough categorization, the distributions belonging to the same type do not necessarily imply that they are identically distributed in the Kolmogorov–Smirnov sense. However, if one only cares for bottom-up replicability of the market sentiment up to a qualitative extent, then, to say the least, one should not consider distributions belonging to different types. For example, if the market sentiment generated by ABS has a U-shape, but that generated by the Ising model has a Bell-shape, then certainly the equivalence of the two is fundamentally rejected or more strongly rejected than that generated by the Kolmogorov–Smirnov test. It is this kind of distinction that makes the proposed qualitative analysis a useful companion of the early quantitative analysis.

Table 4 provides a summary of the type of market-sentiment distributions generated by the ABS machine and the Ising model under different values of the intensity of choice.⁵ First, let us examine the table row by row. The second row of Table 4 shows the type of the distribution of market sentiment generated by the ABS machine. From this row, we can clearly see the effect of the parameter, λ , on the distribution type. When λ is small, say, $\lambda = 100$, the distribution has a concentration on 0.5 (Fig. A.2). As λ increases, it then becomes more divergent as it is first characterized by a bell-shaped distribution ($\lambda = 500, 1000$) (Figs. A.3 and A.4), then by a U-shaped distribution ($\lambda = 5000$) (Fig. A.5) and finally by a 0-or-1 distribution ($\lambda = 10000, 50000$) (Figs. A.6 and A.7).

This pattern, from a high concentration in the center ($x = 0.5$) to the two extremes ($x = 0, 1$) in the tails, is also observed in other similar settings, such as Kirman (1991, 1993); however, it has not been observed in any of our 10 Ising models. In fact, the closest sequence which we can have is from the

⁵ Details can be found in Figs. A.2–A.7.

small-world network with a rewiring rate of 0.5. It starts directly from the bell shape, then gets to the M shape, followed by the U shape, and finally also ends at a zero-or-one distribution. This may help explain why this network topology stands out in Table 3 when λ is rather large. Nevertheless, the M-shaped distribution, which is observed in this sequence and in many others involving Ising models, never appears in the target sequence.

If we also read Table 4 column by column, we can see that for most λ s the type of the market-sentiment distributions of Ising models are way off the type of the target distribution. For example, in the second column, the types of the target distribution is very concentrated on 0.5 (almost degenerated to 0.5). However, this type of distribution is not shared by all Ising models; instead, most of them have bell-shaped distributions. Among them, it is the circle network that has a shape more centered on 0.5, which is most similar to that of the target distribution; hence, as expected, it appears in blue ink in Table 3.

When λ increases up to 500 and 1000, the type of the target distributions is bell-shaped (the 3rd and the 4th columns), but this is again not matched by any of the 10 Ising models. The only case that brings the two models closer is the case when λ is 5,000 (also see Fig. A.5). The fifth column shows that the U-shape of the target is well matched by seven Ising models. Reading this with Table 3, one can find generally higher p-values in this row, compared to other rows with different λ s.

5.2.3. Summary: role of the clustering coefficient

Hence, putting Tables 3 and 4 together, we can see that it is not just the statistical test showing that the ABS machine and the Ising models generate different distributions of market sentiment, but, in many cases, they behave in a completely different manner. Hence, generally speaking, the market-sentiment distribution as demonstrated by the ABS machine has not been well replicated bottom-up through the Ising model.

Despite these deviations, among all 10 network topologies, the circle network and the small-world network with a rewiring rate of 0.5 behave more similarly or more closely to the target under certain values of λ . Specifically, the circle network can generate the sentiment distribution most similar to the target if $\lambda < 10,000$. However, if $\lambda > 10,000$, the small world network with a rewiring (cutting) rate of 0.5 is the best.

To see why these two specific topologies may have the above feature, we also check the statistical properties of the circle network and the small-world network with a rewiring (cutting) rate of 0.5. We find that these two network topologies, compared to others, happen to have very small *cluster coefficients* (see Table 1). In other words, the size of each well-connected subgroup is very small in both of these topologies, which in turn suggests that the interaction behavior in these two networks has to be very local. Despite the fact that the Boltzmann–Gibbs distribution is very much a result of local interaction, a result known in thermodynamics, given the limited trials which we have in this paper, whether the strong local interaction, or any other better formulated property in this direction, is the key to the equivalence between the indirect mesoscopic modeling and the direct microscopic modeling remains an issue for further exploration.

6. Concluding remarks

Since its very beginning, agent-based modeling has been used to show the existence of the *aggregation problem* (the Sonnenschein–Mantel–Debreu theorem) (Kirman, 1992; Stoker, 1993; Blundell and Stoker, 2005; Gallegati et al., 2006) and the limitation of the direct high-level modeling (equation-based modeling). Thomas Schelling's classic work (Schelling, 1971) shows that even a population of agents who have a reasonably good degree of tolerance for people of the unlike can still lead to observed segregation, which may erroneously lead to the conclusion of a strong degree of homophily if one takes direct high-level modeling. In this vein, this paper challenges the direct modeling of the evolution of distribution (mesoscopic structure) using the discrete choice model or the ABS machine.

In the very spirit of bottom-up emergence, we explicitly bring in the local interaction of the heterogeneous agents, even though, in the current setting, we only consider a very simple degree of heterogeneity, namely, two types of agents (optimists vs. pessimists). To make sense of the local interaction, the Ising model is applied to capture the mimetic effect of interaction. After some scale of attempts, we fail to articulate the network topologies which can replicate well the mesoscopic structure generated by the direct high-level discrete-choice model. Our simulation results do, however, provide evidence that network topologies featuring *low clustering coefficients* may behave, to some extent, in ways similar to those of our targeted distribution.

In future work, we can keep on searching for the “right” network topologies which can support the use of the ABS model at the high level. Nevertheless, given the search which we have already made, it should not be hard to see that the ground for this direct high-level modeling could be rather fragile and that it is very sensitively dependent upon the network topologies being considered. Hence, its general applicability is not well warranted. Furthermore, this sensitivity property is not limited to the agent-based version of the DSGE models only, but may be generally extended to a whole class of agent-based models using the same kind of analysis, which, in addition to the ABS model, includes several statistical physical approaches such as the master equation approach, also known as the Chapman–Kolmogorov equation.

In an era of high-performance computing and the ready availability of various data regarding social interactions (Lazer et al., 2009), the research agenda should very naturally be extended to the role of social interactions in macroeconomic behavior through networks and social learning by using these data. Agent-based modeling seems to have a great potential to facilitate this research agenda. However, the essence of this paper is that this research may not be simplified and replaced by only working at the *mesoscopic* level, such as with the agent-based DSGE model and the ABS machine reviewed in this paper. In fact, in a more general context, the following issues have already attracted some discussions in the literature (Vinkovic and Kirman, 2006; Parunak, 2012):

- how much bottom for the bottom-up analysis,
- when to use the mean-field model and when to use multi-agent system,
- when individual interactions can be replaced by their statistical summaries, etc.

The simulation and analysis provided in this paper may, hopefully, help reflect upon these issues.

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Appendix A. Distributions of market sentiment and the intensity of choice

In this appendix, we provide the details of the empirical market-sentiment distributions of both versions of the agent-based DSGE model. The one with the ABS (adaptive belief systems) machine is taken as the *target distribution* (the benchmark), and it is placed on the top of each of the following figures (Figs. A.2–A.7). Then the one with the Ising model is placed underneath the target distribution. Since the Ising model involves 10 different network topologies, there are 10 subfigures, each corresponding to the sentiment distribution of one topology. Hence, as a total, there are eleven distributions drawn for each figure: one for the target (the ABS machine) and 10 for the Ising model. The six figures together corresponds to the six intensities of choice ($\lambda = 100, 500, 1000, 5000, 10,000$ and $50,000$), respectively. Other related parameters can be found in Table 2.

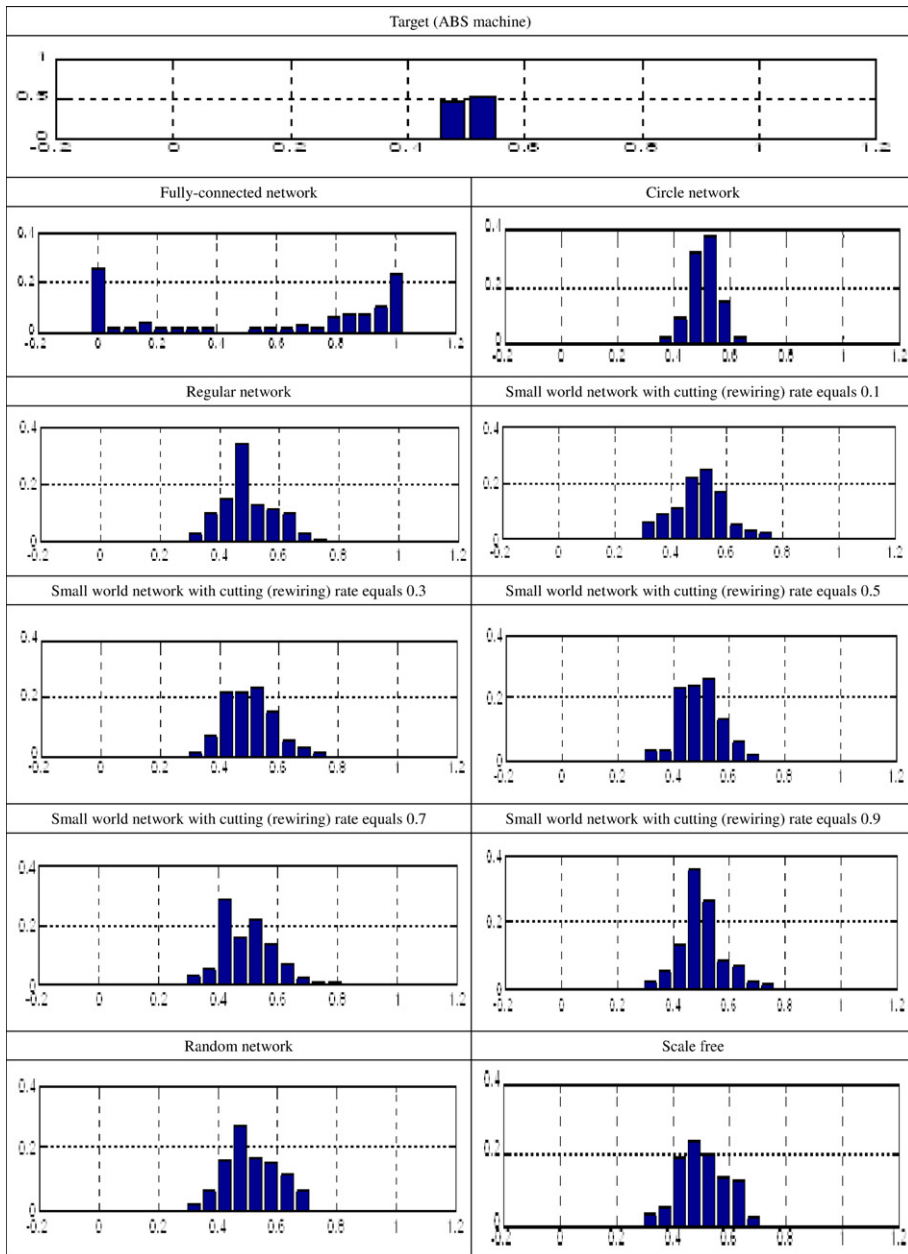


Fig. A.2. Distribution of market sentiment ($\lambda = 100$).

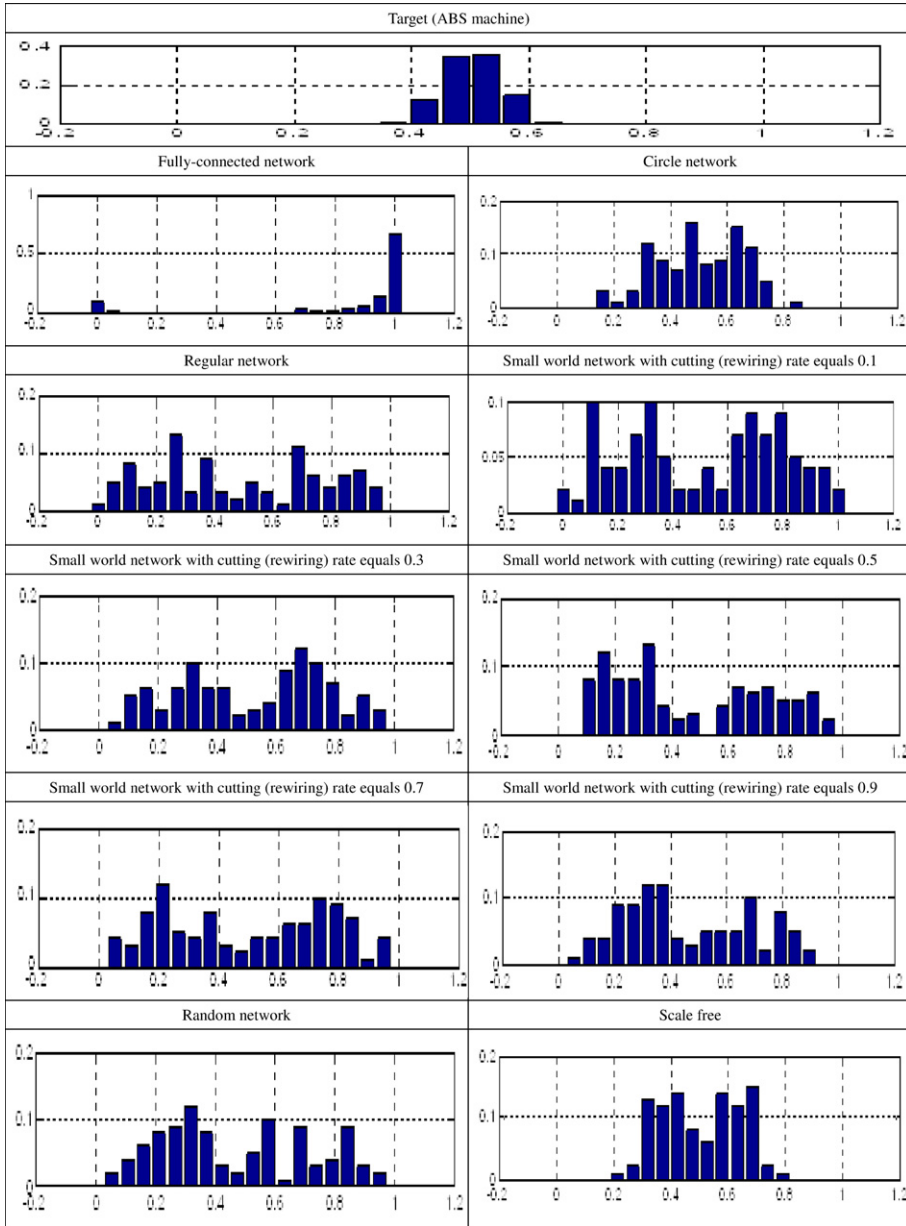


Fig. A.3. Distribution of market sentiment ($\lambda = 500$).

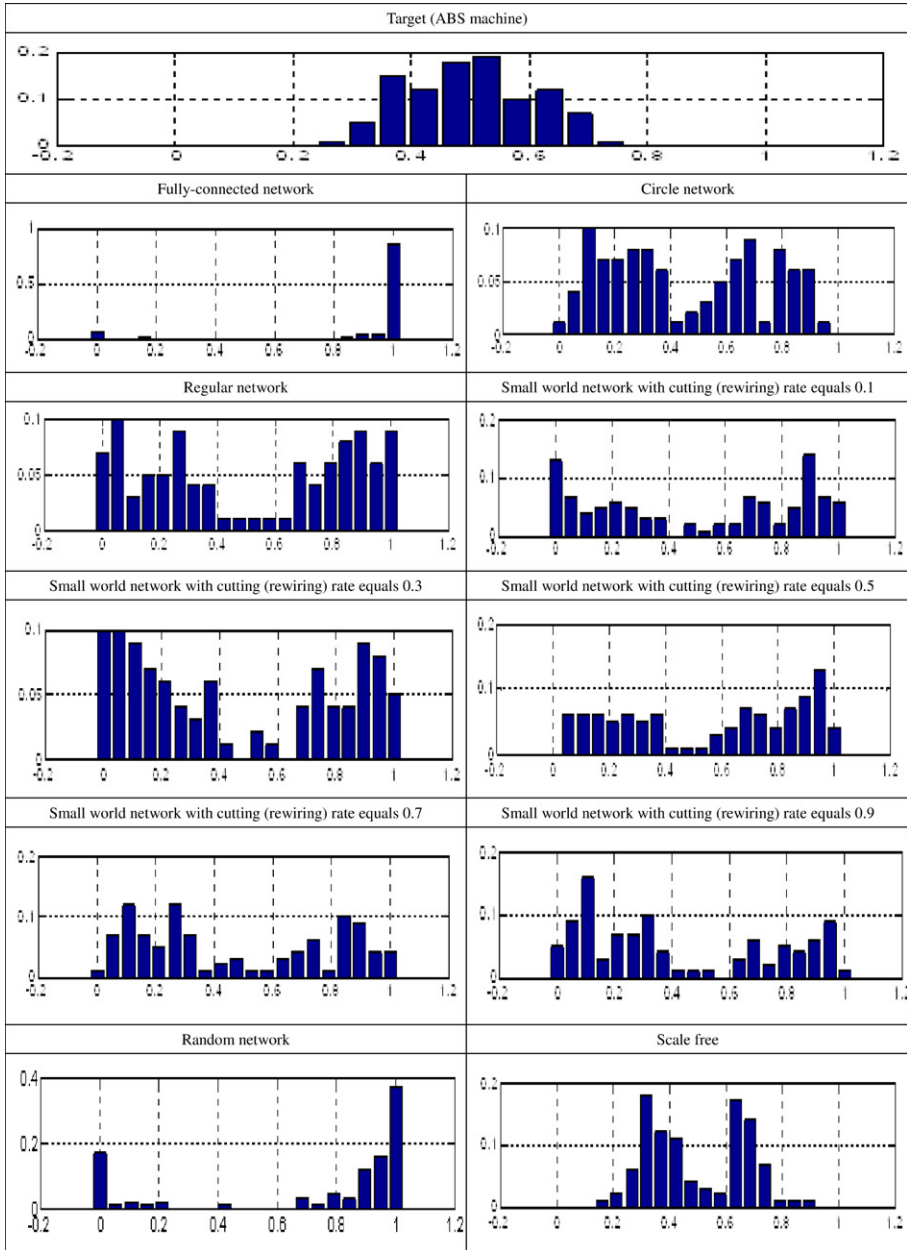


Fig. A.4. Distribution of market sentiment ($\lambda = 1000$).

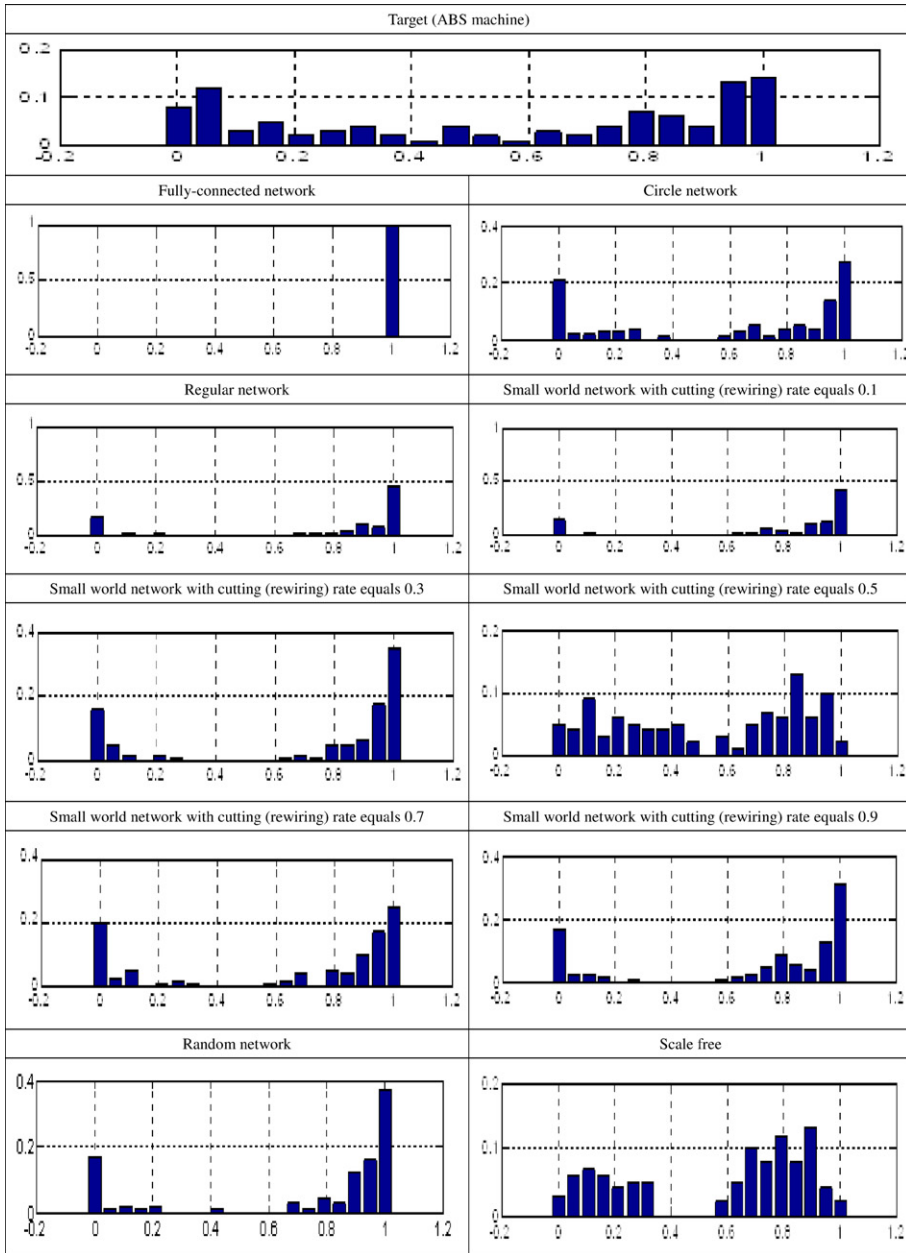


Fig. A.5. Distribution of market sentiment ($\lambda = 5000$).

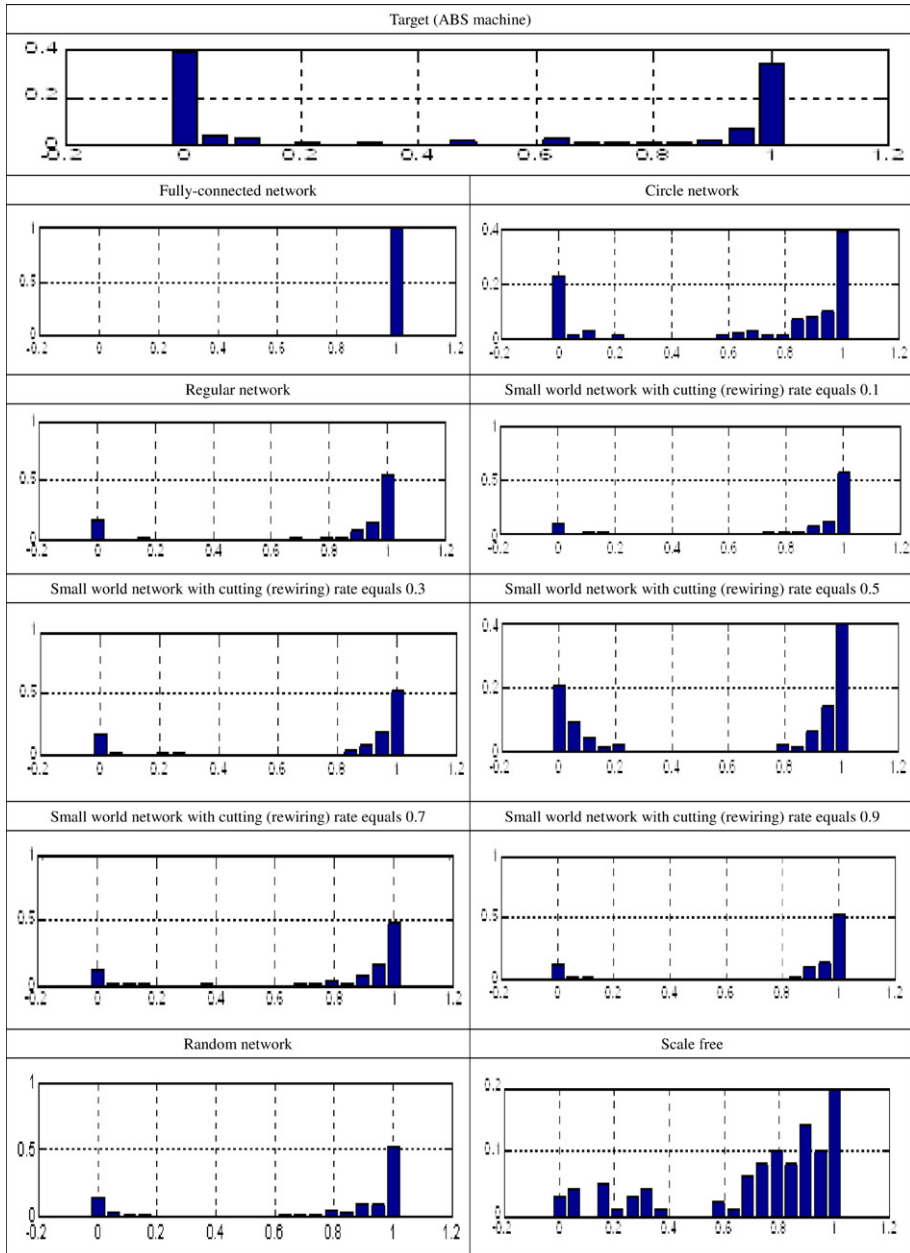


Fig. A.6. Distribution of market sentiment ($\lambda = 10,000$).

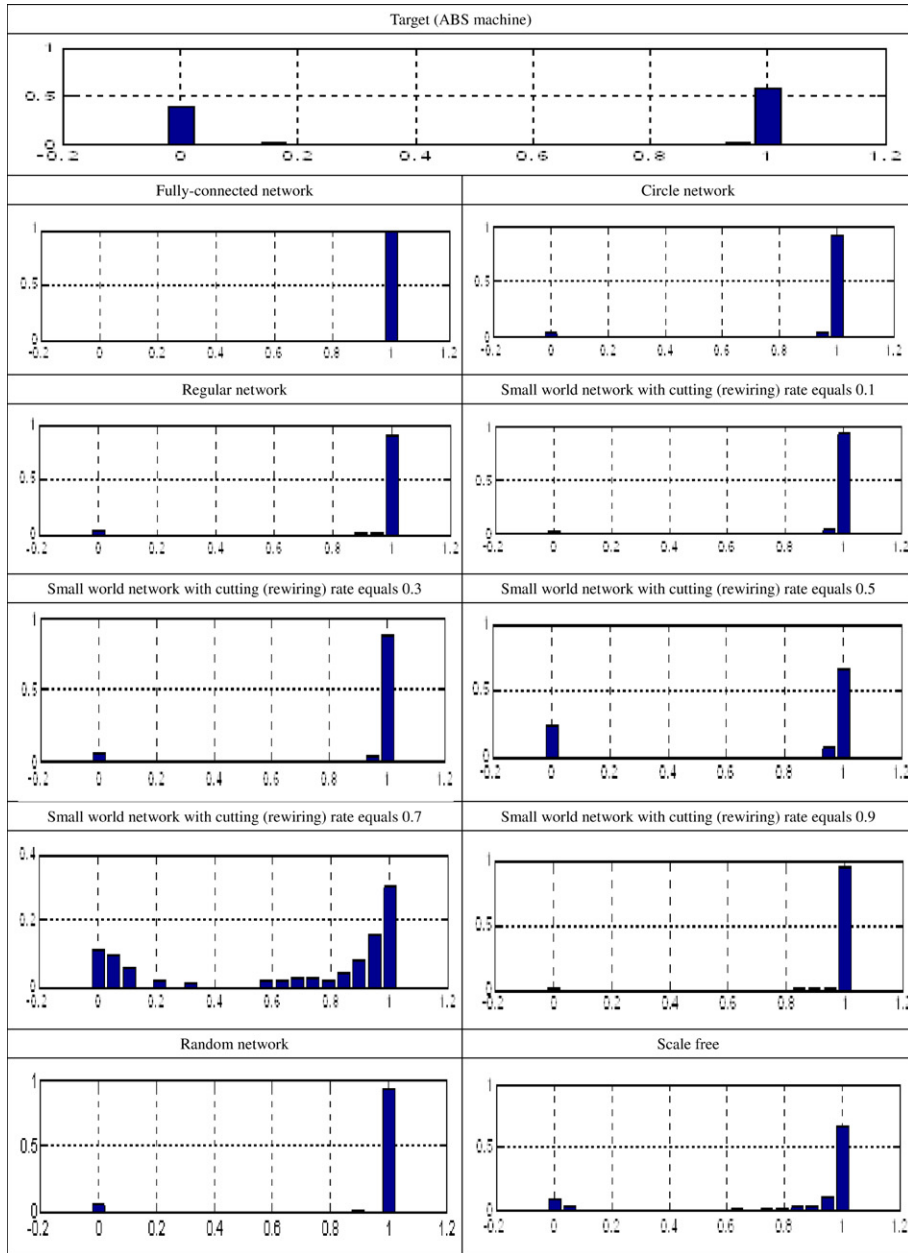


Fig. A.7. Distribution of market sentiment ($\lambda = 50,000$).

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