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Brief Paper

Conservative control policy for weakly dependent siphons in S^{3}PR based on elementary siphons

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Abstract: Siphon-based control suffers from the problem that the number of problematic siphons grows quickly with the size of the system. To reduce the number of monitors, Li and Zhou proposed to divide problematic siphons into elementary and dependent ones. Monitors are added only for elementary siphons; the number of which grows linearly. They adjust the control depth variable for a dependent siphon, if the siphon does not satisfy the controllability and can become unmarked. The control policy for weakly dependent siphons is rather conservative because of some negative terms in the controllability. This study proposes a better estimate of the negative terms and the policy needs no longer be that conservative.

1 Introduction

A flexible manufacturing systems (FMS) consists of several concurrent processes competing for resources to produce different kinds of parts. Each process conducts a sequence of operations to manufacture a part. Circular wait for resources can bring the system into a deadlock where no process can proceed.

Petri nets have been applied to model FMS. Deadlocks are closely related to structure objects called siphons S, which is a set of places containing tokens to make their output transition firable. When S becomes empty of tokens, it remains so permanently and their output transitions can never be fired again.

Control places and related arcs are often added upon emptiable siphons such that they cannot be emptied to avoid deadlocks. The number of emptiable siphons or control elements grows quickly with respect to the size of a Petri net. It is impractical to add a monitor to each emptiable siphon for large systems. Li and Zhou [1–6] tackle this problem by classifying siphons into elementary and dependent ones.

A T-vector η is associated with each siphon S such that $\eta(i)$ is the number of tokens gained in or lost from S by firing transition t_i once. A dependent siphon S_0 strongly depends on elementary siphons $S_1, S_2, \ldots, S_k$ if $\eta_0 = a_1\eta_1 + a_2\eta_2 + \cdots + a_k\eta_k$ with all a_i $(i = 1, 2, 3, \ldots, k)$ are positive. S_0 is a weakly dependent siphon if some a_i is negative. The T-vectors (resp. number) for elementary siphons are mutually independent (linear to the size of the net).

Li and Zhou [1, 2] add monitors to elementary siphons only by greatly reducing the number of control nodes and arcs. It is essential to apply the concept of elementary siphons for the control of large systems. The control depth variable for a dependent siphon may need to be increased, if the siphon does not satisfy the controllability and can become unmarked. The larger the control depth variable, the fewer states the system will reach. The control policy for weakly dependent siphons is rather conservative (such that fewer states are reached) because of some negative terms in the controllability. This paper proposes a better estimate of the negative terms and the policy needs no longer be that conservative.

The rest of the paper is organised as follows. Section 2 presents the basis (elementary siphons and characteristic

T-vectors) to understand the paper. Section 3 reviews the theory on controllability of weakly dependent siphons in [1, 2]. Section 4 presents an example to show that the control policy may be conservative and develops the theory to upgrade the controllability. Section 5 concludes the paper.

2 Preliminaries

A P-vector is a column vector L: $P \to Z$ indexed by P and a T-vector is a column vector J: $T \to Z$ indexed by T, where Z is the set of integers. For economy of space, we use $\sum L(p)p$ (resp. $\sum J(t)t$) to denote a P (resp. T)-vector. P-vector I is a P-invariant I if $I \neq 0$ and $I^{\mathrm{T}}\bullet[N] = \mathbf{0}^{\mathrm{T}}$. $\|I\| = \{p \in P|\ I(p) \neq 0\}$ is called the support of I. A P-invariant is said to be minimal if its support is not a strict superset of that of another, and the greatest common divisor of its elements is one.

For a net system (N, M_0), a non-empty subset S (resp. τ) of places is called a siphon (resp. trap) if ${}^{\bullet}S \subseteq S^{\bullet}$(resp. $\tau^{\bullet} \subseteq {}^{\bullet}\tau$), that is, every transition having an output (resp. input) place in S has an input (resp. output) place in S (resp. τ). S is called an empty siphon at M_0 if $M_0(S) = \sum_{p \in S} M_0(p) = 0$. A minimal siphon does not contain a siphon as a proper subset. It is called a strict minimal siphon (SMS), denoted by S, if it does not contain a trap.

Tokens in a siphon S of an ordinary Petri net (OPN) can either leak out to the complementary set $[S]$ of S or stay in S. Thus, if $S \cup [S]$ forms the support of a minimal P-invariant I, the sum of tokens in $S \cup [S]$ is a constant and $[S] = \|I\| \backslash S$.

Table 1 lists S and $[S]$ of the net in Fig. 1*a*. This paper deals with OPNs only.

Definition 1: $N' = (P', T', F')$ is called a subnet of N where $P' \subseteq P$, $T' \subseteq T$ and $F' = F \cap ((P' \times T') \cup (T' \times P'))$. A net N is strongly connected iff for every node pair (n_i, n_j), n_i, $n_j \in P \cup T$, there is a directed path from n_i to n_j. A subnet $N_i = (P_i, T_i, F_i)$ of N is generated by $X = P_i \cup T_i$, if $F_i = F \cap (X \times X)$.

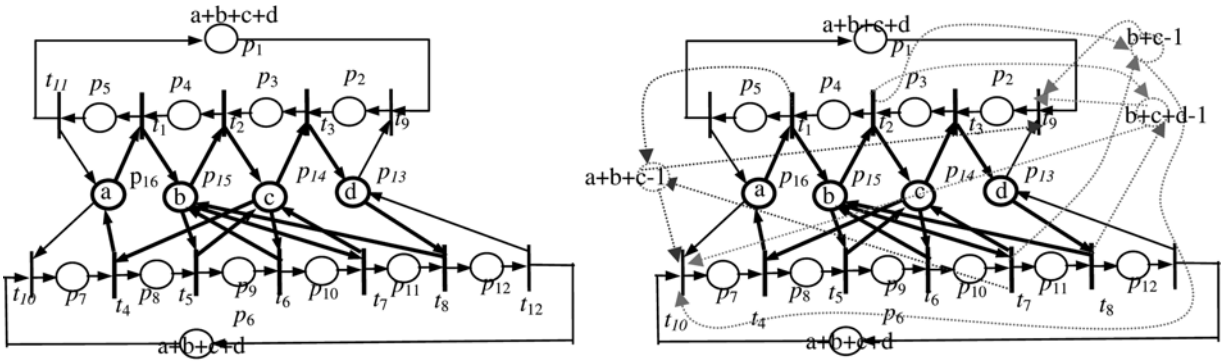

Figure 1 *Weakly dependent siphon and its controlled model*

a Example of weakly dependent siphon [2]
b Controlled model of that in Fig. 1*a*

Table 1 Four SMS in Fig. 1*a* and their η. $\eta_4 = \eta_1 + \eta_2 - \eta_3$

S	η	Set of places	$[S]$
S_1	$+t_2 - t_4 + t_8 - t_9$	$\{p_4, p_{12}, p_{13}, p_{14}, p_{15}\}$	$\{p_2, p_3, p_8, p_9, p_{10}, p_{11}\}$
S_2	$+t_1 - t_3 + t_7 - t_{10}$	$\{p_5, p_{11}, p_{14}, p_{15}, p_{16}\}$	$\{p_3, p_4, p_7, p_8, p_9, p_{10}\}$
S_3	$+t_2 - t_3 - t_4 + t_7$	$\{p_4, p_{11}, p_{14}, p_{15}\}$	$\{p_3, p_8, p_9, p_{10}\}$
S_4	$+t_1 + t_8 - t_9 - t_{10}$	$\{p_5, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$	$\{p_2, p_3, p_4, p_7, p_8, p_9, p_{10}, p_{11}\}$

Definition 2: A siphon is said to be controlled if it is always marked.

An S^3PR is composed of some state machines (with choices) holding and releasing some common resources. Figs. 1*a* and *b* show an example of S^3PR and its controlled model, respectively.

Property 1 [1]: $S \cup [S] = S_R \cup (\cup_{r \in SR} H(r))$ is the support of a P-invariant I.

Definition 3 [1]: Let $\Omega \subseteq P$ be a subset of places of N. P-vector λ_Ω is called the characteristic P-vector of Ω iff $\forall p \in \Omega$, $\lambda_\Omega(p) = 1$; otherwise $\lambda_\Omega(p) = 0$. η is called the characteristic T-vector of Ω, if $\eta^{\mathrm{T}} = \lambda_\Omega^{\mathrm{T}}\bullet[N]$, where $[N]$ is the incidence matrix where '$\bullet$' means a vector or matrix multiplication.

Physically, the firing of a transition t where ($\eta(t) > 0$, $\eta(t) = 0$ and $\eta(t) < 0$) increases, maintains and decreases the number of tokens in S.

Definition 4 [2]: Let $N = (P, T, F)$ be a net with $|P| = m$, which has k siphons $S_1, S_2, \ldots, S_k$, $m, k \in \mathbf{IN}$, where $\mathbf{IN} = \{0, 1, 2, \ldots\}$. Define $[\eta]_{k \times n} = [\eta_1|\eta_2|\ldots|\eta_k]^{\mathrm{T}}$. $[\eta]$ is called the characteristic T-vector matrix $[\eta]$ of the siphons in N.

Let $\eta_{S_\alpha}, \eta_{S_\beta}, \ldots, \eta_{S_\gamma}$ ($\{\alpha, \beta, \ldots, \gamma\} \subseteq \{1, 2, \ldots, k\}$) be a linear independent maximal set of matrix $[\eta]$. Then $\Pi_E = \{S_\alpha, S_\beta, \ldots, S_\gamma\}$ is called a set of elementary siphons. $S \notin \Pi_E$ is called a strongly dependent siphon if $\eta_S = \sum_{S_i \in \Pi_E} a_i \eta_{S_i}$ where $a_i \geq 0$. $S \notin \Pi_E$ is called a weakly dependent siphon if $\exists$ a non-empty A, $B \subset \Pi_E$, such that $A \cap B = \varnothing$ and $\eta_S = \sum_{S_i \in A} a_i \eta_{S_i} - \sum_{S_i \in B} a_i \eta_{S_i}$ where $a_i > 0$.

Note that Definition 4 and the above calculation of linearly independent vectors do not assume N to be an S^3PR and are applicable to arbitrary nets.

Fig. 1*a* shows an example of weakly dependent siphon. Table 1 below lists the four S and their η. $\eta_4 = \eta_1 + \eta_2 - \eta_3$.

3 Controllability by Li and Zhou

This section reviews the control policy and controllability results in [1, 2, 4, 6]. A monitor V_S and some control arcs are added upon each elementary siphon S similar to adding a resource. Thus, V_S plus $H(V_S)$ (the set of holder places of V_S) forms the support of a new minimal P-invariant. To avoid new siphon generation, the output transitions of V_S end at source transitions ($P^{0\bullet}$) of the processes. The input transitions of V_S end at output transitions of places in $[S]$ such that ${}^\bullet V_S = [S]^\bullet \backslash {}^\bullet[S]$. The initial marking of V_S is set to $M_0(S) - \xi_S$ so that the marking of S is always no less than the control depth variable $\xi_S \geq 1$. See Fig. 1*b* for the added monitors and control arcs.

Li and Zhou adjust ξ_S of elementary siphons associated with a dependent siphon to satisfy a marking linear inequality (MLI) as follows:

Theorem 1 (Theorem 1 in [1]): Let (N_0, M_0) be a net system and $S_0, S_1, S_2, \ldots,$ and S_n be its SMS. Assume that S_0 is a strict dependent SMS with respect to elementary siphons $S_1, S_2, \ldots, S_n$ where $\eta_0 = \sum_{i=1}^{n}(a_i\eta_i) = \sigma_a$. S_0 is controlled if (i) N_0 is extended by n additional control places $V_{S_1}, V_{S_2}, \ldots, V_{S_n}$ such that $S_1, S_2, \ldots, S_n$ are controlled and (ii) if $M_0(S_0) > \sum_{i=1}^{n}(a_i M_0(S_i) - a_i \xi_{S_i}) = \sigma_{M_0} - \sigma_\xi$ where ξ_{S_i} is the control depth variables for S_i, where $\sigma_{M_0} = \sum_{i=1}^{n} a_i M_0(S_i)$ and $\sigma_\xi = \sum_{i=1}^{n} a_i \xi_{S_i}$.

This theorem can be proved based on the so-called marking equality (ME) in Theorem 2(iii):

Theorem 2: Let (N_0, M_0) be a net system and S_0 be a dependent SMS with respect to elementary siphons $S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots, S_{n+m}$, where

$$\eta_{S_0} = \sum_{i=1}^{n}(a_i\eta_{S_i}) - \sum_{j=1}^{m}(b_{n+j}\eta_{S_{n+j}}) = \sigma_a - \sigma_b,$$

$$\sigma_a = \sum_{i=1}^{n}(a_i\eta_{S_i}) \quad \text{and} \quad \sigma_b = \sum_{j=1}^{m}(b_{n+j}\eta_{S_j})$$

Then

(i) $\forall S \in \{S_0, S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots, S_{n+m}\}$, $\eta_S = -\eta_{[S]}$ (characteristic T-vector of the complementary set of siphon S equals the negative of that of S).

(ii) $\lambda_{[S_0]} = \sum_{i=1}^{n}(a_i\lambda_{[S_i]}) - \sum_{j=1}^{m}(b_{n+j}\lambda_{[S_{n+j}]})$, where a_i, $b_j \in R$ (set of real numbers), $i \in \{1, 2, \ldots, n\}$ and $j \in [1, 2, \ldots, m]$ (characteristic P-vectors of the complementary sets of siphon $S_0, S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots, S_{n+m}$ follow the same equation as that of the corresponding characteristic T-vectors).

(iii) The ME holds

$$M([S_0]) = \sum_{i=1}^{n}(a_i M([S_i])) - \sum_{j=1}^{m}(b_{n+j}M([S_{n+j}])),$$
$$M \in R(N, M_0) \tag{1}$$

(total tokens in the complementary sets of siphon $S_0, S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots, S_{n+m}$ follow the same equation as that of the corresponding characteristic T-vectors).

Proof of (i): $S \cup [S] = S_R \cup (\cup_{r \in SR} H(r))$ is the support of a P-invariant I based on Property 1 and $S \cap [S] = \varnothing$. Note that $S_R = S \cap P_R. \forall p \in S \cup [S]$, $I(p) = 1$ (valid for OPN); otherwise, $I(p) = 0$. Thus, $I = \lambda_S + \lambda_{[S]}$. $I^T \bullet [N] = \lambda_S^T \bullet [N] + \lambda_{[S]}^T \bullet [N] = \mathbf{0}$ (by the definition of P-invariant), where $\mathbf{0}$ is a vector with all components being 0.

$$\Rightarrow \eta_S = -\eta_{[S]}$$ □

Proof of (ii): Based on equation $\eta_{S_0} = \sigma_a - \sigma_b$, the fact that $\eta_S = -\eta_{[S]}$ and $\eta_S^T = \lambda_S^T \bullet [N]$, we have

$$\eta_{[S_0]} = \sum_{i=1}^{n}(a_i\eta_{[S_i]}) - \sum_{j=1}^{m}(b_{n+j}\eta_{[S_{n+j}]})$$

$$\Rightarrow \lambda_{[S_0]}^T \bullet [N] = \sum_{i=1}^{n}(a_i\lambda_{[S_i]}^T \bullet [N]) - \sum_{j=1}^{m}(b_{n+j}\lambda_{[S_{n+j}]}^T \bullet [N])$$

$$\Rightarrow \left(\lambda_{[S_0]} - \sum_{i=1}^{n} a_i\lambda_{[S_i]} + \sum_{j=1}^{m} b_{n+j}\lambda_{[S_{n+j}]}\right)^T \bullet [N] = \mathbf{0}$$

If $\zeta = \lambda_{[S_0]} - \sum_{i=1}^{n} a_i\lambda_{[S_i]} + \sum_{j=1}^{m} b_{n+j}\lambda_{[S_{n+j}]} \neq \mathbf{0}$, then ζ is a P-invariant. However, all places in $[S_0], [S_1], [S_2], \ldots, [S_{n+m}]$ are not marked in the initial marking of N and hence the union of $[S_0], [S_1], [S_2], \ldots, [S_{n+m}]$ cannot be the support of a P-invariant. This implies that $\zeta = \mathbf{0} \Rightarrow \lambda_{[S_0]} = \sum_{i=1}^{n} a_i\lambda_{[S_i]} + \sum_{j=1}^{m} b_{n+j}\lambda_{[S_{n+j}]}$. □

Proof of (iii): Multiplying both sides of the equation in (1) by M^T, we have

$$\lambda_{[S_0]} \bullet M^T = \sum_{i=1}^{n} a_i\lambda_{[S_i]} \bullet M^T - \sum_{j=1}^{m} b_{n+j}\lambda_{[S_{n+j}]} \bullet M^T$$

$$\Rightarrow M([S_0]) = \sum_{i=1}^{n} a_i M([S_i]) - \sum_{j=1}^{m} b_{n+j} M([S_{n+j}])$$ □

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This theorem holds for FMS modelled by OPN [not general Petri net (GPN)] such as an S^3PMR, since we have assumed $\forall p \in S \cup [S]$, $I(p) = 1$. However, it can be extended to FMS modelled by GPN such as S^4PR and S^3PGR_2 by replacing M with $W((M(A)))$, the weighted sum of tokens in $A = S$ or $[S]$.

This ME says that the total number of tokens trapped in $[S_0]$ and $[S_i]$ follows the same linear algebraic relationship between η_{S_0} and η_{S_i}, $i = 1, 2, \ldots, n, n+1, \ldots, n+m$. This is because physically $-\eta_S(t)$ is the number of tokens removed from S by firing t once. Now, max $M([S_i]) = M_0(S_i) - 1$ (S_i is said to be limit controlled) for S_i to have tokens. In order for S_0 to be controlled, we have $M(S_0) > \max M([S_0])$ or

$$M(S_0) > \max\left(\sum_{i=1}^{n} a_i M([S_i])\right) - \min\left(\sum_{j=1}^{m} b_{n+j} M([S_{n+j}])\right) \tag{2}$$

To be conservative, the term associated with the negative terms is set to zero. That is, if $M(S_0)$ is large enough to be greater than $\max(\sum_{i=1}^{n} a_i\, M([S_i]))$, then (2) necessarily holds.

However, it may not hold that

$$M(S_0) > a_1(M_0(S_1) - 1) + a_2(M_0(S_2) - 1) + \cdots$$
$$+ a_n(M_0(S_n) - 1) = \sum_{i=1}^{n} a_i(M_0(S_i) - 1) = \sigma_{aM_0} - \sum_{i=1}^{n} a_i$$

That is, S_0 may not be controlled when each S_i is limit controlled. After lowering $M([S_i])$ to $M_0(S_i) - \xi_{Si}$, $\xi_{Si} \geq 1$, where ξ_{Si} is the control depth variable mentioned in [1], for each S_i, it may hold that

$$M(S_0) > a_1(M_0(S_1) - \xi_{S_1}) + a_2(M_0(S_2) - \xi_{S_2}) + \cdots$$
$$+ a_n(M_0(S_n) - \xi_{S_n}) = \sum_{i=1}^{n} a_i(M_0(S_i) - \xi_{Si}) = \sigma_{aM_0} - \sigma_{\xi}$$

This is exactly the MLI. Hence $M(S_0) > \max M([S_0])$ and S_0 is controlled.

Li and Zhou further improved the controllability as follows:

Lemma 1 [4]: Let S_0 be a weakly dependent siphon with respect to elementary siphons $S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots, S_{n+m}$ in net system (N, M_0), with $\eta_{S_0} = \sum_{i=1}^{n}(a_i\eta_{S_i}) - \sum_{j=1}^{m}(b_{n+j}\eta_{S_{n+j}})$. S_0 is controlled if inequality $M_0(S_0) > \sum_{i=1}^{n} a_i M_0(S_i) - D_1 - [(\sum_{j=1}^{m} b_{n+i} M_0\ (S_j) - D_2]$ holds, where $D_1 = \min\{\sum_{i=1}^{n} a_i M(S_i) | M = M_0 + [N]\bullet X,\ M \geq \mathbf{0},\ X \geq \mathbf{0}\}$ and $D_2 = \max\{\sum_{j=1}^{m} b_{n+j} M(S_j) | M = M_0 + [N]\bullet X,\ M \geq \mathbf{0},\ X \geq \mathbf{0}\}$, where $\mathbf{0}$ is a vector with all components being 0.

4 Better controllability

This section improves the controllability in last section by upgrading the estimate of D_2 corresponding to the negative terms in η_{S_0} defined in Definition 4 based on the following:

Theorem 3: Let S_0 be a weakly dependent siphons such that $\eta_{S_0} = \sigma_a - \sigma_b$. Let S_0 be unmarked under marking M. Then (i) $M(H(r)) = M_0(r)$, where $r \in S_0$ and $H(r) \subset [S_0]$. (ii) $M(H(r) \cap [S_0]) = M_0(r)$, where $r \in S_0$ and $H(r) \cap [S_0] \neq [S_0]$.

Proof of (i): Since $\{r\} \cup H(r)$ is the support of a minimal P-invariant

$$M(H(r)) + M(r) = M_0(r) \tag{3}$$

$M(r) = 0$ since $r \in S_0$ and S_0 is unmarked, $M(S_0) = 0$. The above equation now reduces to $M(H(r)) = M_0(r)$.

Proof of (ii): Again, $M(H(r)) = M_0(r)$. $M(H(r) \cap [S_0]) + M(H(r) \cap S_0) = M(H(r))$. $M(H(r) \cap S_0) = 0$ since $M(S_0) = 0$. Hence, $(H(r) \cap [S_0]) = M_0(r)$. □

When S_0 is empty under M, $\exists S_j$, where $j \in [1, 2, \ldots, m]$ such that $r \in S_j$ and $H(r) \subset [S_0]$. For instance, $r = p_{14} \in S_3$ in Fig. 1*a*. $H(r) = \{p_3, p_8, p_{10}\} \subset [S_0]$, $S_0 = S_4$. We have $M(H(r)) = M_0(r) = c$ and

$$M([S_3]) \geq c \tag{4}$$

by Theorem 3(i) and the fact that $H(p_{14}) \subset [S_3]$.

Since S_0 is empty under M, $\exists p_1, r_1, p_n, r_n$ (e.g. p_2, p_{13}, p_7, p_{16} in Fig. 1*a*) such that $p_1 \in H(r_1)$, $p_n \in H(r_n)$, $H(r_1) \cap S_0 \neq \varnothing$ and $H(r_n) \cap S_0 \neq \varnothing$. Then $M(p_n) = M_0(r_n)$ and $M(p_1) = M_0(r_1)$ (Theorem 3(ii)). Otherwise, $M(H(r_1) \cap S_0) > 0$ or $M(H(r_n) \cap S_0) > 0$ and S_0 is marked.

In Fig. 1*a*, $D_2 = \max\{\sigma_{bM} | M = M_0 + [N]\bullet X,\ M \geq \mathbf{0},\ X \geq \mathbf{0}\} = \max\{M(S_3) | M = M_0 + [N]\bullet X,\ M \geq \mathbf{0},\ X \geq \mathbf{0}\} = M_0(S_3)$ based on Lemma 1, where $\sigma_{bM} = \sum_{j=1}^{m} b_{n+j} M(S_j)$. That is

$$\max M(S_3) = M_0(S_3) \text{ or } \min M([S_3]) = \mathbf{0} \tag{5}$$

However, max $M(S_3) \leq M_0(S_3) - c$ is based on (4), which contradicts with (5). Note that (5) implies that min $M([S_3]) = 0$, whereas (4) implies that min $M([S_3]) = c$. In general, a better estimate of D_2 is shown in the following:

Theorem 4: Let $\wp(S_j) = \{r | H(r) \subset ([S_0] \cap [S_j])\}$. (i) min $M([S_j]) = \sum_{r\in\wp(S_j)} M_0(r)$. (ii) $D_2 = \max\{\sigma_{bM} | M = M_0 + [N]\bullet X,\ M \geq \mathbf{0},\ X \geq \mathbf{0}\} = \sum_{j=1}^{m} b_{n+j}(M_0(S_j) - \sum_{r\in\wp(Sj)} M_0(r)) - \min\{\sum_{j=1}^{m} b_{n+j} \sum_{r\in(R(Sj)\ -\ \wp(Sj))} M(H(r) \cap [S_j]) | M = M_0 + [N]\bullet X, M \geq \mathbf{0}, X \geq \mathbf{0}\}$, where $R(S_j)$ is the set of resource places in S_j.

Proof of (i): By Theorem 3(i), $M(H(r)) = M_0(r)$, $r \in \wp(S_j)$. $M([S_j]) \geq M(H(r)) = M_0(r)$ since $H(r) \subset ([S_0] \cap [S_j])$. Thus, $M([S_j]) \geq M_0(r)$, $\forall r \in \wp(S_j)$. Similarly, $M([S_j]) \geq \sum_{r\in\wp(Sj)} M(H(r)) = \sum_{r\in\wp(Sj)} M_0(r)$ since $(\cup_{r\in\wp(Sj)} H(r)) \subset [S_j]$. Hence, $\min M([S_j]) = \sum_{r\in\wp(Sj)} M_0(r)$. □

Proof of (ii): In general

$$M([S_j]) = M_0(S_j) - M(S_j) = \sum\nolimits_{r\in\wp(Sj)} M(H(r) \cap [S_j]) + \sum\nolimits_{r\in(R(Sj)-\wp(Sj))} M(H(r) \cap [S_j]) = \sum\nolimits_{r\in\wp(Sj)} M(H(r)) + \sum\nolimits_{r\in(R(Sj)-\wp(Sj))} M(H(r) \cap [S_j]) \quad (6)$$

Equation (6) implies that

$$M(S_j) = M_0(S_j) - M([(S_j)]) = M_0(S_j) - \sum_{r\in\wp(Sj)} M_0(r) - \sum_{r\in(R(Sj)-\wp(Sj))} M(H(r) \cap [S_j])$$

Substituting this equation into D_2, we have

$$D_2 = \max\{\sigma_{bM} | M = M_0 + [N]\bullet X, M \geq \mathbf{0}, X \geq \mathbf{0}\} = \sum_{j=1}^{m} b_{n+j}\left(M_0(S_j) - \sum_{r\in\wp(S_j)} M_0(r)\right) - \min\left\{\sum_{j=1}^{m} b_{n+j} \sum_{r\in(R(S_j)-\wp(S_j))} M(H(r)\cap[S_j]) | M = M_0 + [N]\bullet X, M \geq \boldsymbol{0}, X \geq \boldsymbol{0}\right\}$$

□

For instance, in Fig. 1*a*, there is only one negative b_j term corresponding to S_3. $\wp(S_j) = \{p_{14}\}$. $D_2 = \max\{\sigma_{bM} | M = M_0 + [N]\bullet X,\ M \geq \boldsymbol{0},\ X \geq \boldsymbol{0}\} = (M_0(S_j) - M_0(p_{14}))$. Set $M_0(p_{13}) = a$, $M_0(p_{14}) = b$, $M_0(p_{15}) = c$ and $M_0(p_{16}) = d$. Using Lemma 1, we have $S_0 = S_4$ is controlled if inequality $M_0(S_0) > \sum_{i=1}^{2} M_0(S_i) - D_1 - [(M_0(S_3) - D_2]$ holds. Using $M_0(S_0) = a + b + c + d$, $M_0(S_1) = a + b + c$, $M_0(S_2) = b + c + d$, $M_0(S_3) = b + c$, $D_1 = \min M(S_1) + \min M(S_2) = 2$ and $D_2 = M_0(S_3) - c$. The inequality is now

$$a + b + c + d > a + b + c + b + c + d - 2 - c = a + b + c + d + b - 2 \Rightarrow 2 > b$$

which (i.e. the controllability) holds when $b = 1$.

On the other hand, if we follow Li and Zhou, $D_2 = 0$, the inequality now becomes $2 > b + c$, which (i.e. the controllability) can never hold since $b \geq 1$, $c \geq 1$ and $b + c \geq 2$. A subsequent time-consuming linear integer programming (LIP, Theorem 5 in [1]) may be performed to decide whether S_0 is controlled. This LIP is not required using Theorem 4 when $b = 1$.

Similar to Theorem 2, Theorem 4 holds for FMS modelled by OPN [not GPN] such as an S^3PMR, since we have assumed $\forall p \in S \cup [S]$, $I(p) = 1$. However, it can be extended to FMS modelled by GPN such as S^4PR and S^3PGR_2 by replacing M with $W((M(A)))$, the weighted sum of tokens in $A = S$ or $[S]$.

5 Conclusion

Li and Zhou [1] indicated that the control policy for weakly dependent siphons is rather conservative. As a result, a weakly dependent siphon may be already controlled and needs no monitor even though it fails the controllability. We have improved the controllability by providing a better estimate of D_2 corresponding to the negative terms in $\boldsymbol{\eta}_{S_0}$ defined in Definition 4.

6 References

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