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The portfolio strategy and hedging: A spectrum perspective on mean–variance theory

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ABSTRACT

This paper aims to establish a portfolio strategy using information of lead–lag relationship. The efficient frontier in mean–variance theory has confirmed that the spectrum strategy established by the lead–lag relationship yields superior performance assuming the same volatility. And then we construct the spectrum portfolios based on two approaches: a recursive approach, which uses a recursive method in the lead–lag relationship, and a joint approach, which combines two lead–lag relationships. The effect of the spectrum strategy using mutual fund data from 1999 through 2009 is examined. The results indicate that the spectrum portfolio has a superior performance as compared to the benchmark with both approaches. Furthermore, the spectrum portfolio by recursive approach maintains superior performance in hedging.

1. Introduction

Different asset prices change during various phases of the business cycle, which indicates that levels of aggregate economic activity exhibit expansion and contraction (Narayan & Popp, 2009). Keynes (1976) proposed that asset prices are influenced by the business cycle because investors care more about near-term factors, which are mainly determined by the business cycle. Because asset prices are closely related to the business cycle and different kinds of asset price behavior emerge during various phases of business cycle, there are many lead–lag relationships among different asset prices or portfolios (Kanas & Kouretas, 2005). Return and risk of portfolio depend on the structure of the correlations between assets (for example, Montgomery and Singh (1984), Dowd (2000), Yu and Wu (2001)). Thus, if both a correlation and lead–lag relationship exist for two assets and the information on the lead–lag relationship is properly used, it may increase the efficiency of portfolio strategies.

The idea of increasing portfolio return in this way is based on the concept of making predictions based on the lead–lag relationship (Fernández-Rodríguez, González-Martel, & Sosvilla-Rivero, 2000). Milton and Schwartz (1963) emphasized that the anticipated changes in the money supply should be visible in the market information and in the lead–lag relationship between two markets, the money and stock markets, which can then be used to forecast stock prices. In addition, the information of lead–lag relationship could be regarded as providing signals, e.g., the appropriate time to enter a market. Chiang, Nelling, and Tan (2008) used the lead–lag relationship in a vector autoregressive model to examine the speed of price adjustment in two Chinese stock markets. However, surprisingly, few studies have considered the application of lead–lag relationships in portfolio strategy. In this paper, we will try to use the information of lead–lag relationships to increase portfolio return. We call the portfolio a “spectrum portfolio” under the “spectrum strategy” if the portfolio is constructed using the information of lead–lag relationship.

How do we use the information of lead–lag relationship? When an investor considers this information, he will act as follows. In a bull market (expansion), lag assets are expected to rise because lead assets have already began to rise; thus, he will buy the

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lag asset and sell the lead asset that has already risen. In a bear market (contraction), lag assets are expected to fall because lead assets have already fallen; therefore, he will sell the lag asset and buy the lead asset that has already fallen.

The above argument seems easy; however, can the spectrum portfolio really perform well? In the next section, we will provide a proof that the portfolio's expected return may increase if additional information from the lead–lag relationship can be incorporated. Roughly, the concept may be described in the following manner. H^0 in Fig. 1 represents the efficient frontier derived by the mean–variance theory, where μ and σ are the expected return and standard deviation, respectively. X, Y, and X′ represent the portfolios, comprising assets A, B, or C. If there is information that can increase the portfolio's expected return under the same standard deviation, then the additional information makes H^0 shift up to H^1.

1.1. Existence of information value (EIV)

Under certain conditions, H^1 exists such that $H^1 > H^0$.

We will show for the existence of H^1 under some conditions in the next section.

After analyzing EIV, it is evident that not every lead–lag relationship satisfies the conditions of EIV. Therefore, we examine what types of assets are selected for the spectrum strategy when faced with varying lead–lag relationships. The characteristics of the spectrum strategy will be identified after incorporating the information on the lead–lag relationship. Empirically, substantial individual asset price volatility may offset the efficiency of the spectrum strategy. Hence, the characteristics of the spectrum strategy can help us make a better choice regarding feasible lead–lag relationships. We select some mutual funds according to the characteristics of the spectrum strategy with the expectation that the selected spectrum portfolios will show superior performance. In addition, we adopt the recursive method to identify the dynamic spectrum portfolio, $\widehat{X}$. This action, which is referred to as the recursive approach, seems to strengthen the same lead–lag relationship twice on the benchmark spectrum portfolio, X. Hence, $\widehat{X}$ is expected to perform better than X. In fact, our empirical results support this belief. Another approach (joint approach) indicates that the effect of the lead–lag relationship is the sum of two lead–lag relationships, while strengthening the same lead–lag relationship twice. To check the characteristics of the spectrum strategy, we consider the information of lead–lag relationships among three or more assets. If A leads B, B leads C and A leads C, then both H^1 and H^2 exist such that $H^2 > H^1 > H^0$ (see Fig. 1). We expect that the returns using the information of lead–lag relationship among the three assets will be better than between two assets.

We use data on the bond, equity and energy funds from 1997 to 2009 in this paper. The reason we choose mutual fund data in our paper is that mutual fund prices tend to be less volatile than particular asset prices such as stock prices; therefore, the characteristics of spectrum strategy are less likely to be dominated by volatility. Our results can be extended to products whose underlying is mutual funds, i.e. funds of funds. In addition, spectrum strategy can be used to enhance the efficiency of hedge fund management because hedge funds can be built using long and short positions simultaneously and as well as spectrum portfolios. Spectrum strategy may be regarded as hedge strategy because it comprises both long and short positions. We use the return/risk ratio to measure the hedge level. A comparison of the return/risk ratios of $\widehat{X}$ and X reveals the hedge effects of the spectrum strategy.

The paper is organized as follows. In Section 2, we provide an example to show the *existence of information value* and find the characteristics of spectrum strategy. We discuss the construction of dynamic spectrum strategies in Section 3. Section 4 verifies performance and analyzes the empirical results. The applications of spectrum strategy (for example, to hedging) are introduced in Section 5, and Section 6 presents the conclusion.

2. Setting up the spectrum strategy using lead–lag relationship

First, we present an example which shows information of lead–lag relationship could possibly increase expected return and find the mathematical condition for EIV. In this way, we can more precisely decide when and how we should apply the information of lead–lag relationship. Then, we find the optimal weights of spectrum strategy under the target volatility level using benchmark strategy.

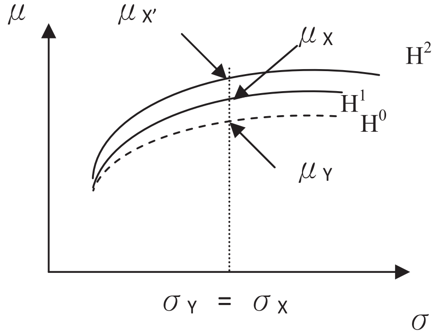

Fig. 1. The efficient frontier under different levels of information.

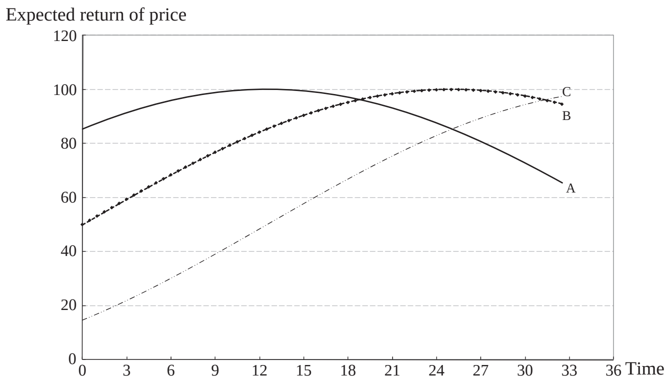

Fig. 2. Three expected returns of lead–lag relationship in terms of waves.

2.1. Value of incorporating lead–lag information in a scenario tree

Suppose that there are two assets, A and B, with prices denoted as $P_A(\cdot)$ and $P_B(\cdot)$, and that the correlation of A and B is given by ρ_{AB}. $P_i(t)=\psi_i(t)+\varepsilon_i(t)$, $i=A,B$, where ψ is the drift term including the trend and cycle and ε is the normal white noise. μ_X and μ_Y are the expected returns of portfolios X and Y, comprising assets A and B, respectively. If the lead–lag relationship is predictable, we expect that $\mu_X>\mu_Y$ in a period. This concept can be expressed in a mathematical manner. Time t_1 belongs to time interval $[T_1,T_2]$. At initial time T_1, Y comprises weight w of A and weight $1-w$ of B until time T_2. Thus, at time T_2, the accumulated expected return of Y, $\mu_Y|_{[T_1,T_2]}$, is

$$\mu_Y\Big|_{[T_1,T_2]}=w\mu_A\Big|_{[T_1,T_2]}+(1-w)\mu_B\Big|_{[T_1,T_2]}=w\frac{P_A(T_2)-P_A(T_1)}{P_A(T_1)}+(1-w)\frac{P_B(T_2)-P_B(T_1)}{P_B(T_1)}.$$

As we do with Y, we set up X with w in A and $1-w$ in B at time T_1 but we only long B in a bull market at time t_1 because we expect that the lag asset will rise. Note that the location of t_1 depends on the lag period and the information of lead–lag relationship that has been incorporated. Based on the idea that we will sell the stock with the high price and buy that with the low price, we sell A and hold only B. In other words, X is a spectrum portfolio. The accumulated expected returns on X, $\mu_X|_{[T_1,T_2]}$, is

$$\mu_X\Big|_{[T_1,T_2]}=\mu_X\Big|_{[T_1,t_1]}+\mu_X|_{[t_1,T_2]},$$

where

$$\mu_X\Big|_{[T_1,t_1]}=w\mu_A\Big|_{[T_1,t_1]}+(1-w)\mu_B|_{[T_1,t_1]};$$

$$\mu_X\Big|_{[t_1,T_2]}=\mu_B\Big|_{[t_1,T_2]}.$$

Then, $\mu_X>\mu_Y$ on $[T_1,T_2]$ depends on the following condition:

$$\begin{aligned}&w\left(\frac{P_A(t_1)}{P_A(T_1)}-\frac{P_B(t_1)}{P_B(T_1)}\right)-w\left(\frac{P_A(T_2)}{P_A(T_1)}-\frac{P_B(T_2)}{P_B(T_1)}\right)\\&+P_B(T_2)\left(\frac{1}{P_B(t_1)}-\frac{1}{P_B(T_1)}\right)>1-\frac{P_B(t_1)}{P_B(T_1)}.\end{aligned} \tag{1}$$

Condition 1 will hold if the first term (term I) and the third term (term III) are positive and the second term (term II) is negative in a bull market. The first and second terms are related to both asset prices and the point in time. As in an upward movement in the market, the price movement by A leads the price of B, meaning that at the start of the period, the rate of increase in the slope of the price changes for A should exceed that of B. However, after some point in time, price A should be less than price B, as the rising trend is about to revert. The above statements indicate that the changes in the relative returns are related to the change in the relative slope of the two waves, which is indicated by the shape of the asset over time. This means that we can use two cycles indicating the lead–lag relationship to generate higher returns.

Suppose that the drift term can be formed in a cycle.[1] The price of A leads the price of B, and the price of B leads the price of C, such that $P_A=\gamma_A\sin(\omega t+\lambda_{AB})+\varepsilon_A$, $P_B=\gamma_B\sin(\omega t)+\varepsilon_B$, and $P_C=\gamma_C\sin(\omega t-\lambda_{BC})+\varepsilon_C$ where the lag period between A and B is λ_{AB} (angular frequency); the lag period between B and C is λ_{BC}; γ represents amplitude; ω represents angular frequency in one

[1] In Harvey and Jaeger (1993) (HJ), they assumed the observed series can be modeled by linear trend, stochastic cycle and normal white-noise disturbances. In our paper the asset price follow HJ model and we reduce the HJ model to simply consider the cycle component alone.

Table 1
A simulated case: the various term values of the left-side of Condition 1.

	Price A vs. price B	Price B vs. price C
Term I	−0.1025	−0.2241
Term II	−0.3241	−0.7920
Term III	−0.1873	−0.6044
Sum	0.0342	−0.0365

The sum is the sum of terms I, II and III.

cycle and t is the current time. We set $\omega = 0.02\pi$, $\lambda_{AB} = \lambda_{BC} = \frac{\pi}{4}$ and $\gamma_A = \gamma_B = \gamma_C = 50$. To avoid negative prices, we let waves (expected return of price) A, B and C shift upward one unit. This does not affect our results. These three waves move in price and time (ignoring noise terms), as shown in Fig. 2.

We can compute the values of terms I, II and III under Condition 1 given the paths in Fig. 2. For example, given $T_1 = 3$, $t_1 = 6$ and $T_2 = 12$, we have the values of terms I, II and III under Condition 1 as in Table 1.

In Table 1, the sum represents the sum of terms I, II and III. It may be a positive return. Consistent with Condition 1, at the start of the period, the first term is more likely to be positive, but the possibility that the second term will be positive is also high; therefore, Condition 1 does not necessarily hold. However, after time t_1, the first term is still positive, but the probability of the second term's being negative increases, which means that Condition 1 is more likely to hold during this period. Thus, we attempt to increase T_2, keeping other parameters fixed, to verify whether Condition 1 is more likely to hold at the end of a down trend and during the upward trend.

Table 2 shows that when T_2 is lengthened, the probability that the return will become positive is higher, and higher returns are likely to result. This means that not only will λ affect the performance of the spectrum strategy; whether it is a full cycle or not but will also affect performance.

2.2. Characteristics of the spectrum strategy

The simulation results in Section 2.1 provide the feasibility of EIV. Next, we want to find the condition of the EIV under the assumption that drift terms A and B are in the form of a sin function: e.g., $\psi = \gamma \sin(\omega t + \lambda)$. Using the relationship between trigonometric functions, the polynomial function $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$ and the first-order approximation $\sin(\omega t + \lambda_{AB}) \approx (\omega t + \lambda_{AB})$, we have

$$\psi_i = \gamma\left[(\omega t + \lambda) - \frac{(\omega t + \lambda)^3}{3!} + \frac{(\omega t + \lambda)^5}{5!} \ldots\right], i = \text{A or B}. \tag{2}$$

Substituting Eq. (2) into Condition 1 and omitting the terms above the third power in the polynomial function, we have

$$\frac{\lambda_{AB}}{\omega} > \frac{T_1 T_2 t_1 - T_1^2 T_2 + T_1^2 t_1 - T_1 t_1^2}{w t_1 (T_2 - t_1) + T_1 T_2 - t_1 (T_1 + T_2) + t_1^2}. \tag{3}$$

Condition 3 indicates the rules for selecting two assets to form a portfolio and also indicates the characteristics of the spectrum strategy. Based on the left-side of Condition 3, it is more possible for the inequality to hold if λ_{AB} and $\frac{1}{\omega}$ are both large. The economic meaning of Eq. (3) is straightforward because when λ_{AB} and $\frac{1}{\omega}$ are larger, the information (including the lag period from asset A to B and the total length of the complete cycle) is more sufficient and clearer. Based on $\frac{1}{\omega}$, a longer cycle will have more clean cyclical pattern (A'Hearn & Woitek, 2001). Then, it is more likely to increase the spectrum portfolio's performance. As for the right side of inequality in Eq. (3), the larger $T_2 - t_1$ is, the more likely the spectrum portfolio performs well. A larger value for $T_2 - t_1$ means that the spectrum strategy is more suitable for long-term investors. The reason why a long-term investor can earn better returns with the spectrum strategy is that business cycle is not a short-term phenomenon; therefore, performance is better when the evaluation period is sufficiently long. Additionally, because investors cannot find the exact cut point for t_1, more time is required for the information of the lead–lag relationship of the two asset classes to emerge.

Table 2
A simulated case: the effects of stretching T_2 in various term values.

	Price A vs. price B			Price B vs. price C		
	$T_2 = 12$	$T_2 = 14$	$T_2 = 16$	$T_2 = 12$	$T_2 = 14$	$T_2 = 16$
Term I	−0.1025	−0.1025	−0.1025	−0.2241	−0.2241	−0.2241
Term II	−0.3241	−0.3986	−0.4714	−0.7920	−1.0059	−1.2267
Term III	−0.1873	−0.1969	−0.2051	−0.6044	−0.6827	−0.7600
Sum	0.0342	0.0992	0.1638	−0.0365	0.0991	0.2425

Sum is the sum of terms I, II and III.

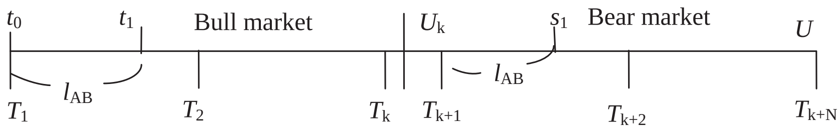

Fig. 3. Partition of the lead–lag periods.

In sum, the spectrum strategy has three characteristics: a larger lag period, a more complete cycle, and a larger T_2-t_1 can enhance spectrum strategy performance. We will construct the spectrum strategy to empirically test these characteristics. Because it is easier for investors to obtain the information of lead–lag relationship than it is for them to determine the exact lag period of lead–lag relationship, we also test whether returns using the information of lead–lag relationships among three assets are better than the same information for two assets.

Now, we want to determine the optimal weight of the spectrum strategy. Suppose we are in a bull market, and we know that A leads B. Incorporating this information, we sell h weight of the lead asset A, and t_1 is decided using the lag period. The target volatility of the benchmark strategy becomes $Var(Y(t_1^{H^0}))$, which is 0.5 weight of asset A and 0.5 weight of asset B conditional on information H^0 at time t_1. The volatility of A based on information H^0 at time t_1 is

$$Var\left(X\left(t_1^{H^1};\lambda_{AB}\right)\right)=(0.5-h)^2\sigma_A^2+(0.5)^2\sigma_B^2+2\rho_{AB}(0.5-h)(0.5)\sigma_A\sigma_B.$$

The restriction is $Var(X(t_1^{H^1};\lambda_{AB}))=Var(Y(t_1^{H^0}))$. Hence, the optimal weight is

$$h=\frac{\sigma_B}{\sigma_A}\rho_{AB}+1. \tag{4}$$

Eq. (4) is independent of the expected returns because we find the optimal weight on the different efficient frontiers (H^0 and H^1). Eq. (4) is dependent on relative coefficients and variance. Once the available information has been exploited, we just need to adjust the variance and relative coefficient for the leading and lagging assets such that variance of the spectrum strategy is consistent with the target variance.

3. Construction of the spectrum strategy

According to Eq. (4), a static portfolio can be established. However, the initial assumption that volatility is a fixed value is apparently inappropriate. Therefore, it is necessary to establish a dynamic portfolio. In the following paragraphs, we present cases with two and three assets corresponding to the recursive and joint approaches, respectively. Both approaches use the spectrum strategy for building the portfolio; thus, we expect that those portfolios will show superior performance.

Suppose there are three assets A, B and C for which A leads B, B leads C and A leads C. The correlations between A, B and C are ρ_{AB}, ρ_{BC}, and ρ_{AC}, respectively. The starting time is t_0, with intervals of 1 month in the series $T_1, T_2, \ldots, T_{k+N}$, and we assume that t_0 to U_k indicates a bull market and that U_k to U indicates a bear market, as shown in Fig. 3.[2]

Remark. In the following, we use the month as the minimum unit of time. If the lead–lag period is less than 1 month, we assume that the volatility for every month is fixed during each cycle and that the spectrum strategy is reestablished at the start of every month. Similarly, if the lead–lag period is less than 2 months, we assume that the volatility is fixed for every two-month period. This can reduce the bias that arises from assuming the volatility to be fixed. One thing to note is that the interval must exceed the number of periods of the lead–lag relationship.

3.1. The dynamic spectrum strategy with two assets and recursive approach

Assume that $w_A^Y(w_A^X)$ and $w_B^Y(w_B^X)$ are the weights of A and B for portfolio $Y(T_i)(X(T_i)), \forall i=1,\ldots,k+N$, and $(w_A^Y, w_B^Y)=(0.5, 0.5)$. Each t_i and s_i denote the timing that causes the difference between X and Y, and they are obtained from the lag period $l_{AB}(=\lambda_{AB}/2\pi\omega)$, i.e. $t_i=T_i+l_{AB}$, $s_i=T_{k+i}+l_{AB}$, as in Fig. 4.

3.1.1. Bull market

In the first partition $[T_1,T_2]$, the portfolio $X(T_1)$ has the same strategy as $Y(T_1)$ at the initial time T_1 because A is expected to rise before B; hence, at t_1 we reduce the h_1 of A until T_2; that is, the weight of portfolio $X(t_1)$ is $(w_A^X, w_B^X)=(0.5-h_1, 0.5)$ in the bull market. Likewise, in the second partition $[T_2,T_3]$, the weight of $X(T_2)$ is $(w_A^X, w_B^X)=(0.5, 0.5)$ at the initial time T_2 in the second partition, as is that of $Y(T_2)$. However, at time t_2, $(w_A^X, w_B^X)=(0.5-h_2, 0.5)$. We repeat and continue this process until we reach T_k. To avoid original benchmark Y has no rebalancing, we use $X(t_1)$ as the updated benchmark. To incorporate and repeat the information of lead–lag relationship repeatedly, we have a new spectrum portfolio, $\widehat{X}$, with respect to X. The weight of portfolio $\widehat{X}(t_1)$ is

[2] T_k may not be equal to U_k.

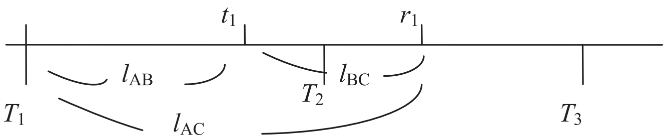

Fig. 4. Time setting in bull market and three assets.

$\left(w_A^{\hat{X}}, w_B^{\hat{X}}\right) = \left(0.5-h_1, 0.5+\ \hat{h}_1\right)$, where $\hat{h} = \frac{-\xi_u+\sqrt{\xi_u^2-4\upsilon_u\kappa_u}}{2\upsilon_u}$.[3] The above process is equivalent to the iteration used by the spectrum strategy. Therefore, $\hat{X}$ exploits the information of the lead–lag relationship strongly than X. Hence we expect the performance of $\hat{X}$ is better than X when X is a benchmark.

Note that we do not place restrictions on the size and sign of h, which means that we allow for leverage tools such as margin trading and shorting. If h is negative in some cases, then this would conflict with our strategy of increasing the weight of asset B. We will deal with this problem by adjusting the data in the following empirical verification process.

3.1.2. Bear market

In the partition $[T_{k+1}, T_{k+2}]$, portfolio $X(T_{k+1})$ has the same strategy as does $Y(T_{k+1})$ at time T_{k+1}. Because A is expected to fall before B, hence, at s_1 we reduce the h_{k+1} of B until T_{k+2}; that is, the weight of portfolio $X(s_1)$ is $(w_A^X, w_B^X) = (0.5, 0.5-h_{k+1})$ in the bull market. Likewise, in the next partition $[T_{2+k}, T_{3+k}]$, $(w_A^X, w_B^X) = (0.5, 0.5)$ at time T_{k+2}, and the same holds for $Y(T_2)$. However, at time s_2, the weight of $X(s_2)$ is $(w_A^X, w_B^X) = (0.5, 0.5-h_{k+2})$. We repeat and continue this process until final partition and T_{k+N}. Here h_i for $i = 1, \ldots, k+N$ can be obtained using Eq. (4). Similar to bull market which $\frown$X is constructed recursively, the weight of portfolio $\hat{X}(t_1)$ is $\left(w_A^{\hat{X}}, w_B^{\hat{X}}\right) = \left(0.5+\ \hat{h}_{k+1}, 0.5-h_{k+1}\right)$, where

$$\hat{h}_{k+1} = \frac{-\xi_d + \sqrt{\xi_d^2 - 4\upsilon_d\kappa_d}}{2\upsilon_d},$$

and so on.[4]

3.2. The dynamic spectrum strategy with three assets and joint approach

Apart from strengthening the same lead–lag relationship twice, integrating two or more lead–lag relationships can also enhance the effect of a spectrum strategy. We provide the following case with three assets. Assume that $Y(T_i)$, $\forall i = 1, \ldots, k+N$, which has equal weights among A, B and C, or $(w_A^Y, w_B^Y, w_C^Y) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Assume that A leads B for l_{AB}, B leads C for l_{BC} and A leads C for l_{AC}. The lead–lag relationship satisfies $l_{AC} \approx l_{AB} + l_{BC}$. The current time is $t_0 (= T_1)$, and for each i, $t_i = T_i + l_{AB}$ and $r_i = t_i + l_{BC}$. The details are shown in Fig. 4.

3.2.1. Bull market

At T_1, when portfolio $X(T_1)$ is at time t_0, it uses the same strategy as $Y(T_1)$. Because A is expected to rise before B, hence, at t_1 we reduce the h_1 of A until r_2; that is, the weight of portfolio $X(t_1)$ is $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}-h_1, \frac{1}{3}, \frac{1}{3}\right)$. At r_1, we reduce the $\theta_1$of B until T_3. The weight of portfolio $X(r_1)$ is $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}-h_1, \frac{1}{3}-\theta_1, \frac{1}{3}\right)$. Again, let us return to the original weight, $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, at time T_3; spectrum portfolio $X(T_3)$ uses the same strategy as $Y(T_3)$. The weight of $X(t_2)$ is $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}-h_2, \frac{1}{3}, \frac{1}{3}\right)$ at time t_2, and the weight of $X(r_2)$ is $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}-h_2, \frac{1}{3}-\theta_2, \frac{1}{3}\right)$ at time r_2. We repeat and continue this process until T_k. The optimal weights h_i^* and θ_i^* are

$$h_i^* = \frac{2}{3}\left(\rho_{AB}\frac{\sigma_B}{\sigma_A} + \rho_{AC}\frac{\sigma_C}{\sigma_A}\right) + \frac{2}{3}$$

and

$$\theta_i^* = 2\rho_{AB}\left(\frac{1}{3} - h_1^*\right)\frac{\sigma_A}{\sigma_B} + \frac{2}{3}\rho_{BC}\frac{\sigma_C}{\sigma_B} + \frac{2}{3}, \forall i = 1, 3, .., k.$$

3.2.2. Bear market

The movements of A, B and C in a bear market are the opposite of the movements in the bull market. At time s_1, we reduce the α_1 of C until r_{k+1}, i.e., the weight of portfolio $X(T_{k+1})$ is $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}-\alpha_1\right)$. At r_{k+2} we reduce the δ_1 of B until T_{k+3}, i.e., the weight of portfolio $X(s_1)$ is $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}, \frac{1}{3}-\delta_1, \frac{1}{3}-\alpha_1\right)$. The same procedure begins at T_3, and the weights of $X(s_2)$ and X

[3] Here $\xi_u = (\sigma_B^2 + \rho_{AB}\sigma_A\sigma_B - 2\rho_{AB}h\sigma_A\sigma_B)$; $\kappa_u = -(h\sigma_A^2 - h^2\sigma_A^2 + \rho_{AB}h\sigma_A\sigma_B)$; $\upsilon_u = \sigma_B^2$.

[4] Here $\xi_d = (\sigma_A^2 + \rho_{BA}\sigma_B\sigma_A - 2\rho_{BA}h\sigma_B\sigma_A)$; $\kappa_d = -(h\sigma_B^2 - h^2\sigma_B^2 + \rho_{BA}h\sigma_B\sigma_A)$; $\upsilon_d = \sigma_A^2$.

Table 3
The partition of the cycles.

	First cycle	Second cycle	Third cycle
Bull market	1999/01–2000/03	2001/11–2004/02	2005/05–2007/08
Bear market	2000/04–2001/10	2004/03–2005/04	2007/09–2009/02

Table 4
Comparing the performance of X, Y and ⌐X in three pairs and three cycles.

		ABd vs. BEy			BEy vs. CEgy			ABd vs. CEgy		
		Y	X	⌐X	Y	X	⌐X	Y	X	⌐X
1st	Re	−0.073	0.084[a]	0.101[b]	0.231	0.387[a]	0.354	0.151	0.422[a]	0.492[b]
2nd	Re	0.116	−0.028	0.106[b]	0.476	0.516[a]	0.717[b]	0.628	0.563	0.717[b]
3rd	Re	−0.263	−0.231[a]	−0.206[b]	0.414	0.646[a]	0.710[b]	−0.021	−0.230	−0.057[b]

If Re of X is larger than Re of Y, we denote 'a' in the upper right corner.
If Re of ⌐X is larger than Re of X, we denote 'b' in the upper right corner.

(r_{k+2}) are $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} - \alpha_2\right)$ and $(w_A^X, w_B^X, w_C^X) = \left(\frac{1}{3}, \frac{1}{3} - \delta_2, \frac{1}{3} - \alpha_2\right)$, respectively. We continue with this procedure until T_{k+N}, where

$$\alpha_i^* = \frac{2}{3}\left(\rho_{BC}\frac{\sigma_B}{\sigma_C} + \rho_{AC}\frac{\sigma_A}{\sigma_C}\right) + \frac{2}{3}$$

and

$$\delta_i^* = 2\rho_{BC}\left(\frac{1}{3} - h_1^*\right)\frac{\sigma_C}{\sigma_B} + \frac{2}{3}\rho_{AB}\frac{\sigma_A}{\sigma_B} + \frac{2}{3}, \forall i = k+1, .., k+N.$$

4. The performance of the spectrum strategy

4.1. The data

We use monthly mutual fund data from January of 1997 to November of 2009. Hsu, Yen, Chang, and Chou (2011) found three lead–lag relationships between nine mutual funds. Here, we use lead–lag relationships in which a bond fund leads a stock fund, a stock fund leads an energy fund and a bond fund leads an energy fund. In this way, we examine the characteristics of the spectrum strategy and the performance of the spectrum portfolio. We select one fund each from among bond funds, stock funds and energy funds and name them ABd, BEy and CEgy.[5,6] The results in Hsu et al. (2011) confirm that ABd leads BEy by 0.355 months (about 7 days), BEy leads CEgy by 1.172 months (about 24 days) and ABd leads CEgy by 1.566 months (about 31 days). Here, more than one piece of information about the lead–lag relationship can be identified. Based on the data on Taiwan's business cycles and the Taiwan Capitalization Weighted Stock Index (1997/01–2009/12), we can divide one period of 12 years into bull markets and bear markets. There are three bull markets and bear markets, as is shown in Table 3. The peak and trough of the business cycle are recognized through official announcements: for example, by the National Bureau of Economic Research (NBER). It is the lag indicator (nominal capital returns) but is still taken into account in the literature, including in Siegel (1991), Abderrezak (1998) and Gonzalez, Powell, Shi, and Wilson (2005). The peak and trough of a stock market can be regarded as a leading indicator. Although the behavior of the stock market as a signal is not correct, investors can easily get this information. We assume that investors who distinguish between bull and bear markets will consider the lead and lag indicators at the same time.

We have indicated that the larger $T_2 - t_1$ under Condition 3 can also enhance spectrum strategy performance; however, we cannot control $T_i - t_i$ under the dynamic spectrum strategy. In addition, a complete cycle pattern can also enhance spectrum strategy performance, but we do not know whether the pattern is complete at the beginning of the spectrum portfolio. The only thing that we can do is to select the large lag period to enhance spectrum strategy performance. Usually, the lag period for funds of the same type is shorter than that of funds of a different type. Therefore, we select three different funds from three groups: bond, stock and energy funds.

5 ABd, BEy, CEgy are Fidelity funds — the International Bond Fund, AIG American Equity Fund Y and the BlackRock World Mining Fund, respectively.
6 When h is negative, h is obtained using the volatility of the previous month and vice versa. Our data do not indicate that there were two consecutive months with a negative h for ABd, BEy or CEgy.

Table 5
The performance of X and Y with two assets and three assets.

	Jo_Y	Jo_X	Sum_Y	Sum_X
1	0.09569	0.09937[a]	0.30933	0.86528
2	0.44444	0.36190	1.21980	1.05046
3	−0.15696	0.32486[a]	0.03710	0.12927

Jo_X (Jo_Y) represents the accumulated returns of X (Y) with joint approach using three assets.
Sum_X (Sum_Y) is the sum of the returns of all the three pairs of X (Y).
[a] Denotes the larger value between Jo_X and Sum_X.

Table 6
Comparing the performance of X, Y and $\widehat{X}$ in three pairs and three cycles.

		ABd vs. BEy			BEy vs. CEgy			ABd vs. CEgy		
		Y	X	$\widehat{X}$	Y	X	$\widehat{X}$	Y	X	$\widehat{X}$
1st	Ra	−1.053	9.974[e]	12.903[f]	20.457	29.022[e]	37.712[f]	15.053	37.642[e]	40.496[f]
2nd	Ra	16.425	16.867[e]	28.146[f]	44.575	45.121[e]	60.127[f]	64.648	53.501	68.573[f]
3rd	Ra	0.443	−7.689	0.331[f]	34.102	42.580[e]	44.321[f]	75.789	38.423	57.154[f]

If Ra of X is larger than Ra of Y, we denote 'e' in the upper right corner.
If Ra of $\widehat{X}$ is larger than Ra of X, we denote 'f' in the upper right corner.

4.2. The performance of dynamic spectrum strategy by recursive approach

With the returns summed for each complete cycle, we obtain the accumulated returns of the spectrum portfolio and benchmark. For example, see the nine accumulated returns in Table 4.[7]

In Table 4, Re denotes the accumulated return in every cycle. If Re of X is larger than Re of Y (e.g. $-0.073<0.084$), we denote 'a' in the upper right corner; otherwise, it is left blank. If Re of $\widehat{X}$ is larger than Re of X (e.g. $0.084<0.101$), we denote 'b' in the upper right corner, otherwise it is left blank. The fact that 'a' accounts for 66% and 'b' for 88% indicates the superior performance of the spectrum portfolio. In addition, the probability of 'b' is larger than that of 'a'. This indicates that the recursive approach indeed strengthens the spectrum strategy and eliminates the possibility that the superior performance of the spectrum portfolio comes from rebalancing. In particular, Re of $\widehat{X}$ ($=0.72$)>Re of Y ($=0.63$)>Re of X ($=0.56$) in the second cycle and in the Abd vs. Egy case. This implies that a spectrum strategy with the recursive approach does not merely improve X's performance but allows X to outdo Y.

4.3. The performance of dynamic spectrum strategy by joint approach

Table 5 presents the function of spectrum strategy in a joint approach. 'Jo_X' (Jo_Y) represents the accumulated returns of a lead–lag relationship using three assets instead of two (benchmark portfolio with three assets), and 'Sum_X' (Sum_Y) is the sum of the returns of all the three pairs of lead–lag relationships (benchmark portfolio with two assets). First, we compare Sum_X and Sum_Y and discover that the spectrum portfolio performs better than the benchmark, except in the second cycle, in which any case between X and Y performs well. Next, we compare Jo_X and Sum_X. Because X is constructed using a joint approach, we expect that Jo_X would be larger than Sum_X. We denote 'c' (which accounts for 66%) in the upper right corner if X is larger than Sum_X.

5. Spectrum strategy in hedging

Our spectrum strategy established using the efficient frontier from mean–variance theory has the property of constant volatility, which makes it convenient to use as a tool for risk control (hedging). Unlike perfect hedging via the replication method, the spectrum strategy cannot fully eliminate risk, but it can enhance portfolio diversification and control risk within a range set by the fund manager, which would be useful to fund of funds. The diversification of a spectrum portfolio can be indirectly observed by both the long and the short weights in a spectrum strategy because such a strategy crosses over two efficient frontiers. Such diversification can be found directly following Brocato and Steed (1998). In 1998, Brocato and Steed reallocated portfolio weights using the lead–lag relationship between equity and bonds in one efficient frontier and showed that the risk ratio of a portfolio can be improved by cyclical reallocation. In order to highlight the effect of hedging further, we use empirical data

[7] The details about the performances of spectrum portfolio and benchmark for each period in the cycles are shown in the Appendix A.

with a recursive approach for calculating the return/volatility ratios (which are measures of hedging effectiveness) of X, Y and $\widehat{X}$ for one period in each cycle, and compare the ratios. The results are presented in Table 6.

For each complete cycle, we obtain the returns and volatilities in a period for the spectrum portfolio and benchmark. Let Ra be equal to the returns divided by volatility. If Ra of X is larger than Ra of Y (e.g. $-0.073<0.084$), we denote ‘e’ in the upper right corner; otherwise, it is left blank. If Ra of $\widehat{X}$ is larger than Ra of X (e.g. $0.084<0.101$), we denote ‘f’ in the upper right corner; otherwise it is left blank. ‘e’ and ‘f’ account for 66% and 100%, respectively. The probabilities of ‘e’ and ‘a’ are the same in Tables 6 and 5. This means that X's performance is better than Y's in both return and risk. However, on a consideration of $\widehat{X}$ and X, ‘b’ and ‘f’ account for 88% and 100%, respectively. This implies that $\widehat{X}$ performs better than X and the risk of $\widehat{X}$ is lower than that of X. Ra of X (=53.5)<Ra of Y(=64.6)<Ra of $\widehat{X}$ (=68.6) in the gray area in Table 6, which indicates that X performs better than Y with no consideration given to volatility. However, Ra of X is worse than Ra of Y when the portfolio's performance is based on risk. This result indicates that we cannot rule out the possibility of higher risk and higher returns. Therefore, we cannot confirm the effect of the spectrum strategy. However, the performance of the spectrum portfolio $\widehat{X}$ is superior to that of X and Y, as indicated in Table 5. Furthermore, Ra of $\widehat{X}$ is higher than Ra of X and Y; that is, the spectrum strategy with a recursive approach can increase performance for enhancing returns and hedging risk.

6. Conclusion

In this paper, we presented a spectrum strategy that incorporates information of lead–lag relationships. We theoretically showed the practical circumstances under which the information of the lead–lag relationship must be applied, and took into account the characteristics of the spectrum strategy. We constructed spectrum portfolios based on two approaches: a recursive approach, which uses a recursive method in the lead–lag relationship, and a joint approach, which combines two lead–lag relationships. We build a dynamic spectrum portfolio using the recursive approach for two assets, and examined the effect of the spectrum strategy using funds data. The results indicate that the spectrum portfolio has a superior performance as compared to the benchmark as well as the spectrum portfolio in which the data of three funds are constructed using the joint approach. Furthermore, we measured hedging effectiveness of the portfolios by the return/volatility ratio. The return/volatility ratios of the spectrum portfolio and the benchmark for two funds are obtained using the recursive approach. We have shown that the spectrum portfolio maintains superior performance in hedging; that is, not only can the spectrum strategy enhance the portfolio's performance, but it also contributes to hedging.

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Appendix A

Table A1
The performance of X and Y in the bull market and the first cycle.

ABd vs. BEy				BEy vs. CEgy				ABd vs. CEgy			
Date	Y	X	$\widehat{X}$	Date	Y	X	$\widehat{X}$	Date	Y	X	$\widehat{X}$
1999/01	0.001	−0.001	0.009	1999/02	0.012	0.017	0.027	1999/02	0.006	0.007	0.000
1999/02	−0.030	−0.015	−0.005	1999/04	0.170	0.144	0.257	1999/04	0.137	0.144	0.237
1999/03	0.021	0.014	0.019	1999/06	0.027	−0.082	−0.059	1999/06	−0.027	−0.015	−0.012
1999/04	0.014	0.018	0.007	1999/08	0.027	0.027	0.052	1999/08	0.034	0.019	−0.012
1999/05	0.003	0.023	0.023	1999/10	0.023	−0.014	−0.037	1999/10	0.015	0.014	−0.006
1999/06	0.028	0.032	0.059	1999/12	0.133	0.087	0.141	1999/12	0.087	0.088	0.124
1999/07	−0.001	−0.020	−0.042	2000/02	−0.113	−0.104	−0.147	2000/02	−0.095	−0.093	−0.132
1999/08	−0.026	−0.044	−0.031	2000/04	0.004	0.021	−0.010	2000/04	−0.042	−0.047	−0.042
1999/09	−0.011	−0.017	−0.045								
1999/10	0.027	0.024	0.034								
1999/11	0.020	0.031	0.042								
1999/12	0.028	0.031	0.053								
2000/01	−0.047	−0.036	−0.069								
2000/02	−0.005	0.000	−0.031								
2000/03	0.064	0.051	0.098								
AR	0.084	0.089	0.121		0.284	0.098	0.224		0.115	0.117	0.157

X is the spectrum and dynamic benchmark portfolio, Y the static benchmark portfolio and $\widehat{X}$ is the dynamic spectrum portfolio by recursive.
AR is the abbreviation for accumulated return.

Table A2
The performances of X and Y in the bear market and the first cycle.

ABd vs. BEy				BEy vs. CEgy				ABd vs. CEgy			
Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$
00/04	−0.024	−0.003	−0.030	00/05	−0.079	−0.035	−0.036	00/05	−0.041	−0.011	0.008
00/05	−0.004	−0.005	0.004	00/07	0.057	0.081	0.003	00/07	0.036	0.067	0.067
00/06	0.023	0.014	0.017	00/09	0.014	0.086	0.082	00/09	−0.017	0.065	0.085
00/07	0.006	0.005	0.007	00/11	−0.068	−0.030	0.071	00/11	−0.015	0.009	0.009
00/08	−0.011	−0.024	−0.032	01/01	0.102	0.086	−0.071	01/01	0.081	0.040	0.040
00/09	−0.010	0.015	0.012	01/03	−0.101	−0.044	0.093	01/03	−0.033	0.008	0.006
00/10	−0.017	−0.055	−0.057	01/05	0.137	0.096	−0.086	01/05	0.090	0.070	0.065
00/11	−0.037	0.024	0.025	01/07	−0.097	−0.002	0.097	01/07	−0.061	−0.005	0.025
00/12	0.035	0.057	0.071	01/09	−0.118	0.017	−0.009	01/09	−0.039	0.028	0.028
01/01	0.022	−0.037	−0.038	01/11	0.101	0.034	−0.013	01/11	0.036	0.006	0.002
01/02	−0.046	0.007	0.008								
01/03	−0.053	0.016	−0.009								
01/04	0.037	−0.031	−0.035								
01/05	0.004	−0.004	−0.004								
01/06	−0.020	0.010	0.006								
01/07	0.002	−0.008	0.004								
01/08	−0.034	0.002	0.019								
01/09	−0.042	0.004	0.004								
01/10	0.012	0.008	0.006								
AR	−0.157	−0.005	−0.021		−0.052	0.290	0.13		0.037	0.277	0.335

X is the spectrum and dynamic benchmark portfolio, Y the static benchmark portfolio and $\hat{X}$ is the dynamic spectrum portfolio by recursive.
AR is the abbreviation for accumulated return.

Table A3
The performance of X and Y in the bull market and the second cycle.

ABd vs. BEy				BEy vs. CEgy				ABd vs. CEgy			
Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$
01/11	0.026	0.047	0.055	01/12	0.108	0.087	0.106	01/12	0.043	0.060	0.072
01/12	−0.003	0.005	0.016	02/02	0.037	0.063	0.113	02/02	0.071	0.077	0.100
02/01	−0.030	−0.015	−0.038	02/04	0.023	0.102	0.109	02/04	0.055	0.038	0.047
02/02	−0.015	−0.005	−0.009	02/06	−0.053	0.045	0.005	02/06	0.032	−0.025	−0.041
02/03	0.023	0.037	0.031	02/08	−0.084	−0.136	−0.102	02/08	−0.057	−0.094	−0.077
02/04	−0.024	−0.056	−0.075	02/10	−0.016	−0.094	−0.083	02/10	0.005	0.001	0.024
02/05	0.011	−0.020	−0.021	02/12	0.053	0.111	0.133	02/12	0.093	0.057	0.074
02/06	−0.026	−0.078	−0.090	03/02	−0.038	−0.015	−0.017	03/02	−0.002	−0.022	−0.025
02/07	−0.037	−0.023	−0.038	03/04	0.019	−0.018	−0.020	03/04	−0.010	−0.030	−0.036
02/08	0.005	−0.053	−0.047	03/06	0.079	0.071	0.071	03/06	0.064	0.087	0.071
02/09	−0.055	−0.058	−0.113	03/08	0.107	0.038	0.071	03/08	0.050	0.076	0.089
02/10	0.028	0.031	0.091	03/10	0.126	0.068	0.072	03/10	0.130	0.135	0.157
02/11	0.035	0.054	0.085	03/12	0.093	0.057	0.089	03/12	0.097	0.080	0.111
02/12	−0.011	−0.053	−0.070	04/02	0.008	0.001	0.026	04/02	−0.001	0.062	0.060
03/01	−0.005	−0.030	−0.067								
03/02	−0.002	−0.020	−0.012								
03/03	0.004	0.017	0.047								
03/04	0.048	0.018	0.049								
03/05	0.045	0.028	0.045								
03/06	0.002	0.027	0.023								
03/07	−0.003	0.024	0.021								
03/08	−0.004	0.004	0.024								
03/09	0.014	−0.031	−0.044								
03/10	0.024	0.026	0.031								
03/11	0.007	−0.015	−0.016								
03/12	0.039	0.019	0.038								
04/01	0.009	0.032	0.033								
04/02	0.006	0.020	0.019								
AR	0.111	−0.067	−0.029		0.462	0.380	0.574		0.567	0.501	0.626

X is the spectrum and dynamic benchmark portfolio, Y the static benchmark portfolio and $\hat{X}$ is the dynamic spectrum portfolio by recursive.
AR is the abbreviation for accumulated return.

Table A4
The performance of X and Y in the bear market and the second cycle.

ABd vs. BEy				BEy vs. CEgy				ABd vs. CEgy			
Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$
04/03	−0.003	0.008	0.012	04/04	−0.106	0.122	0.120	04/04	−0.103	0.078	0.071
04/04	−0.027	−0.001	−0.005	04/06	0.028	−0.021	−0.017	04/06	0.031	−0.023	−0.012
04/05	0.005	−0.015	−0.008	04/08	0.004	−0.042	−0.041	04/08	0.023	−0.024	−0.021
04/06	0.006	0.004	0.006	04/10	0.051	0.071	0.070	04/10	0.062	0.040	0.050
04/07	−0.025	−0.002	−0.010	04/12	0.078	0.072	0.078	04/12	0.067	0.000	0.003
04/08	−0.003	−0.028	−0.008	05/02	0.048	−0.045	−0.038	05/02	0.053	−0.033	−0.032
04/09	0.005	0.018	0.023	05/04	−0.090	−0.019	−0.029	05/04	−0.071	0.023	0.032
04/10	0.021	0.013	0.026								
04/11	0.033	0.025	0.029								
04/12	0.022	0.008	0.010								
05/01	−0.022	−0.017	−0.017								
05/02	0.010	0.000	0.046								
05/03	−0.015	0.006	0.002								
05/04	−0.003	0.021	0.029								
AR	0.006	0.039	0.135		0.013	0.137	0.143		0.061	0.061	0.092

X is the spectrum and dynamic benchmark portfolio, Y the static benchmark portfolio and $\hat{X}$ is the dynamic spectrum portfolio by recursive.
AR is the abbreviation for accumulated return.

Table A5
The performance of X and Y in the bull market and the third cycle.

ABd vs. BEy				BEy vs. CEgy				ABd vs. CEgy			
Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$
05/05	0.004	0.025	0.035	05/06	0.038	0.056	0.044	05/06	0.011	0.005	0.020
05/06	−0.002	0.006	0.001	05/08	0.072	0.084	0.095	05/08	0.045	0.033	0.028
05/07	0.017	0.017	0.026	05/10	0.038	0.057	0.025	05/10	0.026	0.027	0.030
05/08	−0.007	−0.017	−0.018	05/12	0.092	0.121	0.122	05/12	0.070	0.068	0.078
05/09	−0.004	0.016	0.012	06/02	0.054	0.049	0.016	06/02	0.049	0.045	0.031
05/10	−0.017	−0.005	0.001	06/04	0.094	0.092	0.124	06/04	0.091	0.047	0.048
05/11	0.016	0.012	0.022	06/06	−0.038	−0.038	−0.026	06/06	−0.015	−0.014	0.034
05/12	0.003	−0.006	−0.011	06/08	0.039	0.004	0.003	06/08	0.005	−0.015	−0.020
06/01	0.014	0.020	0.011	06/10	0.027	0.020	0.078	06/10	0.021	0.000	0.003
06/02	−0.003	−0.004	−0.001	06/12	0.049	0.043	0.053	06/12	0.043	0.055	0.062
06/03	0.000	−0.002	0.003	07/02	0.019	0.069	0.021	07/02	0.024	−0.004	0.022
06/04	0.015	−0.011	−0.001	07/04	0.093	0.032	0.043	07/04	0.074	0.070	0.067
06/05	−0.013	−0.026	−0.052	07/06	0.060	0.082	0.085	07/06	0.041	0.033	0.053
06/06	−0.006	0.001	0.001	07/08	−0.004	−0.005	−0.011	07/08	0.000	−0.004	0.000
06/07	0.009	0.003	0.004								
06/08	0.015	0.008	0.039								
06/09	−0.001	−0.010	−0.010								
06/10	0.012	−0.004	0.004								
06/11	0.017	−0.005	−0.001								
06/12	−0.001	0.008	0.010								
07/01	0.003	0.008	0.015								
07/02	0.001	−0.007	−0.017								
07/03	0.009	0.008	0.015								
07/04	0.023	0.019	0.028								
07/05	0.008	0.022	0.028								
07/06	−0.011	−0.015	−0.018								
07/07	−0.002	−0.008	−0.023								
07/08	−0.004	−0.010	−0.003								
AR	0.096	0.045	0.102		0.632	0.666	0.672		0.485	0.346	0.457

X is the spectrum and dynamic benchmark portfolio, Y the static benchmark portfolio and $\hat{X}$ is the dynamic spectrum portfolio by recursive.
AR is the abbreviation for accumulated return.

Table A6
The performance of X and Y in the bear market and the third cycle.

ABd vs. BEy				BEy vs. CEgy				ABd vs. CEgy			
Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$	Date	Y	X	$\hat{X}$
07/09	0.029	0.003	0.011	07/10	0.045	−0.049	−0.027	07/10	0.153	0.136	0.155
07/10	0.015	0.025	0.037	07/12	−0.025	−0.011	0.021	07/12	−0.038	−0.024	−0.021
07/11	−0.016	−0.031	−0.036	08/02	0.076	−0.030	−0.111	08/02	0.071	−0.037	−0.036
07/12	−0.008	−0.004	−0.005	08/04	0.057	0.012	0.042	08/04	−0.009	−0.015	−0.020
08/01	−0.021	0.007	0.018	08/06	−0.057	−0.023	−0.024	08/06	0.009	−0.027	−0.010
08/02	−0.001	−0.002	0.013	08/08	−0.055	0.017	0.068	08/08	−0.165	−0.242	−0.247
08/03	0.008	0.006	0.012	08/10	−0.335	0.154	0.204	08/10	−0.483	−0.305	−0.346
08/04	0.016	−0.002	−0.013	08/12	0.031	−0.196	−0.024	08/12	0.022	−0.005	0.090
08/05	0.005	−0.009	−0.018	09/02	−0.046	0.048	−0.091	09/02	−0.067	−0.056	−0.079
08/06	−0.042	0.000	0.010								
08/07	−0.004	−0.021	−0.026								
08/08	−0.022	−0.012	−0.015								
08/09	−0.067	−0.028	−0.063								
08/10	−0.132	−0.198	−0.229								
08/11	−0.033	−0.036	−0.035								
08/12	0.036	0.017	0.046								
09/01	−0.041	−0.014	−0.024								
09/02	−0.081	0.023	0.007								
AR	−0.359	−0.276	−0.308		−0.309	−0.078	0.038		−0.507	−0.575	−0.514

X is the spectrum and dynamic benchmark portfolio, Y the static benchmark portfolio and $\hat{X}$ is the dynamic spectrum portfolio by recursive.
AR is the abbreviation for accumulated return.

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