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Weighted Characteristic P-vector and Deadlock Control of WS^{3}PR

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Current deadlock control approaches for S^3PGR^2 (*systems of simple sequential processes with general resources requirement*) suffer from incorrect or restricted liveness characterization based on the concept of deadly marked siphons (*DMS*) and max-controlled siphons. Dead transitions may exist when there are no DMS and the net model is in livelock states. A new liveness condition is developed based on the so-called max*-controlled siphons to replace that of the restrictive max-controlled siphons. A deadlock control policy is further proposed for *WS^{3}PR* (*weighted S^{3}PR* (*systems of simple sequential processes with resources*)) by adding control nodes and arcs for elementary siphons only, reducing significantly the number of monitors compared with existing methods. A counter example is shown to indicate that Li's characteristic *P*-vector must be weighted. The controlled model for *WS^{3}PR* is proposed and its liveness property is proved.

Keywords: flexible manufacturing systems, deadlock control, Petri nets, siphons, elementary siphons

1. INTRODUCTION

Deadlock in a Flexible Manufacturing Systems (*FMS*) [1] interrupts normal operation schedules degrading significantly system's performance. Hence, it is important to design a *PN* model free of deadlocks. Fanti *et al*. [2] performed a comprehensive review on deadlock control techniques for *FMS*. Deadlock prevention approach [3-10] (as used in this paper) establishes the control policy in a static way based on off-line control mechanisms by building freely a Petri net model first and then adding necessary control to it such that the control model is deadlock-free. Control places and related arcs are often used to achieve such purpose.

1.1 Relevant Literature

Ezpeleta *et al.* proposed a class of PN called systems of simple sequential processes with resources (*S^{3}PR*) [3]. Liveness can be enforced by adding a control place to each *strict minimal siphon* (*SMS*) at the cost of introduction of too many additional places and arcs.

Li *et al.* [4, 5] proposed simpler Petri net controllers based on the concept of the set of elementary siphons (generally much smaller than the set of all *SMS* in large Petri nets) to minimize the new addition of places. The authors added a control place for each elementary siphon while controlling all dependent *SMS* (*i.e.*, always marked), too. This

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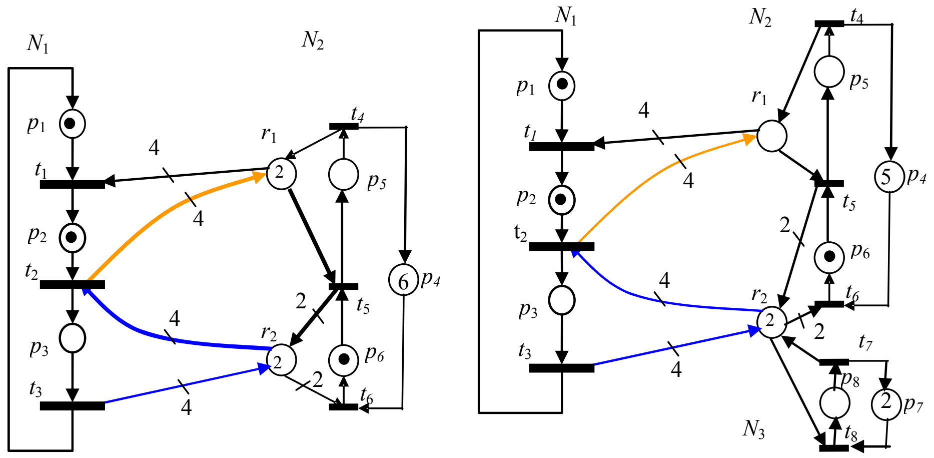

(a) (b)

Fig. 1. (a) Only SMS $S = \{r_1, r_2, p_3, p_5\}$ is max*-controlled, but not max-controlled; (b) $S = \{r_1, r_2, p_3, p_5, p_8\}$. The siphon is neither max*-controlled nor deadly marked (t_7 and t_8 are live).

reduces significantly the number of the control places and makes the method suitable (or applicable) to large-scale Petri nets.

Park and Reveliotis [8] proposed S^3PGR^2 (*systems of simple sequential processes with general resources requirement*, see Fig. 1). Tricas and Martinez [9] proposed a similar system called WS^3PSR (*weighted systems of simple sequential processes with several resources*). The policy is so restrictive that it sequentializes the flow in the siphon. The marking imposed to the control places limits the number of processes that can flow in the problematic areas to minimal.

All these approaches are based on deadly marked siphons (*DMS*). A net with *DMS* is not live; conversely, a live net does not have *DMS*. However the absence of *DMS* only guarantees the liveness of some (not all) transitions in the net. To provide optimum control, one needs first to improve the condition for liveness.

Abdallah *et al.* [10] proposed S^4PR – a generalization of S^3PR nets – to extend S^3PR and production Petri nets (*PPN*) to model systems that not only can use alternative resources, as in S^3PR nets, but also can utilize more than one resource simultaneously. They adopted a deadlock prevention policy by adding a control place for each siphon to remain max-marked for all reachable markings. However, it is only a sufficient condition implying that a non-max-controlled S^4PR may be live.

1.2 Proposed Approach

We show that current deadlock control approaches suffer from incorrect or restricted liveness characterization based on the concept of deadly marked siphons (*DMS*) or max-controlled siphons. A relaxed liveness condition is proposed called max*-controlled siphons.

Li *et al.* [4] said "It should be noted that the proposed elementary siphon and related concepts are applicable to general Petri nets." However, a counter example will be shown

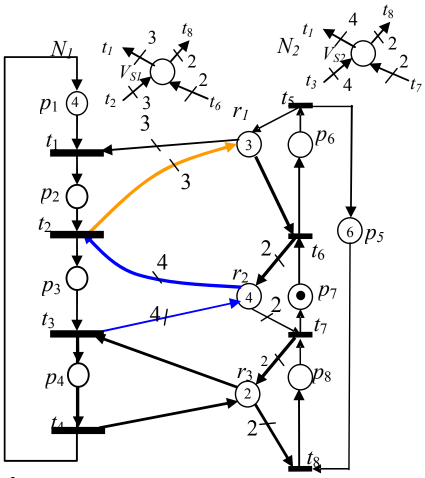

Fig. 2. An example of a WS^3PR with $S_1 = \{r_1, r_2, p_3, p_6\}$, $S_2 = \{r_2, r_3, p_4, p_7\}$, and $S_3 = \{r_1, r_2, r_3, p_4, p_6\}$.

in Fig. 2 where all *SMS* are elementary siphons, while the S^3PR of its ordinary net version has one dependent siphon. To fix the problem, each component of a characteristic *P*-vector is weighted by that of *P*-invariant as shown in section 3. Also Li's [4] approach, like Ezpeleta's, is not maximum permissive because some authorized states may be suppressed.

In this work, a deadlock control policy for WS^3PR is proposed by adding control nodes and arcs for elementary siphons only. The proposed method reduces significantly the number of monitors compared with the existing methods. By adjusting the control depth variables of its elementary siphons, other (dependent) siphons also satisfy the max*-controlled-siphon property. The liveness property of the controlled model is proved.

1.3 Organization of the Manuscript

The rest of the paper is organized as follows: sections 2 and 3 present the basis (WS^3PR, elementary siphons, characteristic *T*-vectors, and max*-controlled siphons) to understand the paper. The control policy (algorithm) for the deadlock prevention of an example will be provided in section 4. Section 5 introduces an FMS example to illustrate the applications of the proposed concepts and policy. Finally, section 6 concludes the paper.

2. PRELIMINARIES

Here only the definitions used in this paper are presented. The reader may refer to [11] for more Petri net details.

Definition 1 A subnet $N_i = (P_i, T_i, F_i)$ of N is generated by $X = P_i \cup T_i$, if $F_i = F \cap (X \times X)$. It is an *I*-subnet, denoted by I, of N if $T_i = {}^\bullet P_i$. I_S is the *I*-subnet (the subnet derived from $(S, {}^\bullet S)$) of an *SMS* S. Note that $S = P(I_S)$; S is the set of places in I_S.

Property 1 [5] The linear combination of Y_1 and Y_2 is a P-invariant if both Y_1 and Y_2 are P-invariants. Furthermore, if Y is a P-invariant of N, then given an initial marking M_0, $\forall M \in R(N, M_0)$, $Y^T \bullet M = Y^T \bullet M_0$ or $W(M) = W(M_0)$, where $W(M) = Y^T \bullet M(W(M_0) = Y^T \bullet M_0)$ is the weighted sum of tokens under $M(M_0)$.

2.1 WS^3PR

A WS^3PR is similar to an S^3PR and is composed of a set of state machines (which are a subclass of ordinary Petri nets) holding and releasing shared resources. Its definition has been provided in this journal [6].

In the sequel, when talking about a marked $WS^3PRN = (P \cup \{p^0\} \cup P_R, T, F)$, denote $P^0 = \{p^0\}$ and refer to N with a full initial marking where, by normal practice, $M_0(p^0)$ is selected so that that the resource can be fully utilized by the S^2PR. N is quasi-live and each transition is potentially firable under M_0 since $M_0(r) \geq \max_{t \in R\bullet} F(r, t)$, $\forall r \in P_R$, $\forall t \in r\bullet$.

A resource subnet is a subnet where all places are resource ones.

2.2 Max*-Controlled Siphons

An S^3PR is live if no siphons ever become empty [3], not necessarily true for WS^3PR since it is a general Petri net (*GPN*). The new condition is called "max-controlled" [12] or the more relaxed "max*-controlled" as defined below.

Definition 2 Let $N = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR, D a siphon, $D_R = D \cap P_R$, $D_P = D \cap P$, and υ the resource subnet in D's *I-subnet* I_D. Let $a_y(p) = \max_{t \in p\bullet} F(p, t)$ ($a_y^*(p) = \max_{t \in p\bullet \cap \upsilon} F(p, t)$, $a_x(p) = \min_{t \in p\bullet} F(p, t)$) be the maximal (minimal for $a_x(p)$) arc weight among all output arcs (which are in υ for $a_y^*(p)$, $\forall p \in \upsilon$ and $a_y^*(p) = 1$, $\forall p \in D_P$). $\forall p \in D$, p is called max-marked (max*-*marked*, min-*marked*) under M if $a_y(p) \leq M(p)$ ($a_y^*(p) \leq M(p)$, $a_x(p) \leq M(p)$). D is said to be max-*controlled* (max*-*controlled*, min-*controlled*), iff $\forall M \in R(N, M_0)$, $\exists p \in D$, p is max-*marked* (max*-*marked*, min-*marked*). N is said to be max-*controlled* (max*-*controlled*, min-*controlled*) *iff* each minimal siphon of (N, M_0) is max-*controlled* (max*-*controlled*, min-*controlled*).

A siphon D is max-controlled (max*-controlled, min-controlled) if for every reachable marking, there exists a max-marked (max*-marked, min-marked) place p in D; *i.e.*, the marking of p, $M(p)$, is greater than the weights of all its output arcs (in the resource subnet where all places are resources). A *GPN* is max-controlled (max*-controlled) iff all its siphons D are max-controlled (max*-controlled).

Physically, when a *GPN* is max-controlled, it behaves like an *OPN* (ordinary *PN*) since $\forall M \in R(N, M_0)$, $\exists p \in P$, $M(p) \geq \max_{t \in p\bullet} F(p, t)$ (all output arcs of p are enabled). However, the condition can be relaxed as shown in Fig. 1 (a) with only one *SMS* $S = \{r_1, r_2, p_3, p_5\}$. None of places in S is max-marked: $r_1(r_2)$ is not max-marked since the output arc (r_1, t_1) $((r_2, t_2))$ is disabled and $M(p_5) = M(p_3) = 0$. Note that while r_1 is max*-marked, namely $a_y^*(r_1) = 1 < M(r_1) = 2$, r_2 is not max*-marked, because $a_y^*(r_2) = 4 < M(r_2) = 2$. Thus, S is not max-controlled − but max*-controlled −, yet the net system is live. The resource subnet υ of I_S is a resource circuit $c_1 = [r_1\ t_5\ r_2\ t_2\ r_1]$. $r_1(r_2)$ is (not) max-marked in υ since output arc (r_1, t_5) $((r_2, t_2))$ in υ is enabled (disabled).

We now give an intuitive explanation. To reach deadlocks when there exists an *SMS S*, tokens in S must be pushed out to (called brink) places to enable transitions in I_S. If every transition of I_S is only disabled by insufficiently marked input places in I_S to reach a so called deadlock-brink state, one can isolate I_S from the rest of the net N. All transitions in I_S are live if the marked S is max-controlled in I_S (*i.e.*, max*-controlled in N) even though it is not so for the marked N.

Definition 3 Let $N = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR and S a siphon in N. $\forall p \in P \cup P^0 \cup P_R$, $p \in {}^\bullet N'$, if $p \notin N'$ and $\exists t$ in N', $p \in {}^\bullet t$. p is called a brink place if $\exists r \in I_S$, $p \in H(r) \cap {}^\bullet I_S$. A cut circuit is a circuit c in I_S, and $\forall p' \in {}^\bullet c$, p' is not a brink place. A *sub-*I_S I'_S is a subnet of I_S obtained by removing all cut circuits (denoted by the set Θ) from I_S (but keeping the resource place in each c); *i.e.*, $I'_S = I_S \backslash \Theta$.

Note that a brink place [*e.g.*, p_2 and p_6 in Fig. 1 (a)] is a holder place of a resource in I_S and also an input place of I_S. In a deadlock-brink state, all brink places p hold tokens so that every arc (p, t) ($t \in I'_S$) is enabled; t may be disabled only by places in I'_S. Note that there is no such t in any cut circuit. Thus, if for any reachable marking, there is a max-marked place p in I'_S (obtained by removing cut circuits from I_S), output transitions t of p ($t \in I'_S$) can be fired since p, by Observation 1, has only two input places and both are marked.

Note that for a reachable marking M such that S is max-marked in I'_S; it may not be so in I_S. That is, arc (r, t) in a cut circuit C may be disabled while r is max-marked in I'_S. This posts no problem since transitions in I'_S are live and there is a firing sequence from M to restore $M(r)$ to $M_0(r)$ to enable (r, t) since (r, t) must be initially (*i.e.*, under M_0) enabled.

Intuitively, $\forall p \in S$, p is max*-marked under $M \in R(N, M_0)$ iff p is max-marked in I'_S. The marking $(p_2 + 2r_1 + r_2 + p_6)$ (*i.e.*, $M(p_2) = 1$, $M(r_1) = 2$, $M(r_2) = 1$, $M(p_6) = 1$, and $M(p) = 0$, for all other places p) in Fig. 1 (a) is a deadlock-brink state for the only *SMS*. In order to reach a deadlock, there exists an *SMS S* in a deadlock-brink state. If S is max*-controlled, at least a transition in I'_S is enabled and hence is deadlock-free. Note that the max*-controlled condition may no longer hold (for GPN) if the weight of the arc (p, t), $p \in P$ (not a resource place), is not one. However, it is true for most Petri net models of FMS.

We now develop the theory for a net system to be max*-controlled. Note that the net system is not live, yet the only siphon S is not deadly marked. Unlike S^3PR, the absence of deadly marked siphons (*DMS*) does not imply the liveness of a WS^3PR. Simple extension of *DMS* is insufficient for the liveness analysis of WS^3PR.

Definition 4 Let S be an *SMS* and Y a P-invariant with components $y_i = Y(p_i)$ satisfying the negative-property: $\forall p_k \in S, y_k > 0$ and $\forall p_i \in (P \cup P^0 \cup P_R)\backslash S$: $y_i \leq 0$, where $(P \cup P^0 \cup P_R)\backslash S = \{x \mid x \in (P \cup P^0 \cup P_R), x \notin S\}$ and $M_S = [0\ 0\ \ldots\ 0\ a_y^*(r_1) - 1\ a_y^*(r_2) - 1\ \ldots\ a_y^*(r_K) - 1\ 0\ 0\ \ldots\ 0]^T$ ($a_y^*(r)$ defined in Def. 2) a marking such that $\forall r_i \in S_R$, $M(r_i) = a_y^*(r_i) - 1$, $i = 1, 2, \ldots, K$ and $\forall p \in (P \cup P^0 \cup P_R)\backslash S_R$, $M(p) = 0$. The weighted sum of tokens under $M_S(M_0)$ is $W(M_S) = W_S = M_S^T \bullet Y = \sum_i (a_y(r_i) - 1) \bullet y_i$ ($W(M_0) = Y^T \bullet M_0$). $W(M(S)) = \sum_{p \in S} M(p) \bullet Y(p)$ is the weighted sum of tokens in S under M. $\Delta W = W(M\ ) - W(M_0)$.

When designing a system, one first selects a Y satisfying the negative-property and then assign an initial marking M_0 such that $W(M_0) > W_S$ for each SMS S to ensure that the system remains max*-controlled under all reachable markings as shown below.

Lemma 1 Let (N, M_0) be a net-system, (1) If $W(M(S)) > W_S$, then S is max*-*marked*; (2) If $W(M_0) > W_S$, then S is max*-*controlled*, where $S \subseteq (P \cup P_R)$ is an SMS of N, Y a P-invariant satisfying the negative-property.

Proof: (1) Assume contrarily thqt $\forall M \in R(N, M_0)$, $\forall p \in S$, $M(p) \leq a_y^*(p) - 1$. One has $W(M(S)) = \sum_{p\in S} M(p) \bullet Y(p) \leq \sum_{p\in S}(a_y^*(p) - 1) \bullet Y(p) = W_S$ – contradiction; (2) By Property 1, one has $W(M_0) = W(M) > W_S$ where $W(M) = W(M(S)) + W(M((P \cup P^0 \cup P_R)\backslash S)) > W_S$. Now, $W(M((P \cup P^0 \cup P_R)\backslash S)) \leq 0$ due to the negative property; thus, $W(M(S)) > W_S - W(M((P \cup P^0 \cup P_R)\backslash S)) \geq W_S$, which leads to the inequality $W(M(S)) > W_S$ satisfying the assumption in (1). And S is max*-marked $\forall M \in R(N, M_0)$. Hence, S is max*-controlled by Def. 2. ❑

By this lemma, if a siphon is initially max*-marked, it remains so for all reachable markings. This facilitates the verification of max*-controlled siphons. If N is ordinary, then $W_S = 0$, the condition $W(M_0) > W_S$ becomes the invariant-controlled one $W(M_0) > 0$ [5], the max*-controlled S is now also invariant-controlled. Similar to the invariant-controlled condition $W(M_0) > 0$, the condition $W(M_0) = Y^T \bullet M_0 > W_S$ in Lemma 1 is also only sufficient (not necessary) for deadlock-freeness. This implies that a WS^3PR may be deadlock-free or even live if $W(M_0) \leq W_S$.

Property 2 [6] Let (N, M_0) be a marked WS^3PR, $M \in R(N, M_0)$ and $t \in T$ be a dead transition under M. Then $M_0 \notin R(N, M)$.

Corollary 1 Let (N, M_0) be a marked WS^3PR, $M \in R(N, M_0)$ and $t \in T$ a dead transition under M. Then there exist $M' \in R(N, M)$ and two subsets $J \subset I_N$ and $H \subset I_N$ such that $I_N = J \cup H$, $I_N = \{1, 2, \ldots, k\}$, $J \cap H = \varnothing$, $J \neq \varnothing$ and: (1) $\forall h \in H$, $M'(p_h^0) = M_0(p_h^0)$; (2) $\forall j \in J$, $M'(p_j^0) < M_0(p_j^0)$ and $\{p^\bullet \mid p \in P$, and $M'(p) > 0\}$ is a set of dead transitions.

The above lemma and corollary were taken from [3]. Property 2 states that if there exists a dead transition t, then M_0 is not reachable. Otherwise, t is potentially firable and not dead – contradiction. Corollary 1 states that if there exists a dead transition t, then some WPs are blocked and cannot proceed to complete operations. If they can return to the initial idle state, then t is potentially firable and not dead – contradiction.

Based on Corollary 1, the main property about siphon is now proved: If there is a dead transition t under M, then there exists a non-max*-marked siphon under $M' \in R(N, M)$. The basic idea behind the proof is the construction of a non-max*-marked siphon. It consists of two sets of places: (1) resource places r such that one of its output arcs is disabled; (2) unmarked holders of these r. The proof is similar to that for S^3PR with some differences due to the condition of "non-max*-marked" instead of "empty siphons".

Theorem 1 Let (N, M_0) be a marked WS^3PR, where $N = (P \cup P^0 \cup P_R, T, F)$, and $t' \in T$ a dead transition under M. Then $\exists M' \in R(N, M)$, $\exists S$ a siphon so that S is nonempty (*i.e.*, not a null set) and *non*-max'-*marked*.

Proof**:** See the proof of Theorem 2 in [15]. ❑

Corollary 2 Let (N, M_0) be a marked WS^3PR, where $N = (P \cup P^0 \cup P_R, T, F)$, (N, M_0) is live if every siphon is max*-*controlled*.

Proof**:** See the proof of Theorem 3 in [15]. ❑

We propose a deadlock prevention policy in the next section based on Theorem 1. Note that the net in Fig. 1 (a) remains live, unlike S^3PR, even when $M_0(p_1)$ is very large – no longer so if $M(r_1)$ increases from 2 to 4.

3. ELEMENTARY SIPHONS AND CHARACTERISTIC VECTORS

This section first redefines characteristic *P*-vector and others and illustrate a counter-example with respect to the one in Li *et al.* It is desired that an S^3PR and its weighted S^3PR have the same set of elementary and dependent siphons and the same system of equations of characteristic *T*-vectors. To find elementary siphons, Li *et al.* [4] have to find all SMS; the number of which grows exponentially with the size of the net. Thus, Li *et al.* have to start all over for each new weighted S^3PR – quite time consuming.

One would get away with such troubles for a simple basic subclass of S^3PR (called BS^3PR in [13]), where the set of elementary and dependent siphons correspond to that synthesized from elementary and compound (*i.e.*, multiple interconnected) resource circuits (containing only resource places) respectively. There is no need for the above duplicate computation for a weighted S^3PR.

Definitions of characteristic *P*-vector and characteristic *T*-vector, elementary and dependent siphons have been defined in [4]. We first formally redefine characteristic *P*-vector and characteristic *T*-vector followed by a counter example.

Definition 5 Let $\Omega \subseteq P$ be a subset of places of N and Y a *P*-invariant with components $y_i = Y(p_i)$ satisfying the negative-property: $\forall p_k \in \Omega$, $y_k > 0$ and $\forall p_i \in P\backslash\Omega$: $y_i \leq 0$, where $P\backslash\Omega = \{x \mid x \in P, x \notin \Omega\}$. *P*-vector λ_Ω is called the characteristic *P*-vector of Ω iff $\forall p \in \Omega$, $\lambda_\Omega(p) = Y(p)$; otherwise $\lambda_\Omega(p) = 0$. η is called the characteristic *T*-vector of Ω, if $\eta^T = \lambda_\Omega^T \bullet [N]$ and Ω is an *SMS S*, where $[N]$ is the incidence matrix.

For economy of space and to avoid long vectors, $\sum L(p)p(\sum J(t)t)$ will be used to denote a $P(T)$-vector. The following definition helps to illustrate a counter example.

Definition 6 Let N^w and N^r be a WS^3PR and its reduced S^3PR (with characteristic *T*-vectors η^w and η respectively) where $(\eta^w)^T = (\lambda_S^w)^T \bullet [N^w]$, $\lambda_S^w(\forall p \in S, \lambda_S(p) = Y(p)$; otherwise $\lambda_S(p) = 0$; Y is a *P*-invariant) is the characteristic *P*-vector of an *SMS S* and $[N^w]$ is the incidence matrix.

However, a counter example is shown in Fig. 2 where N^w and N^r have the same set of *SMS*: $S_1 = \{r_1, r_2, p_3, p_6\}$, $S_2 = \{r_2, r_3, p_4, p_7\}$, and $S_3 = \{r_1, r_2, r_3, p_4, p_6\}$. $\eta_1 = [-1\ 1\ 0\ 0\ 0\ 1\ -1\ 0]^T$, $\eta_2 = [0\ -1\ 1\ 0\ 0\ 0\ 1\ -1]^T$, $\eta_3 = [-1\ 0\ 1\ 0\ 0\ 1\ 0\ -1]^T$, $\eta_1^w = [-3\ 0\ 3\ 0\ 0\ 2\ -2\ 0]^T$, $\eta_2^w = [0\ -4\ 4\ 0\ 0\ 1\ 1\ -2]^T$, and $\eta_3^w = [-3\ -1\ 4\ 0\ 0\ 1\ 0\ -2]^T$. It is easy to see, considering a linear combination of characteristic T-vectors, that $\eta_3 = \eta_1 + \eta_2$, $\eta_3{}^w \neq \eta_1{}^w + \eta_2{}^w$, and no $a_1, a_2 \in \boldsymbol{R}$ such that $\eta_3{}^w = a_1\eta_1{}^w + a_2\eta_2{}^w$. Thus, if one follows the method in [5], there is no dependent siphon (defined in [6]).

However, if one multiplies each component $\lambda_S{}^w(p)$ of a characteristic P-vector λ_S in N^w by $Y(p)$, the component of a minimal P-invariant $Y = [Y(r_1)\ Y(r_2)\ Y(r_3)\ Y(p_1)\ Y(p_2) \ldots Y(p_8)]^T = [1\ 1\ 1\ 0\ 3\ 4\ 1\ 0\ 1\ 2\ 2]^T$, then $\eta_1{}^w = [-3\ 3\ 0\ 0\ 0\ 1\ -2\ 0]^T$, $\eta_2{}^w = [0\ -4\ 4\ 0\ 0\ 0\ 2\ -2]^T$, $\eta_3{}^w = [-3\ -1\ 4\ 0\ 0\ 1\ 0\ -2]^T$ and $\eta_3{}^w = \eta_1{}^w + \eta_2{}^w$ – the same linear expression as that for N^r.

Definition 7 An *SMS* S is said to be *ξ-controlled* iff Y satisfies the negative property and $\forall M \in R(N, M_0)$, $W(M(S)) > W_S + \xi$, where $\xi > 0$ is called a control depth variable [4], $W(M(S))$ the sum of weighted token in S under M and W_S defined in Def. 4.

All elementary siphons must be ξ-controlled to keep all dependent siphons max*-controlled since only elementary siphons receive control places and arcs.

4. CONTROL POLICY FOR DEADLOCK PREVENTION IN WS^3PR

This section extends the deadlock prevention technique for S^3PR by Ezpeleta *et al.* to WS^3PR. For each elementary siphon S, one adds a control place V_S and control arcs as in [3, 4] (except the initial marking at V_S and the weights of the control arcs). We show how to assign P-invariants to achieve the negative property defined in Def. 4. Based on Lemma 1.2, one can then determine the initial markings to make S max*-controlled. The following definitions help subsequent discussions.

Definition 8 Let $N = O_{i=1}{}^k N_i = (P \cup P^0 \cup P_R, T, F)$ be a WS^3PR and $\prod$ be the set of *SMS* in N. Given $S \in \prod$, where $S = S_P \cup S_R$, $S_R = S \cap P_R$, $S_P = S \backslash P_R$, the S's complementary set $[S] = (\cup_{R \in} S_R H(r)) \backslash S$ denotes the set of holders of resources in S, which do not belong to S. $[S^i] = [S] \cap P_i$, $i \in I_N = \{1, 2, \ldots, k\}$.

Definition 9 Let V_S be the control place associated with elementary siphon S. Y^S is the P-invariant associated with S. Y^S: $Y^S(p_j) = 1$, $\forall p_j \in S_R$, or $Y^S(p_j) = F(t, r)$, $t \in p_j{}^\bullet \cap {}^\bullet r$, $\forall p_j \in H(r)$, $r \in S_R$, and $Y^S(p_j) = 0$ for all other p_j.

To disable transitions in an I_S to move toward deadlocks, tokens in S should be moved to $[S]$ (the complementary set) as many as possible via paths of the form $[r_i\ t_{ij}\ p_{ij}\ t'_{ij}]$ or $[t_{ij}\ p_{ij}\ t'_{ij}]$. Note that the operation place p_{ij} in $H(r_i)$ is in $[S]$. Tokens in r_i can be trapped by firing t_{ij}. When the amount of tokens trapped in all p_{ij} from r_i is larger than $M_0(r_i) - a_y(r_i)$, r_i will become non-max*-marked.

Thus, one adds a circuit $[V_S\ t_{ij}\ p_{ij}\ t'_{ij}\ V_S]$ ($[V_{S1}\ t_1\ p_2\ t_2\ V_{S1}]$ or $[V_{S1}\ t_7\ p_7\ t_6\ V_{S1}]$ in Fig. 2) to prevent the above token trapping where $F(V_S, t_{ij}) = F(t'_{ij}, V_S) = F(r, t_{ij})$ (= 3 or 2) ; *i.e.*, identical arc weights. This results in a new P-invariant whose support covers $[S]$ and V_S only, while the approach in [3, 4] covers $[S]$ and V_S as a proper subset. The former is better than the latter in view of the fact that it disturbs less on the uncontrolled model. However, it may create new *SMS* while the latter does not.

We follow the approach in [3, 4] to construct control arcs first followed by assignments of P-invariant Y^V and $M_0(V_S)$.

4.1 Control Arcs

Definition 10 With $[S^i]$ defined in Def. 8, $t_\alpha = {}^\bullet[S^i]\backslash[S^i]^\bullet$ (*resp.* $t_\beta = [S^i]^\bullet\backslash{}^\bullet[S^i]$) is a start (end) transition in ${}^\bullet[S^i]$ ($[S^i]^\bullet$) such that ${}^\bullet t_\alpha \notin [S^i]$ ($t_\beta^\bullet \notin [S^i]$). $C(x, y)$ denotes the circuit containing nodes x and y and there exists a path in this from x to y (denoted by $\Gamma(x, y)$), $x \neq y$, and the path does not pass a $p^0 \in P^0$. Let $t^*_1 \in P^{0\bullet}$, $P_S(t) = \{p \mid \exists C, s.t.\ C(t^*_1, p)\ \&\ C(p, t)\}$ ($P_S(t)$ is the set of places p such that t^*_1, p and t are in a circuit that contains a p_0), $P_S = [V_S] = \cup_{t\alpha} P_S(t_\beta)$ (P_S is the controller region or the set of places p in the region between $V_S^\bullet$ and ${}^\bullet V_S$), $T_\gamma(t_\alpha) = \{t \mid t \in T\backslash P^{0\bullet}, \exists\Gamma(t^*_1, t_\alpha), s.t.\ {}^\bullet t \cap P(\Gamma(t^*_1, t_\alpha)) \subseteq P_S, t \notin \Gamma'(t^*_1, t_\alpha), \forall\Gamma'(t^*_1, t_\alpha)\}$ [$P(\Gamma(t^*_1, t_\alpha)$ is the set of places on the path from t^*_1 to t_α; ${}^\bullet t$ (resp. t) is (resp. *not*) on some (resp. any) path from t^*_1 to t_α], and $y_{\max} = \max Y^S(p)$, $p \in P_S(t_\beta)$ where Y^S was defined in Def. 9.

Note that $T_\gamma(t_\alpha)$ is the set of transitions where tokens can leak out from $[V_S]$ (= P_S, the controller region) if there is no control arc from $t \in T_\gamma(t_\alpha)$ to V_S. $y_{\max}$ is defined such that the P-invariant Y defined in Lemma 3 below will satisfy the negative property.

Definition 11 $\forall t_\alpha$ and its corresponding t_β, $F_A(V_S, t^*_1) = F_A(t_\beta, V_S) = y_{\max} = F_A(t_\mu, V_S)$, $\forall t_\mu \in T_\gamma(t_\alpha)$.

In Fig. 2, for S_2, $t_\alpha(t_\beta)$ in $[S_2^1] = \{p_3\}$: $t_2(t_3)$, $t^*_1 = t_1$, $P_S(t_2) = \{p_2\}$, $P_S(t_3) = \{p_2, p_3\}$, $T_\gamma(t_2) = \varnothing$; and $y_{\max} = 4$; $t_\alpha(t_\beta)$ in $[S_2^2] = \{p_8\}$: $t_8(t_7)$, $P_S(t_7) = \{p_8\}$, $T_\gamma(t_8) = \varnothing$; and $y_{\max} = 2$. More examples will be shown in section 5. The control arcs (denoted by F_A in Def. 11) are added exactly the same as in [4] except that they are weighted as shown in Table 3. Ideally, ${}^\bullet V_S = [S]^\bullet$ and $V_S^\bullet = {}^\bullet[S]$ for less disturbance to the uncontrolled model. $V_S^\bullet$ was set to a subset of $P^{0\bullet}$ in [3, 5] to prevent new *SMS* generation. The support of Y^V includes a set of places (called $P_S(t_\beta)$ in Def. 10) on paths from transitions in $V_S^\bullet$ to t_β. It contains $[S]$ plus a set of places called $P_S(t_\alpha)$) on paths from transitions in $V_S^\bullet$ to t_α. To avoid leakage of tokens from places $p_c \in P_S(t_\alpha)$ (*e.g.* p_6 in Fig. 3) which has more than one output transition, control arcs must be added from $p_c^\bullet$ (called $T_\gamma(t_\alpha)$) to V_S. The following observation is helpful for the proof of Lemma 5.

Observation 1 If $P_S(t_\alpha) \neq \varnothing$, then moving tokens from V_S into $P_S(t_\alpha)$ may not consume the tokens in S. Only tokens moving to $P_S(t_\beta)$ consume the tokens in S.

4.2 Initial Markings of V_S

Based on the above P-invariant, one can determine the initial markings of V_S (Lemma 4) so that each elementary siphon S is ξ-controlled and every dependent siphon is max*-controlled. Afterwards one can define the final controlled model followed by the proof of its liveness.

Lemma 2 (Lemma 3) is useful to prove Lemma 3 (Lemma 4).

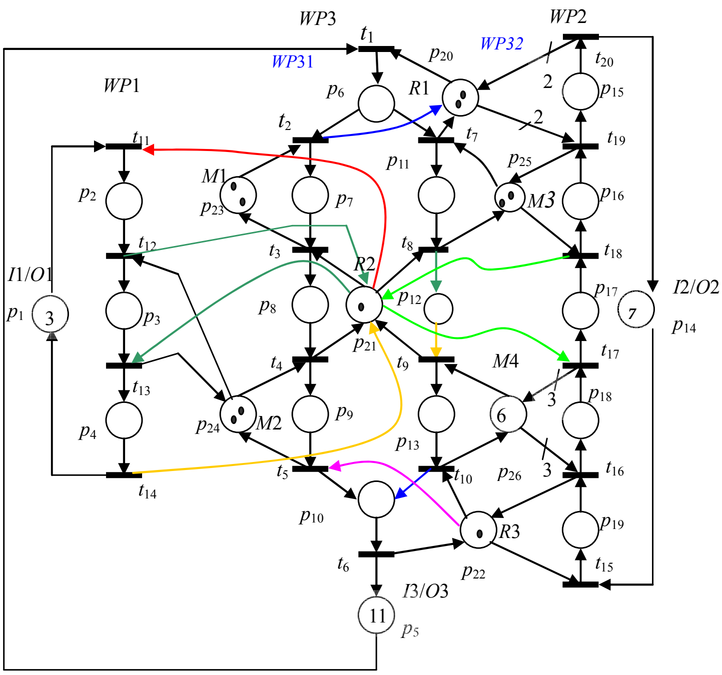

Fig. 3. WS^3PR, a weighted version of the S^3PR in [3, 5].

Lemma 2 Let $Y = Y_\pi + Y_\varphi$ be a P-invariant where $\Omega = \pi \cup \varphi = ||Y||$. (1) $\eta_\pi = -\eta_\varphi$; (2) $\Delta W = \lambda_\Omega{}^T \bullet M - \lambda_\Omega{}^T \bullet M_0 = \lambda_\Omega{}^T \bullet \Delta M = \eta^T \bullet x$ where $\Delta M = M - M_0$; (3) Given a firing vector x, $\Delta W_\pi = -\Delta W_\varphi = (\eta_\pi)^T \bullet x$, where $W_\pi = Y^T{}_\pi \bullet M$ and $W_\varphi = Y^T{}_\varphi \bullet M$.

Proof: (1) By the definition of P-invariant, one has $[N]^T \bullet Y = 0 = [N]^T \bullet (Y_\pi + Y_\varphi)$. Using $\Omega = ||Y||$, one has $\eta = \lambda_\Omega{}^T \bullet [N] = Y^T \bullet [N]$; hence $\eta_\pi = -\eta_\varphi$; (2) Multiplying both sides of equation $M = M_0 + [N] \bullet x$ by $\lambda_\Omega{}^T$, one has $\lambda_\Omega{}^T \bullet M = \lambda_\Omega{}^T \bullet M_0 + \lambda_\Omega{}^T \bullet ([N] \bullet x) = \lambda_\Omega{}^T \bullet M_0 + (\lambda_\Omega{}^T \bullet [N]) \bullet x = \lambda_\Omega{}^T \bullet M_0 + \eta^T \bullet x, \lambda_\Omega{}^T \bullet (M - M_0) = \lambda_\Omega{}^T \bullet \Delta M = \Delta W = \eta^T \bullet x$; (3) From (1) and (2), one has $\Delta W_\pi = (\eta_\pi)^T \bullet x = -(\eta_\varphi)^T \bullet x = -\Delta W_\varphi$. ❑

Based on Def. 11, one can assign P-invariant Y^V as follows.

Definition 12 Let V_S be the control place associated with elementary siphon S and Y^V the P-invariant associated with V_S. Y^V: $Y^V(p_j) = -F_A(t_\beta, V_S)$ (Def. 11), $\forall p_j \in P_S(t_\beta)$, $Y^V(p_j) = -1$, for $p_j = V_S$, and $Y^V(p_j) = 0$ for all other p_j.

Lemma 3 (1) $\forall S \in \prod_E$, $Y = Y^S + Y^V$ satisfies the negative property where $Y^S(Y^V)$ is defined in Def. 9 (Def. 12); (2) For any firing vector x, $-\Delta W_S \leq \Delta W_V$.

Proof: (1) $\forall p_j \in S$, $Y^V(p_j) = 0$ and hence $Y(p_j) = Y^S(p_j) + Y^V(p_j) > 0$. $\forall p_j \in P_S(t_\beta)$, one has $Y^V(p_j) = -F_A(t_\beta, V_S) = -y_{\max} = -\max Y^S(p)$, $p \in P_S(t_\beta)$ by Defs. 10 and 12 respectively. Thus,

$$Y(p_j) = Y^S(p_j) + Y^V(p_j) = Y^S(p_j) - \max Y^S(p) \leq 0. \tag{1}$$

If $p_j = V_S$, then $Y(p_j) = Y^S(p_j) + Y^V(p_j) = -1 < 0$ since $Y^S(p_j) = 0$. For all other p_j, $Y^V(p_j) = Y^S(p_j) = 0$; thus $Y(p_j) = 0$. In summary, $\forall p_j \in (P \cup P^0 \cup P_R \cup P_A)\backslash S$, $Y(p_j) \leq 0$ and Y satisfies the negative property; (2) Using Lemma 2.3, one has

$$\Delta W_S = -\Delta W_{[S]} \text{ and } \Delta W_V = -\Delta W_{[V]} \text{ where } [V] = \|Y^V\|\backslash\{V_S\} \text{ and } V = \{V_S\}. \quad (2)$$

Because $Y^S(p_j) \leq -Y^V(p_j)$ due to the fact that $Y(p_j) \leq 0$ based on inequality (1) and $M(p_j) \geq 0$, $\forall p_j \in [V] \cup [S]$, one has

$$\Delta W_{[S]} \leq -\Delta W_{[V]}. \quad (3)$$

Substituting inequality (3) back into (2), one has $-\Delta W_S \leq \Delta W_V$. ❑

Lemma 4 Let $Y = Y^S + Y^V$ with $Y^S(Y^V)$ defined in Def. 9 (Def. 12) and $M_0(V_S) = M_0(S) - W_S - \xi_S$; (1) $\forall S \in \prod_E$, it is *ξ-controlled*; (2) $M_0(S) - W_S - \delta_S \geq \xi_S \geq 1$ where $\delta_S = \max F(V_S, t)$, $t \in T$.

Proof: (1) The maximal ΔW_V occurs when $M(V_S) = 0$ implying $(\Delta W_V)_{\max} = -Y(V_S) \bullet M_0(V_S) = M_0(V_S) = M_0(S) - W_S - \xi_S$. By Lemma 3.2, one has $(\Delta W_S) \leq (\Delta W_V)_{\max} = M_0(V_S) = M_0(S) - W_S - \xi_S \Rightarrow M_0(S) - W(M(S)) \leq M_0(S) - W_S - \xi_S$, implying $W(M(S)) \geq W_S + \xi_S$, *i.e.*, S is ξ-controlled by Def. 7, where $\Delta W_S = (Y^S)^T \bullet \Delta M = (Y^S)^T \bullet M - (Y^S)^T \bullet M_0 = (Y^S)^T \bullet M - M_0(S)$, and $(Y^S)^T \bullet M_0 = \sum_{R\in S} Y^S(r)M_0(r) = \sum_{R\in S} M_0(r) = M_0(S)$. (2) Note that the above ξ_S is upper bounded by $M_0(S) - W_S$ so that $M_0(V_S) \geq 1$, which may cause some output transitions of V_S to become dead. To avoid this, one sets $M_0(V_S) \geq \delta_S = \max F(V_S, t)$, $t \in T$. This sets $M_0(S) - W_S - \delta_S \geq \xi_S \geq 1$. ❑

ξ_S must be such that each dependent siphon is max*-controlled as in the following lemma.

Lemma 5 Y and $M_0(V_S)$ are defined as in Lemma 4 and $\lambda_\Omega^T = (Y^S)^T$ for λ_Ω with $\Omega = S_0$ and Def. 9 for Y^S. S_0 is max*-*controlled* if the inequality $M_0(S_0) > W_{S0} + \sum^n_{i=1} a_i(M_0(S_i) - W_{Si} - \xi_{Si})$ holds where each S_i is an elementary siphon.

Proof: Multiplying both sides of equation $M = M_0 + [N] \bullet x$ by λ_{S0}^T, one has $\lambda_{S0}^T \bullet M = \lambda_{S0}^T \bullet M_0 + \lambda_{S0}^T \bullet ([N] \bullet x) = \lambda_{S0}^T \bullet M_0 + (\lambda_{S0}^T \bullet [N]) \bullet x = \lambda_{S0}^T \bullet M_0 + \eta_0^T \bullet x$.

Using the fact that $\lambda_{S0}^T = (Y^S)^T$, $\lambda_{S0}^T \bullet M_0 = \sum_{R\in S} Y^S(r)M_0(r)$ and $\eta_0 = \sum^n_{i=1}(a_i\eta_i) - \sum^m_{j=1}(b_j\eta_j)$ (S_0 is a weakly dependent siphon if each $b_j > 0$), one has $\lambda_{S0}^T \bullet M = (Y^S)^T \bullet M = M_0(S_0) + \sum^n_{i=1}(a_i\eta_i)^T \bullet x - \sum^m_{j=1}(b_j\eta_j)^T \bullet x$.

Defining $\Delta W_{S0} = W(M(S_0)) - W(M_0(S_0))$ (Recall $\Delta W = W(M) - W(M_0)$ in Def. 4.), one has

$$\Delta W_{S0} = W(M(S_0)) - M_0(S_0) = \sum^n_{i=1}(a_i\eta_i)^T \bullet x - \sum^m_{j=1}(b_j\eta_j)^T \bullet x \quad (3)$$

where $W(M_0(S_0)) = \sum_{R\in S0} Y(r)M_0(r) = \sum_{R\in S0} M_0(r) = M_0(S_0)$ and $Y(r) = 1$, $\forall r \in S_0$.

To maximize $|\Delta W_{S0}|$, $|\sum^n_{i=1}(a_i\eta_i)^T \bullet x|$ ($|\sum^m_{j=n+1}(b_j\eta_j)^T \bullet x|$), should be maximized (minimized). $|(a_i\eta_i)^T \bullet x|$ ($|(b_j\eta_j)^T \bullet x|$) is the amount of weighted tokens trapped in $[S_i]$ ($[S_j]$). In the presence of P_S, the tokens in $V_{Sj}(V_{Si})$ may be completely trapped in $P_{Sj}\backslash[S_j]$

($[S_i]$) rather than $[S_j]$ ($P_{Si}\backslash[S_i]$) to minimize $|(b_j\eta_j)^T \bullet x|$ (maximize $|(a_i\eta_i)^T \bullet x|$). (Note that $\forall k \in I_N = \{1, 2, \ldots, n\}$, $[S_k]$ is a subset of P_{Sk}. The above thesis does not hold when $[S_k] = P_{Sk}$; however, no *SMS* depends on S_k and $a_k = 0$.)

Thus, $|\Delta W_{S0}|_{\max} = |\sum^n_{i=1}(a_i\eta_i)^T \bullet x|_{\max} = \sum^n_{i=1}a_i|\Delta W_{Si}|_{\max} \leq \sum^n_{i=1}a_i|\Delta W_{VSi}|_{\max} = \sum^n_{i=1}a_iM_0(V_{Si})$ (Lemma 3.2). Now $|\Delta W_{S0}| \leq |\Delta W_{S0}|_{\max}$; thus, $\sum^n_{i=1}a_iM_0(V_{Si}) \geq |\Delta W_{S0}| = M_0(S_0) - W(M(S_0))$ by Eq. (3) and one has $M_0(S_0) - \sum^n_{i=1}a_iM_0(V_{Si}) \leq W(M(S_0))$. With $M_0(V_{Si}) = M_0(S_i) - W_{Si} - \xi_{Si}$ and the assumption $M_0(S_0) > W_{S0} + \sum^n_{i=1}a_i(M_0(S_i) - W_{Si} - \xi_{Si})$ of the lemma, one has $W_{S0} < W(M(S_0))$ and S_0 is max*-controlled. ❑

Note that the b_j terms do not show up in the inequality of Lemma 5. This is because the consumption of tokens in V_{Sj} tends to increase the tokens in S_0 in the absence of $P_S(t_\alpha)$, which by Observation 1 may capture the above tokens resulting in no token increase in S_0.

Definition 13 $\forall S \in \prod_E$, $M_0(V_S)$ must be such that (1) $\sum^{|\Pi E|}_{i=1}\xi_{Si}$ is minimized via integer programming, (2) $M_0(S) - W_S - \delta_S > \xi_S \geq 1$, and (3) For every dependent siphon S_0, $M_0(S_0) > W_{S0} + \sum^n_{i=1}a_i(M_0(S_i) - W_{Si} - \xi_{Si})$ as in Lemma 5.

Definition 14 Let (N, M_0) be a marked WS^3PR ($P \cup P^0 \cup P_R$, T, F). The net $(N_A, M_{0A}) = (P \cup P^0 \cup P_R \cup P_A, T, F \cup F_A, M_{0A})$ is the controlled system of (N, M_0) iff (1) $P_A = \{V_S \mid S \in \prod_E\}$ is the set of places such that there exists a bijective mapping from $\prod_E$ into it; (2) F_A is defined in Def. 11; (3) M_{0A} is defined as follows: (a) $\forall p \in P \cup P^0 \cup P_R$, $M_{0A}(p) = M_0(p)$; (b) $\forall V_S \in P_A$, $M_{0A}(V_S)$ is determined in Def. 12.

Theorem 2 [6] Let (N_A, M_{0A}) be the controlled system of a marked WS^3PR (N, M_0). Then (N_A, M_{0A}) is live.

5. FMS EXAMPLE

The net system in Fig. 3 is a WS^3PR and contains deadlocks. There are 6 elementary siphons and 12 dependent siphons as shown in Tables 1 and 2 respectively. For example, S_3 is a dependent *SMS* w.r.t. to S_4 and S_{18}. Let us apply our deadlock prevention algorithm to this net system. First add 6 control places V_{S1}, V_{S4}, V_{S10}, V_{S16}, V_{S17}, and V_{S18} which correspond to six elementary siphons S_1, S_4, S_{10}, S_{16}, S_{17}, and S_{18}, respectively.

Table 3 and Fig. 4 show the new places, arcs as well as $M_0(V_{Si})$, added using the control policy for the WS^3PR in Fig. 3. Note that $t_{15}(3)$ in the 2nd column for S_4 indicates that $F_A(V_S, t_{15}) = 3$. For S_4, there are two $t_\alpha(t_\beta)$ in $[S_4^1]$: $t_3(t_5)$ and $t_8(t_{10})$, $t^*_1 = t_1$, $P_S(t_3) = \{p_6, p_7\}$, and $P_S(t_5) = \{p_6, p_7, p_8, p_9\}$, $T_\gamma(t_3) = \varnothing$ and $y_{\max} = 2$. There is one $t_\alpha(t_\beta)$ in $[S_4^2]$: $t_{11}(t_{13})$, $t^*_1 = t_{11}$, $T_\gamma(t_{11}) = \varnothing$ and $y_{\max} = 1$. There is one $t_\alpha(t_\beta)$ in $[S_4^3]$: $t_{15}(t_{17})$, $t^*_1 = t_{15}$, $T_\gamma(t_{15}) = \varnothing$ and $y_{\max} = 3$. For S_{18}, there is one $t_\alpha(t_\beta)$ in $[S_{18}^1]$: $t_7(t_8)$, $t^*_1 = t_1$, $T_\gamma(t_7) = \{t_2\}$, $y_{\max} = 1$; one $t_\alpha(t_\beta)$ in $[S_{18}^3]$: $t_{17}(t_{18})$, $t^*_1 = t_{15}$, $T_\gamma(t_{17}) = \varnothing$, $y_{\max} = 1$.

Note that the method proposed in [3] is used to add a control place for each elementary siphon and no new *SMS* will be generated due to these new additional places.

Note that all control arcs, *e.g.* $F_A(V_{S4}, t_{15}) = F_A(t_{17}, V_{S4}) = 3$, associated with the same (different) *WP* (may) carry the same (different, *e.g.* $F_A(V_{S4}, t_{15}) \neq F_A(V_{S4}, t_1)$) weights. t_{15} and $t_{17}(t_1)$ are in the same (different) *WP*. This is the reason that one defines control arcs relative to each $[S^i]$ in Defs. 10 and 11.

Table 1. Elementary siphons, resource circuits, and η^w for the net in Fig. 3.

Elem. siphons	places	c	η^w
S_1	$p_{10}, p_{18}, p_{22}, p_{26}$	$[p_{22}\ t_{10}\ p_{26}\ t_{16}\ p_{22}]$	$-t_9 + t_{10} - t_{15} + t_{16}$
S_4	$p_4, p_{10}, p_{17}, p_{21}, p_{22}, p_{24}, p_{26}$	$[p_{21}\ t_{17}\ p_{26}\ t_{16}\ p_{22}\ t_5\ p_{24}\ t_4\ p_{21}]$	$-t_3 + t_5 - t_8 + t_{10} - t_{11}$ $+ t_{13} - t_{15} - 2t_{16} + 3t_{17}$
S_{10}	$p_4, p_9, p_{12}, p_{17}, p_{21}, p_{24}$	$[p_{21}\ t_{13}\ p_{24}\ t_4 p_{21}]$	$-t_3 + t_4 - t_{11} + t_{13}$
S_{16}	$p_2, p_4, p_8, p_{13}, p_{17}, p_{21}, p_{26}$	$[p_{21}\ t_{17}\ p_{26}\ t_9\ p_{21}]$	$+ t_9 - t_8 + 3t_{17} - 3t_{16}$
S_{17}	$p_2, p_4, p_8, p_{12}, p_{15}, p_{20}, p_{21}, p_{23}, p_{25}$	$[p_{21}\ t_3\ p_{23}\ t_2\ p_{20}\ t_{19}\ p_{25}\ t_{18}\ p_{21}]$	$-t_1 + t_3 + t_8 - t_{17} - t_{19}$
S_{18}	$p_2, p_4, p_8, p_{12}, p_{16}, p_{21}, p_{25}$	$[p_{21}\ t_8\ p_{25}\ t_{18}\ p_{21}]$	$-t_7 + t_8 - t_{17} + t_{18}$

Table 2. Dependent siphons, their η^w, $M_0(S)$, and W_S for the net in Fig. 3.

dependent siphons	places	η^w relationship	$M_0(S)$	W_S(Def. 4)
S_2	$p_4p_{10}p_{15}p_{20}p_{21}p_{22}p_{23}\ p_{24}p_{25}p_{26}$	$\eta^w{}_2 = \eta^w{}_4 + \eta^w{}_{17}$	16	3
S_3	$p_4p_{10}p_{16}p_{21}p_{22}p_{24},p_{25}p_{26}$	$\eta^w{}_3 = \eta^w{}_4 + \eta^w{}_{18}$	12	2
S_5	$p_4p_9p_{13}p_{15}p_{20}p_{21}p_{23}p_{24}p_{25}p_{26}$	$\eta^w{}_5 = \eta^w{}_{10} + \eta^w{}_{16} + \eta^w{}_{17}$	15	1
S_6	$p_4p_9p_{13}p_{16}p_{21}p_{24}p_{25}p_{26}$	$\eta^w{}_6 = \eta^w{}_{10} + \eta^w{}_{16} + \eta^w{}_{18}$	11	0
S_7	$p_4p_9p_{13}p_{17}p_{21}p_{24}p_{26}$	$\eta^w{}_7 = \eta^w{}_{10} + \eta^w{}_{16}$	9	0
S_8	$p_4p_9p_{12}p_{15}p_{20}p_{21}p_{23}p_{24}p_{25}$	$\eta^w{}_8 = \eta^w{}_{10} + \eta^w{}_{17}$	9	1
S_9	$p_4p_9p_{12}p_{16}p_{21}p_{24}p_{25}$	$\eta^w{}_9 = \eta^w{}_{10} + \eta^w{}_{18}$	5	0
S_{11}	$p_2p_4p_8p_{10}p_{15}p_{20}p_{21}p_{22}p_{23}p_{25}p_{26}$	$\eta^w{}_{11} = \eta^w{}_1 + \eta^w{}_{16} + \eta^w{}_{17}$	14	3
S_{12}	$p_2p_4p_8p_{13}p_{15}p_{20}p_{21}p_{23}p_{25}p_{26}$	$\eta^w{}_{12} = \eta^w{}_{16} + \eta^w{}_{17}$	13	1
S_{13}	$p_2p_4p_8p_{10}p_{16}p_{21}p_{22}p_{25}p_{26}$	$\eta^w{}_{13} = \eta^w{}_1 + \eta^w{}_{16} + \eta^w{}_{18}$	10	2
S_{14}	$p_2p_4p_8p_{13}p_{16}p_{21}p_{25}p_{26}$	$\eta^w{}_{14} = \eta^w{}_{16} + \eta^w{}_{18}$	9	0
S_{15}	$p_2p_4p_8p_{10}p_{17}p_{21}p_{22}p_{26}$	$\eta^w{}_{15} = \eta^w{}_1 + \eta^w{}_{16}$	8	2

Table 3. Elementary siphons and the control model.

Elementary siphons	$V_S{}^\bullet$	${}^\bullet V_S$	$M_0(V_S)$	W_S(Def. 4)
S_1	$\{t_1, t_{15}\}$	$\{t_{16}, t_{10}, t_2\}$	4	2
S_4	$\{t_1, t_{11}, t_{15}(3)\}$	$\{t_5, t_{10}, t_{13}, t_{17}(3)\}$	7	2
S_{10}	$\{t_1, t_{11}\}$	$\{t_4, t_7, t_{13}\}$	2	0
S_{16}	$\{t_1, t_{15}(3)\}$	$\{t_2, t_9, t_{17}(3)\}$	6	0
S_{17}	$\{t_1, t_{15}\}$	$\{t_3, t_8, t_{19}\}$	5	1
S_{18}	$\{t_1, t_{15}\}$	$\{t_2, t_8, t_{18}\}$	2	0

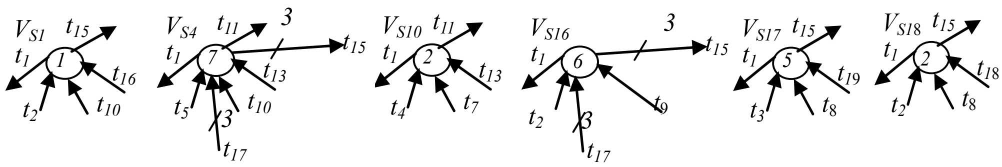

Fig. 4. Control model for the FMS in Fig. 3.

Note that the controlled model in Fig. 4 is the same as that in [3, 4] except that some arcs are weighted.

Table 4. Marking relationships between dependent and elementary siphons for the net in Fig. 3.

dependent siphons	Initial marking relationships	Controlled?
S_2	$(M_0(S_2) - W_{S2}) - (M_0(S_4) + M_0(S_{17}) - W_{S4} - W_{S17} - 2) = 1$	yes
S_3	$(M_0(S_3) - W_{S3}) - (M_0(S_4) + M_0(S_{18}) - W_{S4} - W_{S18} - 2) = 1$	yes
S_5	$(M_0(S_5) - W_{S5}) - (M_0(S_{10}) + M_0(S_{16}) + M_0(S_{17}) - W_{S10} - W_{S16} - W_{S17} - 3) = 1$	yes
S_6	$(M_0(S_6) - W_{S6}) - (M_0(S_{10}) + M_0(S_{16}) + M_0(S_{18}) - W_{S10} - W_{S16} - W_{S18} - 3) = 1$	yes
S_7	$(M_0(S_7) - W_{S7}) - (M_0(S_{10}) + M_0(S_{16}) - W_{S10} - W_{S16} - 2) = 1$	yes
S_8	$(M_0(S_8) - W_{S8}) - (M_0(S_{10}) + M_0(S_{17}) - W_{S10} - W_{S17} - 2) = 1$	yes
S_9	$(M_0(S_9) - W_{S9}) - (M_0(S_{10}) + M_0(S_{18}) - W_{S10} - WS_{18} - 2) = 1$	yes
S_{11}	$(M_0(S_{11}) - W_{S11}) - (M_0(S_1) + M_0(S_{16}) + M_0(S_{17}) - W_{S1} - W_{S16} - W_{S17} - 3) = -4$	no
S_{12}	$(M_0(S_{12}) - W_{S12}) - (M_0(S_{16}) + M_0(S_{17}) - W_{S16} - W_{S17} - 2) = 1$	yes
S_{13}	$(M_0(S_{13}) - W_{S13}) - (M_0(S_1) + M_0(S_{16}) + M_0(S_{18}) - W_{S1} - W_{S16} - W_{S18} - 3) = -4$	no
S_{14}	$(M_0(S_{14}) - W_{S14}) - (M_0(S_{16}) + M_0(S_{18}) - W_{S16} - W_{S18} - 2) = 1$	yes
S_{15}	$(M_0(S_{15}) - W_{S15}) - (M_0(S_1) + M_0(S_{16}) - W_{S1} - W_{S16} - 2) = -4$	no

Consider the least restrictive case when $\xi_{Si} = 1$, $i = 1, 4, 10, 16, 17, 18$. One has $M_0(V_{S1}) = 4$, $M_0(V_{S4}) = 7$, $M_0(V_{S10}) = 2$, $M_0(V_{S16}) = 6$, $M_0(V_{S17}) = 5$, and $M_0(V_{S18}) = 2$. The dependent siphons, their elementary siphons, the initial marking relationships between dependent and elementary siphons and the controllability of dependent siphons due to Lemma 5 are shown in Table 4.

Note that there are three "no" in Table 4 (three equalities with – 4 on the right-hand sides) and hence S_{11}, S_{13}, and S_{15} may not be controlled. The controllability can be met by setting $\xi_{S1} = 4$ for the above three equalities to have values of one since $M_0(S_1)$ appears only in the three associated initial marking relationships. The resulting controlled net reaches 3054 states out of the total 69536 states of the uncontrolled model using the INA (Integrated Net Analyzer).

However, this does not mean that the controlled net with $\xi_{S1} = 1$ is dead similar to that in [4]. Without adjusting any control depth variable, the controlled WS^3PR reaches 10304 states and is live. This is of great significance because one only adds six control places and 32 arcs – amazingly exactly the same as that in [4] – and the resultant Petri net is live. In [3], 18 control places and 106 arcs are added, which makes the final controller three times more complex than that of ours where the model is a weighted version and hence more complicated than the S^3PR in [4].

6. CONCLUSION

We have improved the liveness condition of WS^3PR.with max*-controlled siphons. Exploiting this, a better control technique for WS^3PR has been pursued.

The condition of max*-controlled siphons may be extended to more complicated models (*e.g.*, multiple types of resources used at a job stage) where (1) Any *SMS* can be synthesized from strongly connected resource subnets, and (2) The weight of any output arc from an operation place is unity.

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