Economic Determinants of Default Risks and Their Impacts on Credit Derivative Pricing

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This study constructs a credit derivative pricing model using economic fundamentals to evaluate CDX indices and quantify the relationship between credit conditions and the economic environment. Instead of selecting specific economic variables, numerous economic and financial variables have been condensed into a few explanatory factors to summarize the noisy economic system. The impacts on default intensity processes are then examined based on no-arbitrage pricing constraints. The approximated results show that economic factors indicated credit problems even before the recent subprime mortgage crisis, and economic fundamentals strongly influenced credit conditions. Testing of out-of-sample data shows that credit evolution can be identified by dynamic explanatory factors.

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Consequently, the factor-based pricing model can either facilitate the evaluation of default probabilities or manage default risks more effectively by quantifying the relationship between economic environment and credit conditions. © 2010 Wiley Periodicals, Inc. Jrl Fut Mark 30:1058–1081, 2010

INTRODUCTION

This study modifies the reduced-form model to evaluate credit derivatives with economic fundamentals. To identify actual credit evolution, the influences of economic conditions on default risks are quantified through no-arbitrage pricing constraints. In a changing economic environment, the proposed credit derivative pricing model facilitates the evaluation of credit risks and raises awareness of aggravated credit conditions for effective risk management.

This study has three objectives: identifying relevant explanatory factors for default risks, quantifying the influences of these economic factors on default risks, and pricing credit derivatives based on economic conditions. First, a dynamic model is constructed to summarize information from relevant economic and financial data to assess changes in the economic environment. Instead of using specific macroeconomic variables, we choose dynamic explanatory factors from a large number of variables to represent the complex economic system. Because firms are exposed to the same macroeconomic conditions, systematic factors, and financial markets, contagious default intensities lead to temporal clustering of defaults. Macroeconomic indicators were incorporated into real activity and inflationary groups as in previous studies (Ang, Dong, & Piazzesi, 2004; Ang & Piazzesi, 2003; Wu & Zhang, 2008). Moreover, mortgage-related derivatives have a dominant market share,[1] which contributed to the subprime mortgage bubble in the US that led to a general economic recession. Housing market financial data are used to obtain a housing factor, as the US housing market was the first to be affected by the current credit crunch, and losses in that market have caused ripple effects throughout the world economy. To obtain useful information from the US housing market regarding rising delinquency and foreclosure risk, relevant indicators from default risks are included as the third explanatory factor. In summary, this study combines various economic and financial variables into three explanatory factors: the real economy, inflation and housing.

Second, survival probabilities are modeled using these explanatory factors to quantify how they influence default conditions. By imposing no-arbitrage pricing constraints and constructing default intensity processes as an affine

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[1]U.S. credit markets include corporate bonds, municipal bonds, commercial paper, asset-backed securities, CDOs, and mortgage-related securities. From the statistics provided by the Securities Industry and Financial Markets Association (SIFMA), at the end of 2005, mortgage-related credit derivatives reached a 63% market share in U.S. credit markets, with the issuances of $3.546 trillion notional.

model of fundamental economic factors, unobservable default intensities are taken from credit derivative spreads. This approach makes it possible to derive survival probabilities from default intensities and quantify the influence of various economic and financial factors on default risks.

Third, if explanatory factors influence default probabilities in a credit derivative pricing model, analyzing the changes of these fundamental economic factors can provide useful indicators of credit evolution. After deriving the impacts of various economic variables on survival probabilities through the no-arbitrage dynamic model, they are applied to price CDX index spreads. The resulting spreads reveal clear changes in credit conditions before 2007.

Changes in the economic environment have some impact on total credit risk. Bhansali, Gingrich, and Longstaff (2008) separated credit risk into idiosyncratic, sector wide and economy wide defaults. Longstaff and Rajan (2008) demonstrated that economy-wide credit risk had been rising markedly since 2007 and was the main cause of increased credit spreads during 2007. This is consistent with the recognition of many studies that defaults clustered and were contagious (Das, Freed, Geng, & Kapadia, 2006; Davis & Lo, 2001; Giesecke & Weber, 2004, 2006; Haworth, Reisinger, & Shaw, 2008; Jarrow & Yu, 2001; Jorion & Zhang, 2007; Lonstaff & Rajan, 2008; Rösch & Winterfeldt, 2008). Numerous studies have also proposed that corporate defaults and bankruptcies can be better understood using systematic components and macroeconomic indicators, such as gross domestic product (GDP) and personal income growth (Altman, Brady, Resti, & Sironi, 2005; Collin-Dufresne, Goldstein, & Martin, 2001; Couderc & Renault, 2004; Das, Duffie, Kapadia, & Saita, 2007; Duffie, Leandro, & Wang, 2007; Lennox, 1999; Lo, 1986; McDonald & Van de Gucht, 1999). Some researchers have focused on the relationship between theoretical determinants of CDS spreads and default risks (Amato, 2005; Ericsson, Jacobs, & Oviedo, 2009). This study examines the relationship between economic variables and credit evolution with the combination of several economic indicators to provide more insight into movements of default probabilities driven by economic determinants.

Unlike previous research (Amato & Luisi, 2006; Wu & Zhang, 2008), this study analyzes and models the influences of economic indicators on the credit spreads of CDX indices, which comprise various entities and are well diversified. As the current credit crunch was triggered by the subprime mortgage crisis, it spilts over through credit derivative instruments eventually resulting in clustered defaults of financial institutions. Estimated results indicate that all parameters are statistically significant and factor dynamics coincide with actual economic phenomenon. Both inflation and housing factors strongly and positively affected default intensities. On the contrary, the real economic factor exerts a significant negative influence. After pricing with default intensity

processes, the credit spreads show that economic indicators revealed the extent of the credit disarray before 2007. Examining the economic and financial data and quantifying the linkage between the explanatory factors and default risks can improve understanding of the credit crunch and manage credit risks more effectively in the future.

METHODOLOGY

Credit default swaps (CDSs) are the most popular instruments in the credit derivative market. To facilitate trading, standard CDS indices (CDX) are used as benchmarks for credit risks. In the literature, both structural and reduced-form models have been used to price credit derivatives. Owing to the difficulties of calibrating the specific dynamic model to individual credit entities in structural models and the disadvantage of determining the default environment based on a single factor model, a reduced-form model examining fundamental factors is proposed.

The economic environment changes stochastically with the release of new information. To simplify a noisy economic system, explanatory factors are combined from numerous variables using the Kalman filter. These factors are used rather than specific variables, and each factor is updated once new observations become available.

Compressing Economic and Financial Variables into Three Explanatory Factors

Instead of examining the potential role of economic variables in default intensities and using regression method with specific choice of some explanatory variables, we divide variables into three dynamic factors. The dynamic factor model can extract information from economic and financial data and suppress noise. Real activities and inflation variables are classified to identify specific effects on default intensities. Housing market variables are employed to determine the relationship between the housing bubble and the credit crunch. At the end of 2005, mortgage-related credit derivatives reached a 63% market share, with 3.546 trillion issuances. Since mortgage-related derivatives dominated the markets, it is necessary to properly consider the systematic risk from credit markets. Moreover, the 2008 credit crunch was caused by the subprime mortgage crisis, which in turn was triggered by the housing market downturn, excessively loose lending standards and overly complex structured credit derivatives. When managing risk it is important to quantify the magnitude of housing market performance.

The dynamics of explanatory factors in physical measure $\mathbb{P}$ are represented as

$$dN_t = -\varphi N_t dt + dB_t^P \tag{1}$$

where φ denotes an $n \times n$ transition matrix, and B_t^P represents a vector of standard Brownian motions under the physical measurement $\mathbb{P}$. The matrix φ is restricted to a diagonal matrix yielding independent explanatory factors. By Euler approximation, we obtain the discrete-time version of the factor dynamics in Equation (1) as

$$N_t = \Phi N_{t-\Delta t} + \varepsilon_t \tag{2}$$

where vector N_t denotes the explanatory factors with dimension $n \times 1$, $N_t \in \mathbb{R}^n$. The autoregressive coefficient matrix Φ is an $n \times n$ matrix, Δt is the time interval, and $\varepsilon_t \sim N(0, Q)$ is an $n \times 1$ normal innovation vector where Q is a diagonal covariance matrix. This study groups economic and financial variables into three explanatory factors, $n = 3$. Observable economic and financial data series are set as affine functions of explanatory factors N_t through the following measurement equation:

$$M_t = CN_t + \varepsilon_t^M \tag{3}$$

where M_t denotes the observable economic and financial variables with dimension $m \times 1$, $M_t \in \mathbb{R}^m$. C represents the coefficient matrix with dimension $m \times n$, and the disturbance $\varepsilon_t^M \sim N(0, R^M)$ is an m-vector with zero mean and measurement error covariance matrix R^M. This disturbance can be seen as the residual effects not measured by explanatory factors. Finally, ε_t and ε_t^M are assumed to be independent.

Between observations, the priori estimation of explanatory factors and their covariance before time t are denoted as $\hat{N}_{t^-}$ and $\hat{P}_{t^-}$, respectively:

$$\hat{N}_{t^-} = \Phi \hat{N}_{t-\Delta t}$$

$$\hat{P}_{t^-} = \Phi \hat{P}_{t-\Delta t} \Phi^T + Q$$

where Φ^T denotes the transposition of Φ. The one-step ahead prediction of measurement variable $\hat{M}_{t^-}$, and its covariance $\hat{\Sigma}_{t^-}$ are

$$\hat{M}_{t^-} = C\hat{N}_{t^-}$$

$$\hat{\Sigma}_{t^-} = C\hat{P}_{t^-}C^T + R^M.$$

After new observations become available, the Kalman filter procedure is used to refine the priori predictions of explanatory factors and their covariance to derive *posteriori* predictions and their covariance:

$$\hat{N}_t = \hat{N}_{t^-} + K_t(M_t - C\hat{N}_{t^-})$$

$$\hat{P}_t = \hat{P}_{t^-} - K_t C\hat{P}_{t^-}$$

where

$$K_t = \hat{P}_{t^-}C^T(C\hat{P}_{t^-}C^T + R^M)^{-1}$$

denotes the Kalman gain. Consequently, all estimates were improved with the availability of additional observations. As we assume that prediction errors follow a normal distribution, the log likelihood function can be defined as

$$L_t(\varphi, C, R^M) = -\frac{1}{2}\log|\hat{\Sigma}_{t^-}| - \frac{1}{2}\left[(M_t - \hat{M}_{t^-})^T(\hat{\Sigma}_{t^-})^{-1}(M_t - \hat{M}_{t^-})\right]. \qquad (4)$$

Then we obtain the parameter estimates by maximizing the sum of the log likelihood values of prediction errors from all sample periods.

Influence of Economic Environment on Default Intensity

To investigate the effect of the economic environment on default intensity, and evolution of the credit environment with changing economic and financial conditions, it was assumed that default intensity is an affine function of the three dynamic explanatory factors extracted from economic and financial data, where:

$$\lambda(N_t) = \alpha + \beta^T N_t. \qquad (5)$$

The coefficient vectors, α and β, represent the simultaneous effects on default intensity from changes in explanatory factors, thus linking the dynamics of default intensity to shocks on economic variables. The measurement equation in the Kalman filter procedure is then defined as:

$$\lambda_t = \alpha + \beta^T N_t + \varepsilon_t^\lambda. \qquad (6)$$

Measurement error ε_t^λ is identified as disturbances that are not measured by explanatory factors and are independent of each explanatory factor N_t.

Dynamic Pricing Model for Credit Derivatives and Default Intensity Processes with No-arbitrage Constraints

As this study utilizes data from the Dow Jones CDX North America Investment Grade (DJ CDX NA IG) index to find the relationship between default intensities and economic conditions, it is necessary to calibrate CDX market quotes to obtain parameter estimates of default intensities. The DJ CDX NA IG index is a standard credit default index designed to facilitate trading and improve the liquidity of CDSs. Valuation for CDX index contracts differs slightly from single-name CDSs. For single-name CDS contracts the payment for swap premium ceases following default events. On the contrary, on the CDX, default entities are removed from the index and swap premium payments continue at a decreased notional amount until maturity.

Under risk neutral measure $\mathbb{Q}$, investors will receive payments at times t_1 to t_T with the present value of these regular payments being denoted as the first part of the premium leg:

$$PL_1 = s \sum_{c=1}^{T} (t_c - t_{c-1}) E^{\mathbb{Q}}(t_c) D(t_c)$$

where s denotes CDX spread, t_c represents payment dates, $E^Q(t_c)$ is the expected principal at time t_c, and $D(t_c)$ denotes the discount factor. Assuming defaults on average occur during the middle of payment dates, the present value of accrual payments in default comprises the other part of the premium leg:

$$PL_2 = s \left[0.5 \sum_{c=1}^{T} (t_c - t_{c-1})(E^{\mathbb{Q}}(t_{c-1}) - E^{\mathbb{Q}}(t_c)) D(t_c^d) \right]$$

where $t_c^d = 0.5(t_{c-1} + t_c)$.[2]

The present value of the premium leg is represented as

$$PL = PL_1 + PL_2.$$

The present value of the default leg is

$$DL = \sum_{c=1}^{T} (E^{\mathbb{Q}}(t_{c-1}) - E^{\mathbb{Q}}(t_c)) D(t_c^d).$$

Since the CDX index is defined as the breakeven spread, the credit spread is obtained while the present value of the default leg equals the premium leg and leaves no arbitrage opportunities:

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[2]Consistent with Hull and White (2008).

$$s = \frac{\sum_{c=1}^{T}(E^{\mathbb{Q}}(t_{c-1}) - E^{\mathbb{Q}}(t_c))D(t_c^d)}{\sum_{c=1}^{T}(t_c - t_{c-1})E^{\mathbb{Q}}(t_c)D(t_c) + 0.5\sum_{c=1}^{T}(t_c - t_{c-1})(E^{\mathbb{Q}}(t_{c-1}) - E^{\mathbb{Q}}(t_c))D(t_c^d)}.$$

(7)

Without loss of generality, we assume the principal $V = 1$. The expected value at each payment time is

$$E^{\mathbb{Q}}(t_c) = V \cdot E^{\mathbb{Q}}[S(t_c)|\mathbb{F}_t] = E^{\mathbb{Q}}[S(t_c)|\mathbb{F}_t]$$

(8)

where $E^{\mathbb{Q}}[S(t_c)|\mathbb{F}_t]$ denotes the expected cumulative survival probability at time t_c, $t_c > t$, conditional on time-t information $\mathbb{F}_t$ under measure $\mathbb{Q}$. For simplicity, the expected cumulative survival probabilities at time τ, $\tau \geq t$, are denoted as

$$S_t(\tau) \equiv E^{\mathbb{Q}}(S(\tau)|\mathbb{F}_t).$$

According to the default intensity model, the cumulative survival probability is defined as a conditional function on the path of default intensity λ_t. It can be represented as

$$S_t(\tau) = E^{\mathbb{Q}}\left[exp\left(-\int_t^{t+\tau}\lambda_s ds\right)\middle|\mathbb{F}_t\right].$$

Since the physical survival probabilities of explanatory factors are not relevant for the pricing of financial derivatives, by using Girsanov's theorem, we obtain the Radon-Nikodym derivative of $\mathbb{Q}$ with respect to $\mathbb{P}$:

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = exp\left(-\int_t^{t+\tau}\eta_s dB_s + \frac{1}{2}\int_t^{t+\tau}\eta_s^2 ds\right)$$

where η_t is defined as the difference between actual and risk-neutral default probability reflecting the market price of the default risk premium. Then, from Equation (6):

$$S_t(\tau) = E^{\mathbb{Q}}\left[exp\left(-\int_t^{t+\tau}(\alpha + \beta^T N_s + \varepsilon_s^\lambda)\,ds\right)|\mathbb{F}_t\right]$$

$$= E^{\mathbb{Q}}\left[exp\left(-\int_t^{t+\tau}(\alpha + \beta^T N_s)ds\right)|\mathbb{F}_t\right] \cdot E^{\mathbb{Q}}\left[exp\left(-\int_t^{t+\tau}\varepsilon_s^\lambda ds\right)|\mathbb{F}_t\right], \quad (9)$$

where ε_s^λ is the disturbance of the cumulative survival probability, not attributed to explanatory factors. This study only discusses the portion that can be determined by explanatory factors. Under some technical conditions described in Duffie, Pan, and Singleton (2000), $S_t(\tau)$ in Equation (9) can be derived as

$$S_t(\tau) = exp[\alpha_\lambda(\tau) + \beta_\lambda^T(\tau)N_t^\mathbb{Q}] \tag{10}$$

where $N_t^\mathbb{Q}$ denotes the explanatory factors under $\mathbb{Q}$ measure. To obtain the expected survival probabilities under risk-neutral measure $\mathbb{Q}$ for credit derivative pricing, it is necessary to change the dynamics of the three explanatory factors from $\mathbb{P}$ to $\mathbb{Q}$. Since η_t denotes the market price of default risk, without loss of generality, it can be specified as an affine model of explanatory factors:

$$\eta_t = \alpha_\eta + \beta_\eta N_t.$$

The dynamics of explanatory factors under the risk neutral measure can then be expressed as:

$$\begin{aligned} dN_t^\mathbb{Q} &= [-\alpha_\eta - (\varphi + \beta_\eta)N_t]dt + dB_t^\mathbb{Q} \\ &= (\varphi + \beta_\eta)[-\alpha_\eta(\varphi + \beta_\eta)^{-1} - N_t]dt + dB_t^\mathbb{Q}. \end{aligned} \tag{11}$$

From Equations (10) and (11), $\alpha_\lambda(\tau)$ and $\beta_\lambda(\tau)$ are given as solutions to the following Riccati ordinary differential equations:

$$\frac{d\alpha_\lambda(\tau)}{dt} = \alpha - \beta_\lambda^T(\tau) \cdot \alpha_\eta - \frac{1}{2}\beta_\lambda^T(\tau)\beta_\lambda(\tau)$$

$$\frac{d\beta_\lambda(\tau)}{dt} = \beta - (\varphi + \beta_\eta)^T\beta_\lambda(\tau)$$

with boundary conditions $\alpha_\lambda(0) = 0$ and $\beta_\lambda(0) = 0$. The coefficients α and β in Equation (6) are obtained by solving these differential equations through numerical procedure.

Estimating Correlations Between Default Intensities and Explanatory Factors

As discussed in Section 2.1, this study derives three explanatory factors from a set of economic and financial variables from measurement Equation (3) using Kalman filter approach. Another measurement equation derives the linkage between default intensities and these explanatory factors:

$$X_t = X_{Model}(N_t, j) + \varepsilon_t^X$$

where X_t denotes the observable market CDX index spread at time t, $X_{Model}(N_t, j)$ represents the model spread determined by functions of survival probabilities and explanatory factors as mentioned in Section 2.3, j is the maturity of CDX, and $\varepsilon_t^X \sim N(0, R^X)$ denotes measurement errors and is assumed to be normally distributed with zero mean and covariance matrix R^X. Because the CDX index

spread is not linearly related to these explanatory factors N_t, we apply the extended Kalman filter approach to approximate the following linear measurement equation for estimating the parameters by maximizing the sum of the log likelihood values:

$$X_t \approx x_t \cdot N_t + \varepsilon_t^X$$

where $x_t = \left. \dfrac{\partial X_{Model}(N_t, j)}{\partial N_t} \right|_{N_t = \hat{N}_t}$.

The log likelihood function is defined as

$$L_t(\alpha_\eta, \varphi + \beta_\eta, \alpha_\lambda, \beta_\lambda, R^X) = -\frac{1}{2} \log |\hat{\Sigma}_{t^-}^X| - \frac{1}{2} \left[(X_t - \hat{X}_{t^-})^T (\hat{\Sigma}_{t^-}^X)^{-1} (X_t - \hat{X}_{t^-}) \right]$$

(12)

where $\hat{X}_{t^-}$ and $\hat{\Sigma}_{t^-}^X$ denote the priori estimated state and the variance of this estimation error, respectively, which are defined as

$$\hat{X}_{t^-} = X_{Model}(\hat{N}_{t^-}, j)$$
$$\hat{\Sigma}_{t^-}^X = x_t \hat{P}_{t^-} x_t^T + R^X.$$

The Kalman filter procedure improves all model spreads as additional market CDX spreads become available, thus the default intensities are derived using the no-arbitrage CDX pricing model mentioned in Section 2.3.

DATA

Credit Spreads

As CDX indices closely reflect broad credit markets, this study uses the spreads of the DJ CDX NA IG index to derive credit default intensities and quantify their relationship with economic conditions. The two primary indices for CDSs are the DJ CDX NA IG index for the US and the Dow Jones iTraxx Europe index. Because the 2008 credit crunch started with overexpansion of credit in the US housing market, our estimation is based on DJ CDX NA IG rather than iTraxx.

The default intensities are estimated using spreads of the DJ CDX NA IG index for maturities of 5 and 10 years over the period Sep. 28, 2004 to Dec. 31, 2006. The estimated results are applied to derive theoretical CDX indices from extracted dynamic explanatory factors for Sep. 28, 2004 through Mar. 31, 2009. The in-sample period is from Sep. 28, 2004 through Dec. 31, 2006 and the out-of-sample period is from Jan. 1, 2007 to Mar. 31, 2009. The standard
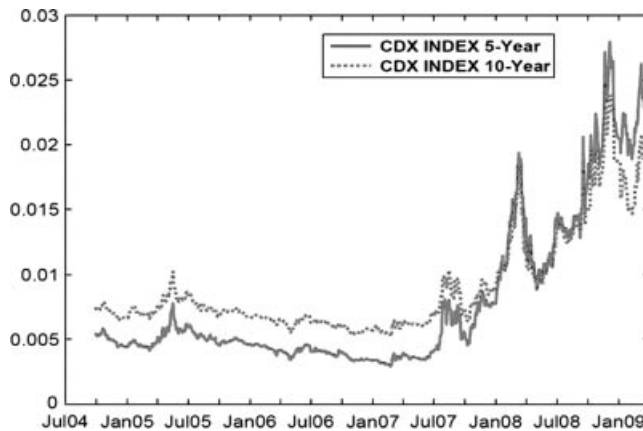
FIGURE 1
DJ CDX NA IG index spreads. This figure plots the time series of the CDX index from Sep. 2004
to Mar. 2009.

maturities for the CDX are 1, 2, 3, 4, 5, 7, and 10 years, the highest trading vol-
ume is for the 5-year index, followed by the 10-year index. This investigation
examines the 5-year index, which is the most liquid and the 10-year index,
which is the longest for considering the whole yield curve of credit spreads.
The DJ CDX NA IG index comprises equal weights of the 125 most liquid
investment grade CDS entities traded in North America. Each reference entity
thus has a weight of 0.8% in the index. New series of DJ CDX NA IG are recre-
ated every six months (Mar. 20 and Sep. 20), and the underlying CDS entities
are reconstituted. Similarly, the data roll over every six months marks the start
date of a new version index. Figure 1 plots the spreads of the 5- and 10-year
indices.

Economic and Financial Variables

We extract dynamic explanatory factors from 11 monthly and quarterly eco-
nomic variables from Mar. 1998 to Mar. 2009.[3] All data are from the
DataStream except the house price index (HPI), which is from Standard and
Poor's Case-Shiller Home Price Indices. The economic and financial variables
are sorted into three components, which represent the fundamental explanato-
ry factors: real economy, inflation, and housing factors.

The investigation incorporates macroeconomic indicators from real activi-
ty and inflation as in previous studies (Ang & Piazzesi, 2003; Ang et al., 2004;

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[3]The market statistics of prime and sub-prime mortgages are treated separately after 1998. Handling the sub-
prime mortgage variables separately can distinguish their influences from that of prime mortgage variables
and identify their influences on default intensities.

TABLE I
Data Summary

Variables	Frequency	Mean	Standard Deviation
Factor 1. Real Activity			
GDP Deflator	Quarterly	1.7900	0.3994
Industrial Production (IP)	Monthly	1.6284	3.3213
Personal Income(PI)	Monthly	4.3291	1.5230
Unemployment	Monthly	4.2855	16.1102
Factor 2. Inflation			
Consumer Price Index (CPI)	Monthly	2.6872	1.0307
Producer Price Index (PPI)	Monthly	3.6599	4.8500
Factor 3. Housing			
House Price Index (HPI)	Monthly	8.2480	9.7153
Delinquency-All	Quarterly	2.1268	1.0127
Delinquency-Subprime	Quarterly	13.2133	3.0945
Foreclosure-All	Quarterly	0.5058	0.2202
Foreclosure-Subprime	Quarterly	2.1247	0.8730

Wu & Zhang, 2008). Considering mortgage-related derivatives are the main components in credit markets with a 63% market share in 2005, it is necessary to carefully quantify mortgage-related shocks to the pricing of CDX index-based credit derivatives. Consequently, the housing factor is included to adequately consider the systematic risks from credit markets. Each variable has been selected through comprehensive examination. After carefully reviewing credit markets and previous literature, characteristics of credit markets are analyzed and numerous variables are surveyed to dissect influences on default intensities. Then correlations are examined with CDX indices to select economic and financial series with potentially useful information on credit evolution. Finally, factor analysis is used to test whether these selected variables could be reduced to a few meaningful important dimensions. Table I summarizes the statistics of all variables.

The real economic component includes four variables: GDP deflator (GDP deflator), industrial production, unemployment, and personal income. This study calculates the year-on-year percentage change of each variable and standardizes it by sample mean and sample standard deviation.

The two variables contained in the inflation component are consumer price index (CPI) and producer price index (PPI). These two variables are also converted into year-on-year percentage changes and then standardized as with the real economic component.

The remaining five variables which make up the housing factor include HPI, delinquency of all US real estate mortgage loans (Delinquency-All),

Variable Extraction Statistics

Economic Variables	C_1	C_2	C_3	$(\varepsilon_{it}^M)^T \varepsilon_{it}^M$
Factor 1. Real Activity				
GDP Deflator	0.2981	–	–	0.705
	(18.3241)			
Industrial Production (IP)	0.5319	–	–	0.091
	(21.5861)			
Personal Income(PI)	0.2535	–	–	0.790
	(16.8867)			
Unemployment	−0.5359	–	–	0.085
	(21.6190)			
Factor 2. Inflation				
Consumer Price Index (CPI)	–	0.9699	–	0.035
		(28.8421)		
Producer Price Index (PPI)	–	0.9730	–	0.100
		(28.9400)		
Factor 3. Housing				
House Price Index (HPI)	–	–	−0.3430	0.096
			(18.7524)	
Delinquency-All	–	–	0.3385	0.143
			(18.7173)	
Delinquency-Subprime	–	–	0.3392	0.141
			(18.6569)	
Foreclosure-All	–	–	0.3504	0.020
			(18.7064)	
Foreclosure-Subprime	–	–	0.3415	0.111
			(18.6391)	

Note. The parameters are estimated by the measurement equation $M_t = CN_t + \varepsilon_t^M$ with the Kalman filter approach, and the dynamics of explaining factor are $N_t = \Phi N_{t-\Delta t} + \varepsilon_t$. The absolute t-statistic values of these estimations are reported in parentheses.

delinquency of US real estate subprime mortgage loans (Delinquency-Subprime), foreclosure of all US real estate mortgage loans (Foreclosure-All), and foreclosure of US real estate subprime mortgage loans (Foreclosure-Subprime). Since subprime mortgage loan data are available from Mar. 1998, the sample period is from Mar. 1998 to Mar. 2009. In the housing factor, only the HPI is converted into year-on-year percentage changes and standardized because the other four variables are ratio indicators.

RESULTS

Explanatory Factors

Table II lists the estimated results of explanatory factors. The coefficients, C_1, C_2, and C_3, represent the factor loadings of economic variables in the real

economy, inflation, and housing factors, respectively. The absolute *t*-statistic values of these estimations are reported in parentheses. The last column shows the variance of prediction error. Lower prediction error variance indicates the explanatory factor provides better prediction for this economic variable and this economic variable is useful as an explanatory factor for a complex economic system.

Table II shows the statistical significance for estimates of all variables. Variables from the real economic factor are significantly positive, except for the standardized year-on-year unemployment rate, which is significantly negative. The results correspond closely with actual economic situations. During a recession, the output and personal income decrease while unemployment increases. Variables comprising the real economy factor have low prediction error variance, with the exception of the GDP deflator and personal income variables, which are significantly positive, also providing useful information.

The inflation factor contains CPI and PPI. The coefficients of both these variables are significantly positive with low prediction error variance, useful for providing information about nominal economic activities.

All financial series of the housing factor have significantly positive coefficients with low prediction error variances, except for the HPI which has a significantly negative coefficient. The results indicate that the housing factor responds negatively to shocks in the HPI. This phenomenon coincides with the reality that lower house prices create difficulties in refinancing mortgage loans and drive up foreclosure and delinquency rates. Conversely, some true mortgage risks are masked by house price appreciation. Table II also shows that all variables of the housing factor have low prediction error variances, and thus provide useful housing market information. The lowest prediction error variance is reached by Foreclosure-All followed by HPI.

According to the parameters obtained using the Kalman filter, the three dynamic explanatory factors are derived from the 11 economic and financial series. Figure 2 illustrates how factor dynamics can effectively represent economic conditions. The real economy is characterized by a solid line and denoted as factor 1 in Figure 2. Real economic activity markedly decreased after 2000, gradually increasing after falling to a fresh low in 2002, then slid again at the end of 2006. The first recession corresponded to the bursting of the dotcom bubble, while the second recession coincided with the current credit crunch. The dashed line represents inflation, factor 2, and appears relatively smooth. However, its steep slope after 2007 coincided with the crude oil price reaching a record high in 2008 while the drop at the end of 2008 corresponded with the decline of crude oil prices. The other line characterizes housing, factor 3, which reached a record high after 2007 as the HPI dropped and subprime loan delinquency and foreclosure further increased. Moreover, Figure 2
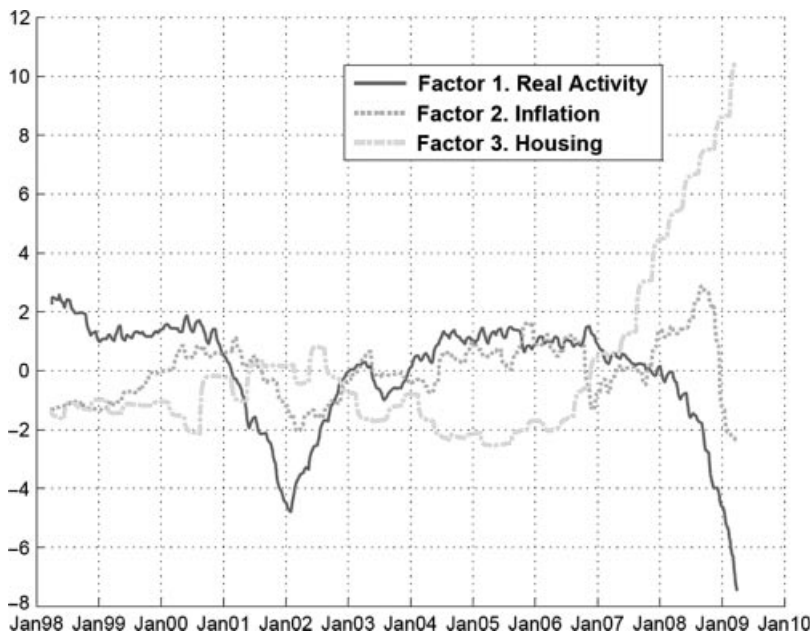
FIGURE 2
Explaining factors. This figure displays the series of explaining factors extracted from 11 economic and
financial variables. The real economic, inflation, and housing factors are denoted as factors 1, 2,
and 3, respectively.

shows that, except for the inflation factor, the economic recession and housing
market upheaval continued until the end of sample period.

Response of Credit Conditions to Economic Environment

After estimating default intensity responses to explanatory factors using the
Kalman filter approach, the CDX is priced. To test out-of-sample data, a CDX
evaluation model is constructed using observations from Sep. 2004 to Dec.
2006, and the holdout period from 2007 to Mar. 31, 2009. The results in Table III
show that impacts on default intensities, denoted as β, are all statistically sig-
nificant. The effects of both inflation and housing on default intensities are
strongly positive while the real economy factor is significantly negative. The
results mostly coincide with the nature of the explanatory factors shown in
Table IV. The table displays that the real economy is negatively correlated with
other explanatory factors and CDX index for different maturities. This negative
correlation corresponds to the results indicating that the real economy factor
negatively impacts the default intensities, and thus the spreads of CDX indices
rise with the depression in real economic activities.

TABLE III

The Estimation Results and the Impacts on Default Intensities from Explaining Factors

α_η	$\varphi + \beta_\eta$			α	β
$\begin{bmatrix} -0.5000 \\ (54.2509) \\ 0.1891 \\ (0.2462) \\ 0.3285 \\ (0.5322) \end{bmatrix}$	$\begin{bmatrix} 0.1844 \\ (11.3817) \\ - \\ \\ - \end{bmatrix}$	$\begin{matrix} - \\ \\ 0.4665 \\ (18.2456) \\ - \end{matrix}$	$\begin{matrix} - \\ \\ - \\ \\ 0.7547 \\ (0.1453) \end{matrix}$	$\begin{bmatrix} 0.0095 \\ (32.255) \end{bmatrix}$	$\begin{bmatrix} -0.0040 \\ (6.505) \\ 0.0020 \\ (2.286) \\ 0.0035 \\ (3.152) \end{bmatrix}$

Note.   This table lists the parameters that represent the impacts of dynamic explaining factors on default intensities. The parameters are derived from the dynamics of explaining factors and the historical data of CDX indices by using maximum likelihood method and Kalman filter approach. The absolute *t*-statistic values of these estimations are listed in parentheses.

TABLE IV

The Correlation of All Variables and Indices

	N_{real}	$N_{inflation}$	$N_{housing}$	CDX-5 year	CDX-10 year
N_{real}	1	−0.0134	−0.8808	−0.9029	−0.8608
$N_{inflation}$	−0.0134	1	0.1404	0.2116	0.2485
$N_{housing}$	−0.8808	0.1404	1	0.8506	0.8112
CDX-5 year	−0.9029	0.2116	0.8506	1	0.9877
CDX-10 year	−0.8608	0.2485	0.8112	0.9877	1

Since the parameters α and β determine the impact on default intensity from explanatory factors as shown in Equation (5), $\lambda_t(N_t) = \alpha + \beta^T N_t$, and since economic variables are affine functions of explanatory factors, the effects of individual variable shocks on default intensities can be calculated from

$$\lambda_t = \alpha + \beta^T (C^T C)^{-1} C^T M_t. \tag{13}$$

Therefore, survival probabilities of credit derivatives are linked to economic variables, listed in Figure 3.

The responses of survival probabilities to all economic and financial shocks appear reasonable and generally coincide with economic reality. For the real economy factor, since the top row of β in Table III is −0.004, this factor negatively influences default intensities. The default intensities simultaneously rise as the real economy undergoes a contraction. From Figure 3, the responses of survival probabilities to unemployment shocks differ from those of other variables. The GDP deflator and unemployment variables are used for illustrative purposes. The GDP deflator variable has a positive coefficient. Meanwhile,
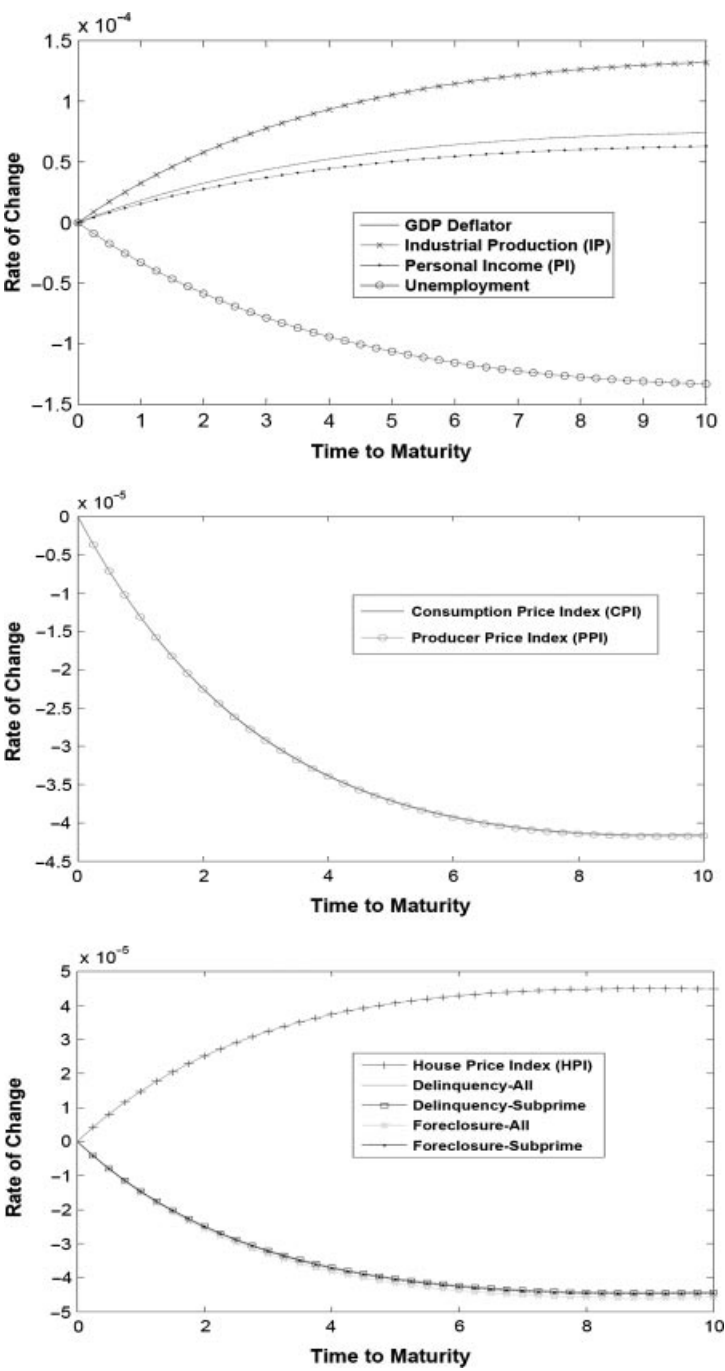
FIGURE 3
Responses of survival probabilities to shocks related to individual economic and financial series. This figure graphs the changes in survival probability with a 1% increase in variables, including the real activity factor, inflation factor, and housing factor from the top to the bottom panel, respectively.

this explanatory factor negatively affects default intensities. The GDP deflator negatively impacts default intensities and raises survival probabilities. On the contrary, the unemployment variable has a negative coefficient with the real economy factor, shocks involving the unemployment variable positively affect default intensities and negatively affect survival probabilities. According to the top panel of Figure 3, the results of this study indicate that survival probability responds negatively to unemployment, but positively to other variables.

For housing, the coefficient of HPI is negative. Although the parameter determines that housing positively influences default intensities, the HPI negatively impacts default intensities through its negative coefficient. Since the coefficients of the other variables are positive, they positively impact default intensities. As shown in the bottom panel of Figure 3, all variables in housing negatively influence survival probabilities except for the HPI.

For inflation, both CPI and PPI have positive coefficients, both inflation series positively affect default intensities. Therefore, default risk positively responds to inflation pressure and survival probabilities are negatively influenced by inflation variables, as illustrated in the middle panel of Figure 3.

Pricing Credit Derivatives

To measure how the credit environment is affected by economic conditions, the default intensity process is linked to the dynamics of various economic variables. Estimated results coincide with credit market characteristics. The default intensities across different maturities simultaneously rise with stressed markets. We use the estimated parameters in Table III to price CDX spreads for the two most popular trading maturities (5 and 10 years). Figure 4 plots market and estimated spreads of the CDX for the selected maturities from Sep. 2004 to Mar. 2009. The parameters are estimated for the sample period, Sep. 2004 to Dec. 2006. Estimated spreads for Sep. 2004 to Dec. 2006 are in-sample valuations while those from Jan. 2007–Mar. 2009 are the out-of-sample valuations. The market for credit spreads began to rise substantially in the middle of 2007, peaked in Mar. 2008, and broke the record high after the third quarter of 2008 before turning down in late 2008 and peaking again in early 2009. These credit spreads jumped in response to credit events.

We now price CDX index using the estimated default intensities. The resulting credit spreads are listed in the bottom panel of Figure 4. Before the market quotes markedly increased after the second quarter of 2007, theoretical spreads strongly increased in late 2006 and recorded a new high in early 2007. As the market quotes sharply increased in Mar. 2008 and peaked in late 2008, the evaluated spreads increased steadily, corresponding to weak economic activity, intense inflationary pressure, and extreme values in housing. From late
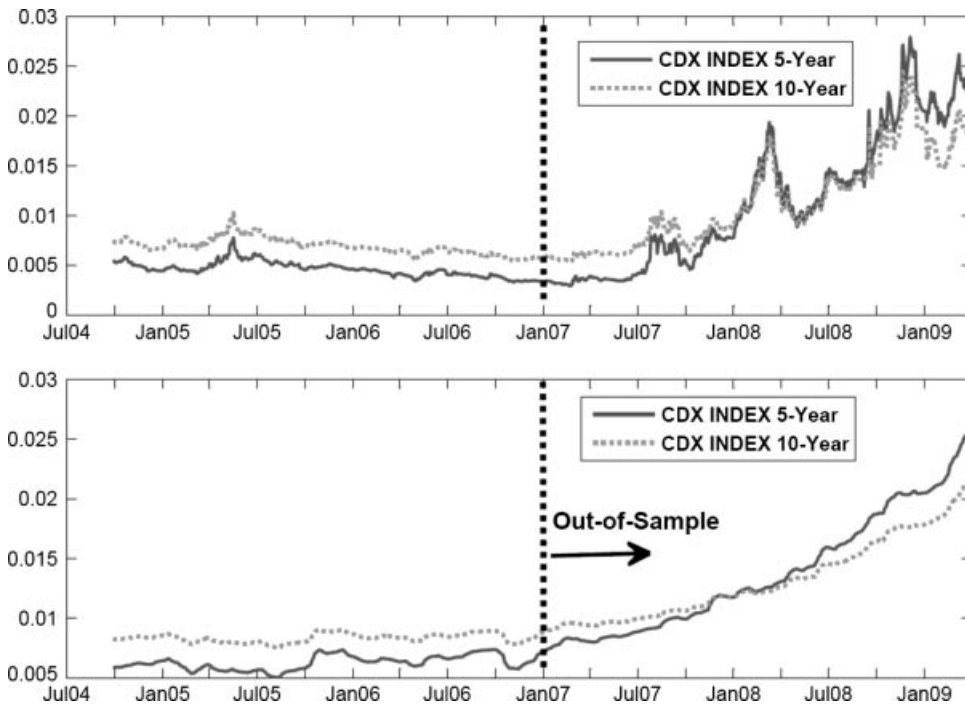
FIGURE 4
Market and model estimated credit spreads of the DJ CDX NA IG index. The top panel displays the market quotes of the DJ CDX NA IG index, and the bottom panel lists the theoretical spreads derived from the proposed model. The time series of market quotes and theoretical spreads cover the period from September 2004 through March 2009. Furthermore, the theoretical spreads during Sep. 2004 to Dec. 2006 are the in-sample valuations while those during Jan. 2007 to Mar. 2009 are out-of-sample valuations.

2008 through early 2009, market quotes rapidly fell off and then peaked again, while the theoretical spreads stopped increasing before rising again. During this period, the theoretical spreads reflected the improvement of inflation conditions in late 2008 and then displayed the worsening conditions in real activity and housing until the end of valuation period.

The estimated spreads show that the economic environment already revealed credit disarray. As shown in Figure 2, at the end of 2006, real economic activity decreased and the high delinquency and foreclosure rates from the housing factor steepened. Following the unfolding of the subprime mortgage crisis, the market lost liquidity, and the cost of borrowing money increased. Thus, the housing factor reached a record high. With the lack of improvement in the fundamental economic environment, panic persisted.[4]

[4]In a declining housing market, homeowners will find it more difficult to refinance loans or sell houses and fewer subprime mortgage clients will be able to afford house payments, leading to defaults on loans, accelerating delinquencies and foreclosures. Since the fundamental economic environment has failed to improve, the model presented in this study shows that the credit crunch did not recede until early 2009.

Market quotes often jump in response to credit events. The market spreads of the CDX index increased markedly during the unfolding of the subprime mortgage crisis in mid-2007, and then once again broke record highs in Mar. 2008 as the credit crisis worsened and Bear Stearns was sold. Owing to the monthly or quarterly release of economic observations, the theoretical spreads are smoother than market quotes. Although the theoretical spreads are not affected by credit events as rapidly as actual market spreads, the out-of-sample valuations show that the theoretical spreads exhibit the same dynamic behavior as market quotes and also reflect improvements in economic fundamentals. Despite the difficulties in forecasting credit spreads, the linkage between economic conditions and default risks portended the subprime mortgage meltdown in late 2006, which was the inevitable result of weak economic activity and housing market disarray. Consequently, examining economic and financial data series and quantifying the linkage between the explanatory factors and default risks could have helped raise awareness of the credit crunch as early as late 2006.

Subsample Analysis with Quarterly Updated Parameters

In previous sections, we have used the parameters estimated in the sample period from Sep. 28, 2004 through Dec. 31, 2006 to derive the theoretical CDX indices. Thus, we have a long out-of-sample valuation period from Jan. 1, 2007 to Mar. 31, 2009. In this section, to incorporate new information released from economic and financial observations after 2007, we update estimated parameters quarterly. This process yields nine subsamples and out-of-sample valuations. Table V lists the subsample periods.

TABLE V
Subsample Summary

| Subsample	Sample Period for Deriving Estimated Parameters	Out-of-Sample Valuation Period
Subsample 1	2004/09/28–2006/12/31	2007/01/01–2007/03/31
Subsample 2	2004/09/28–2007/03/31	2007/04/01–2007/06/30
Subsample 3	2004/09/28–2007/06/30	2007/07/01–2007/09/30
Subsample 4	2004/09/28–2007/09/30	2007/10/01–2007/12/31
Subsample 5	2004/09/28–2007/12/31	2008/01/01–2008/03/31
Subsample 6	2004/09/28–2008/03/31	2008/04/01–2008/06/30
Subsample 7	2004/09/28–2008/06/30	2008/07/01–2008/09/30
Subsample 8	2004/09/28–2008/09/30	2008/10/01–2008/12/31
Subsample 9	2004/09/28–2008/12/31	2009/01/01–2009/03/31
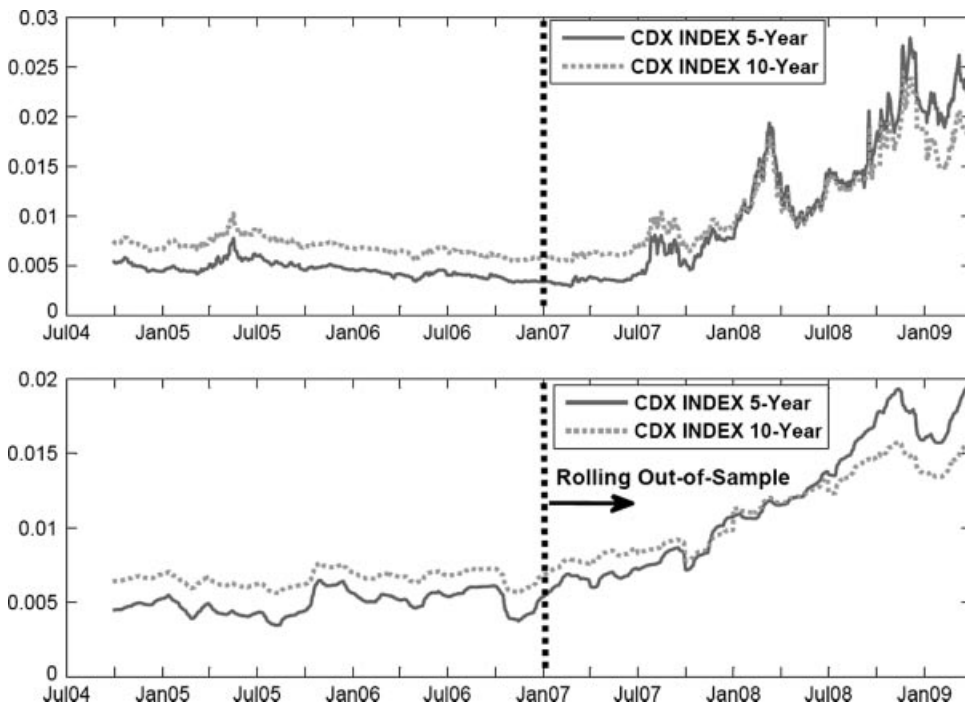
FIGURE 5
Market and model estimation credit spreads of the DJ CDX NA IG index with quarterly updated parameters (rolling valuations). The top panel displays the market quotes of the DJ CDX NA IG index, and the bottom panel lists the theoretical spreads derived from the proposed model. The time series of market quotes and theoretical spreads cover the period from September 2004 through March 2009. Moreover, the theoretical spreads during Sep. 2004 to Dec. 2006 are the in-sample valuations while those during Jan. 2007 to Mar. 2009 are rolling out-of-sample valuations.

Figure 5 plots market quotes in the top panel and displays theoretical spreads with quarterly updated parameters in the bottom panel. Similar to Figure 4 and Figure 5 shows that theoretical spreads spiked at the end of 2006. Moreover, the valuations with updated parameters and rolling out-of-sample periods provide more accurate valuations and cause the theoretical spreads to trend up and down like the actual credit spreads. After 2007, the out-of-sample valuations of credit spreads with updated parameters gradually increased, before peaking in mid 2007. The valuations continued to rise until peaking again in Nov. 2008, declining dramatically in late 2008, and finally increasing until the end of the valuation period. Correspondingly, the market quotes sharply increased in mid 2007, and peaked in Mar. 2008 with the collapse of Bear Stearns. The market recorded a fresh high after the third quarter of 2008 with the collapse of Lehman Brothers and the liquidity crisis of AIG, decreasing after these credit events in late 2008, and finally peaking one last time in

early 2009. Because the actual credit environment failed to improve, the proposed default panic persisted until the end of the valuation period with the continued economic downturn and housing collapse.

In summary, testing the model using out-of-sample data identifies credit evolution through the link between economic conditions and default intensities. Relative to the valuations based on a single estimation period shown in Figure 4, Figure 5 demonstrates that the valuation performances are improved with the updated links between economic conditions and credit risks, and thus the theoretical spreads can more accurately depict credit evolution.

CONCLUSION

This study provides more complete insight into the movements of default probabilities driven by economic determinants through the no-arbitrage dynamic factor model, and prices credit derivatives by applying these explanatory factors to the reduced-form pricing model. Since the study defines the default intensities as affine functions of explanatory factors, the default risks vary with economic conditions. The estimated results and out-of-sample valuations show that the link between economic conditions and default risks can depict credit evolution and more effectively manage default risks.

This study summarizes relevant information through continuously updated observations from many economic and financial variables, and then sorts these variables into three components. The extracted factors include the real economy, inflation, and housing. Because mortgage-related derivatives are the main components in credit markets, it is necessary to carefully quantify mortgage-related shocks to credit derivative pricing. Housing is another indicator, in addition to real economy and inflation factors. Subsequently, this investigation condenses various economic and financial indicators into three dynamic explanatory factors, and updates each factor as new observations arrive.

By imposing no-arbitrage restrictions, the credit environment is linked with economic conditions by setting default intensity process as an affine function of explanatory factors. The results indicate that explanatory factors significantly affect default intensities. Corresponding to actual economic conditions, the inflation and housing factors both strongly and positively affect default risks, while the real economic factor exerts a significant and negative influence. The estimated results are then used to derive the responses of survival probabilities to the shocks of individual economic and financial series, and applied to value out-of-sample CDX spreads.

Extending traditional credit derivative pricing formulas, estimated parameters of economic fundamentals are used to value CDX spreads across maturities by adopting a reduced-form model. Our pricing results support that economic

conditions have a considerable impact on credit risks. Although the market spreads markedly increased during mid-2007, the out-of-sample valuations of credit spreads reveal the economic environment had already displayed aggravated credit conditions at the end of 2006. Therefore, identifying the economic environment from different economic and financial series and quantifying their relationship with default risks can raise awareness of credit evolution and contribute to managing credit risks more effectively.

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