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Variable control scheme in the cascade processes

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ABSTRACT

The article considers the variables process control scheme for cascade processes. We construct variable sample sizes and sampling intervals (VSSI) control charts to effectively monitor the input variable and the output variable produced by a cascade process. The performance of the proposed VSSI control charts is measured by the adjusted average time to signal derived by a Markov chain approach. An example of the metallic film thickness of the computer connectors system shows the application and the performance of the proposed VSSI control charts in detecting shifts in means of the cascade process. Furthermore, the performance of the proposed VSSI control charts and the fixed sample sizes and sampling intervals control charts are compared by numerical analysis results. These demonstrate that the former is much faster in detecting small and medium shifts. The optimum VSSI control charts are also proposed using optimization technique when quality engineers cannot specify the values of the variable sample sizes and sampling intervals. It has been found that the optimum VSSI control charts work and are thus suggested whenever quality engineers cannot specify the values of variable sample sizes and sampling intervals. Furthermore, the impacts of misusing Shewhart charts to monitoring the process means on the cascade process are also investigated.

1. Introduction

Control charts are important tools in statistical quality control and are used to effectively monitor a process if it is in-control or out-of-control. However, even though Shewhart (1931) $\overline{X}$ control charts are used to monitor a process by taking samples of equal size at a fixed sampling interval (FSSI), they are usually slow in signaling small to moderate shifts in the process mean. Consequently, in recent years several alternatives have been developed to improve the performance of $\overline{X}$ control charts. One of the useful approaches to improve the detecting ability is to use a variable sample sizes and sampling intervals (VSSI) control chart instead of the traditional FSSI. Whenever there is some indication that a process parameter may be changed, the next sampling interval should be shorter and the sample size should be larger. On the other hand, if there were no indication, then the next sampling interval should be longer and the sample size should be smaller.

The exponential weighted moving average (EWMA) control chart is an effective alternative to the Shewhart control chart when small process shifts are of interest. Some properties of EWMA control schemes have been discussed (see Tagaras (1998)). Very little work has been down on VSSI EWMA control charts for monitoring process mean. The properties of EWMA charts with variable sample sizes and sampling intervals were studied by Reynolds and Arnold (2001), Park, Lee, and Kim (2004). Tagaras (1998) reviewed the literature on adaptive control charts.

However, these articles assume that there is only a single process step, whereas many products are currently produced in the cascade process. Consequently, it is not appropriate to monitor the cascade process by utilizing a control chart for each quality variable. Zhang (1984) proposed the simple cause-selecting control chart to control the specific quality by adjusting the effect of input quality variable (X) on output quality variable (Y) since the input quality variable influences the output quality variable on the cascade process. The cause-selecting values (e) are Y minus the effect of X, and the cause-selecting control chart is constructed accordingly. Wade and Woodall (1993) reviewed and analyzed the cause-selecting control chart and examined the relationship between the cause-selecting control chart and the Hotelling T^2 control chart. In their opinion the cause-selecting control chart outperforms Hotelling T^2 control chart since it is easy to distinguish whether the last step of the cascade process is out-of-control. Therefore, it seems reasonable to develop variables control schemes to control the cascade process. Yang (1998) designed the X and cause-selecting control charts to monitor the dependent process steps with minimal process cost using renewal equation approach. Yang and Chen (2003) proposed monitoring approach for dependent process steps with Weibull shock model. Yang (2005) addressed the control approach for dependent steps with over-adjusted means. Yang and Yu (2009) proposed VSI EWMA charts to monitor dependent process steps. However, the

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properties of the VSSI control charts used to control the small shift in process means on the cascade process have not yet been discussed. Therefore, a need to study the performance of the joint VSSI EWMA control charts on the cascade process has arisen. In this paper, the joint VSSI EWMA control charts are proposed to control the process means on the cascade process. In the next section, the performance of the proposed EWMA control charts is measured by the adjusted average time to signal (AATS) by a Markov chain approach. Finally, we use an example of computer connectors system to show the application of the proposed EWMA control charts. We also compare the performance between the VSSI EWMA control charts and FSSI EWMA control charts. In case the variable sample sizes and sampling intervals cannot be specified the optimum VSSI EWMA control charts should be used. Furthermore, engineers may not know the using of cause-selecting control charts correctly. The impacts of misusing Shewhart $Z_{\overline{X}}$ and $Z_{\overline{Y}}$ charts to monitoring the process means on the cascade process are also investigated.

2. Description of the joint VSSI $EWMA_{Z_{\bar{x}}}$ and $EWMA_{Z_e}$ control charts

Consider a cascade process controlled by the joint VSSI $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$ control charts. Let X be the measurable input quality variable on the first step of the cascade process. Assume further that this process starts in a state of statistical control, that is, X follows a normal distribution with the mean at its target value, μ_X, and the standard deviation at its target value σ_X. Let Y be the measurable output quality variable on the second step of the cascade process, and follow a normal distribution conditional on X. Since the two steps are dependent, and Y is affected by X, following Wade and Woodall (1993), the relationship between Y and X is generally expressed as

$$Y_i|X_i = f(X_i) + \varepsilon_i, \quad i = 1,2,3,\ldots,m \tag{2-1}$$

where $\varepsilon_i \sim NID(0,\sigma^2)$. Let Y represent $Y|X$. If the function $f(X_i)$ is known, the values of the standardized error term $\varepsilon_i^* = \frac{Y_i - f(X_i)}{\sigma}$ are called the cause-selecting values since they are the values of Y_i adjusted for the effects of X_i. In practice, the true function $f(X_i)$ is usually unknown and thus must be estimated using the data of the initial m samples of size one. Thus the estimate for $f(X_i)$ will be $\hat{Y}_i$. The residuals, $e_i = Y_i - \hat{Y}_i \sim NID(0,\sigma_e^2)$. The standardized residuals $e_i^* = \frac{e_i}{\sigma_e}$ are called the cause-selecting values.

In our study the chosen sample size is variable, not one again, and taken from the end of the last step of the cascade process; when the process steps are all in-control, the standardized samples, $Z_{\overline{X}}$ and $Z_{\bar{e}}$ are

$$Z_{\overline{X}i} = \frac{\overline{X}_i - \mu_X}{\frac{\sigma_X}{\sqrt{n_q}}} \sim N(0,1), \quad i = 1,2,3,\ldots,\ m = 1,2,3,\ldots, \text{ and}$$

$$Z_{\bar{e}i} = \frac{\bar{e}_i - \mu_e}{\frac{\sigma_e}{\sqrt{n_q}}} \sim N(0,1), \quad i = 1,2,3,\ldots,\ m = 1,2,3,\ldots \tag{2-2}$$

Assume that the first step is subject to the special cause 1 such that the mean of $\overline{X_i}$ shifts from μ_X to $\mu_X + \delta_1\sigma_X/\sqrt{n_q}(\delta_1 \neq 0)$ and the variance is unchanged; and the second step is subject to the special cause 2 such that the mean of $\overline{e_i}$ shifts from 0 to $\delta_2\sigma_e/\sqrt{n_q}(\delta_2 \neq 0)$ and the variance is unchanged. That is, $Z_{\overline{X},i} = \frac{\overline{X}_i - \mu_X}{\frac{\sigma_X}{\sqrt{n_q}}} \sim N(\delta_1, 1)$ and/or $Z_{\bar{e},i} = \frac{\bar{e}_i - \mu_e}{\frac{\sigma_e}{\sqrt{n_q}}} \sim N(\delta_2, 1)$ for out-of-control cascade process. The out-of-control distribution of $Z_{\overline{X_i}}$ or $Z_{\overline{e_i}}$ will be adjusted to in-control state, once one true signal is obtained from the proposed control charts. Let T_{sci} be the time until the occurrence of special cause i, where $i = 1, 2$, and follow an exponential distribution with the probability density function

$$f(t) = \gamma_i \exp(-\gamma_i t) \quad t_{sci} > 0, \mathrm{i} = 1,2 \tag{2-3}$$

where $1/\gamma_i$ is the mean time that the step i of the cascade process remains in a state of statistical control.

The $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$ control charts are constructed to detect the small shifts in process means faster. Thus, we need to derive the distributions of the statistics $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$

$$EWMA_{Z_{x,i}} = \lambda_1 Z_{\overline{X}_i} + (1-\lambda_1)EWMA_{Z_{\overline{X},i-1}}, \quad i = 1,2,\ldots,$$

$$\text{where } EWMA_{Z_{x,i}} \sim N\left(0, \frac{\lambda_1}{2-\lambda_1}\right) \quad \text{if } i \to \infty$$

$$EWMA_{Z_{\bar{e},i}} = \lambda_2 Z_{\bar{e}_i} + (1-\lambda_2)EWMA_{Z_{\bar{e},i-1}}, \quad i = 1,2,\ldots,$$

$$\text{where } EWMA_{Z_{e,i}} \sim N\left(0, \frac{\lambda_2}{2-\lambda_2}\right) \quad \text{if } i \to \infty \tag{2-4}$$

An in-control state analysis for the joint VSSI $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$ control charts is performed since the shifts in the mean on the cascade process do not occur when the process is just starting, but occur at some time in the future. The samples $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$ are plotted on the joint VSSI $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$ control charts with warning limits of the form $\pm w_{Z_{\overline{X}}}$ and $\pm w_{Z_e}$, and control limits of the form $\pm k_{Z_{\overline{X}}}$ and $\pm k_{Z_e}$, respectively, where $0 < w_{Z_{\overline{X}}} < k_{Z_{\overline{X}}}$ and $0 < w_{Z_e} < k_{Z_e}$ (see Fig. 2.1).

The search for the special cause 1 and adjustment in the first step is undertaken when the sample $EWMA_{Z_{\overline{X}}}$ falls outside the interval $\left(-k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}},\ k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right)$, that is when the $EWMA_{Z_{\overline{X}}}$ chart produces a true signal. The search for the special cause 2 and adjustment in the last step is undertaken when the sample $EWMA_{Z_e}$ falls outside the interval $\left(k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}},\ -k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right)$, that is when the $EWMA_{Z_e}$ chart produces a true signal. For a discontinuous process, the cascade process is stopped to search for the special causes and adjustment after a true signal is obtained from the proposed control charts, and the process is brought back to an in-control state.

The position of the current sample in each control chart constructs the sampling interval of the next sample.

We divide the joint VSSI $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$ control charts into the following three regions.

$$I_{Z_{\overline{X1}}} = \left(-w_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \quad w_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right) \quad \text{(central region)}$$

$$I_{Z_{\overline{X_2}}} = \left(-k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \quad -w_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right)$$

$$\cup\left(w_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \quad k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right) \quad \text{(warning region)}$$

$$I_{Z_{\overline{X_3}}} = \left(-k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \quad k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right) \quad \text{(control region)}$$

$$I_{Z_{\overline{e1}}} = \left(-w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \quad w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right) \quad \text{(central region)}$$

$$I_{Z_{\overline{e_2}}} = \left(-k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \quad -w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right)$$

$$\cup\left(w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \quad k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right) \quad \text{(warning region)}$$

$$I_{Z_{\overline{e_3}}} = \left(-k_{Z_{\overline{X}}}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \quad k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right) \quad \text{(control region)}$$

Three VSSIs are adopted, $0 < t_1 < t_2 < t_3 < \infty$, $0 < n_3 < n_2 < n_1 < \infty$. If the samples, $EWMA_{Z_{\overline{X}}}$ and $EWMA_{Z_e}$, all fell within the central regions, then the next sampling interval should be long (t_3) but the sample size should be small (n_3). If one sample fell

$$UCL_{EWMA_{Z_{\bar{X}}}} = k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad UCL_{EWMA_{Z_{\bar{e}}}} = k_{Z_{\bar{e}}}\sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

$$UWL_{EWMA_{Z_{\bar{X}}}} = w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad UWL_{EWMA_{Z_{\bar{e}}}} = w_{Z_{\bar{e}}}\sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

$$CL_{EWMA_{Z_{\bar{X}}}} = 0 \qquad CL_{EWMA_{Z_{\bar{e}}}} = 0$$

$$LWL_{EWMA_{Z_{\bar{X}}}} = -w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad LWL_{EWMA_{Z_{\bar{e}}}} = -w_{Z_{\bar{e}}}\sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

$$LCL_{EWMA_{Z_{\bar{X}}}} = -k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \qquad LCL_{EWMA_{Z_{\bar{e}}}} = -k_{Z_{\bar{e}}}\sqrt{\frac{\lambda_2}{2-\lambda_2}}$$

(1) $EWMA_{Z_{\bar{X}}}$ chart (2) $EWMA_{Z_{\bar{e}}}$ chart

Fig. 2.1. The control limits of VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts.

within the central region but another fell within the warning region, then the next sampling interval should be median (t_2) and the sample size should be median (n_2). If all samples fell within the warning regions, then the next sampling interval should be short (t_1) but the sample size should be large (n_1).

The relationship between the next sampling interval $(t_q, q = 1,2,3)$ and the position of the current samples is expressed as follows.

$$(t_q, n_q) = \begin{cases} (t_3, n_3) & \text{if } Z_{\bar{X}_i} \in I_{Z_{\bar{X}1}},\ Z_{\bar{e}_i} \in I_{Z_{e1}}, \\ (t_2, n_2) & \text{if } Z_{\bar{X}_i} \in I_{Z_{\bar{X}2}},\ Z_{\bar{e}_i} \in I_{Z_{e1}} \\ (t_2, n_2) & \text{if } Z_{\bar{X}_i} \in I_{Z_{\bar{X}1}},\ Z_{\bar{e}_i} \in I_{Z_{e2}} \\ (t_1, n_1) & \text{if } Z_{\bar{X}_i} \in I_{Z_{\bar{X}2}},\ Z_{\bar{e}_i} \in I_{Z_{e2}} \end{cases} \tag{2-5}$$

The first sample size and sampling interval taken from the process when it is just starting is assumed chosen randomly. When the process is in control, all sample sizes and sampling intervals, including the first one, should have a probability of p_{01} of being (t_3, n_3), a probability of $p_{02} + p_{03}$ of being (t_2, n_2), and a probability of p_{04} of being (t_1, n_1), where $\sum_{i=1}^{4} p_{0i} = 1, p_{01}, p_{02}, p_{03}$ and p_{04} are given by

$$\begin{aligned} p_{01} &= P\left(|EWMA_{Z_{\bar{X}}}| < w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \middle| |EWMA_{Z_{\bar{X}}}| < k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \delta_1 = 0\right) \\ &\cdot P\left(|EWMA_{Z_e}| < w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} \middle| |EWMA_{Z_e}| < k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \delta_2 = 0\right) \\ &= \left(\frac{2\Phi(w_{Z_{\bar{X}}}) - 1}{2\Phi(k_{Z_{\bar{X}}}) - 1}\right)\left(\frac{2\Phi(w_{Z_e}) - 1}{2\Phi(k_{Z_e}) - 1}\right) \end{aligned} \tag{2-6}$$

$$\begin{aligned} p_{02} &= P\left(|EWMA_{Z_{\bar{X}}}| < w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \middle| |EWMA_{Z_{\bar{X}}}| < k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \delta_1 = 0\right) \\ &\cdot P\left(-k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} < EWMA_{Z_e} < w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} \text{ or } w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right. \\ &\left. < EWMA_{Z_e} < k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} \middle| |EWMA_{Z_e}| < k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \delta_2 = 0\right) \\ &= \frac{(2\Phi(w_{Z_{\bar{X}}}) - 1)\cdot(2\Phi(k_{Z_e}) - 2\Phi(w_{Z_e}))}{(2\Phi(k_{Z_{\bar{X}}}) - 1)(2\Phi(k_{Z_e}) - 1)} \end{aligned}$$

$$\begin{aligned} p_{03} &= P\left(-k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} < EWMA_{Z_{\bar{X}}} < -w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \text{ or } w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right. \\ &\left. < EWMA_{Z_{\bar{X}}} < k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \middle| |EWMA_{Z_{\bar{X}}}| < k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \delta_1 = 0\right) \\ &\cdot P\left(|EWMA_{Z_e}| < w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} \middle| |EWMA_{Z_e}| < k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \delta_2 = 0\right) \\ &= \frac{(2\Phi(w_{Z_{\bar{X}}}) - 1)\cdot(2\Phi(k_{Z_e}) - 2\Phi(w_{Z_e}))}{(2\Phi(k_{Z_{\bar{X}}}) - 1)(2\Phi(k_{Z_e}) - 1)} = P_{02} \end{aligned}$$

$$\begin{aligned} p_{04} &= P\left(-k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} < EWMA_{Z_{\bar{X}}} < -w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \text{ or } w_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}\right. \\ &\left. < EWMA_{Z_{\bar{X}}} < k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}} \middle| |EWMA_{Z_{\bar{X}}}| < k_{Z_{\bar{X}}}\sqrt{\frac{\lambda_1}{2-\lambda_1}}, \delta_1 = 0\right) \\ &\cdot P\left(-k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} < EWMA_{Z_e} < -w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} \text{ or } w_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}\right. \\ &\left. < EWMA_{Z_e} < k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}} \middle| |EWMA_{Z_e}| < k_{Z_e}\sqrt{\frac{\lambda_2}{2-\lambda_2}}, \delta_2 = 0\right) \\ &= \left(\frac{2\Phi(k_{Z_{\bar{X}}}) - 2\Phi(w_{Z_{\bar{X}}})}{2\Phi(k_{Z_{\bar{X}}}) - 1}\right)\left(\frac{2\Phi(k_{Z_e}) - 2(w_{Z_e})}{2\Phi(k_{Z_e}) - 1)}\right) \end{aligned}$$

To facilitate the computation of the performance measures, $w_{Z_{\bar{X}}}, k_{Z_{\bar{X}}}, w_{Z_e}$ and k_{Z_e} will be specified with the constraint that the probability of a sample falling in the central region is same for both the $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts when the process is in-control. Thus,

$$\begin{aligned} P_r(|EWMA_{Z_{\bar{X}}}| < w_{Z_{\bar{X}}} \| EWMA_{Z_{\bar{X}}}| < k_{Z_{\bar{X}}}, \delta_1 = 0)\cdot &= P_r(|EWMA_{Z_e}| \\ &< w_{Z_e} \| EWMA_{Z_e}| < k_{Z_e}, \delta_2 = 0) \end{aligned} \tag{2-7}$$

implying, $w_{Z_{\bar{X}}} = w_{Z_e} = w, k_{Z_{\bar{X}}} = k_{Z_e} = k$ and $\lambda_1 = \lambda_2 = \lambda$.

If $t_1 = t_2 = t_3 = t_0$ then the joint VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts reduce to the joint $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts with VSSs

(n_q, t_0). If $n_1 = n_2 = n_3 = n_0$ then the joint VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\hat{e}}}$ charts reduce to the joint $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\hat{e}}}$ charts with VSIs (n_0, t_q). If $w_{Z_{\bar{X}}} = w_{Z_{\hat{e}}} = 0, t_1 = t_2 = t_3 = t_0$ and $n_1 = n_2 = n_3 = n_0$, then the joint VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\hat{e}}}$ charts reduce to the joint $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\hat{e}}}$ charts with FSSI (n_0, t_0).

$$\begin{aligned}
&t_3 P(EWMA_{Z_{\bar{X},i-1}} \in I_{Z_{\bar{X}1}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{Z_{e1}} | \delta_1 = 0, \delta_2 = 0) \\
&+ t_2 P(EWMA_{Z_{\bar{X},i-1}} \in I_{Z_{\bar{X}1}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{Z_{e2}} | \delta_1 = 0, \delta_2 = 0) \\
&+ t_2 P(EWMA_{Z_{\bar{X},i-1}} \in I_{Z_{\bar{X}2}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{Z_{e1}} | \delta_1 = 0, \delta_2 = 0) \\
&+ t_1 P(EWMA_{Z_{\bar{X},i-1}} \in I_{Z_{\bar{X}2}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{Z_{e2}} | \delta_1 = 0, \delta_2 = 0) \\
&= t_0 P\left(-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{\bar{X},i-1}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0, \delta_2 = 0\right) \\
&\quad P\left(-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{\hat{e},i-1}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0, \delta_2 = 0\right)
\end{aligned}$$

3. Comparison of control charts

Sampling schemes should be compared under equal conditions; that is, VSSI and FSSI schemes should demand the same average sample size and average sampling interval under the in-control period. That is,

$$E[n_q || EWMA_{Z_{\bar{X}}}| < k, |EWMA_{Z_{\hat{e}}}| < k, \delta_1 = 0, \delta_2 = 0] = n_0 \quad (3\text{-}1)$$

$$E[t_q || EWMA_{Z_{\bar{X}}}| < k, |EWMA_{Z_{\hat{e}}}| < k, \delta_1 = 0, \delta_2 = 0] = t_0 \quad (3\text{-}2)$$

Based on Eq. (3-1), the following equation can be formulated as

$$\begin{aligned}
&n_1 P(EWMA_{Z_{\bar{X},i-1}} \in I_{EWMA_{Z_{\bar{X}1}}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{EWMA_{Z_{e1}}} | \delta_1 = 0, \delta_2 = 0) \\
&+ n_2 P(EWMA_{Z_{\bar{X},i-1}} \in I_{EWMA_{Z_{\bar{X}1}}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{EWMA_{Z_{e2}}} | \delta_1 = 0, \delta_2 = 0) \\
&+ n_2 P(EWMA_{Z_{\bar{X},i-1}} \in I_{EWMA_{Z_{\bar{X}2}}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{EWMA_{Z_{e1}}} | \delta_1 = 0, \delta_2 = 0) \\
&+ n_3 P(EWMA_{Z_{\bar{X},i-1}} \in I_{EWMA_{Z_{\bar{X}2}}} | \delta_1 = 0, \delta_2 = 0) P(EWMA_{Z_{\hat{e},i-1}} \in I_{EWMA_{Ze2}} | \delta_1 = 0, \delta_2 = 0) \\
&= n_0 P\left(-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{\bar{X},i-1}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0, \delta_2 = 0\right) \\
&\quad P\left(-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{\hat{e},i-1}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0, \delta_2 = 0\right)
\end{aligned}$$

Simplifying,

$$\begin{aligned}
&4\Phi(w)^2[n_1 - 2n_2 + n_3] + 4\Phi(w)[-n_1 + 2n_2\Phi(k) + n_2 - 2n_3\Phi(k)] \\
&\quad - t_0(2\Phi(k) - 1)^2 + n_1 - 4n_2\Phi(k) + 4n_3(\Phi(k))^2 = 0
\end{aligned}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative function.

It follows

$$w = \Phi^{-1}\left[\frac{-4B_1 \pm \sqrt{16B_1^2 - 16A_1C_1}}{8A}\right]. \quad (3\text{-}3)$$

where

$A_1 = n_1 - 2n_2 + n_3$
$B_1 = -n_1 + 2n_2\Phi(k) + n_2 - 2n_3\Phi(k)$
$C_1 = -[n_0(2\Phi(k) - 1)^2 - n_1 + 4n_2\Phi(k) - 4n_3(\Phi(k))^2]$

Based on Eq. (3-2), the following equation can be formulated as

Simplifying,

$$\begin{aligned}
&4\Phi(w)^2[t_3 - 2t_2 + t_1] + 4\Phi(w)[-t_3 + 2t_2\Phi(k) + t_2 - 2t_1\Phi(k)] \\
&\quad - t_0(2\Phi(k) - 1)^2 + t_3 - 4t_2\Phi(k) + 4t_1(\Phi(k))^2 = 0
\end{aligned}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative function.

The warning limit is

$$w = \Phi^{-1}\left[\frac{-4B_2 \pm \sqrt{16B_2^2 - 16A_2C_2}}{8A_2}\right] \quad (3\text{-}4)$$

where

$A_2 = t_3 - 2t_2 + t_1$
$B_2 = -t_3 + 2t_2\Phi(k) + t_2 - 2t_1\Phi(k)$
$C_2 = -[t_0(2\Phi(k) - 1)^2 - t_3 + 4t_2\Phi(k) - 4t_1(\Phi(k))^2]$

From Eqs. (3-3) and (3-4), t_3 can be obtained by the following formula:

$$t_3 = \frac{-4(\Phi(w))^2(-2t_2 + t_1) - 4\Phi(w)(2\Phi(k)t_2 + t_2 - 2\Phi(k)t_1) + t_0(2\Phi(k) - 1)^2 + 4\Phi(k)t_2 - 4(\Phi(k))^2 t_1}{4(\Phi(w))^2 - 4\Phi(w) + 1} \quad (3\text{-}5)$$

However, to obtain w and let $0 < w < k$, the constraints $0 < n_3 < n_2 < n_0 < n_1 < \infty$ and $1 < t_1 < t_2 < t_0 < t_3 < \infty$ are required. Thus, the warning limit can be obtained by using Eq. (3-3) and choosing a combination of the three specified, (n_1, n_2, n_3) and n_0 or Eq. (3-4) and choosing a combination of the specified (t_1, t_2, t_3) and t_0.

In this paper, the VSSI scheme is compared with the FSSI scheme and one adaptive control scheme was considered to be better than another when it allows the joint VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts to detect changes in mean of the cascade process faster.

4. Performance measurement

The speed with which a control chart detects process shifts measures the chart's statistical efficiency. For a VSSI, the detection speed is measured by the average time from either mean shifting until either $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts or both signal, which is known as the AATS. That is, the AATS is the mean time that the process remains out of control.

Since $T_{SCi} \sim \exp(-\gamma_i t), t > 0, i = 1, 2$, the occurrence time, $T_{(1)}$, until the first special cause occurs is

$T_{(1)} \sim \exp(\gamma_1 + \gamma_2)$ where $T_{(1)} = \min(T_{SC1}, T_{SC2})$

Hence,

$$AATS = ATC - \frac{1}{\gamma_1 + \gamma_2} \tag{4-1}$$

The average time of the cycle (ATC) is defined as the average time from the start of a process until a true signal is obtained from the proposed charts and the out-of-control step 1 and/or step 2 are correctly adjusted. The ATC is the sum of the average in-control time and average out-of-control time (see Duncan (1956)). The Markov chain approach is applied to compute the ATC due to the memory-less property of the exponential distribution. Thus, at each sampling, one of the 28 states is assigned based on whether the process step is in-control or out-of-control and the position of samples (see Table 4.1 for the 28 states of the process). The status of the process when the $(i+1)^{th}$ sample is taken, and the position of the i^{th} sample on the joint $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts define the transition states of the Markov chain. The joint VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts produce a signal when at least one of the samples falls outside its control limits. When the true signal comes from the out-of-control step i, then the step will be adjusted, $i = 1, 2$. However, the in-control step i will not be adjusted and continued when the signal is false, $i = 1, 2$. The transition situation among the states is described as follows.

If the current state is any one of the States 1–4 then the in-control cascade process will not be adjusted and the state may transit to any one of the States 1–28 after sampling time interval t_q and adopting the sample of size $n_q, q = 1, 2, 3$. If the current state is any one of the States 5–8 then it may transit to any one of the States 13–16, 19–23 and 26–28 after sampling time interval t_q and adopting the sample of size $n_q, q = 1, 2, 3$. States 9–12 are similar to States 5–8. If the current state is any one of the States 13–16 then it may transit to any one of the States 22–23 and 26–28 after sampling time interval t_q and sample size $n_q, q = 1, 2, 3$. If the current state is State 17, it indicates at least one false signal comes from the in-control cascade process then the process is not adjusted and continued, and it instantly becomes any one of the States 1–4 with probability $P_{17=j}, j = 1-4$, and $\sum_{j=1}^{4} P_{17=j} = 1$. Any one of the States 1–4 thus transits to any one of the States 1–28 after a sampling time interval t_q and sample size $n_q, q = 1, 2, 3$. States 18 and 19 are similar to State 17. If the current state is any one of the States 20–23, then it instantly becomes any one of the States 5–8. Any one of the States 5–8 thus transits to any one of the States 5–8, 13–16, 19–23 and 26–28 after a sampling time interval t_q and sample size $n_q, q = 1, 2, 3$. If the current state is any one of the States 24–27, then it instantly becomes any one of the States 9–12. Any one of the States 9–12 thus transits to any one of the States 9–12, 18 and 24–28 after a sampling time interval t_q and sample size $n_q, q = 1, 2, 3$. Based on Markov chain properties, all states may be classified into transient states and absorbing state. If the

Table 4.1
Definition of 28 process states.

State	Does SC1 occur?	The location of sample statistic $EWMA_{Z_{\bar{X}}}$	Is an alarm in the first step?	Does SC2 occur?	The location of sample statistic $EWMA_{Z_e}$	Is an alarm in the second step?	Transient state or absorbing state?
1	No	$I_{EWMA_{Z_{\bar{X}}1}}$	No	No	$I_{EWMA_{Z_{\bar{e}}1}}$	No	Transient state
2	No	$I_{EWMA_{Z_{\bar{X}}1}}$	No	No	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
3	No	$I_{EWMA_{Z_{\bar{X}}2}}$	No	No	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
4	No	$I_{EWMA_{Z_{\bar{X}}2}}$	No	No	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
5	Yes	$I_{EWMA_{Z_{\bar{X}}1}}$	No	No	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
6	Yes	$I_{EWMA_{Z_{\bar{X}}1}}$	No	No	$I_{EWMA_{Z_{e_2}}}$	No	
7	Yes	$I_{EWMA_{Z_{\bar{X}}2}}$	No	No	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
8	Yes	$I_{EWMA_{Z_{\bar{X}}2}}$	No	No	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
9	No	$I_{EWMA_{Z_{\bar{X}}1}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
10	No	$I_{EWMA_{Z_{\bar{X}}1}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
11	No	$I_{EWMA_{Z_{\bar{X}}2}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
12	No	$I_{EWMA_{Z_{\bar{X}}2}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
13	Yes	$I_{EWMA_{Z_{\bar{X}}1}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
14	Yes	$I_{EWMA_{Z_{\bar{X}}1}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
15	Yes	$I_{EWMA_{Z_{\bar{X}}2}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
16	Yes	$I_{EWMA_{Z_{\bar{X}}2}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
17	At least one false alarm under the in-control process						
18	No	$I_{EWMA_{Z_{\bar{X}}3}}$	Yes (false alarm)	Yes	$I_{EWMA_{Z_{\bar{e}}3}}$	Yes (true alarm)	
19	Yes	$I_{EWMA_{Z_{\bar{X}}3}}$	Yes (true alarm)	No	$I_{EWMA_{Z_{\bar{e}}3}}$	Yes (false alarm)	
20	Yes	$I_{EWMA_{Z_{\bar{X}}1}}$	No	No	$I_{EWMA_{Z_{\bar{e}}3}}$	Yes (false alarm)	
21	Yes	$I_{EWMA_{Z_{\bar{X}}2}}$	No	No	$I_{EWMA_{Z_{\bar{e}}3}}$	Yes (false alarm)	
22	Yes	$I_{EWMA_{Z_{\bar{X}}1}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}3}}$	Yes (true alarm)	
23	Yes	$I_{EWMA_{Z_{\bar{X}}2}}$	No	Yes	$I_{EWMA_{Z_{\bar{e}}3}}$	Yes (true alarm)	
24	No	$I_{EWMA_{Z_{\bar{X}}3}}$	Yes (false alarm)	Yes	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
25	No	$I_{EWMA_{Z_{\bar{X}}3}}$	Yes (false alarm)	Yes	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
26	Yes	$I_{EWMA_{Z_{\bar{X}}3}}$	Yes (true alarm)	Yes	$I_{EWMA_{Z_{\bar{e}}1}}$	No	
27	Yes	$I_{EWMA_{Z_{\bar{X}}3}}$	Yes (true alarm)	Yes	$I_{EWMA_{Z_{\bar{e}}2}}$	No	
28	True alarms on process steps 1 and 2						Absorbing state

Table 5.1
$EWMA_{Z_{\bar{x}}}$ and $EWMA_{Z_{\bar{e}}}$ values for the samples $1 \sim 41$.

Sample no.	$\overline{X_i}$	$Z_{\overline{X_i}}$	$EWMA_{Z_{\overline{X_i}}}$	$\overline{e_i}$	$Z_{\overline{e_i}}$	$EWMA_{Z_{\overline{e_i}}}$
1	209.2	−1.847	−0.092	−0.295	−0.686	−0.034
2	211.0	1.451	−0.015	0.484	1.125	0.024
3	210.0	−0.381	−0.033	0.340	0.790	0.062
4	210.6	0.718	0.0041	0.066	0.154	0.067
5	210.4	0.352	0.022	0.158	0.366	0.082
6	210.4	0.352	0.038	−0.642	−1.493	0.003
7	210.2	−0.015	0.035	0.049	0.113	0.009
8	209.8	−0.748	−0.004	−0.569	−1.322	−0.058
9	209.8	−0.748	−0.041	0.631	1.467	0.018
10	210.4	0.352	−0.021	−0.042	−0.099	0.012
11	210.2	−0.015	−0.021	−0.351	−0.816	−0.029
12	211.2	1.8185	0.071	−0.007	−0.017	−0.029
13	210.0	−0.3815	0.048	−0.260	−0.604	−0.057
14	210.4	0.352	0.064	−0.242	−0.563	−0.083
15	211.4	2.185	0.170	0.102	0.236	−0.067
16	210.8	1.085	0.215	−0.225	−0.522	−0.089
17	209.6	−1.114	0.149	0.922	2.143	0.022
18	210.0	−0.381	0.122	−0.060	−0.139	0.014
19	209.6	−1.114	0.061	0.322	0.749	0.051
20	210.8	1.085	0.112	−0.425	−0.987	−0.001
21	210.0	−0.381	0.087	−0.260	−0.604	−0.031
22	209.0	−2.214	−0.028	−0.204	−0.474	−0.053
23	209.8	−0.748	−0.064	0.431	1.002	−0.001
24	210.0	−0.381	−0.080	0.740	1.719	0.085
25	210.6	0.718	−0.040	−0.934	−2.169	−0.027
Monitored sample no.	$\overline{X_i}$	$Z_{\overline{X_i}}$	$EWMA_{Z_{\overline{X_i}}}$	$\overline{e_i}$	$Z_{\overline{e_i}}$	$EWMA_{Z_{\overline{e_i}}}$
26	211.2	1.818	0.053	−0.607	−1.411	−0.097
27	212	3.284	0.215	0.428	0.995	−0.042
28	211	1.451	0.277	0.484	1.125	0.016
29	212	3.284	0.427	1.228	2.855	0.158
30	210.2	−0.015	0.405	1.049	2.4375	0.272
31	209.2	−1.847	0.293	−0.095	−0.222	0.248
32	210.6	0.718	0.314	1.666	3.872	0.429
33	209.6	−1.114	0.243	−0.678	−1.575	0.329
34	210.4	0.352	0.248	0.558	1.296	0.377
35	209.8	−0.748	0.198	0.631	1.467	0.431
36	209.8	−0.748	0.151	−0.169	−0.392	0.390
37	210.4	0.352	0.161	0.358	0.831	0.412
38	210.2	−0.015	0.152	−0.751	−1.745	0.304
39	208.8	−2.580	0.015	−0.913	−2.121	0.183
40	210.4	0.352	0.032	−0.842	−1.957	0.076
41	209	−2.214	−0.080	−0.004	−0.009	0.072

current state is any one of the States 1–27, then it may transit to other state. Hence, States 1–27 are transient states. State 28 is reached when at least one true signal is obtained from the cascade process. State 28 cannot transit to any other states, hence it is an absorbing state.

Denote $\mathbf{P}$ be the transition probability matrix, where $\mathbf{P}$ is a square matrix of order 28. Let $P_{i,j}(t_q, n_q)$ be the transition probability from the prior state i to the current state j with sampling interval t_q and sample size n_q, where t_q and n_q are determined by the prior state i, $i = 1, 2, \ldots, 28$, $j = 1, 2, \ldots, 28$, $q = 1, 2, 3$. The transition probability, for example, from State 1 to State 4 with sampling interval t_3 and sample size n_3 is calculated as

$$\begin{aligned} P_{1,4}(t_3, n_3) &= e^{-\gamma_1 t_3} \cdot e^{-\gamma_2 t_3} \\ &\quad \cdot P\left[-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{\bar{x}}} < -w\sqrt{\frac{\lambda}{2-\lambda}} \quad \text{or} \quad w\sqrt{\frac{\lambda}{2-\lambda}} \leqslant EWMA_{Z_{\bar{x}}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0, \delta_2 = 0\right] \\ &\quad \cdot P\left[-k\sqrt{\frac{\lambda}{2-\lambda}} < EWMA_{Z_{\bar{e}}} < -w\sqrt{\frac{\lambda}{2-\lambda}} \quad \text{or} \quad w\sqrt{\frac{\lambda}{2-\lambda}} \leqslant EWMA_{Z_{\bar{e}}} < k\sqrt{\frac{\lambda}{2-\lambda}} | \delta_1 = 0, \delta_2 = 0\right] \\ &= e^{-\gamma_1 t_3} \cdot e^{-\gamma_2 t_3} \cdot (2\Phi(k) - 2\Phi(w))^2 \end{aligned}$$

From the elementary properties of Markov chains (see Cinlar (1975) or Yang (2005)), the ATC is derived as follows:

$$\text{ATC} = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{t} + \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}M_f + \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1}(A)T_r \tag{4-2}$$

where $\mathbf{b}' = (p_{01}, p_{02}, p_{03}, p_{04}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \ldots, 0)$ is the vector of starting probabilities for States 1,2,...,27, where the first sample size and sampling interval have probability p_{01} (Eq. (2-6)) of being long sampling interval and small sample size (or State 1 with probability p_{01}), the probability $p_{02} (= p_{03})$ of being median sampling interval and median sample size (or State 2/State 3 with probability p_{02}) and the probability p_{04} of being short sampling interval and large sample size (or State 4 probability p_{04}); $\mathbf{I}$ is the identity matrix of order 27; $\mathbf{Q}$ is the transition probability matrix where elements represent the transition probability, $P_{i,j}(t_q, n_q)$, from transient State i, $i = 1, \ldots, 27$, to transient State j, $j = 1, \ldots, 27$; $M'_f = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, T_f, T_f + T_r, T_f + T_r, T_f, T_f, T_r, T_r, T_f, T_f, T_r, T_r)$ is the vector of searching time of false alarm (T_f) and/or searching and adjust time of true alarm (T_r) for State 1–27; $t' = (t_3, t_2, t_2, t_1, t_3, t_2, t_2, t_1, t_3, t_2, t_2, \ t_1, t_3, t_2, t_2, t_1, t_1^*, t_1^*, t_1^*, t_2^*, t_2^*, t_2^*, t_2^*, t_3^*, t_3^*, t_3^*, t_3^*)$ is the vector of the variables sampling intervals for States 1–27, where t_1^* is the average time of sampling interval for States 17–19, t_2^* is the average time of sampling interval for States 20–23, t_3^* is the average time of sampling interval for States 24–27. A is the vector of transition probability, $P_{i,28}(t_q)$, form transition State ***i***,***i***=1,...,27, to absorbing State 28.

$$UCL_{EWMA_{Z_{\bar{X}}}} = 0.3990 \qquad UCL_{EWMA_{Z_e}} = 0.3990$$

$$UWL_{EWMA_{Z_{\bar{X}}}} = 0.175 \qquad UWL_{EWMA_{Z_e}} = 0.175$$

$$CL_{EWMA_{Z_{\bar{X}}}} = 0 \qquad CL_{EWMA_{Z_e}} = 0$$

$$LWL_{EWMA_{Z_{\bar{X}}}} = -0.175 \qquad LWL_{EWMA_{Z_e}} = -0.175$$

$$LCL_{EWMA_{Z_{\bar{X}}}} = -0.3990 \qquad LCL_{EWMA_{Z_e}} = -0.3990$$

Fig. 5.1. The VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control limits.

5. An example

Consider a data set of measurements of the metallic film thickness of the computer connectors system. Let X = gold concentration in the electroplating tank and Y = metallic film thickness be measured from the end of the second step. Y produced in the second step is influenced by X produced in the first step. Two machines are assumed to be used in the process steps. One machine could only fail in the first step and shift the mean of X distribution, and another machine could only fail in the second step, and shift the mean of Y distribution. Presently, the joint FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts are used to monitor the shift in mean on the two steps of the cascade process every hour. When the control charts indicate that at least one of the process steps is out-of-control, it requires adjustment. To construct the control charts, 25 samples of size 5 (X, Y) are taken randomly from the process to analyze their statistical relationship. The QQ plot (Johnson, 1992) of the 25 samples indicates that the data follows bivariate normal distribution. The relationship of X and Y is expressed by a linear regression model. The fitted model is

$$\hat{Y}|X = 105.3 + 0.456X \tag{5-1}$$

Thus, the residuals or specific quality, $e = Y - \hat{Y}|X$. The estimated means and standard deviations of independent variables X and e are $(\hat{\mu}_X = 210.208, \hat{\sigma}_X = 1.489)$, and $(\hat{\mu}_e = 0, \hat{\sigma}_e = 0.926)$, respectively. That is, when both steps are in-control, $X \sim N(210.208, 1.489^2)$ and $e \sim N(0, 0.926^2)$. From historical data, the estimated failure frequency is 0.04 times per hour (or $\gamma_1 = 0.04$) for machine 1 and 0.2 times per hour (or $\gamma_2 = 0.2$) for machine 2. The failures of machine1 and 2 are independent and only influence the means of X and e, but the standard deviations are unaffected. The failure machine 1 would shift the mean of X to $\hat{\mu}_X + \delta_1 \hat{\sigma}_X$ where $\delta_1 = 0.5$. The failure machine 2 would shift the mean of e to $\delta_2 \hat{\sigma}_e$ where $\delta_2 = 0.25$. Hence, for out-of-control step 1,$X \sim N(210.208 + 0.5 \cdot 1.489, 1.435^2)$; for out-of-control step 2, $e \sim N(0.25 \cdot 0.926, 0.926^2)$.

The FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ charts have control limits placed at ± 2.492 and$\lambda = 0.05$ with average run length 370 (Montgomery (2005)) when $T_f = T_r = 0$, respectively. The average time of

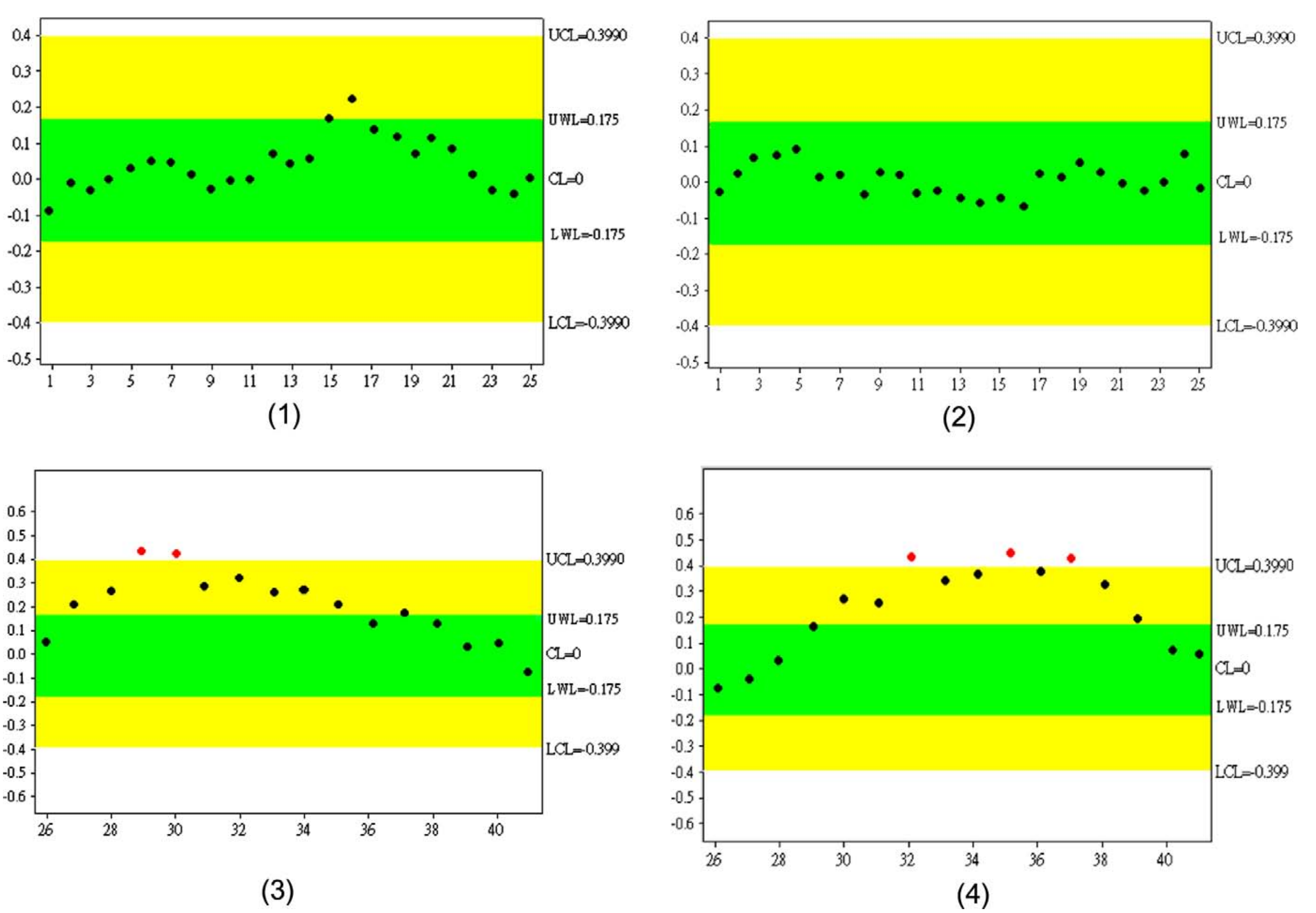

Fig. 5.2. (1) Trial VSSI$EWMA_{Z_{\bar{X}}}$ control chart. (2) Trial VSSI $EWMA_{Z_e}$ control chart. (3) Monitoring result of VSSI $EWMA_{Z_{\bar{X}}}$ control chart. (4) Monitoring result of VSSI $EWMA_{Z_e}$ control chart.

Table 6.1
AATS of VSSI and FSSI charts under various combinations of parameters.

No.	t_1	t_2	t_3	n_3	n_2	n_1	δ_1	δ_2	γ_1	γ_2	(T_f, T_r)	λ_1	λ_2	k	t_0	n_0	w	AATS of VSSI	AATS of FSSI	Saving time (%)
1.	0.01	0.1	2.61	2	4	15	0.15	0.15	0.08	0.1	(0.01, 0.05)	0.05	0.05	2.492	1	5	0.8343	18.6922	28.1439	33.58
2.	0.01	0.1	2.24	3	5	12	0.15	0.15	0.15	0.2	(0.05,0.1)	0.05	0.05	2.492	1	5	0.9218	23.7257	39.1303	39.37
3.	0.01	0.1	1.42	4	6	18	0.15	0.15	0.04	0.05	(0.1,0.5)	0.05	0.05	2.492	1	5	1.3265	18.4732	23.1721	20.23
4.	0.01	0.5	1.82	3	5	12	0.25	0.25	0.08	0.1	(0.01,0.05)	0.05	0.05	2.492	1	5	0.9218	4.3714	6.5972	33.74
5.	0.01	0.5	1.25	4	6	18	0.25	0.25	0.15	0.2	(0.05,0.1)	0.05	0.05	2.492	1	5	1.3265	5.5081	7.7933	29.32
6.	0.01	0.5	6.11	2	4	15	0.25	0.25	0.04	0.05	(0.1,0.5)	0.05	0.05	2.492	1	5	0.4534	5.7765	6.6317	12.90
7.	0.01	1.0	1.04	4	6	18	0.50	0.50	0.08	0.1	(0.01,0.05)	0.05	0.05	2.492	1	5	1.3265	0.9852	1.020	4.16
8.	0.01	1.0	1.43	2	4	15	0.50	0.50	0.15	0.2	(0.05,0.1)	0.05	0.05	2.492	1	5	0.8343	1.0524	1.0844	295
9.	0.01	1.0	1.28	3	5	12	0.50	0.50	0.04	0.05	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9218	1.5239	1.5651	2.63
10.	0.05	0.1	2.08	2	5	18	0.25	0.50	0.08	0.2	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9675	2.6327	3.0879	14.74
11.	0.05	0.1	1.77	3	6	15	0.25	0.50	0.15	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	1.0959	3.1134	4.4495	30.03
12.	0.05	0.1	2.27	4	4	12	0.25	0.50	0.04	0.1	(0.05,0.1)	0.05	0.05	2.492	1	5	0.9120	2.1545	2.4680	12.70
13.	0.05	0.5	1.48	3	6	15	0.50	0.15	0.08	0.2	(0.1,0.5)	0.05	0.05	2.492	1	5	1.0959	11.9352	13.7129	12.96
14.	0.05	0.5	1.83	4	4	12	0.50	0.15	0.15	0.05	(0.01,005)	0.05	0.05	2.492	1	5	0.9120	4.7020	5.4075	13.05
15	0.05	0.5	1.70	2	5	18	0.50	0.15	0.04	0.1	(005,0.1)	0.05	0.05	2.492	1	5	0.9675	10.8298	12.5450	13.67
16.	0.05	1.0	1.28	4	4	12	0.15	0.25	0.08	0.2	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9120	13.9668	14.3816	2.88
17	0.05	1.0	1.22	2	5	18	0.15	0.25	0.15	0.05	(001,005)	0.05	0.05	2.492	1	5	0.9679	14.7928	16.8565	12.24
18.	0.05	1.0	1.12	3	6	15	0.15	0.25	0.04	0.1	(0.05,0.1)	0.05	0.05	2.492	1	5	1.0959	9.8003	11.0843	11.58
19.	0.09	0.1	2.20	2	6	12	0.50	0.25	0.08	0.05	(005,0.1)	0.05	0.05	2.492	1	5	0.9277	2.3842	2.9177	18.28
20.	0.09	0.1	2.06	3	4	18	0.50	0.25	0.15	0.1	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9713	2.9231	3.6100	19.03
21.	0.09	0.1	1.66	4	5	15	0.50	0.25	0.04	0.2	(0.01,0.05)	0.05	0.05	2.492	1	5	1.1505	3.3419	4.7882	30.21
22.	0.09	0.5	1.68	3	4	18	0.15	0.50	0.08	0.05	(0.05,0.1)	0.05	0.05	2.492	1	5	0.9713	9.5811	11.0589	13.36
23.	0.09	0.5	1.41	4	5	15	0.15	0.50	0.15	0.1	(0.1,0.5)	0.05	0.05	2.492	1	5	1.1505	10.5207	12.0184	12.46
24.	0.09	0.5	1.78	2	6	12	0.15	0.50	0.04	0.2	(0.01,0.05)	0.05	0.05	2.492	1	5	0.9277	3.5641	4.0563	12.13
25.	0.09	1.0	1.09	4	5	15	0.25	0.15	0.08	0.05	(0.05,0.1)	0.05	0.05	2.492	1	5	1.1505	11.2069	12.3731	9.43
26.	0.09	1.0	1.25	2	6	12	0.25	0.15	0.15	0.1	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9277	13.1914	15.9154	17.12
27.	0.09	1.0	1.21	3	4	18	0.25	0.15	0.04	0.2	(0.01,0.05)	0.05	0.05	2.492	1	5	0.9713	15.5334	17.4152	10.81
28.	0.01	0.1	2.00	3	5	15	0.75	0.75	0.08	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	0.9993	1.1548	0.6801	
29.	0.01	0.1	2.00	3	5	15	1.0	1.0	0.08	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	0.9993	1.1533	0.6651	
30.	0.01	0.1	2.00	3	5	15	1.25	1.25	0.08	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	0.9993	1.1533	0.6650	

Saving time (%) = [AATS(FSSI) − AATS(VSSI)]/AATS(FSSI)%.

searching any process step is 0.01 h (or $T_f = 0.01$) when at least one false signal occurs. The average correct adjustment time of any process step is 0.05 h (or $T_r = 0.05$) when at least one true signal occurs. The AATS of the FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ charts is 4.788 h. The slowness with which the FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts detect shifts in the process ($\delta_1 = 0.5, \delta_2 = 0.25$) has led the quality manager to proposing building the $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts with VSSIs. The construction and the application of the proposed VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts are illustrated. The following are the guidelines for using the proposed charts:

Step 1. Let the factor of control limits, $k = 2.492$ and $\lambda = 0.05$, to maintain the in-control average run length is 370 for $EWMA_{Z_{\bar{X}}}$ or $EWMA_{Z_{\bar{e}}}$ control chart (see Montgomery (2005)).

Step 2. Since $0 < t_1 < t_2 < t_0 < t_3 < \infty$ and $0 < n_3 < n_2 < n_0 < n_1 < \infty$ are required, and for performance of process control engineers adopt the specified combination ($t_1 = 0.09$ h, $t_2 = 0.1$ h, $t_3 = 1.66$ h, $n_3 = 4, n_2 = 5$ and $n_1 = 15$).

Step 3. Letting $t_1 = 0.09$ h, $t_2 = 0.1$ h, $n_3 = 4, n_2 = 5, n_1 = 15, k = 2.492$ and $\lambda = 0.05$ in Eq. (3-4) leads to $w = 1.096$. From Eq. (3-5), $t_3 = 1.66$ h.

Consequently, the structures of the proposed VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts are as Fig. 5.1.

With the design parameters determined, the VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts can be used for controlling the metallic film thickness of computer connectors cascade process. According to the VSSI scheme, if both samples ($EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$) fell in central regions, then the long sampling interval $t_3 = 1.66$ h and small sample size $n_3 = 4$ are adopted. If one of the samples fells in the central region but the other fell in warning region, then a medium sampling interval $t_2 = 0.1$ h and medium sample size $n_2 = 5$ are adopted. If both samples fell in warning regions, then the short sampling interval $t_1 = 0.09$ h and large sample size $n_1 = 15$ are adopted. If at least one sample fell outside the control limits of any proposed control chart, then the process steps are stopped to search the occurred special cause and adjusted. The AATS is used to measure the performance of the proposed VSSI control charts. The proposed Markov chain approach is used to obtain the ATC and calculate the AATS. There are 28 possible states, as presented in Table 4.1. The AATS is 3.113 h according to Eq. (4-1).

The VSSI scheme improves the sensitivity of the joint FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ charts. From the example, in order to detect a shift in the process mean, the AATS of the VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ charts has been reduced from 4.788 h to only 3.113 h. The percentage of saving time is 30.03%.

An example using the VSSIs is introduced now. When the process starts, a random procedure decides the first sampling interval $t_2 = 0.1$ h with medium sample of size $n_2 = 5$, and the average is ($\bar{x} = 209.2, \bar{e} = -0.295$). The first sample with the values of $Z_{\bar{X}}$ and $Z_{\bar{e}}$ are ($Z_{\bar{X}} = -1.848, Z_{\bar{e}} = -0.686$). Thus, their $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ values are calculated as follows:

$$EWMA_{Z_{\bar{x}},1} = \lambda Z_{\bar{X}_1} + (1-\lambda)EWMA_{Z_{\bar{X},0}} = 0.05 \cdot 1.848 + 0.95 \cdot 0 = -0.092$$

$$EWMA_{Z_{\bar{e}},1} = \lambda Z_{\bar{e}_1} + (1-\lambda)EWMA_{Z_{\bar{e}},0} = 0.05 \cdot (-0.686) + 0.95 \cdot 0 = -0.034$$

Since both samples fall in the central regions, the second sample will be observed adopting a small sample of size $n_3 = 4$ after long sampling interval $t_3 = 1.66$ h. The second sample is ($\bar{x} = 211, \bar{e} = 0.484$). Since $Z_{\bar{X}} = 1.451$ and $Z_{\bar{e}} = 1.125$, so $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ values are calculated as follows:

$$EWMA_{Z_{\bar{x}},2} = \lambda Z_{\bar{X}_2} + (1-\lambda)EWMA_{Z_{\bar{X},1}} = 0.05 \cdot 1.451 + 0.95 \cdot (-0.092) = -0.015$$

$$EWMA_{Z_{\bar{e}},2} = \lambda Z_{\bar{e}_2} + (1-\lambda)EWMA_{Z_{\bar{e}},1} = 0.05 \cdot 1.125 + 0.95 \cdot (-0.034) = 0.024$$

Both samples fall in the central regions, the third sample will be observed adopting a small sample of size $n_3 = 4$ after the long sampling interval $t_3 = 1.66$ h. The $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ values for samples $1 \sim 25$ are illustrated in Table 5.1, and plotted on the trial VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts (Fig. 5.2(1)–(2)). All the 25 samples ($EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{X}}}$) are within the charts. Hence the VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_{\bar{e}}}$ control charts are used to monitor the samples $26 \sim 41$. We find that the 29th and the 30th $EWMA_{Z_{\bar{X}}}$ values fell outside the $EWMA_{Z_{\bar{X}}}$ chart (Fig. 5.2(3)), but the 32th,35th and 37th$EWMA_{Z_{\bar{e}}}$ values fell outside the VSSI $EWMA_{Z_{\bar{e}}}$ chart (Fig. 5.2(4)). It indicates that the step 1 is out-of-control on the 29th and 30th samples, but the step 2 is out-of-control on the 32th, 35th and 37th samples. Hence, the steps 1 and 2 are stopped and machines 1 and 2 are adjusted.

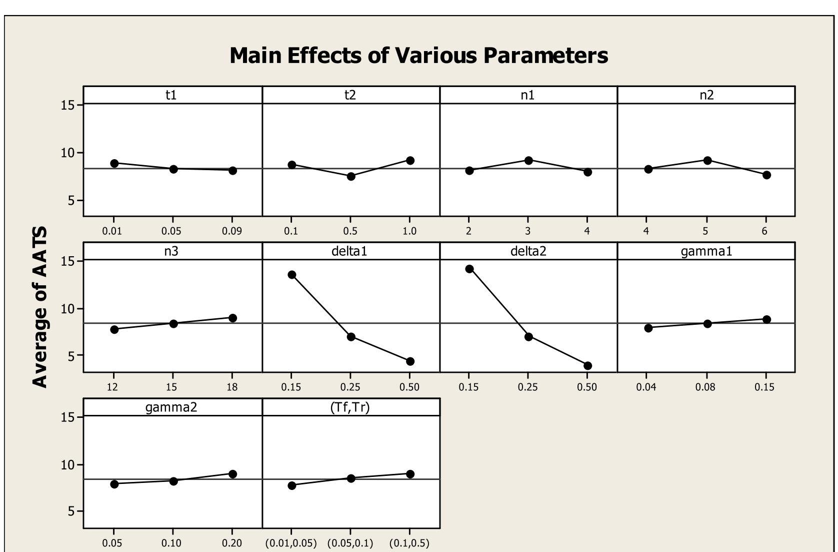

Fig. 6.1. The main effects for average AATS under various parameters.

Table 6.2
AATS of optimum VSSI and FSSI charts under various combinations of parameters.

No.	t_1^*	t_2^*	t_3^*	n_3^*	n_2^*	n_1^*	δ_1	δ_2	γ_1	γ_2	$(T_f T_r)$	λ_1	λ_2	k	t_0	n_0	W^*	Optimum AATS of VSSI chats	AATs of VSSI charts	AATS of FSSI charts	(1) Saving time (%)	(2) Saving time (%)
1	0.001	0.101	2.371	3	5	11	0.15	0.15	0.08	0.1	(0.01,0.05)	0.05	0.05	2.492	1	5	0.8888	18.521	18.6922	28.1439	34.19	33.58
2.	0.001	0.101	4.701	2	5	7	0.15	0.15	0.15	0.2	(0.05,0.01)	0.05	0.05	2.492	1	5	0.5885	22.285	23.7257	39.1303	43.05	3937
3.	0.001	0.101	2.371	3	5	11	0.15	0.15	0.04	0.05	(0.1,0.5)	0.05	0.05	2.452	1	5	0.8888	16.871	18.4732	23.1721	27.19	20.28
4.	0.001	0.101	1.527	3	8	12	0.25	0.25	0.08	0.1	(0.01.0.05)	0.05	0.05	2.492	1	5	1.2418	3.920	4.3714	6.5972	40.583	33.74
5.	0.001	0.101	2.320	2	6	11	0.25	0.25	0.15	0.2	(0.05.0.1)	0.05	0.05	2.492	1	5	0.9020	4.246	5.5081	7.7933	45.52	29.32
6.	0.001	0.101	2.048	3	6	10	0.25	0.25	0.04	0.05	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9836	4.383	5.7765	6.6317	33.91	12.90
7.	0.001	0.101	1.129	4	12	12	0.50	0.50	0.08	0.1	(0.01,0.05)	0.05	0.05	2.492	1	5	1.7716	0.864	0.9852	1.0280	15.95	4.16
8.	0.001	0.101	1.129	4	12	12	0.50	0.50	0.15	0.2	(0.05,0.1)	0.05	0.05	2.492	1	5	1.7716	0.908	1.0524	1.0844	16.27	29.5
9.	0.001	0.101	1.129	4	12	12	0.50	0.50	0.04	0.05	(0.1,0.5)	0.05	0.05	2.492	1	5	1.7716	1.406	1.5239	1.5651	10.17	2.63
10.	0.001	0.101	1.433	3	9	11	0.25	0.50	0.008	0.2	(0.1,0.5)	0.05	0.05	2.492	1	5	1.3718	2.419	2.6327	3.0879	21.66	14.74
11.	0.001	0.101	1.424	3	9	12	0.25	0.50	0.15	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	1.3263	3.096	3.1134	4.4495	30.42	30.03
12.	0.001	0.101	1.424	3	9	12	0.25	0.50	0.04	0.1	(0.05,0.1)	0.05	0.05	2.492	1	5	1.3263	1.898	2.1545	2.4680	23.10	12.70
13.	0.001	0.101	2.007	2	7	11	0.50	0.15	0.08	0.2	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9978	11.356	11.9352	13.7129	17.19	12.96
14.	0.001	0.101	1.433	3	9	11	0.50	0.15	0.15	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	1.3178	4.527	4.7020	5.4075	16.28	13.05
15.	0.001	0.101	2.048	3	6	10	0.50	0.15	0.04	0.1	(0.05,0.1)	0.05	0.05	2.492	1	5	0.9836	10.376	10.8298	12.5450	17.29	13.67.
16.	0.001	0.101	2.320	2	6	11	0.15	0.25	0.08	0.2	(0.1,0.5)	0.05	0.05	2.492	1	5	0.9020	8.680	13.9668	14.3816	39.65	2.88
17.	0.001	0.101	2.771	2	5	11	0.15	0.25	0.15	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	0.8036	11.838	14.7928	16.8565	29.77	12.24
18.	0.001	0.101	2.246	3	5	12	0.15	0.25	0.04	0.1	(0.05,0.1)	0.05	0.05	2.492	1	5	0.9218	7.254	9.8003	11.0843	34.56	11.58
19.	0.001	0.101	1.527	3	8	12	0.50	0.25	0.08	0.05	(0.05,0.1)	0.05	0.05	2.492	1	5	1.2419	2.191	2.3842	2.9177	24.91	18.28
20.	0.001	0.101	1.527	3	8	12	0.50	0.25	0.15	0.1	(0.1,0.5)	0.05	0.05	2.492	1	5	1.2419	2.758	2.9231	3.6100	23.60	19.03
21.	0.001	0.101	1.444	3	9	10	0.50	0.25	0.04	0.2	(0.01,0.05)	0.05	0.05	2.492	1	5	1.3087	3.300	3.3419	4.7882	31.08	30.21
22.	0.001	0.101	1.789	3	7	9	0.15	0.50	0.08	0.05	(0.05,0.1)	0.05	0.05	2.492	1	5	1.080	9.191	9.5811	11.0589	16.89	13.36
23.	0.001	0.101	1.444	3	9	10	0.15	0.50	0.15	0.1	(0.1,0.5)	0.05	0.05	2.492	1	5	1.3087	9.983	10.5207	12.0184	16.94	12.46
24.	0.001	0.101	1.351	3	10	12	0.15	0.50	0.04	0.2	(0.01,0.05)	0.05	0.05	2.492	1	5	1.3991	3.394	3.5641	4.0563	16.33	12.13
25.	0.001	0.101	2.371	3	5	11	0.25	0.15	0.08	0.05	(0.05,0.1)	0.05	0.05	2.492	1	5	0.8889	8.267	11.2069	12.3731	33.19	9.43
26.	0.001	0.101	2.537	3	5	10	0.25	0.15	0.15	0.1	(0.1,0.5)	0.05	0.05	2.452	1	5	0.8502	9.850	13.1914	15.9154	38.11	17.12.
27.	0.001	0.101	2.537	3	5	10	0.25	0.15	0.04	0.2	(0.01,0.05)	0.05	0.05	2.492	1	5	0.8502	12.538	15.5334	17.4152	28.01	10.81
28.	0.001	0.999	1.005	4	12	12	*0.75*	*0.75*	0.08	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	1.7716	0.681	1.3747	0.6801		
29.	0.001	0.999	1.005	4	12	12	*1.0*	*1.0*	0.08	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	1.7716	0.667	1.0763	0.6651		
30.	0.001	0.999	1.005	4	12	12	*1.25*	*1.25*	0.08	0.05	(0.01,0.05)	0.05	0.05	2.492	1	5	1.7716	0.667	0.6887	0.6650		

(1) Saving time (%) = [AATS of FSSI − optimum AATS of VSSI]/AATS of FSSI.
(2) Saving time (%) = [AATS of FSSI − AATS of VSSI]/AATS of FSSI.

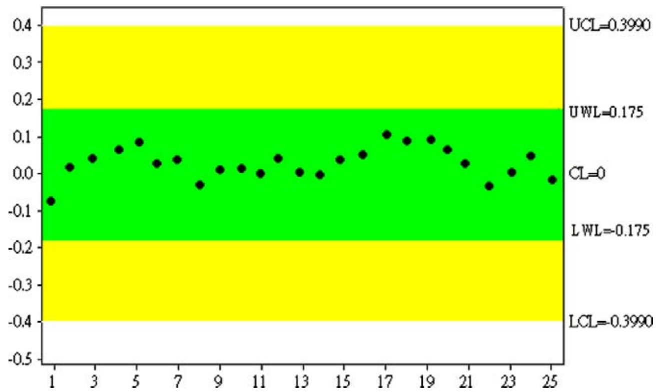

Fig. 7.1. Trial $EWMA_{Z_e}$ control chart (Nos. 1–25).

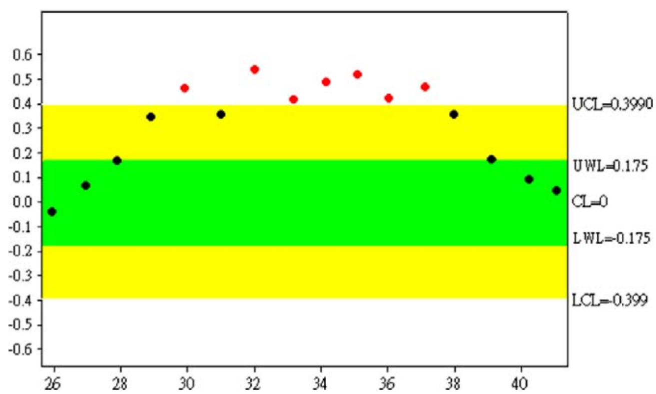

Fig. 7.2. Monitoring result of $EWMA_{Z_e}$ control chart.

6. Performance comparison between VSSI and FSSI schemes

Table 6.1 provides the AATS of the VSSI and FSSI schemes, which are obtained under various combinations of parameters based on orthogonal array $L_{27}(3^{13})$ table. The specified parameters are $\gamma_1 = (0.04, 0.08, 0.15), \gamma_2 = (0.05, 0.10, 0.20)$, $\delta_1 = (0.15, 025, 0.5)$, $\delta_2 = (0.15, 025, 0.5), t_0 = 1.0, (t_1, t_2) = (0.01, 0.05, 0.09), (0.1, 0.5, 1.0)$, $(n_1, n_2, n_3) = (15, 4, 2), (12, 5, 3), (18, 6, 4)$, and $(T_f, T_r) = (0.01, 0.05), (0.05, 0.1)$.

Comparing the AATS between the FSSI and VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts, it can be seen that the performance of the VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts is better for detecting small shifts ($0.15 < \delta_1 \leqslant 0.5$ and $0.15 < \delta_2 \leqslant 0.5$) in process means (see Table 6.1, Nos. 1–27). The VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts save detection time from 2.63% to 33.74% compared to the FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts. If the shift scales $\delta_1 > 0.5$ and $\delta_2 > 0.5$ then the performance of FSSI control charts is slightly better than VSSI control charts (see Table 6.1, Nos. 28–30). To examine the effects of various parameters on the AATS, the main effect plots show the significant parameters are δ_1 and δ_2 (Fig. 6.1). As δ_1, δ_2 increases, AATS decreases.

When quality engineers cannot specify the VSSIs the optimal VSSIs of the proposed charts are thus suggested. The optimal VSSI of the proposed charts are determined using optimization technique (Fortran IMSL BCONF subroutine) to minimize AATS under the same constraints and parameters as described before. The optimum VSSI and AATS under various combinations of parameters are illustrated in Table 6.2. We find that the optimum VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts save detection time from 10.17% to 45.52% compared to the FSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts, and the optimum VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts also work better than the $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts with specific variable sampling intervals. However, the VSSI and FSSI control charts almost have same performance when the shift scales $\delta_1 > 0.5$ and $\delta_2 > 0.5$ (see Table 6.2, Nos. 28–30). Hence, the optimum VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts are suggested whenever engineers cannot specify the VSSIs. After comparing the three kinds of the control charts, we recommend an approach for various δ_1 and δ_2. (1) When δ_1 and δ_2 are all small ($\delta_1 \leqslant 0.5$ and $\delta_2 \leqslant 0.5$), since the AATS of the optimum VSSI control charts is always the smallest, we recommend to take the optimum VSSI control charts as the control scheme of a process. (2) When δ_1 and δ_2 are larger ($\delta_1 > 0.5$ and $\delta_2 > 0.5$), since the optimum VSSI control charts and FSSI control charts almost have same AATS, we recommend to take the VSSI or FSSI control charts as the control scheme of a process.

7. Misusing $EWMA_{Z_{\bar{Y}}}$ control chart

In many real situations, engineers may misuse $EWMA_{Z_{\bar{Y}}}$ control chart to monitor mean in the second step. Fig. 7.1 shows the trial $EWMA_{Z_{\bar{Y}}}$ control chart using the samples 1–25, and Fig. 7.2 shows the monitoring results (from samples 26–41) of using $EWMA_{Z_{\bar{Y}}}$ control chart. On the second step, there are seven outliers, sample 30, 32, 33, 34, 35, 36 and 37, occur on the $EWMA_{Z_{\bar{Y}}}$ chart. Compare to $EWMA_{Z_e}$ control chart (outliers on sample 32, 35 and 37), it indicates that misusing $EWMA_{Z_{\bar{Y}}}$ control chart will lead to unnecessarily adjust the mean on the second step. Incorrect adjustment of a process will increase in variability of the quality of products and cost (see Woodall (1986)).

8. Conclusions

The proposed VSSI scheme controlling two dependent steps of cascade process substantially improves the performance of the FSSI scheme by increasing the speed when small shifts in the means of cascade process are detected. We have found that the optimum VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts always work better (in the cases examined) than the specified VSSI $EWMA_{Z_{\bar{X}}}$ and $EWMA_{Z_e}$ control charts. The optimum VSSI scheme controlling the cascade process is thus suggested when quality engineers cannot specify the VSSIs.

This paper considered two steps of the cascade process. However, a study of the variable parameters (VP) control charts for the cascade process, whereby one of the double special causes shifts the process means and the other changes the process variances is an interesting topic for future research.

The extension of the proposed model to study VP control charts on multiple process steps or other control charts, such as attributes charts, CUSUM charts or multivariate cases, is straightforward.

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