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A New Chart for Monitoring Service Process Mean

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Control charts are demonstrated effective in monitoring not only manufacturing processes but also service processes. In service processes, many data came from a process with nonnormal distribution or unknown distribution. Hence, the commonly used Shewhart variable control charts are not suitable because they could not be properly constructed. In this article, we proposed a new mean chart on the basis of a simple statistic to monitor the shifts of the process mean. We explored the sampling properties of the new monitoring statistic and calculated the average run lengths of the proposed chart. Furthermore, an arcsine transformed exponentially weighted moving average chart was proposed because the average run lengths of this modified chart are more intuitive and reasonable than those of the mean chart. We would recommend the arcsine transformed exponentially weighted moving average chart if we were concerned with the proper values of the average run length. A numerical example of service times with skewed distribution from a service system of a bank branch in Taiwan is used to illustrate the proposed charts. Copyright © 2011 John Wiley & Sons, Ltd.

Keywords: mean chart; process mean; binomial distribution; skewed distribution; average run length

1. Introduction

Control charts are commonly used tools to improve the quality of manufacturing processes. In the past few years, more and more statistical process control techniques are applied to service industry, and control charts are also becoming an effective tool in improving the service quality. There were a few studies in this area, like those of Maccarthy and Wasusri[1], Tsung *et al*.,[2] and Ning *et al*.[3] Many service process data do not come from a process with normal distribution or known distribution, some are from unknown population. Hence, the commonly used Shewhart variable control charts are not suitable because they cannot be properly constructed and their performance could not be properly evaluated. In most cases, normality was assumed for variable data, some with other known distributions. When we had no knowledge of the underlying distribution, it is not possible to derive the necessary sampling properties to construct the chart and evaluate its performance. Hence, we need to find an alternative. Using nonparametric approach seems to be a good alternate way. Some research had been carried out in this area, like those of Ferrell[4], Bakir and Reynolds[5], Amin *et al*.,[6] Chakraborti *et al*.,[7] Altukife[8,9], Bakir[10,11], Chakraborti and Eryilmaz[12], Chakraborti and Graham[13], Chakraborti and Van der Wiel[14], Das and Bhattacharya[15], and Li *et al*.[16] A major drawback of the previous nonparametric approaches is that they are not easy for practitioners to apply because they are not statisticians and do not quite understand the proper way to implement the scheme.

In this article, we propose a new control chart for variable data to monitor the process mean, without assuming a process distribution. The approach is simple to understand and easy to use. The article is organized as follows: in Section 2, we discuss the construction of a newly proposed mean chart and its performance. In Section 3, we propose an arcsine transformed exponentially weighted moving average (EWMA) chart. In Section 4, we apply the proposed EWMA chart to monitor the service quality of a service system in a bank branch and compare its performance with the mean chart and three existing charts. Finally, in Section 5, we summarize the findings.

2. Proposed mean chart

Assume that a critical quality characteristic, X, has a mean μ. Let $Y=X-\mu$ and $p=P(Y>0)=$the "process proportion." If the process was in control, then $p=p_0$; if the process was out of control, that is, μ had shifted, then $p=p_1\neq p_0$. If p_0 is not given, it will be estimated using the preliminary data set (i.e. the phase I of the statistical process control).

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A random sample of size n, $X_1, X_2, \ldots, X_n$, is taken from X to monitor the process mean. Define $Y_j = X_j - \mu$ and $I_j = \begin{cases} 1, & \text{if } Y_j > 0, \\ 0, & \text{otherwise,} \end{cases}$ $j = 1, 2, \ldots, n$. Let M be the total number of $Y_j > 0$, then $M = \sum_{j=1}^{n} I_j$ would follow a binomial distribution with parameters (n, p_0) for an in-control process, where p0=probability of success.

2.1. *Construction of the mean chart*

Monitoring the process mean shifts is equivalent to monitoring the changes in process proportion. For the in-control process, we defined the monitoring statistics M_t as the number of ($Y_j > 0$) at time t, $M_t \sim B(n, p_0)$. The center line (CL), the lower control limit (LCL), and the upper control limit (UCL) of the proposed mean chart are CL = np_0, LCL = $np_0 - 3\sqrt{np_0(1-p_0)}$, and UCL = $np_0 + 3\sqrt{np_0(1-p_0)}$, and then plot M_t.

If any $M_t \geq$UCL or $M_t \leq$LCL, the process is deemed to be out of control.

Note that although the resulting chart is a "binomial chart (np chart)", this is a new chart in that the binomial variable is not the count of nonconforming units in the sample but rather the number of X_j values in a sample that are above the in-control process mean.

To measure the performance of the proposed mean chart, we calculated the average run length (ARL). The in-control ARL, ARL_0, of the mean chart depends on the values of n and p_0.

2.2. *ARL of the mean chart*

The chance of observing a false signal is

$$\begin{aligned} Q &= P\left(M_t \leq \text{LCL or } M_t \geq \text{UCL} \mid M_t \sim \text{B}(n, p_0)\right) \\ &= P\left(M_t \leq np_0 - 3\sqrt{np_0(1-p_0)} \text{ or } M_t \geq np_0 + 3\sqrt{np_0(1-p_0)}\right) \\ &= \sum_{i=0}^{\left[np_0 - 3\sqrt{np_0(1-p_0)}\right]} \binom{n}{i} p_0^{\,i}(1-p_0)^{n-i} + \sum_{i=np_0+3\sqrt{np_0(1-p_0)}}^{n} \binom{n}{i} p_0(1-p_0)^{n-i} \end{aligned}$$

where [a] is Gauss' symbol, that is, the largest integer$\leq a$, and $\text{ARL}_0 = 1 / Q$. Table I lists the values of ARL_0 for n=9(1)20 and p_0 = 0.25 (0.05)0.5.

When the process is out of control, the chance of observing a true signal is

$$\begin{aligned} Q_1 &= P\left(M_t \leq \text{LCL or } M_t \geq \text{UCL} \mid M_t \sim \text{B}(n, p_1)\right) \\ &= P\left(M_t \leq np_0 - 3\sqrt{np_0(1-p_0)} \text{ or } M_t \geq np_0 + 3\sqrt{np_0(1-p_0)}\right) \\ &= \sum_{i=0}^{\left[np_0 - 3\sqrt{np_0(1-p_0)}\right]} \binom{n}{i} p_1^{\,i}(1-p_1)^{n-i} + \sum_{i=np_0+3\sqrt{np_0(1-p_0)}}^{n} \binom{n}{i} p_1(1-p_1)^{n-i} \end{aligned}$$

and the out-of-control ARL, $\text{ARL}_1 = 1 / Q_1$.

Table II lists the values of ARL_1 for p_1 = 0.05(0.05)0.45 under the in-control proportion p_0 = 0.5. Similar calculations can easily be performed for other in-control values of p_0.

Table I. ARL_0 of the mean chart

	p_0					
n	0.25	0.3	0.35	0.4	0.45	0.5
9	745	233	716	3815	1322	256
10	285	629	1852	596	2937	512
11	842	233	491	1362	277	1024
12	360	591	1179	356	542	2048
13	177	248	398	760	1058	293
14	464	600	904	718	420	546
15	238	274	353	417	810	1024
16	608	638	768	819	644	239
17	322	309	272	372	410	426
18	804	699	536	724	752	762
19	437	354	297	679	486	1372
20	254	782	588	468	407	388

ARL_0 (p_0)=ARL_0 (1–p_0) for $p_0 > 0.5$.

Table II. ARL$_1$ of the mean chart under $p_0 = 0.5$

	p_1								
n	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
9	2	3	4	7	13	25	48	97	186
10	2	3	5	9	18	35	74	163	348
11	2	3	6	12	24	51	114	272	647
12	2	4	7	15	32	72	176	456	1197
13	1	2	3	4	8	16	34	78	184
14	1	2	3	5	10	21	49	123	319
15	1	2	3	6	12	28	71	192	551
16	1	1	2	3	5	10	22	54	139
17	1	1	2	3	6	13	31	81	229
18	1	1	2	4	7	17	42	121	377
19	1	1	2	4	9	22	59	183	625
20	1	1	2	2	4	9	23	62	192

ARL$_1$ (p_1)=ARL$_1$(1–p_1), for $p_1 > 0.5$.

From Table I, we found that ARL$_0$s are quite different from the supposed value of 370. The reason is that the binomial distribution is asymmetric for $p \neq 0.5$. In Table II, it is found that the values of out-of-control ARL$_1$ did not change with n inversely as it normally should.

To rectify this problem, we proposed an "arcsine transformed EWMA chart" because EWMA chart would monitor the small shifts of the process mean quickly and effectively.

Let $T = \sin^{-1}\left(\sqrt{\frac{M}{n}}\right)$, then the distribution of T would be approximately normal with a mean $\sin^{-1}\left(\sqrt{p}\right)$ and variance 1/(4n) (see Mosteller and Youtz[17]).

3. The new EWMA chart

We define the new EWMA statistic as

$$\mathrm{EWMA}_{T_i} = \lambda T_i + (1-\lambda)\mathrm{EWMA}_{T_{i-1}} \quad 0 < \lambda \leq 1$$

Let the starting value, EWMA$_{T_0}$, be the mean of T; that is, $\mathrm{EWMA}_{T_0} = \sin^{-1}\sqrt{p_0}$ for an in-control process. Hence, the mean and the variance of EWMA$_{T_i}$ are $E(\mathrm{EWMA}_{T_i}) = \sin^{-1}\sqrt{p_0}$ and $\mathrm{Var}(\mathrm{EWMA}_{T_i}) = \frac{\lambda\left[1-(1-\lambda)^{2i}\right]}{2-\lambda}(1/4n)$.

The asymptotic variance of EWMA$_{T_i}$ is $\mathrm{Var}(\mathrm{EWMA}_{T_i}) = \frac{\lambda}{2-\lambda}(1/4n)$.

We could now construct the new EWMA chart as follows:

$$\mathrm{UCL} = \sin^{-1}(\sqrt{p_0}) + k\sqrt{\frac{\lambda}{4n(2-\lambda)}}$$

$$\mathrm{CL} = \sin^{-1}(\sqrt{p_0})$$

$$\mathrm{LCL} = \sin^{-1}(\sqrt{p_0}) - k\sqrt{\frac{\lambda}{4n(2-\lambda)}}$$

and plot EWMA$_{T_i}$.

The two parameters, k and λ, are chosen to satisfy certain required in-control ARL (ARL$_0$).

3.1. *In-control ARLs of the new EWMA chart*

We used the ARL to measure the performance of the proposed chart. Following Lucas and Saccucci[18], the ARLs of the new EWMA chart are evaluated by the Markov chain approach.

Table III lists the ARL$_0$ = 370.5 under various combinations of (n,p_0), for n=9(1)20 and p_0 = 0.25(0.05)0.75, with the combination (λ = 0.2, k=2.86) in the new EWMA chart.

3.2. *Out-of-control ARLs of the new EWMA chart*

The values of ARL$_1$ of the new EWMA chart are a function of (n, k, λ). Adopting the in-control process proportion p_0=0.613, ARL$_0$= 370.5, with λ=0.2 and k=2.86, the values of ARL$_1$ of the EWMA chart for n=9(1)20 and p_1 = 0.25(0.05)0.95 are listed in Table IV. The values of ARL$_1$ of the mean chart for n=9(1)20 and p_1 = 0.25(0.05)0.95 under p_0 = 0.613 are listed in Table V. Now the values

Table III. ARL_0 of the $EWMA_T$ chart (λ=0.2, k=2.86)

	p_0										
n	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
9	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
10	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
11	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
12	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
13	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
14	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
15	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
16	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
17	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
18	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
19	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5
20	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5	370.5

Table IV. ARL_1 of the EWMA chart (λ = 0.2, k=2.86, p_0=0.613)

	p_1								
n	0.25	0.35	0.45	0.55	0.613	0.65	0.75	0.85	0.95
9	6.0	10.5	25.9	132.0	370.5	153.8	15.1	3.4	1.5
10	5.6	9.6	23.4	123.0	370.5	146.8	13.3	3.1	1.4
11	5.2	8.9	21.3	115.1	370.5	140.3	11.8	2.8	1.4
12	4.9	8.2	19.6	108.1	370.5	134.3	10.6	2.6	1.4
13	4.7	7.3	17.0	101.8	370.5	128.8	9.6	2.4	1.3
14	4.4	7.3	17.0	96.2	370.5	123.6	8.7	2.3	1.3
15	4.2	6.9	15.9	91.1	370.5	118.8	8.0	2.1	1.2
16	4.1	6.6	15.0	86.5	370.5	114.3	7.4	2.0	1.2
17	3.9	6.3	14.1	82.4	370.5	106.1	6.4	1.9	1.1
18	3.8	6.0	13.4	78.5	370.5	102.4	6.0	1.8	1.1
19	3.6	5.8	12.8	75.1	370.5	99.0	6.0	1.8	1.1
20	3.5	5.6	12.2	71.8	370.5	96.1	5.4	1.7	1.1

Table V. ARL_1 of the mean chart under p_0 = 0.613

	p_1							
n	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
9	13.3	48.3	217.1	1321.6	12687.8	262144	26012295	5.12E+11
10	17.8	74.3	394.8	2936.8	36251.0	1048576	173415306	1.024E+13
11	23.7	114.3	717.8	6526.2	103574.2	4194304	1.156E+9	2.048E+14
12	6.3	23.6	120.6	925.7	12708.5	453438	1.117E+8	1.788E+13
13	7.9	33.8	203.9	1908.3	33628.0	1677722	6.882E+8	3.303E+14
14	3.6	11.9	58.8	464.9	7087.2	311410	1.141E+8	4947E+13
15	4.2	16.2	93.8	810.0	617.8	74.8	11.4	2.2
16	5.1	22.2	151.0	1574.6	963.5	99.8	13.5	2.3
17	2.8	9.7	54.2	525.1	1340.2	133.0	15.8	2.4
18	3.3	12.8	83.4	982.1	2150.8	177.4	18.6	2.5
19	2.1	6.7	35.7	361.3	2538.5	236.4	21.9	2.7
20	2.4	8.5	53.0	609.0	458.1	41.1	5.7	1.4

Table VI. The service times from 10 counters in a bank branch

m	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	$\widetilde{X}_j$	$\frac{M}{10}$	$EWMA_{T_i}$	$EWMA_{X\text{-bar}}$
1	0.88	0.78	5.06	5.45	2.93	6.11	11.59	1.20	0.89	3.21	3.81	0.2	0.63	5.38
2	3.82	13.4	5.16	3.20	32.27	3.68	3.14	1.58	2.72	7.71	7.67	0.3	0.62	5.84
3	1.40	3.89	10.88	30.85	0.54	8.40	5.10	2.63	9.17	3.94	7.68	0.4	0.63	6.21
4	16.8	8.77	8.36	3.55	7.76	1.81	1.11	5.91	8.26	7.19	6.95	0.7	0.71	6.36
5	0.24	9.57	0.66	1.15	2.34	0.57	8.94	5.54	11.69	6.58	4.73	0.4	0.70	6.03
6	4.21	8.73	11.44	2.89	19.49	1.20	8.01	6.19	7.48	0.07	6.97	0.6	0.74	6.22
7	15.08	7.43	4.31	6.14	10.37	2.33	1.97	1.08	4.27	14.08	6.71	0.5	0.74	6.32
8	13.89	0.30	3.21	11.32	9.90	4.39	10.5	1.70	10.74	1.46	6.74	0.5	0.76	6.40
9	0.03	12.76	2.41	7.41	1.67	3.70	4.31	2.45	3.57	3.33	4.16	0.2	0.70	5.95
10	12.89	17.96	2.78	3.21	1.12	12.61	4.23	6.18	2.33	6.92	7.02	0.5	0.71	6.16
11	7.71	1.05	1.11	0.22	3.53	0.81	0.41	3.73	0.08	2.55	2.12	0.1	0.64	5.35
12	5.81	6.29	3.46	2.66	4.02	10.95	1.59	5.58	0.55	4.10	4.50	0.3	0.62	4.70
13	2.89	1.61	1.30	2.58	18.65	10.77	18.23	3.13	3.38	6.34	6.89	0.4	0.64	4.66
14	1.36	1.92	0.12	11.08	8.85	3.99	4.32	1.71	1.77	1.94	3.71	0.2	0.60	5.11
15	21.52	0.63	8.54	3.37	6.94	3.44	3.37	6.37	1.28	12.83	6.83	0.5	0.64	5.45
											$\bar{\bar{x}} = 5.77$	$\bar{p} = 0.39$		

of ARL_1 behave normally; that is, it changes inversely with n. As shown in Table V, the values of ARL_1 in the EWMA chart are smaller than those in the mean chart. The results suggest that the detection ability of the EWMA chart is better than the mean chart. Hence, we would recommend the EWMA chart if we were concerned with the proper values of ARL_0.

4. When population parameters are unknown

When the in-control process mean, μ, and hence the in-control process proportion, p_0, are unknown, we would use the following preliminary sample data

$$X_{i1}, X_{i2}, \dots, X_{in}, \ i = 1, 2, \dots, m,$$

from m sampling periods, each with n observations, to estimate them, that is,

$$\hat{\mu} = \bar{\bar{X}} = \frac{\sum_{i=1}^{m} \bar{X}_i}{m}; \quad \bar{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}, \ i = 1, 2, \dots, m,$$

and

$$\hat{p} = \bar{p} = \frac{\sum_{i=1}^{m} M_i/n}{m}$$

The mean chart and the EWMA chart are thus constructed. The M_i and the $EWMA_{T_i}$ for the m samples are plotted on the resulting mean chart and EWMA chart, respectively. If all points fell inside the control limits, then we conclude that the process seemed to be in control.

4.1. Example

The service time is an important quality characteristic for a bank branch in Taiwan. To measure the efficiency in the service system of a bank branch, the sampling service times (in minutes) are measured from 10 counters every 2 days for 30 days; that is, 15 samples of size n=10. These data have been analyzed and have a right-skewed distribution, as shown in Table VI.

Here, sample size=10, number of samples=15, $\bar{\bar{x}} = 5.77$, M_i=sum of positive differences (X_j–5.77), i=1, 2,. . ., 15, $\bar{p} = \sum_{i=1}^{15} \frac{M_i}{10} / 15 = 0.39$, adopting ARL_0=370.5.0 with λ=0.2 and k=2.86 based on Table III. The EWMA chart is shown in Figure 1 (UCL=0.85, CL=0.66, and LCL=0.47). There is no out-of-control signals, so the process seems to be in control.

For comparison, the corresponding X-bar chart, the $EWMA_{X\text{-bar}}$ chart, and the transformed X-bar chart by applying $X^{0.25}$ transformation because X is a right-skewed distribution (follows an exponential distribution) (see, e.g. Montgomery[19]), which requires normality, were constructed and plotted in Figures 2–4. Both the X-bar chart and the $EWMA_{X\text{-bar}}$ chart (Figures 2 and 3) gave the same conclusion, so did the mean chart (Figure 5). However, the transformed X-bar chart gave a false alarm (sample 11) after searching the cause.

To show the detection ability of the proposed EWMA chart for the data set of service times from a new automatic service system of the bank branch, 10 new samples of size 10, samples 16–25, were collected and listed in Table VII.

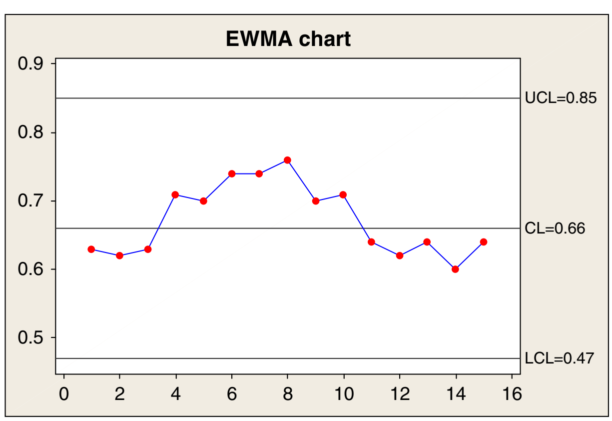

Figure 1. The EWMA chart

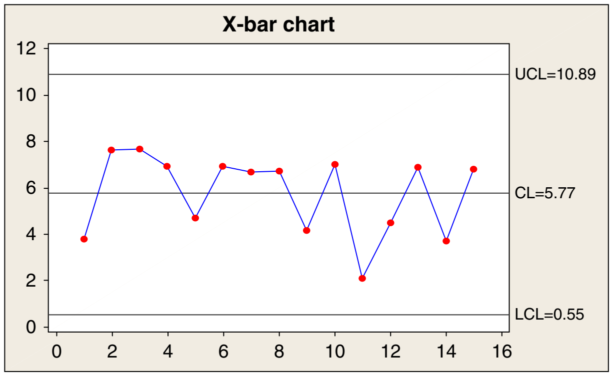

Figure 2. The *X*-bar chart

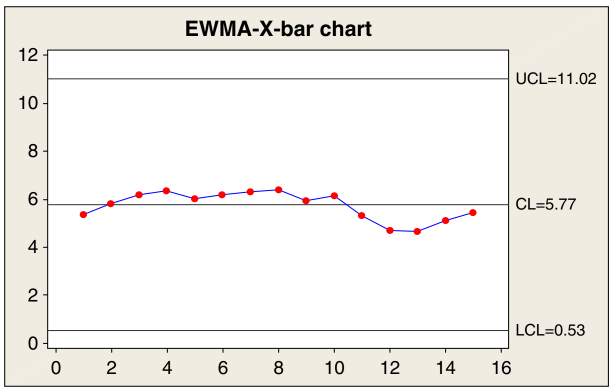

Figure 3. The EWMA$_{X\text{-bar}}$ chart

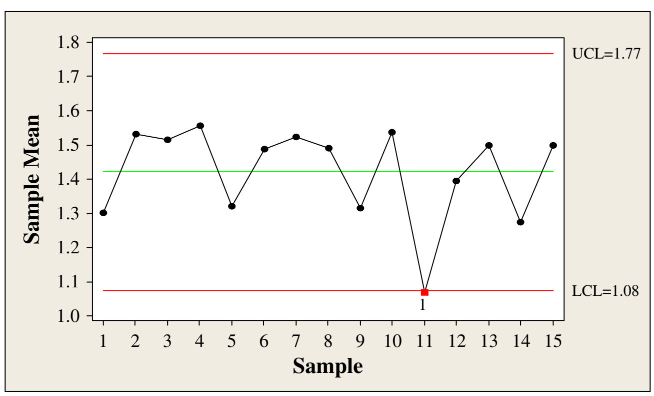

Figure 4. The transformed *X*-bar chart

The plots of samples 17 to 25 fell below the LCL of the EWMA$_T$ chart (see Figure 6). It signaled that the process was not in control. That is, the service times are significantly reduced because of the improved new automatic service system. However, the corresponding *X*-bar chart and EWMA$_{X\text{-bar}}$ chart showed no signals (see Figures 7 and 8). The transformed *X*-bar chart detected signals at samples 17 and 18 (see Figure 9). The proposed EWMA$_T$ chart detects the small shifts of the process mean quickly and effectively than those of the *X*-bar chart, the EWMA$_{X\text{-bar}}$ chart, and the transformed *X*-bar chart. The mean chart in Figure 10 also detected signals at samples 17, 18, 20, 22, 23, and 24.

To construct the *X*-bar chart, the EWMA *X*-bar chart and the transformed *X*-bar chart require the normality assumption but not the mean chart and the arcsine transformed EWMA chart. In this example, neither *X*-bar chart nor EWMA *X*-bar chart detected out-of-control signals, and the transformed *X*-bar chart detected only two out-of-control signals. However, the proposed simple charts—the mean chart and the transformed EWMA chart—did, as it should.

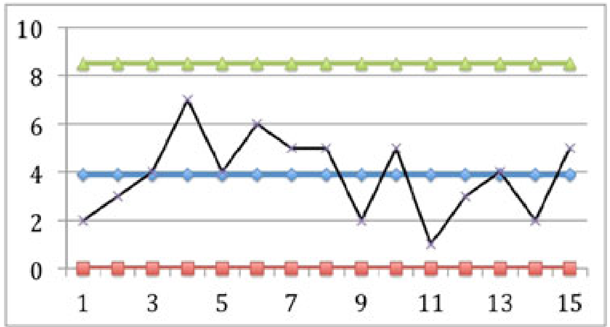

Figure 5. The mean chart

Table VII. The new service times from 10 counters in a bank branch

m	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	$\bar{X}_j$	$M/10$	$EWMA_{T_i}$	$EWMA_{X\text{-bar}}$
16	3.54	0.01	1.33	7.27	5.52	0.09	1.84	1.04	2.91	0.63	2.28	0.1	0.58	4.82
17	0.86	1.61	1.15	0.96	0.54	3.05	4.11	0.63	2.37	0.05	2.03	0.0	0.46	4.26
18	1.45	0.19	4.18	0.18	0.02	0.70	0.80	0.97	3.60	2.94	1.80	0.0	0.37	3.77
19	1.37	0.14	1.54	1.58	0.45	6.01	4.59	1.74	3.92	4.82	2.73	0.1	0.36	3.56
20	3.00	2.46	0.06	1.80	3.25	2.13	2.22	1.37	2.13	0.25	1.79	0.0	0.29	3.21
21	1.59	3.88	0.39	0.54	1.58	1.70	0.68	1.25	6.83	0.31	2.48	0.1	0.29	3.06
22	5.01	1.85	3.10	1.00	0.09	1.16	2.69	2.79	1.84	2.62	2.28	0.0	0.24	2.90
23	4.96	0.55	1.43	4.12	4.06	1.42	1.43	0.86	0.67	0.13	2.38	0.0	0.19	2.80
24	1.08	0.65	0.91	0.88	2.02	2.88	1.76	2.87	1.97	0.62	1.76	0.0	0.15	2.59
25	4.56	0.44	5.61	2.79	1.73	2.46	0.53	1.73	7.02	2.13	3.21	0.1	0.18	2.72

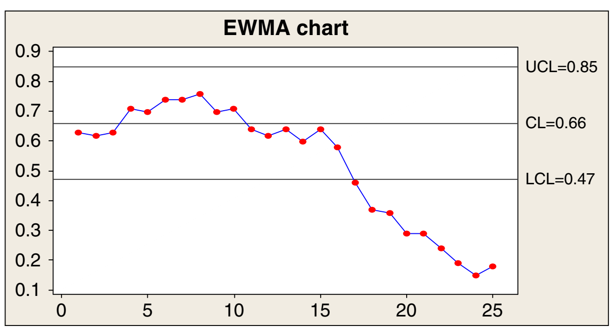

Figure 6. The new EWMA chart

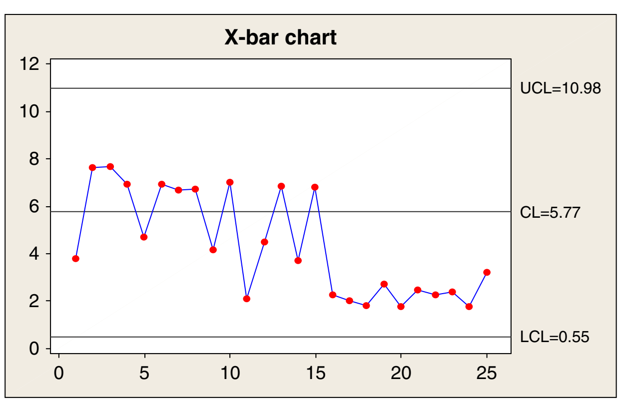

Figure 7. The *X*-bar chart

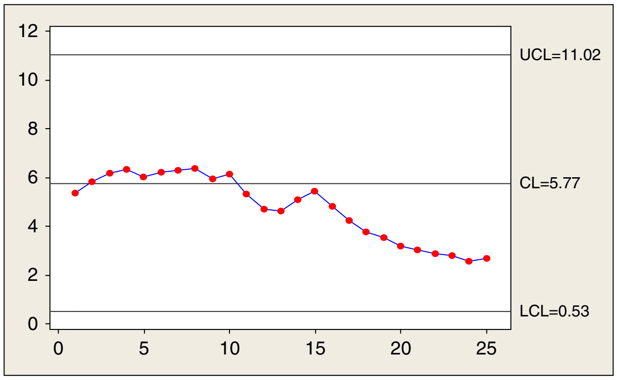

Figure 8. The EWMA$_{X\text{-bar}}$ chart

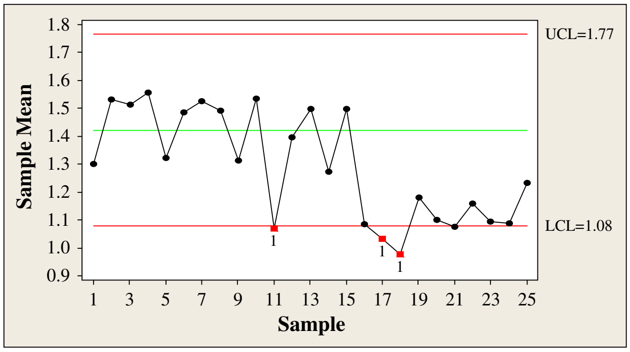

Figure 9. The transformed *X*-bar chart

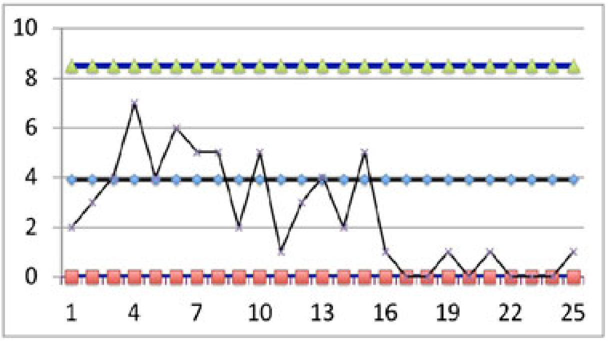

Figure 10. The new mean chart

5. Conclusion

In this article, we propose the mean chart on the basis of a simple statistic to monitor the mean shifts in the process. The sampling properties of the new monitoring statistic are explored and the ARLs of the proposed chart are calculated. Furthermore, a new EWMA chart is proposed because it provides more intuitive and reasonable in-control ARLs. A numerical example of service times with a skewed distribution from a bank branch is used to illustrate the application of the new EWMA chart, and its detection ability was compared with three existing charts. The new EWMA chart showed better detection ability than those three charts. The new EWMA chart is thus recommended.

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