# Mitigating the systematic errors of e-GPS leveling

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> The test results show that the proposed method can mitigate the systematic errors of orthometric height from e-GPS leveling efficiently. In the last issue, we published the first part of the paper. We present here the concluding part

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## Test results and discussion

### Tainan e-GPS System

Tainan City Government has established its e-GPS system in September 2007. The e-GPS system contains 6 reference stations, and covers the whole city. Five reference stations, SCES, NJES, RFES, WHES, BKBL, evenly distributed in Tainan city's borders, forming a nearly regular pentagon network; and the approximate geographic center in Tainan City setting of the sixth reference station, KAWN, its location just in the pentagonal-shaped center. And, it makes all the distances between the reference stations less than 30 km. In order to improve the accuracy and effi ciency of e-GPS surveying in the mountain area, the seventh reference station, YJLO, was installed in April 2010. Hence, the Tainan e-GPS system has 7 reference stations since then. All reference stations are equipped with Trimble NetR5, and the mobile stations are equipped with Trimble R8. Both types of receivers, Trimble NetR5 and Trimble R8, can track signals from GPS satellites and GLONASS satellites. The distribution map of 7 reference stations of Tainan e-GPS system is shown in Figure 1.

Tainan e-GPS system, through the fi eld testing, achieving the following accuracies: ±2cm in plane coordinates (x, y), and ±5cm in ellipsoidal height h, its accuracy is sufficient to be applied to the cadastral surveying, engineering surveying, etc. (Tainan, 2012).


Figure 1: The distribution map of 7 reference stations of Tainan e-GPS system.

### Test data

Three data sets of Tainan area (with total area of about 2,192 square kilometers or 219,200 hectares) are used to test the proposed methods. The data sets including: (1) data set 1 of 145 fi rst-order benchmarks, with orthometric height H from fi rstorder leveling and plane coordinates (x, y) and ellipsoidal height h from static GPS surveying of 2003, provided by the Ministry of the Interior, Republic of China; (2) data set 2 of 145 fi rst-order benchmarks, with orthometric height H only from fi rst-order leveling of 2009, provided by the Ministry of the Interior, Republic of China; (3) data set 3 of 118 fi rst-order benchmarks, with plane coordinates (x, y) and ellipsoidal height h from Tainan e-GPS system of 2011, provided by Tainan City Government.

**Test results and discussion**

**Accuracy Analysis of e-GPS Leveling:**

The following procedures are performed to evaluate the accuracy of e-GPS leveling:

(1) Train a 2 x p, x 1BP neural network

(2) represents the input layer has two elements, plane coordinates (x, y) of each benchmark; $p_1$ denotes the number of neurons in the hidden layer; 1 represents the output layer has 1 element, undulation N of each benchmark), in order to construct a regional geoid model of Tainan City, with 145 first-order benchmarks of data set 1; (2) Estimate undulation of all 118 first-order benchmarks of data set 3, using the trained 2 x $p_1$ x 1 BP neural network and the plane coordinates (x, y) of each benchmark; (3) Calculate the orthometric height , using the formula of $\hat{H} = h - \hat{N}$ , with the ellipsoidal height h from e-GPS system and the estimated $\hat{N}$ undulation from the above procedure, of all 118 first-order benchmarks of data set 3; (4) Compute the height difference $\Delta H,$ , using the formula of $\Delta H = H - \hat{H}$ (H denotes the orthometric height from data set 2, and $\hat{H}$: represents the estimated orthometric height from procedure 3), of all 118 first-order benchmarks of data set 3.

Table 1. The statistics of $\Delta N$ of 36 check points of data set 1, using a geoid model from $2 \times 35 \times 1$ BP neural network

| m (m) | σ (m) | Mean (m) | Maximum (m) | Minimum (m) |
|---|---|---|---|---|
| ±0.029 | ±0.028 | 0.009 | 0.089 | -0.054 |

Table 2. The statistics of $\Delta H$ of 118 first-order benchmarks of Tainan City

| m (m) | σ (m) | Mean (m) | Maximum (m) | Minimum (m) |
|---|---|---|---|---|
| ±0.072 | ±0.050 | -0.051 | 0.061 | -0.213 |

According to the preceding procedure 1, in order to construct a regional geoid model of Tainan City with BP neural network, 145 fi rst-order benchmarks of data set 1 are divided into two groups, one group as the reference point (109 points) to train a BP neural network; another group as a check point (36 points) to assess the accuracy of the regional geoid model.

Since orthometric height H and ellipsoidal height h of each benchmark of data set 1 are known, the undulation N of each benchmark can be calculated using the formula N = h – H . And, it is assuming that N is the true value. Suppose further that the undulation of each benchmark estimated by the trained BP neural network is , then, the undulation difference $\Delta N$ of each benchmark, is defi ned by the following equation.

$$\Delta N_i = N_i - \hat{N}_i, i = 1,2,\cdots,n \qquad (17)$$

where i = 1,2,...,n stands for the sequential number of check points; n is the total number of check points.

After trial and error tests, it is found that a 2 x 35 x 1BP neural network can offer better regional geoid model accuracy (Lin, 2007; Lin, 2012). The statistics of $\Delta N$ of 36 check points of data set 1 are shown in Table 1. In Table 1, 'm (m)' indicates mean square error in units of meter; ' (m)' indicates standard deviation in units of meter; 'Mean (m)' indicates mean value in units of meter; 'Maximum (m)' indicates maximum value in units of meter; 'Minimum (m)' indicates minimum value in units of meter.

Based on the previously mentioned procedures 2 to 4, compute the height difference $\Delta H_i$ of all 118 first-order benchmarks of data set 3. The statistics of $\Delta H_i$ of all 118 first-order benchmarks are shown in Table 2. It can be seen from the results of Table 2 that the standard deviation of $\Delta H_i$ is ± 0.050m.

The accuracy of h from e-GPS system is ± 0.050m (Tainan, 2012). Besides, the accuracy of estimated undulation $\hat{N}$ is ± 0.028m, according to the results of Table 1. Based on the formula $\hat{H} = h - \hat{N}$ and according to the principle of error propagation, the accuracy $\hat{H}$: of from e-GPS leveling is ±0.057m.

By definition of $\Delta H = H - \hat{H}$, where the accuracy of H is ±0.009m (Yang et al., 2003); the accuracy of $\hat{H}$: is ±0.057m. According to the principle of error propagation, the accuracy of $\Delta H_i$ from e-GPS leveling is ±0.058m. Therefore, further examining the results of Table 2, it is found that (1) the standard deviation and mean square error of ΔH varies considerably (0.022m), and (2) the mean value of $\Delta H_i$ is -0.051m (not 0.000m). Therefore, judging the test results of the e-GPS leveling, it may still have some systematic errors to be corrected.

**Test results of proposed methods:**

$P = \{P_1, P_2, \cdots, P_n\}$ and $Q = \{Q_1, Q_2, \cdots, Q_n$ and data of 118 first-order benchmarks of data set 3, will be used to test the three proposed methods. The number of reference points , check points and validation point n of data set 3 are 89, 29, and 118 respectively.

**Test results of CFM**

Based on the above-mentioned procedures of CFM, data of 118 fi rstorder benchmarks are used to test the performances of 4-parameter, 6-parameter, and 10-parameter CFM. The statistics of ΔH of 118 firstorder benchmarks, before and after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM, are shown in Table 3. In Table 3,

$\Delta H_i$ (N/A)denotes the value of ΔH before correcting systematic errors;ΔH(4-par) , ΔH(6-par) , ΔH(10-par)and denote the value of ΔH after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM respectively.

Can be seen from the results in Table 3, after correcting systematic errors estimated by 4-parameter CFM, the standard deviation of $\Delta\tilde{H}$ decreased ±0.037m (close to the mean square error value), and the mean of $\Delta\tilde{H}$ dropped to 0.000m; after correcting systematic errors estimated by 6-parameter CFM, the standard deviation of $\Delta\tilde{H}$ decreased ±0.034m (close to mean square error ), and the mean of dropped to 0.000m; after correcting systematic errors estimated by 10-parameter CFM, the standard deviation of $\Delta\tilde{H}$ decreased ±0.028m (With mean square error differ by ± 0.002m), and the mean of $\Delta\tilde{H}$ dropped to- 0.011m.

**Test results of BP&BP**

Based on the specific procedures of BP&BP, systematic errors of e-GPS leveling, $\delta\hat{H}$ and $\delta\hat{\hat{H}}$, should be estimated by 2 x $p_1$ x 1 and 2 x $p_2$ x 1 BP neural networks respectively. In order to determine the number of neurons p1 and p2 using trial and error method, $P=\{P_1,P_2,\cdots,P_n\}$ and $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first-order benchmarks are used to train a 2 x $p_1$ x 1BP neural networks respectively. In order to demonstrate the procedures of determining the number of neurons p1 with trial and error method, the statistics of $\sigma_{set}$, $\sigma_{chk}$ and $\sigma_{val}$ of $\delta\hat{H}$ of 118 first-order benchmarks, after changing the number of neurons ($p_1$=1,2,...,15) in the hidden layer of BP neural network, are shown in Table 4. It can be seen from Table 4 that when the number of neurons $p_1$ is 8 the results are best. Hence, a 2 x 8 x 1BP neural network will be used to estimate values of $\delta\hat{H}$ , $\tilde{H}$ , $\Delta\tilde{H}$ of 118 first-order benchmarks.

Then, the number of neurons p2 of 2 x $p_2$ x 1BP neural network is determined, using trial and error method, with $P=\{P_1,P_2,\cdots,P_n\}$ and $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first order benchmarks. It is found that the number of neurons $p_2$ is 5 the results are best. Hence, a 2X5X1BP neural network will be used to estimate values of $\delta\hat{\hat{H}}$ , $\tilde{\tilde{H}}$ , and $\Delta\tilde{\tilde{H}}$ of 118 first-order benchmarks. The statistics of $\sigma_{set}$, $\sigma_{chk}$ and $\sigma_{val}$ of $\delta\hat{\hat{H}}$ of of 118 first-order benchmarks, 2 x 5 x 1 using a BP neural network are shown in Table 5.

Table 6 shows that the statistics of $\Delta H$ of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&BP. In Table 6, $\Delta H(N/A)$ denotes the value of $\Delta H$ before correcting systematic errors; $\Delta\tilde{H}(BP1-2\times8\times1)$ and $\Delta\tilde{H}(BP2-2\times5\times1)$ denote values of after correcting systematic errors estimated 2x 8 x 1 by BP neural network and 2X5X1BP neural network and 2 x 5 x 1 respectively. Be seen from the results in Table 6, after correcting systematic errors estimated by a 2 x 8 x 1BP neural network, the standard deviation of $\Delta\tilde{H}$ decreases to ±0.030m (difference between the standard deviation and the mean square error is ±0.014m), the mean of $\Delta\tilde{H}$ declines to 0.032m. After correcting systematic errors estimated by a 2 x 5 x 1BP neural network, the standard deviation of $\Delta\tilde{H}$ decreases to ±0.029m (equal to the mean square error), and the mean of $\Delta\tilde{H}$ declines to -0.007 m.

Table 3. The statistics of $\Delta H$ of 118 first-order benchmarks, before and after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM

| $\Delta H$ | m (m) | σ (m) | Mean (m) | Maximum (m) | Minimum (m) |
|---|---|---|---|---|---|
| $\Delta H$(N/A) | ±0.072 | ±0.050 | -0.051 | 0.061 | -0.213 |
| $\Delta\tilde{H}$ (4–par) | ±0.036 | ±0.037 | 0.000 | 0.091 | -0.136 |
| $\Delta\tilde{H}$ (6–par) | ±0.034 | ±0.034 | 0.000 | 0.070 | -0.122 |
| $\Delta\tilde{H}$ (10–par) | ±0.030 | ±0.028 | -0.011 | 0.098 | -0.087 |

Table 4. The statistics of $\sigma_{ref}$, $\sigma_{chk}$ and $\sigma_{val}$ of $\delta\hat{H}$ of 118 first-order benchmarks, after changing the number of neurons ($p_1$=1,2,...,15) in the hidden layer of 2x$p_1$x1 BP neural network

| $p_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\sigma_{ref}$(m) | 0.032 | 0.028 | 0.028 | 0.027 | 0.028 | 0.028 | 0.033 | 0.027 |
| $\sigma_{chk}$(m) | 0.039 | 0.034 | 0.033 | 0.034 | 0.034 | 0.036 | 0.041 | 0.032 |
| $\sigma_{val}$(m) | 0.035 | 0.032 | 0.030 | 0.030 | 0.031 | 0.032 | 0.038 | 0.029 |
| $p_1$ | 9 | 10 | 11 | 12 | 13 | 14 | 15 | |
| $\sigma_{ref}$(m) | 0.029 | 0.028 | 0.029 | 0.029 | 0.028 | 0.030 | 0.029 | |
| $\sigma_{chk}$(m) | 0.036 | 0.034 | 0.033 | 0.033 | 0.033 | 0.033 | 0.034 | |
| $\sigma_{val}$(m) | 0.033 | 0.030 | 0.030 | 0.031 | 0.031 | 0.030 | 0.031 | |

Table 5. The statistics of $\sigma_{ref}$, $\sigma_{chk}$ and $\sigma_{val}$ of $\delta\hat{\hat{H}}$ of 118 first-order benchmarks, using a 2x5x1BP neural network

| $\sigma_{ref}$(m) | $\sigma_{chk}$(m) | $\sigma_{val}$(m) |
|---|---|---|
| 0.029 | 0.032 | 0.029 |

Table 6. The statistics of $\Delta H$ of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&BP

| $\Delta H$ | m (m) | σ (m) | Mean (m) | Maximum (m) | Minimum (m) |
|---|---|---|---|---|---|
| $\Delta H$(N/A) | ±0.072 | ±0.050 | -0.051 | 0.061 | -0.213 |
| $\Delta\tilde{H}$(BP1–2x8x1) | ±0.044 | ±0.030 | 0.032 | 0.119 | -0.063 |
| $\Delta\tilde{\tilde{H}}$(BP2–2x5x1) | ±0.029 | ±0.029 | -0.007 | 0.077 | -0.105 |

Table 7. The statistics of $\Delta H$ of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&CFM algorithm.

| $\Delta H$ | m (m) | σ (m) | Mean (m) | Maximum (m) | Minimum (m) |
|---|---|---|---|---|---|
| $\Delta H$(N/A) | ±0.072 | ±0.050 | -0.051 | 0.061 | -0.213 |
| $\Delta\tilde{H}$(BP1–2x2x1) | ±0.042 | ±0.031 | 0.029 | 0.119 | -0.079 |
| $\Delta\tilde{\tilde{H}}$(CFM2–6–par) | ±0.029 | ±0.029 | 0.000 | 0.066 | -0.105 |

## Test results of BP&CFM

Based on the specific procedures of BP& CFM, systematic errors of e-GPS leveling, $\delta\hat{H}$ and $\delta\hat{\hat{H}}$, should be estimated by a 2 x $p_1$ X 1BP neural network and a 6-parameter CFM respectively. In order to determine the number of neurons $p_1$ using trial and error method, $P=\{P_1,P_2,\cdots,P_n\}$ data of 118 first-order benchmarks are used to train a 2 x $p_1$ 1BP neural network. It is found that the number of neurons $p_1$ is 2 the results are best. Hence, a 2 x 2 x 1BP neural network will be used to estimate values of $\delta\hat{H}$, $\tilde{H}$, $\Delta\tilde{H}$ of 118 first-order benchmarks. Then, 6 parameters of CFM are estimated, using least squares method, with $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first order benchmarks. Finally, values of $\delta\hat{\hat{H}}$, $\tilde{\tilde{H}}$ and $\Delta\tilde{\tilde{H}}$ of 118 first-order benchmarks are estimated by a 6-parameter CFM.

Table 7 shows statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&CFM. In

Table 7, ΔH(N/A) denotes the value of ΔH before correcting systematic errors; $\Delta\tilde{H}$ (BP1-2X2X1) and denote values of $\Delta\tilde{\tilde{H}}$ (CFM2-6-par) after correcting systematic errors estimated by 2 x 2 x 1BP neural network and 6-parameter CFM respectively.

It can be seen from the results in Table 7, after correcting systematic errors estimated by a 2 x 2 x 1BP neural network, the standard deviation of $\Delta\tilde{H}$ decreases to ±0.031m (difference between the standard deviation and the mean square error is ±0.011m), the mean of $\Delta\tilde{\tilde{H}}$ declines to 0.029m. Then, after correcting systematic errors estimated by a 6-parameter CFM, the standard deviation of decreases to ±0.029m (equal to the mean square error), and the mean of $\Delta\tilde{\tilde{H}}$ declines to 0.000 m.

Table 8. The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by CFM, BP&BP, and BP&CFM respectively

| ΔH | m (m) | σ (m) | Mean (m) | Maximum (m) | Minimum (m) |
|---|---|---|---|---|---|
| N/A | ±0.072 | ±0.050 | -0.051 | 0.061 | -0.213 |
| 4-CFM | ±0.036 | ±0.037 | 0.000 | 0.091 | -0.136 |
| 6-CFM | ±0.034 | ±0.034 | 0.000 | 0.070 | -0.122 |
| 10-CFM | ±0.030 | ±0.029 | -0.011 | 0.098 | -0.087 |
| BP&BP | ±0.029 | ±0.029 | -0.007 | 0.077 | -0.105 |
| **BP&CFM** | **±0.029** | **±0.029** | **0.000** | **0.066** | **-0.105** |

### Summary

The statistics of ΔH of 118 fi rst-order benchmarks, before and after correcting systematic errors estimated by CFM, BP&BP, and BP&CFM respectively, are summarized and shown in Table 8. In Table 8, N/A stands for the value of ΔH without any systematic error correction; 4-CFM, 6-CFM and 10-CFM denote values of $\Delta\tilde{H}$ after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM respectively; BP&BP denotes the value of $\Delta\tilde{\tilde{H}}$ after correcting systematic errors estimated by a 2 x 2 x 1BP neural network and a 2 x 5 x 1BP neural network respectively; BP&CFM indicates the value of $\Delta\tilde{\tilde{H}}$ after correcting systematic errors estimated by a 2 x 2 x 1BP neural network and 6-parameter CFM respectively.

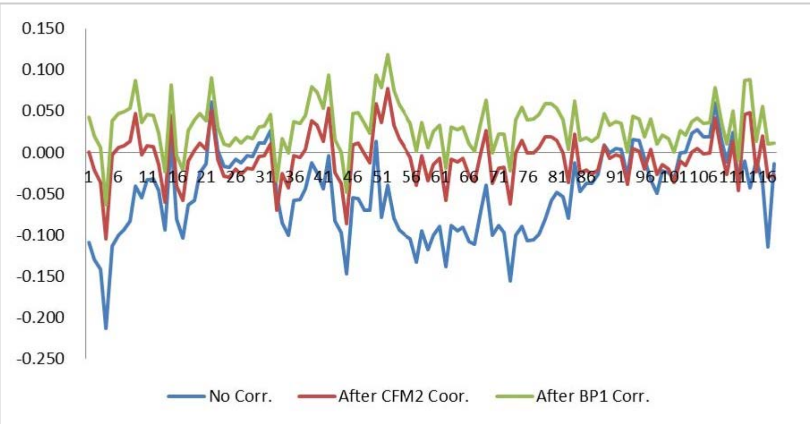


Figure 2. The ΔH comparison charts of 118 fi rst-order benchmarks, before and after correcting systematic error estimated by BP&CFM.

Can be seen from the results of Table 8, after systematic error correction estimated by CFM, BP&BP, and BP&CFM, the standard deviation of ΔH can be reduced considerably. Among them, the performances of BP&CFM and BP&BP are the best. In terms of reduced the mean of ΔH, BP&CFM, 4-CFM and 6-CFM perform the best. Then, it is checked that whether the mean square error of ΔH is equal to the standard deviation of ΔH or not? It is found that

BP&CFM, BP&BP and 6-CFM meet the requirements. Therefore, on three aspects into consideration, i.e. (1) Is the standard deviation of ΔH the smallest? (2) Is the standard deviation of ΔH equal to the mean square error of ? (3) Is the mean of ΔH is equal to 0.000m? It is found that the performance of BP & CFM is the best, and followed by BP&BP. The ΔH comparison charts of 118 fi rst-order benchmarks, before and after correcting systematic errors estimated by BP&CFM, are shown in Figure 2. In Figure 2, "No Corr." denotes the value of ΔH before correcting systematic errors; "After BP1 Corr." and "After CFM2 Corr." denote values of ΔH after correcting systematic errors estimated by 2X2X1BP neural network and 6-parameter CFM respectively; the vertical axis expresses the value of ΔH (m); and the horizontal axis stands for the sequential number of 118 fi rst-order benchmarks.

## Conclusions

Address the systematic errors of estimated orthometric height of e-GPS leveling, three methods, i.e. CFM, BP&BP, and BP&CFM, are proposed in this paper. Three data sets of Tainan City are used to test the proposed methods. The test results show that, among the three methods, BP & CFM is the most effective way to mitigate the systematic errors of e-GPS leveling, followed BP&BP, and 6-parameter CFM. Using BP & CFM algorithm, for example, the standard deviation of is reduced to ±0.029m from ±0.050m and the mean of ΔH is equal to 0.000m.

In this paper, it is found that the systematic errors of e-GPS leveling can be mitigated effectively if BP&CFM is applied, using the data sets from Tainan City. However, if the test area is increased, such as the southern region of Taiwan, and even extended to the entire island of Taiwan, the BP & CFM algorithm, is still valid? Remains to be further validated in the future.

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