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The Distributions of Policy Reserves Considering the Policy-Year Structures of Surrender Rates and Expense Ratios

Chenghsien Tsai
Weiyu Kuo
Derek Mi-Hsiu Chiang

Abstract

In this article, we examine how the policy-year structures of expense ratios and surrender rates affect the distributions of policy reserves. Our results show that a convex expense ratio curve, though reduces the mean and the uncertainty of reserves, could make the beneficial impact of surrenders on insurers become detrimental. Our results also show that the convexity of the surrender rate curve is favorable to insurers while the volatilities of surrender rates are unfavorable. We further find that neglecting the policy-year structures of surrender rates and expense ratios may result in overestimation of the mean and the uncertainty of reserves.

Introduction

Policy reserves are the largest liabilities of life insurers. Estimating policy reserves and quantifying the associated uncertainty require a proper estimation for the probability distribution of policy reserves that in turn entails an explicit modeling of uncertain future cash flows and their associated discount rates. The reserving methods of Bowers et al. (1997) considered a probabilistic future lifetime but assumed a constant discount rate. Panjer and Bellhouse (1980), Bellhouse and Panjer (1981), Giaccotto (1986), Beekman and Fuelling (1990, 1991, 1993), and De Schepper and Goovaerts (1992) incorporated stochastic interest rates into the reserving for individual policies.

Chenghsien Tsai is at the Department of Risk Management and Insurance, National Chengchi University. Weiyu Kuo is at the Department of International Business, National Chengchi University. Derek Mi-Hsiu Chiang is at the Department of Money and Banking, National Chengchi University. Chenghsien Tsai can be contacted via e-mail: ctsai@nccu.edu.tw. The authors are grateful to two anonymous referees for their valuable comments, to Stephen P. D’Arcy and Richard MacMinn for their helpful suggestions, to Zn-Hong Chen and Fang-Shu Chan for their programming assistance, to the National Science Council of Taiwan for its financial support (project numbers NSC 96-2918-I-004-006 and NSC 96-2416-H-004-026-MY3), and to Georgia State University (GSU) as well as Santa Clara University (SCU) for the kind support during the visit of the corresponding author. A portion of the research was completed while the author was visiting GSU and SCU.

Further extensions that considered a pool of policies can be found in Frees (1990), Parker (1994a, 1996), and Marceau and Gaillardetz (1999). In addition, Tsai, Kuo, and Chen (2002) considered the reserving for a pool of policies and introduced surrender rate, a risk factor that captures the sensitivity of the cash flows of life insurance policies to interest rates.

The results of Tsai, Kuo, and Chen (2002), albeit insightful, contradict the actual practice in the life insurance industry. They show that surrender options are beneficial to life insurers since higher surrender rates reduce both the mean reserves and the uncertainty of policy reserves. In reality, the life insurance industry has endeavored to restrain surrender rates in most times, and life insurers are often encouraged to keep surrender rates low (cf. American Council of Life Insurers [ACLI], 2000, p. 11).

We conjecture that the contradiction is due to the omission of the policy-year structures of expense ratios and surrender rates when estimating policy reserves. Expense ratio, defined as the ratio of expenses paid to the premiums received by insurers, can be seen to decrease with the policy year. As a consequence of initial commissions and fixed costs being incurred, the expense ratio achieves its maximum when a policy commences and recedes thereafter as the policy year progresses. We describe this pattern of expense ratios as "the convex policy-year structure of expense ratios" or as "the convex expense ratio curve." Surrender rates exhibit a convex policy-year structure as well. The voluntary termination rates reported in the *Life Insurers Fact Book* by the ACLI (2000) show that the termination rates of policies in force less than 2 years are much higher than the rates of policies in force 2 years or more. Specifically, the average of the former rates from 1965 to 2000 is 18.2 percent while the average of the latter is 5.4 percent. As suggested by the asset-share calculation example in Black and Skipper (2000, pp. 779-786), a convex expense ratio curve, when coupled with a convex surrender rate curve, is harmful to insurers when surrender rates are higher than expected. We therefore speculate that incorporating both policy-year structures into reserving may reconcile the results of Tsai, Kuo, and Chen (2002) with the actual practice in the life insurance industry.

To understand how the policy-year structure of expense ratios affects reserve distributions, we experiment with three expense ratio patterns: flat, moderately convex, and significantly convex. To analyze how surrender rate curves affect reserve distributions, we first specify the convexity and volatility pattern of the surrender rate curve based on empirical observations in Taiwan.[1] Then we experiment with alternative specifications to investigate how the convexity and volatility pattern affect reserve distributions, respectively and jointly. The final part of our analysis considers both policy-year structures simultaneously.

We find that a convex expense ratio curve reduces the exposures of reserves to interest rate fluctuations. Moreover, we find that the convexity of the expense ratio curve determines how surrenders affect reserve distributions. Surrenders are beneficial to life insurers when the expense ratio curve is flat, yet they increase the mean reserves when the curve is convex. The damaging effect of surrenders to life insurers increases

[1] What we mean by "convexity" in this article is the convex curvature of a curve. The volatility pattern is used to describe the shifts, especially the nonparallel shifts, of a curve.

with the convexity of the expense ratio curve and usually outweighs the benefits of surrenders identified in Tsai, Kuo, and Chen (2002).

With regard to the policy-year structure of surrender rates, we find that the convexity of the surrender rate curve benefits life insurers. This is due to the interest rate sensitivity of surrender rates being diminished. The volatilities of surrender rates are, on the other hand, unfavorable to life insurers. This is because they amplify the interest rate sensitivity of surrender rates.

Our results on the joint effect of both policy-year structures on reserve distributions indicate that neglecting these policy-year structures may result in overestimating the mean and the uncertainty of reserves. The results of Tsai, Kuo, and Chen (2002) imply the demand for high capital-to-premium ratios to maintain an acceptable level of solvency probability. In contrast, our results are more in line with the high leverage ratio prevalent in the life insurance industry and the long-run solvency record of the industry. Our results also confirm that higher than expected surrender rates are damaging to insurers, which justifies the common insurance industry practice in minimizing surrender rates.

The remainder of this article is organized as follows. The section on "Simulation Settings" outlines the basic settings of Monte Carlo simulations to generate reserve distributions under the consideration of the two policy-year structures. In the next section "The Policy-Year Structures of Expense Ratios" and the subsequent section "Consideration for the Policy-Year Structures of Surrender Rates," we analyze how the policy-year structures of expense ratios and surrender rates affect reserve distributions, respectively. The joint effect of both policy-year structures on reserve distributions is analyzed in next section. The last section concludes our findings.

SIMULATION SETTINGS

The policy pool considered consists of 100,000 20-year endowment policies that are issued to 30-year-old males and are subject to two causes of decrement: death and surrender. We assume that the policy pays \$1,000 at the end of the death year or the 20th year. For each policy the termination probabilities due to deaths during the age interval of x and $x + 1$ are denoted by $q_x^{(m)}$, where x is a positive integer and $30 \leq x < 50$. Assuming that $q_x^{(m)}$ has the same distribution as the 1980 CSO male mortality table and premiums are received at the beginning of the year, we obtain a net level premium of \$27.133 at the fixed pricing rate of 6 percent.

Denote the surrender rate at time t ($t \in Z$) for policies in the ith policy year as $SR_t(i)$. If policyowners surrender their policies during the ith policy year, they receive S_i at the end of that year, where S_i equals ${}_iV_{30}$ that is the benefit reserve calculated according to the method described in Bowers et al. (1997).

Let $D_x^{(m)}$ denote the number of lives who leave the group between ages x and $x + 1$ for the death decrement, $D_i^{(s)}$ denote the number of lives who leave the group during the ith policy year due to surrenders, and $C^{(\tau)}(x)$ denote the number of survivors at age x out of the original 100,000 lives.[2] The present value of the cash flows generated from this policy pool, denoted by L, then equals

$$\sum_{x=30}^{49}\left(1{,}000 \times D_x^{(m)} \times v_{x-30+1}\right) + \sum_{i=1}^{20}\left(S_i \times D_i^{(s)} \times v_i\right) + 1{,}000 \times C^{(\tau)}(50) \times v_{20}$$
$$- \left[\sum_{x=30}^{49} 27.133 \times (1 + \mathit{Loading}) \times (1 - \mathit{Exp}_{x-30+1}) \times C^{(\tau)}(x) \times v_{x-30}\right], \qquad (1)$$

where $\nu_{x-30} = \{ \begin{array}{ll} 1 & \text{if } x = 30, \\ \frac{1}{(1+r_1)(1+r_2)\cdots(1+r_{x-30})} & \text{if } 30 < x \leq 50', \end{array}$ r_{x-30} is the 1-year spot rate prevailing in policy year $x - 30$, *Loading* represents the charged loading rate as a percentage of the net level premium, and Exp_i is the expense ratio of policy year i.[3] The random variable L represents the present value of the liabilities associated with the policy pool that is of primary concerns to insurers. We will simulate the distribution of L based on the scenarios that we propose for surrender behaviors and expense patterns.

To speed up the simulations, we assume that $D_x^{(m)} = C^{(\tau)}(x) \times q_x^{(m)}$ and $D_i^{(s)} = [C^{(\tau)}(x) - D_x^{(m)}] \times SR_i(i)$.[4] This assumption leads to negligible difference from the results obtained under the alternative assumption that $D_x^{(m)}$ and $D_i^{(s)}$ are binomially distributed with parameters $(C^{(\tau)}(x), q_x^{(m)})$ and $(C^{(\tau)}(x) - D_x^{(m)}, SR_i(i))$ due to the large size of the pool and the independence among policyholders' decrements given decrement rates.

With regard to the specifications on Exp_i and $SR_t(i)$, we assume several patterns of Exp_i and alternative models for $SR_t(i)$ to analyze how they affect the distribution of L. We propose three patterns of expense ratio curves in the section on "The Policy-Year Structures of Expense Ratios" to investigate how the policy-year structures of expense ratios affect reserve distributions. To investigate the effects of surrender rate curves on reserve distributions, we make assumptions on the relations among $\Delta SR_t(i)$, the behavior of one policy-year surrender rate, and the shape of the initial surrender rate curve (i.e., $SR_0(i)$) to simulate arrays of $SR_t(i)$. In particular, we assume that $\Delta SR_t(i) = f(i) \times \Delta SR_t(4)$.[5] Then we assume that $\Delta SR_t(4)$ and Δr_t behave

[2] Note that the ith policy year and the age interval of x and $x + 1$ have a one-to-one correspondence relation: the ith policy year corresponds to the age interval of $30 + i - 1$ and $30 + i$.

[3] We assume that all expenses are paid at the beginning of the policy year.

[4] One of the referees pointed out that this assumption is indeed equivalent to assuming that $q_x'^{(m)}$ has the same distribution as the CSO male mortality table, that $SR_i'(i)$ follows the model assumed later in the text, and that mortality occur before withdrawal, where $q_x'^{(m)}$ and $SR_i'(i)$ are called the absolute rate of decrement in Bowers et al. (1997).

[5] Equivalent ideas of assuming proportional changes can be found in the finance literature such as Schaefer (1984) and the appendix of chapter 9 in Saunders and Cornett (2006) the appendix of chapter 9) where they assume the term structure to possess such behavior in their analysis. On the other hand, choosing the 4th policy year as the anchor point is arbitrary and

according to the vector autoregression model in Tsai, Kuo, and Chen (2002) (TKC) as follows:

$$\begin{bmatrix} \Delta SR_t(4) \\ \Delta r_t \end{bmatrix} = \begin{bmatrix} \underset{(-5.193)}{-0.243^{***}} \\ \underset{(-0.890)}{-0.199} \end{bmatrix} \begin{bmatrix} 1 & \underset{(-9.819)}{-1.053^{***}} & \underset{(-1.148)}{-0.008} \end{bmatrix} \begin{bmatrix} SR_{t-1}(4) \\ r_{t-1} \\ 1 \end{bmatrix} + \begin{bmatrix} \underset{(1.650)}{0.240} & \underset{(-0.881)}{-0.046} \\ \underset{(-0.210)}{-0.146} & \underset{(0.597)}{0.149} \end{bmatrix}$$

$$\times \begin{bmatrix} \Delta SR_{t-1}(4) \\ \Delta r_{t-1} \end{bmatrix} + \begin{bmatrix} \underset{(-0.094)}{-0.012} & \underset{(-2.934)}{-0.151^{***}} \\ \underset{(-1.037)}{-0.642} & \underset{(-2.085)}{-0.514^{*}} \end{bmatrix} \begin{bmatrix} \Delta SR_{t-2}(4) \\ \Delta r_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^L \\ \varepsilon_t^I \end{bmatrix}, \quad (2)$$

where $\mathbf{E} = [\,\varepsilon_t^{SR} \;\; \varepsilon_t^r\,]' \sim N(\mathbf{0}, \hat{\Sigma})$ and $\hat{\Sigma} = [\begin{smallmatrix} 7.28 \times 10^{-6} & 8.09 \times 10^{-6} \\ 8.09 \times 10^{-6} & 1.67 \times 10^{-4} \end{smallmatrix}]$.[6] With further assumptions on $f(i)$ and $SR_0(i)$, we will be able to simulate $SR_t(i)$ for $t \in N$ and $i = 1, 2, \ldots, 20$ given initial values of interest rates.[7]

The benchmark case where reserve distributions are estimated without considering the policy-year structures of expense ratios and surrender rates is the one that assume $Exp_i = 0$, $Loading = 0$, $f(i) = 1$, and the initial shape of the surrender rate curve being flat (i.e., $SR_0(i) = SR_0(j)$ for $i \neq j$). Given that $r_0 = r_{-1} = 6$ percent and $SR_0(i) = SR_{-1}(i) = 7$ percent, we simulate the reserve distribution estimated without considering both policy-year structures and report the summary statistics in Table 1.

THE POLICY-YEAR STRUCTURES OF EXPENSE RATIOS

In this section, we investigate how the policy-year structure of expense ratios affect reserve distributions by proposing three patterns of the expense ratio curve as shown in Table 2.[8] These patterns are designed to have an actuarially fair loading rate of 42.86 percent in an environment where a constant interest rate of 6 percent and surrender

different choices will give rise to different mean reserves and other statistics. The argument with respect to how the surrender rate curve affects reserve distributions, however, would remain invariant since the convexity and the volatility pattern of the curve are the determining factors as we will see from later analyses. We also experimented with choosing the 2nd and 7th year as the reference year, respectively, and the results that we obtained are consistent with those that are based on the 4th year as the reference year.

[6] We recognize the "abuse" of the TKC model since the model is based on the year average instead of a reference year. However, the literature provides no empirical models about the policy-year structure of surrender rates and the lack of adequate data prevents us from establishing one. We will make an alternative assumption about the behavior of $\Delta SR_t(4)$ in the section on "Robustness Checks" to immunize our underlying stories from the subjectiveness of any particular assumption on $\Delta SR_t(i)$.

[7] In this article, we use the 1-year spot rate and the interest rate interchangeably. Given the insignificant coefficient of the change in the 1-year spot rate of the cointegrating vector, $SR_{t-1}(4) - 1.053r_{t-1} - 0.008$, Equation System (2) implies an AR(2) process for the change in the interest rate.

[8] We report three expense ration patterns out of a total of five being experimented. The two retained patterns are similar to pattern 2, with one being more convex than pattern 2 and the other being less.

TABLE 1
Summary Statistics of the Policy Reserve Distributions Estimated Without Considering the Policy-Year Structures of Expense Ratios and Surrender Rates

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Case 1: The Interest Rate and Lapse Rate Are Generated by Equation System (2)						
1,086,994	−8,961	4,299,088	1.01	3.56	9,838,873	2.04
Case 2: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 0%						
1,515,129	28,158	9,425,174	0.51	2.48	18,914,225	1.85

TABLE 2
The Three Tested Expense Ratio Patterns

Policy Year	Expense Ratio Pattern 1	Expense Ratio Pattern 2	Expense Ratio Pattern 3
1	0.300000	0.561899	1.275151
2	0.300000	0.480313	0.618502
3	0.300000	0.410573	0.300000
4	0.300000	0.350958	0.145513
5	0.300000	0.300000	0.070580
6	0.300000	0.256441	0.034234
7	0.300000	0.219206	0.016605
8	0.300000	0.187378	0.008054
9	0.300000	0.160171	0.003907
10	0.300000	0.136915	0.001895
11	0.300000	0.117035	0.000919
12	0.300000	0.100042	0.000446
13	0.300000	0.085516	0.000216
14	0.300000	0.073099	0.000105
15	0.300000	0.062485	0.000051
16	0.300000	0.053413	0.000025
17	0.300000	0.045657	0.000012
18	0.300000	0.039028	0.000006
19	0.300000	0.033361	0.000003
20	0.300000	0.028517	0.000001

Note: Pattern 3 is called the most convex curve in the text because the expense ratio has the highest decreasing rate of 2.0617 ($= \frac{Exp_i}{Exp_{i+1}}$) across policy years.

rate of 7.2 percent are assumed. The 6 percent of interest rate is chosen so that it is consistent with the pricing rate of the net level premium, and the 7.2 percent selected for the surrender rate reflects the average termination rate in United States from 1959 to 1995. Throughout this section, the policy-year structure of surrender rates is assumed to be flat and shifts in a parallel fashion so that we can focus on the effects of expense ratio curve on reserve distributions.[9]

[9] In other words, we assume that $SR_0(i) = SR_0(j)$ for $i \neq j$ and that $f(i) = 1$ throughout this section.

TABLE 3
Summary Statistics of the Policy Reserve Distributions Under the Flat Policy-Year Structure of Expense Ratios

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Case 1: The Interest Rate and Lapse Rate Are Generated by Equation System (2)						
1,086,994	−8,961	4,299,088	1.01	3.56	9,838,873	2.04
Case 2: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 0%						
1,515,129	28,158	9,425,174	0.51	2.48	18,914,225	1.85
Case 3: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 7.2%						
549,451	58,911	3,796,097	0.45	2.49	7,545,066	1.84
Case 4: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Randomly Drawn From the Normal Distribution (0.072, 0.0225)						
551,926	53,602	3,800,941	0.46	2.55	7,505,415	1.83

The Flat Policy-Year Structure of Expense Ratios

Consider the policies with a loading rate of 42.86 percent and the first pattern of expense ratios in Table 2. If the 1-year spot rate and surrender rates behave according to Equation System (2), the mean reserve would increase from \$0 to \$1,086,994 as shown in Case 1 of Table 3.

The observed increase in the mean reserve is a combined effect of two forces.[10] First, changing the interest rate from constant to stochastic increases the mean from \$0 to \$549,451, as the third case of Table 3 indicates. The increase is due to the convexity of the present value function with respect to interest rates.[11] Second, the interest rate sensitivity of surrender rates further increases the mean by \$535,068 (= \$1,086,994 − \$551,926). This can be inferred from the resemblance between Case 3 and Case 4 and the difference between Case 4 and Case 1.[12] This detrimental effect of interest-rate-sensitive surrenders is consistent with the findings of Tsai, Kuo, and Chen (2002).[13]

The difference between Case 2 and Case 3 of Table 3 demonstrates that early surrenders are beneficial to life insurers when the expense ratio curve is flat.[14] Higher surrender rates reduce not only the mean of reserves but also the uncertainty of

[10] The explanation for the increase in the uncertainty of reserves follows the same decomposition of two forces.

[11] For a detailed discussion on this convexity effect, please refer to "Robustness Checks" of Tsai, Kuo, and Chen (2002).

[12] The resemblance between Case 3 and Case 4 indicates that the randomness of the surrender rate itself does not matter.

[13] Although the main finding of Tsai, Kuo, and Chen (2002) is that the aggregate value of the surrender options offered to policyholders is negative to insurers, the harmful effect of the interest rate sensitivity of surrender rates can be inferred from the difference between Figure 5 and Figure 7 of their paper.

[14] These two cases differ from each other in the surrender rate levels only. The surrender rate in Case 2 is fixed at 0 percent while the surrender rate level in Case 3 is set at 7.2 percent. We indeed have two other cases in hand to help deduce how reserve distributions change with surrender rate levels. These two cases have the same specifications as Cases 2 and 3 do but

reserves, for example, the standard deviation and 95th percentile of the reserve distribution. Surrenders are favorable to insurers because a higher surrender rate implies that policyowners on average leave the pool at an earlier stage, thus shortening the effective maturity of policies. A shortened maturity reduces a policy's exposure to interest rate fluctuations. The interest rate risk of reserves and the adverse impact of the present value function's convexity on the mean reserve are mitigated therefore.[15]

The fact that reserve distributions have positive means provides another ground for understanding the beneficial effect of surrenders on insurers observed from Case 2 and Case 3 in Table 3. Positive mean reserves indicate that the present value of the expected cash outflows exceeds that of the inflows and imply that these policies have negative expected net present values to insurers. The policyholders who choose to surrender their policies at the times when reserve distributions have positive means indeed relinquish their expected gains from insurers. Insurers benefit from these surrenders as a result, and higher surrender rates thus lead to more benefits. The above results and arguments are consistent with the cases in Tsai, Kuo, and Chen (2002) when the surrender option is shown to have a negative value.

Table 3 also illustrates that the reserve distribution obtained under a flat expense ratio curve with an actuarially fair loading rate is the same as the distribution obtained assuming neither loadings nor expenses. Case 1 and Case 2 of Table 3 have the same statistics as those of Table 1, respectively. This is because the loadings and expenses are netted out in each period.[16] Therefore, the cases in which the expense ratio curve is flat with a fair loading rate produce results that reconcile with the case of no expenses and loadings.

The Most Convex Policy-Year Structure of Expense Ratios

Here we consider the policies having the third pattern of expense ratios in Table 2 and a loading rate of 42.86 percent. When the interest rates and surrender rates behave in accordance with Equation System (2), the mean reserve is $815,929 as shown in Case 1 of Table 4.

Both the mean reserve and the interest rate risk of reserves are smaller than the ones in Case 1 of Table 3. This is because a convex expense ratio curve induces a natural hedging mechanism against the policy's exposure to interest rate fluctuations. When the interest rate is in line with the pricing rate, net premiums and loading charges would balance out the discounted benefit payments and expenses, respectively. Any discrepancy between the interest rate and the pricing rate shall introduce a deficiency or surplus in the net premiums and loading charges. When the expense

assume different surrender levels. One assumes that the surrender rate is fixed at 3 percent while the other fixes the surrender rate at 10 percent. The changing trends/directions of the reserve distributions in these cases enable us to infer the impacts of the surrender rate levels on the reserve distributions.

[15] Interest rate fluctuations have two harmful effects on insurers: increasing the mean of reserves (due to the convexity of the present value function) and increasing the uncertainty of reserves. The term "the interest rate risk of reserves" in this article refers to the latter case.

[16] More specifically, $(1 + \text{loading rate}) * (1 - \text{expense ratio}) = (1 + 0.4286) * (1 - 0.3) = 1$ in Equation (1).

TABLE 4
Summary Statistics of the Policy Reserve Distributions Under the Most Convex Policy-Year Structure of Expense Ratios

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Case 1: The Interest Rate and Lapse Rate Are Generated by Equation System (2)						
815,929	−85,602	3,178,986	1.04	3.62	7,265,546	2.03
Case 2: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 0%						
−3,448,981	−4,774,549	7,882,993	0.53	2.47	11,121,313	1.85
Case 3: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 7.2%						
485,247	33,600	3,149,051	0.48	2.47	6,253,339	1.83
Case 4: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Randomly Drawn From the Normal Distribution (0.072, 0.0225)						
493,037	49,353	3,157,494	0.45	2.49	6,239,733	1.82

ratio curve is convex and the mortality rate increases with age, the net premium deficiency/surplus will bear an opposite sign to the loading charge deficiency/surplus. For instance, decreasing the interest rate to a level below 6 percent will make the present value of the expected net premiums become lower than that of the expected benefit payments. Yet the decrease will make the loadings achieve a higher expected present value than the expenses. The "overcharged" loadings offset a portion of the loss from the "undercharged" net premiums. Therefore, a convex expense ratio curve makes the expenses/loadings a natural hedging pair against the exposure of the net premiums/benefit payments to interest rate fluctuations.

Despite of the aforementioned hedging benefit, a convex expense ratio curve turns the beneficial effect of surrenders on the mean reserve into a detrimental one. The difference between Case 2 and Case 3 of Table 4 demonstrates how higher surrender rates significantly increase mean reserves. This difference indicates that a 7.2 percent underestimation on the surrender rate would result in an increase of \$3,934,228 (= \$485,247 − \$−3,448,981) in the mean reserve. This implies over half a million dollar increase in the mean reserve for every 1 percent of underestimation on the surrender rate.

The mean reserve increases with the surrender rate for the following reason. During the early stages when the paid expenses are higher than the charged loadings, the surrendered policies cause losses of insurers. These losses are recoverable from the loadings charged on persisting policyowners if the surrender rate is equal to the rate assumed in pricing. Should the surrender rate be higher than the assumed, losses resulted from the surrendered policies will be higher than expected and the recoveries will be smaller in magnitude. The mean reserve therefore increases when the surrender rate is higher than the assumed. Similarly, we can deduce that the mean reserve decreases as the surrender rate gets lower. Surrenders therefore have damaging effect to insurers when the expense ratio curve is convex.

The considerable increases in the mean reserves suggest that the damage caused by higher surrender rates under the third expense ratio pattern outweighs the benefit

TABLE 5
Summary Statistics of the Policy Reserve Distributions Under a Moderately Convex Policy-Year Structure of Expense Ratios

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S. D.
Case 1: The Interest Rate and Lapse Rate Are Generated by Equation System (2)						
898,045	−43,041	3,533,242	1.04	3.60	8,048,880	2.02
Case 2: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 0%						
−1,493,452	−2,866,559	8,321,296	0.52	2.48	13,858,606	1.84
Case 3: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Fixed at 7.2%						
505,187	51,999	3,386,493	0.47	2.48	6,709,905	1.83
Case 4: The Interest Rate Is Generated by Equation (2) but the Lapse Rate Is Randomly Drawn From the Normal Distribution (0.072, 0.0225)						
510,647	51,587	3,394,095	0.46	2.53	6,738,363	1.83

of higher surrender rates identified in the section "The Flat Policy-Year Structure of Expense Ratios." We further observe that underestimating surrender rates causes more severe damages to insurers than does the interest rate sensitivity of surrenders. For instance, the increase of \$546,421 (= \$3,934,228/7.2) in the mean reserve caused by a mere 1 percent underestimation on the surrender rate is higher than the \$322,892 increase caused by the interest rate sensitivity of surrender rates. The convexity of the expense ratio curve is therefore a determining factor on how surrender options affect mean reserves.

On the other hand, we find that surrenders reduce the risk of reserves not only in the case of a flat expense ratio curve but also in the convex case.[17] By comparing the standard deviations and 95th percentiles in Cases 2 and 3 of Table 4, we see that the risk of reserves decreases with surrender rates. This is justifiable because the way in which surrenders shorten the effective maturities of policies and diminish the uncertainty of future liabilities is unaffected by the convexity of the expense ratio curve.

Robustness Checks

To establish the robustness of the above results, we conduct similar analyses based on the second pattern of expense ratios in Table 2. This expense ratio pattern exhibits the convexity that is between those of pattern 1 and pattern 3. As shown in Table 5, the results are consistent with our findings in the section "The Most Convex Policy-Year Structure of Expense Ratios." The mean, standard deviation, and the 95th percentile

[17] The primary risk of reserving analyzed in this article is the interest rate risk as defined in footnote 15. The relation between interest rates and surrender rates may aggravate or mitigate the interest rate risk. What we mean by "the risk of reserves" from now on is the "aggregate/net" interest rate risk after considering the surrender rate sensitivity to the interest rate. The magnitude of the risk is reflected by the size of the standard deviation and the 95th percentile of the reserve distribution.

of reserves in Case 1 of Table 5 are smaller than the ones in Case 1 of Table 3. These results confirm that a convex expense ratio curve, when coupled with a level loading rate, produces a hedging effect for the exposure of net premiums/benefit payments to interest rate fluctuations.

Comparing the mean reserves of Case 3 across Tables 3–5, we find that the mitigation effect of the hedging mechanism increases with the convexity of the expense ratio curve. In particular, Case 3 of Table 5 is more favorable to insurers than Case 3 of Table 3 yet less favorable than the corresponding case in Table 4. The beneficial effects of the convexity of the expense ratio curve are further confirmed.

Table 5 also confirms the damaging effect of early surrenders on mean reserves under a convex expense ratio curve. The comparison of Case 2 with Case 3 of Table 5 shows that higher surrender rates lead to higher mean reserves. An increase in the surrender rate could impair life insurers more severely than the interest rate sensitivity of surrender rates (e.g., \$505,187 − \$−1,493,452 = \$1,998,639 vs. \$898,045 − \$510,647 = \$387,398).

The changes in the mean reserve from Case 2 to Case 3 in Tables 3–5 further illustrate that the overall impact of early surrenders on mean reserves depends critically on the convexity of the expense ratio curve. In Table 3, surrenders are beneficial to life insurers. This is consistent with the findings of Tsai, Kuo, and Chen (2002). Yet as shown in Tables 4 and 5, surrenders become harmful to insurers and this harmful effect increases with the convexity of the expense ratio curve. These results imply that the beneficial effect of surrenders identified in Tsai, Kuo, and Chen can in fact be overshadowed by the harmful effect of surrenders under a convex expense ratio curve. This interesting implication is new to the existent literature. Therefore, the policy-year structure of expense ratios is an important factor in understanding the influence of early surrenders on policy reserves.

CONSIDERATIONS FOR THE POLICY-YEAR STRUCTURES OF SURRENDER RATES

Empirical Data on the Policy-Year Structures of Surrender Rates

We extract data from the Taipei Life Insurer Association to construct the policy-year structure of surrender rates. Since 1993 the association has calculated the termination rate of the life insurance policies sold in Taiwan by policy year, and the rate is traced up to the 15th policy year.[18] It calculates the termination rates by gender, age, and physical examination requirement group using both the number of policies and the face amounts. The most recent report that we get contains the policy-year termination rates observed in 2003. We therefore have 11 "termination rate curves" as shown in Table 6.

The data in Table 6 display two essential features for the policy-year structure of termination rates. First, both the termination rate and its volatility decrease with the policy year in general. Second, the termination rate and its volatility drop dramatically from the 1st policy year to the 2nd policy year and decreases gradually thereafter. In

[18] The policies consist of endowment, term-life, and whole-life insurance policies. The association calculates only the aggregate surrender rate without reporting individual surrender rates by types of policies.

TABLE 6
The Policy-Year Termination Rates (%) in Terms of Policy Numbers Observed From 1993 to 2003 by the Taipei Life Insurer Association

Observed \ Policy Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	Mean	Standard Deviation
1	27.84	24.91	24.05	21.31	15.60	11.90	12.22	11.25	13.02	10.31	11.74	16.74	6.48
2	7.86	7.73	7.80	6.83	6.71	5.44	4.64	5.77	5.64	6.19	6.13	6.43	1.06
3	7.37	7.13	7.37	6.14	5.51	5.53	6.43	6.87	6.49	5.11	5.73	6.33	0.80
4	5.12	5.53	6.62	5.79	4.64	4.10	4.50	4.88	5.26	4.49	4.48	5.04	0.73
5	4.21	4.68	4.95	4.51	3.87	3.32	3.25	4.59	4.46	3.72	3.74	4.12	0.57
6	4.08	4.66	4.70	3.90	3.51	3.17	2.89	3.10	3.62	3.03	2.51	3.56	0.71
7	3.53	3.81	4.20	3.43	2.95	2.91	2.58	2.71	3.06	2.74	2.95	3.17	0.51
8	3.20	3.44	3.53	2.95	2.67	2.47	2.31	2.30	2.54	2.14	2.19	2.70	0.50
9	3.22	3.20	3.18	2.55	2.44	2.32	2.04	2.13	2.20	3.46	2.45	2.65	0.51
10	2.68	3.10	2.88	2.38	2.21	2.15	1.93	1.95	2.08	1.66	1.77	2.25	0.46
11	2.74	3.51	3.62	2.92	2.48	2.26	1.95	2.02	2.08	1.99	1.54	2.46	0.67
12	2.04	2.43	2.81	2.42	2.05	1.96	1.61	1.67	1.80	1.59	1.51	1.99	0.42
13	1.79	1.80	2.35	2.17	2.16	1.75	1.44	1.56	1.75	1.43	1.36	1.78	0.33
14	1.45	1.54	2.11	1.66	1.80	1.75	1.35	1.39	1.38	1.31	1.30	1.55	0.26
15	1.05	1.51	1.69	1.32	1.27	1.49	1.31	1.37	1.42	1.34	1.21	1.36	0.17
16	0.93	0.98	1.19	1.10	1.00	1.04	1.39	1.19	1.34	1.25	1.06	1.13	0.15

particular, the average termination rate is 16.74 percent with a standard deviation of 6.48 percent in the 1st policy year, whereas it is 6.43 percent with a standard deviation of 1.06 percent in the 2nd policy year. In other consecutive policy years, the decreases in the means and the standard deviations are less than 1 percent and 0.5 percent, respectively. The termination rates reported in the *Life Insurers Fact Book* by ACLI (2000) display similar characteristics. Specifically, the termination rates of the policies in force less than 2 years have a significantly higher mean and standard deviation than those of the policies in force 2 years or more. The former rates have a mean of 18.19 percent and a standard deviation of 2.36 percent during the period from 1965 to 2000, while the latter has a mean of 5.43 percent with a standard deviation of 1.69 percent during the same period.

Based on the features displayed by Table 6, we specify $f(i)$ and $SR_0(i)$ (%) as follows:

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16+
$f(i)$	7.0	1.5	1.0	1.0	0.7	0.6	0.5	0.4	0.4	0.4	0.4	0.4	0.3	0.2	0.2	0.2
$SR_0(i)$ (%)	28.0	16.0	12.5	10.0	8.3	7.0	6.0	5.0	4.5	4.5	4.5	3.0	3.0	3.0	2.5	2.0

The specification of $f(i)$ is derived, with minor smoothing adjustments, by scaling the standard deviations in Table 6 with respect to $SR_t(4)$. $SR_0(i)$ are set by two criteria. First, the simulated $SR_t(i)$ should have the means consistent with the relative scale of the means in Table 6. Second, they should generate a comparable number of surrendered policies over the entire policy period to the number used to generate Table 1.

Simulation Results and Analyses

The simulated $SR_i(i)$ in the section on "Empirical Data on the Policy-Year Structures of Surrender Rates" is used to calculate the $D_i^{(s)}$ in Equation (1). We exclude expenses and loadings in this section to focus on the impact of the surrender rate curve on policy reserves. The simulation results are reported in Table 7.

Two factors contribute to the differences between Table 7 and Table 1: the convexity of the surrender rate curve and the policy-year structure of the surrender rate volatilities. To understand how these two factors affect policy reserves, respectively, we compare the resulting changes in reserve distributions under four different specifications for the convexity and the volatilities of the surrender rate curve as listed in Table 8.

TABLE 7
Summary Statistics of the Reserve Distribution When Surrender Rates Exhibit a Convex Policy-Year Structure and Have Volatilities Decreasing With the Policy Year

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
632,448	21,476	3,074,582	0.75	2.96	6,590,657	1.94

TABLE 8
Decomposition of the Differences Between the Flat Specification of the Surrender Rate Curve and the Specification Used in Table 7

	Flat Specification		Convexity Specification		Volatility Specification		Table 7 Specification	
i	$f(i)$	Initial Value (%)	$f(i)$	Initial Value (%)	$f(i)$	Initial Value (%)	$f(i)$	Initial Value (%)
1	1.0	7.0	1.0	28.0	7.0	7.0	7.0	28.0
2	1.0	7.0	1.0	16.0	1.5	7.0	1.5	16.0
3	1.0	7.0	1.0	12.5	1.0	7.0	1.0	12.5
4	1.0	7.0	1.0	10.0	1.0	7.0	1.0	10.0
5	1.0	7.0	1.0	8.3	0.7	7.0	0.7	8.3
6	1.0	7.0	1.0	7.0	0.6	7.0	0.6	7.0
7	1.0	7.0	1.0	6.0	0.5	7.0	0.5	6.0
8	1.0	7.0	1.0	5.0	0.4	7.0	0.4	5.0
9	1.0	7.0	1.0	4.5	0.4	7.0	0.4	4.5
10	1.0	7.0	1.0	4.5	0.4	7.0	0.4	4.5
11	1.0	7.0	1.0	4.5	0.4	7.0	0.4	4.5
12	1.0	7.0	1.0	3.0	0.4	7.0	0.4	3.0
13	1.0	7.0	1.0	3.0	0.3	7.0	0.3	3.0
14	1.0	7.0	1.0	3.0	0.2	7.0	0.2	3.0
15	1.0	7.0	1.0	2.5	0.2	7.0	0.2	2.5
16+	1.0	7.0	1.0	2.0	0.2	7.0	0.2	2.0

The first specification is named "Flat Specification" because it assumes that the initial surrender rate curve is flat and makes parallel shifts only. The fourth specification is named "Table 7 Specification" because it is the specification used to generate Table 7. The other two specifications are designed to represent the two major differences that contrast the Table 7 Specification to the Flat Specification. "Convexity Specification" reflects the convexity of the curve and assumes a convex surrender rate curve that gives rise to parallel shifts. "Volatility Specification" reflects the curve's volatility pattern with respect to the policy year. It assumes a flat surrender rate curve at the beginning, but the surrender rate of different policy years will change in different magnitudes. Table 9 reports the results under the Convexity Specification and the Volatility Specification.[19]

Since some of the volatilities specified under the Volatility Specification are larger than those under the Flat Specification but some are smaller, additional volatility specifications are required to properly interpret the results obtained under the Volatility Specification. We thus experiment with four flat volatility patterns as presented in Table 10.

These four volatility patterns differ from the Flat Specification only in terms of the volatility scale. The surrender rate curves under these specifications are all flat

[19] The results for Flat Specification and Table 7 Specification have been presented in Tables 1 and 7.

TABLE 9
Summary Statistics of the Reserve Distributions Under the Convexity and Volatility Specifications in Table 8

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Panel A: Results of Convexity Specification						
878,265	−5,559	3,314,000	1.06	3.68	7,620,705	2.03
Panel B: Results of Volatility Specification						
790,679	25,739	4,020,102	0.71	2.91	8,601,953	1.94

TABLE 10
Specifications for Flat Surrender Rate Curves That Move in a Parallel Fashion With Different Magnitudes

	Quarter	Half	Flat Specification		Double	Quadruple
i	$f(i)$	$f(i)$	$f(i)$	Initial Value (%)	$f(i)$	$f(i)$
1	0.25	0.5	1.0	7.0	2.0	4.0
2	0.25	0.5	1.0	7.0	2.0	4.0
3	0.25	0.5	1.0	7.0	2.0	4.0
4	0.25	0.5	1.0	7.0	2.0	4.0
5	0.25	0.5	1.0	7.0	2.0	4.0
6	0.25	0.5	1.0	7.0	2.0	4.0
7	0.25	0.5	1.0	7.0	2.0	4.0
8	0.25	0.5	1.0	7.0	2.0	4.0
9	0.25	0.5	1.0	7.0	2.0	4.0
10	0.25	0.5	1.0	7.0	2.0	4.0
11	0.25	0.5	1.0	7.0	2.0	4.0
12	0.25	0.5	1.0	7.0	2.0	4.0
13	0.25	0.5	1.0	7.0	2.0	4.0
14	0.25	0.5	1.0	7.0	2.0	4.0
15	0.25	0.5	1.0	7.0	2.0	4.0
16+	0.25	0.5	1.0	7.0	2.0	4.0

initially but then shift in different magnitudes. By comparing reserve distributions under these different specifications, we will be able to understand the effects of increasing/decreasing surrender rate volatilities on policy reserves. The summary statistics of the reserves distributions under these specifications are shown in Table 11.

Table 11 reveals that increasing the surrender rate volatility will increase the mean as well as the risk of the reserve distribution. This is easily justified because amplifying surrender rate volatilities when changes in the surrender rate are affected by interest rate fluctuations means that the interest rate sensitivity of surrender rates is increased and the sensitivity is harmful to life insurers as we have seen in the section on "The Flat Policy-Year Structure of Expense Ratios." In addition, by comparing Panel B of

TABLE 11
Summary Statistics of the Reserve Distributions Under the Specifications in Table 10

Specification	Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Quarter	692,159	42,971	3,955,181	0.59	2.67	8,196,001	1.90
Half	820,944	27,037	4,045,079	0.73	2.91	8,705,857	1.95
Flat	1,086,994	−8,961	4,299,088	1.01	3.56	9,838,873	2.04
Double	1,606,493	−73,881	4,931,486	1.41	4.69	12,289,972	2.17
Quadruple	2,133,678	−168,821	5,409,896	1.55	4.93	14,243,903	2.24

Table 9 with that of Table 11, we find that most statistics under the Volatility Specification of Table 8 lie between those of the Quarter Specification and the Half Specification of Table 10. The Volatility Specification therefore may be regarded as a specification that has smaller surrender rate volatilities on average than the Flat Specification.

Having recognized how the volatility pattern of the surrender rate curve affects reserve distributions, we proceed to investigate the effects of the curve's convexity patterns on reserve distributions. By comparing Panel A of Table 9 with Case 1 of Table 1, we find that changing the surrender rate curve from flat to convex generates results that are favorable to insurers. The mean, standard deviation, and 95th percentile of reserves in Panel A of Table 9 are all smaller than those in Case 1 of Table 1. These results are understandable because an increase in convexity in fact diminishes the interest rate sensitivity of surrender rates. Surrender rates are high in the early stage of the policy life and become low in the later stage irrespective of the interest rate levels. Such "irrationality" benefits life insurers.

The above analyses demonstrate how the convexity and the volatility pattern of surrender rate curves affect reserve distributions, and they provide us with a rationale behind the difference between Table 7 and Case 1 of Table 1. Table 7 reports a much smaller mean (\$632,448 vs. \$1,086,994), a smaller standard deviation (\$3,074,582 vs. \$4,299,088), and a smaller 95th percentile of the reserve distribution (\$6,590,657 vs. \$9,838,873) because the surrender rate curve is convex and has smaller volatilities on average.[20] Both features reduce the interest rate sensitivity of surrender rates and thus benefit life insurers.

Incorporating surrender charges will make our results more significant because surrender charges enhance the beneficial impact of surrenders to insurers. Policyholders get a smaller amount of money back when insurers impose charges on surrenders. Considering surrender charges will therefore provide stronger supports for our finding that convex surrender rate curves with volatilities decreasing with the policy year are favorable to insurers. We conduct simulations in which surrender charges are taken into account and observe an enlarged difference between the mean reserves resulted from a flat surrender rate curve and a convex one.[21]

[20] Recall that the volatility pattern specified in Table 7 is the same as the Volatility Specification in Table 8 that is equivalent to a specification between the Quarter specification and the Half specification in Table 11.

[21] For the sake of the article's length, the results are not reported in the article.

TABLE 12
Specifications of a Surrender Rate Curve That Is Less Convex and Has a Less Convex Volatility Pattern Than the Table 7 Specification With the Two Decompositions

	Curve Specification		Convexity Specification		Volatility Specification	
i	$f(i)$	Initial Value (%)	$f(i)$	Initial Value (%)	$f(i)$	Initial Value (%)
1	4.0	14.0	1.0	14.0	4.0	7.0
2	3.0	12.0	1.0	12.0	3.0	7.0
3	2.0	11.0	1.0	11.0	2.0	7.0
4	1.0	10.0	1.0	10.0	1.0	7.0
5	1.0	9.0	1.0	9.0	1.0	7.0
6	0.9	8.0	1.0	8.0	0.9	7.0
7	0.9	7.0	1.0	7.0	0.9	7.0
8	0.8	7.0	1.0	7.0	0.8	7.0
9	0.8	6.5	1.0	6.5	0.8	7.0
10	0.7	6.5	1.0	6.5	0.7	7.0
11	0.7	6.0	1.0	6.0	0.7	7.0
12	0.7	6.0	1.0	6.0	0.7	7.0
13	0.7	5.5	1.0	5.5	0.7	7.0
14	0.5	5.0	1.0	5.0	0.5	7.0
15	0.5	4.5	1.0	4.5	0.5	7.0
16+	0.5	4.0	1.0	4.0	0.5	7.0

Robustness Checks

Robustness Across Different Specifications of the Surrender Rate Curve. To establish the robustness of the above findings, we first experiment with several other specifications of the surrender rate curve. In Table 12, "Curve Specification" represents a less convex case than the Table 7 Specification in terms of the convexity and the policy-year structure of volatilities. "Convexity Specification" and "Volatility Specification" of Table 12 represent the two major differences that contrast the Curve Specification to the Flat Specification.

The simulation results based on the specifications of Table 12 are reported in Table 13. They confirm our previous findings. By comparing Panel B of Table 13 with Case 1 of Table 1, we find that the Convexity Specification of Table 12 generates results that are more favorable to life insurers. The observation that a convex surrender rate curve is more favorable to life insurers than a flat curve is confirmed. Moreover, the Convexity Specification of Table 8 leads to more favorable results than the Convexity Specification of Table 12, which implies that a more convex surrender rate curve is more appealing to life insurers. The above comparisons demonstrate that the convexity of the surrender rate curve is favorable to life insurers.

The Curve Specification of Table 12, as a combination of the Convexity Specification and the Volatility Specification of Table 12, is expected to generate better results for life insurer than the Flat Specification but worse results than the Table 7 Specification. It should lead to better results than the Flat Specification since both of its compos-

TABLE 13
Summary Statistics of the Reserve Distributions Under the Specifications in Table 12

Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Panel A: Results for Curve Specification						
821,174	143	3,511,444	0.92	3.35	7,857,440	2.00
Panel B: Results for Convexity Specification						
936,018	−9,688	3,631,820	1.03	3.62	8,314,118	2.03
Panel C: Results for Volatility Specification						
960,847	3,618	4,167,510	0.90	3.31	9,301,129	2.00

ing specifications are more favorable.[22] This is confirmed by the summary statistics presented in Panel A of Table 13. The mean, standard deviation, and 95th percentile of reserves in Panel A of Table 13 are all smaller than those of Flat Specification in Table 11. On the other hand, the Curve Specification should lead to worse results than the Table 7 Specification because both of its composing specifications are more adverse than those of the Table 7 Specification.[23] This can be confirmed by comparing Panel A of Table 13 with Table 7. We see that the mean and the risk measures of the reserve distribution are larger in Panel A than in Table 7. All results are as expected.

Robustness Across Different Surrender Rate Models. One major assumption in the above simulations is that $\Delta SR_t(4)$ behaves in the same way as the termination rate in the TKC model. This assumption is not realistic since the TKC model is based on a proper statistical analysis using the year average instead of a reference year. To further secure the robustness of our findings, we propose an arc-tangent function to replace the surrender rate function in Equation System (2).[24] More specifically, we assume that $SR_t(4)$ has the following relation to the spread between market interest rates and the pricing rate:[25]

$$SR_t(4) = \min\{0.12, \max[0.07 + 0.045 \times \tan^{-1}(50 \times \text{Spread}), 0.05]\}. \qquad (3)$$

Then we rerun the simulations that generate Tables 1, 7, 9, 11, and 13.[26]

22 Comparisons between Panel C of Table 13 and Table 11 show that the Volatility Specification in Table 12 leads to the results that are more favorable to life insurers than the Flat Specification.

23 Panel B and Panel C of Table 13 indicate more adverse reserve distributions to life insurer than the reserve distributions indicated by Panel A and Panel B of Table 9, respectively.

24 The idea of using an arc-tangent function to model the surrender rate is borrowed from Babbel and Merrill (1998) and Babbel, Gold, and Merrill (2002).

25 Based on the historical surrender rates of the United States from 1965 to 2000, we set the maximum and minimum surrender rates at 12 percent and 5 percent, respectively, in Equation (3). The maximum and minimum are reached at the spreads of 6 percent and −3 percent, respectively. We further set the surrender rate to be 7 percent when the spread is 0 percent. This parameter set generates a similar average surrender rate to the one resulting from Equation System (2).

26 The results are not reported in the article for the sake of its length.

With the reference-year surrender rate being assumed of an arc-tangent function, the obtained results confirm the stories that were told under Equation System (2). They reveal that increasing the surrender rate volatility increases the mean and the risk of the reserve distribution. They also show that changing the surrender rate curve from flat to convex gives rise to results that are favorable to insurers. We further vary the parameters of the arc-tangent function to make the surrender rate be more and less sensitive to the spreads between the pricing rate and interest rates. The simulation results are also consistent with our previous findings.

Sectional Conclusions. In this section we have shown that our findings are robust across different surrender rate curve specifications, and in addition, we have shown that our findings are robust across different surrender rate models. The robustness holds in both cases because the benefits due to the surrender rate curve's convexity and the damages caused by its volatilities are both direct consequences of the interest rate sensitivity of surrenders. Any setting or model that reflects such interest rate dependency of surrender rates is likely to tell the same story. Should future empirical models of policy-year surrender rates be available, we expect to see reserve distributions thus generated to further support our rationale above.

CONSIDERATIONS FOR BOTH POLICY-YEAR STRUCTURES

The natural step to take, after considering the individual policy-year structure of expense ratios and surrender rates, is to analyze the reserve distribution under the joint influence of both structures. We simulate the reserve distribution based on the policy-year structure of surrender rate as specified in Table 7 together with the second and third expense ratio patterns of Table 2. The results are reported in Panel A of Table 14.[27] For robustness checks, we report in Panel B the results based on the Curve Specification of Table 12 and the two expense ratio patterns.[28]

All results are as expected. The natural hedging benefits of the convex expense ratio curve and the benefits of the convex surrender rate curve with decreasing volatilities result in lower mean reserves and less risk of reserves than the corresponding cases in Tables 4, 5, 7, and 13. For instance, the mean of $533,010 and the 95th percentile of $4,855,222 in Panel A of Table 14 are smaller than those of Case 1 in Table 4 (the third expense ratio pattern with a flat surrender rate curve) and those of Table 7 (the same policy-year structure of surrender rates with a flat expense ratio pattern).

The figures of Table 14 are considerably smaller than those of Case 1 in Table 1. This indicates that neglecting the convex expense ratio patterns and the convex surrender rate curves with lower volatilities would significantly overestimate the mean and the risk of policy reserves. Our findings show that the actuaries using deterministic pricing method would underestimate the contract value by $533,010 instead of $1,086,994. The risk is still substantial in contrast to the annual premium, but not to the extent

[27] The corresponding actuarially fair loading rates under the pricing rate of 6 percent to the two expense ratio patterns are 44.91 percent and 54.94 percent, respectively. We do not report the results with the first expense ratio pattern because the case and results are the same as those of Table 7.

[28] The fair loading rates under 6 percent pricing rate for the two expense ratio patterns are 44.61 percent and 49.26 percent.

TABLE 14
Summary Statistics of the Reserve Distributions Under Two Curve Specifications and Expense Ratio Patterns

Expense Ratio Pattern	Mean	Median	Standard Deviation	Skewness	Kurtosis	95th Percentile	(95% − Mean)/ S.D.
Panel A: Results Under the Table 7 Specification and the Two Expense Ratio Patterns							
2	566,724	3,335	2,598,712	0.76	2.95	5,600,901	1.94
3	533,010	66,842	2,246,199	0.72	2.90	4,855,222	1.92
3*	720,795	278,326	2,132,663	0.72	2.90	4,825,513	1.92
Panel B: Results Under the Curve Specification in Table 12 and the Two Expense Ratio Patterns							
2	722,148	12,796	2,899,580	0.93	3.35	6,518,028	2.00
3	682,865	71,247	2,531,204	0.89	3.30	5,684,992	1.98
3*	847,229	261,992	2,417,026	0.89	3.30	5,621,103	1.98

*Denotes the situation where the surrenders rates in the first 2/3 policy years are higher than the expected ones for 2 percent.

of the severity suggested by Case 1 in Table 1. These results support the importance of the policy-year structures in estimating reserve distributions. Since Tsai, Kuo, and Chen (2002) not only assume the surrender rate curve to be flat and shift in a parallel fashion but also neglect expense ratio patterns, they are likely to overestimate the mean and the risk of policy reserves to a significant extent.

We further experiment with the cases of unexpected surrenders. The expense ratio pattern 3* in Panel A of Table 14 represents the case in which surrender rates are higher than the expected ones by 2 percent during the first 2 policy years while the pattern 3* in Panel B represents a 2 percent increase of surrender rates in the first 3 policy years. Surrender rates, when they are higher than expected, result in higher mean reserves and less risk of reserves. This is in line with the results obtained in the sections on "The Most Convex Policy-Year Structure of Expense Ratios" and "Robustness Checks." On the other hand, the adverse effects of higher surrender rates suggested by the higher means differ from the findings of Tsai, Kuo, and Chen (2002). Surrenders turn from being favorable to insurers to detrimental when both policy-year structures are joined in force. This demonstrates the determinant role of these two policy-year structures in quantifying the effects of surrenders on reserve distributions.

CONCLUSIONS

Naive estimations on policy reserves could impair the solvency of life insurers. Estimating reserve distributions requires one to identify the determinant factors, which constitute the risk profile of policy reserves. Research attempts that follow this line of thinking so far have considered random mortality, stochastic interest rates, and interest-rate-sensitive surrender rates. In this article we identify two additional factors that have significant impacts: the policy-year structures of expense ratios and surrender rates.

We find that a convex expense ratio curve prevalent in the life insurance industry is beneficial to insurers. The benefit comes from the fact that the gain/loss resulted from the expenses/loadings pair counterbalances a portion of the loss/gain from the benefit payments/premiums pair in the environment of stochastic interest rates. This gives rise to a natural hedging mechanism that mitigates the policy's exposure to interest rate fluctuations. In addition, we find that this hedging effect increases with the convexity of the expense ratio curve.

We also find that the policy-year structure of expense ratios determines how early surrenders affect reserves. When the expense ratio curve is flat, higher surrender rates are beneficial to life insurers since they result in lower means and less risk of reserves. When the curve is convex, surrenders become harmful to insurers in terms of mean liabilities. The increases in mean reserves suggest that the detrimental effect outweighs the beneficial effect of surrenders established in the findings of Tsai, Kuo, and Chen (2002). Furthermore, the net damage increases with the convexity of the expense ratio curve. The damage can be severer than that caused by the interest rate sensitivity of surrenders.

With regard to the policy-year structure of surrender rates, we find that the structure affects reserve distributions via two routes: its convexity and its volatility pattern. A convex surrender rate curve is beneficial to insurers because it alleviates the interest rate sensitivity of surrender rates. For the same reason, smaller volatilities of surrender rates are favorable to life insurers. Convex surrender rate curves with smaller average volatilities are therefore found to be favorable to insurers. The benefits increase with the convexity of the surrender rate curve but decrease with the volatilities of surrender rates.

The final part of our analysis focuses on the reserve distributions under the influences of stochastic interest rates, interest-rate-sensitive surrender rates, the policy-year structure of expense ratios, and the policy-year structure of surrender rates. By doing so, we add to the existing literature a generalized multifactor setting previously unseen. Our results imply that Tsai, Kuo, and Chen (2002) likely overestimate the mean and the risk of reserves. Such overestimation is a direct consequence of neglecting the policy-year structures that we have identified in this article. Insurers, however, still have to be cautious about the unexpected surrenders. Our results reveal that higher than expected surrender rates during the first several policy years can raise the mean reserve to a certain extent.

The results of this article should be robust across the maturities of endowment policies and should be applicable to n-year term-life insurance policies.[29] The expense ratio curve renders insurers the natural hedging benefits as long as it is convex. Such benefits are not derived from the maturity or the pure endowment component of the policy. Furthermore, the critical role played by the expense ratio curve in determining whether surrenders are beneficial or harmful to insurers is not derived from the maturity or the pure endowment component of the policy. We speculate

[29] Our results are also applicable to whole-life insurance policies since a whole-life insurance policy can be deemed as an endowment policy with a maturity of $\omega - x$ years in which ω is the limiting age of the life table.

that the extent of such benefits shall increase with contract maturities,[30] and for term-life policies this beneficial effect shall become even more prominent.[31] We also speculate the net losses caused by surrenders to increase with the policy maturity. With regard to the policy-year structure of surrender rates, the benefits of the convexity and the damages of the volatilities shall exist across the maturities of endowment and be applicable to term-life insurance policies. This is so because these effects are derived from the interest rate sensitivity of surrenders. We speculate that the interest rate sensitivity shall increase with the importance of the savings component of a policy. Therefore, the effects of the surrender rate curve are expected to be less significant for term-life insurance and decrease with policy maturities.

As was shown in the two "Robustness Checks" sections, the findings in this article are robust across different specifications of the expense ratio and surrender rate curves, and they are robust across different surrender rate models as well. However, quantifying the exact extent of the impact of surrenders on reserve distributions requires a model that captures the relation between the interest rate and the policy-year surrender rate. This will be of interest to scholars in the future.

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[30] The first reason is that the loading rate often increases with the maturity of a policy, and a higher loading rate enhances the natural hedging benefits. The second reason is that the potential over- and undercharged loadings increase with the maturity, given the same annual premium, loading rate, and convexity of the expense ratio curve. The beneficial extent of the natural hedging mechanism therefore increases with the policy maturity.

[31] First, term-life insurance policies usually have higher loading rates than endowment policies since their savings components are less. Second, the natural hedging benefits are expected to be more prominent for term-life policies than that of endowment policies because their exposure of the net premiums/benefit payments to interest rate fluctuations is smaller, given the same annual premium, loading rate, and convexity of the expense ratio curve.

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