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Crossover behavior of stock returns and mean square displacements of particles governed by the Langevin equation

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Abstract – It is found that the mean square log-returns calculated from the high-frequency one-day moving average of US and Taiwan stocks with the time internal τ show ballistic behavior $\theta\tau^{\alpha_1}$ with the exponent $\alpha_1 \approx 2$ for small τ and show diffusion-like behavior $D\tau^{\alpha_2}$ with the exponent $\alpha_2 \approx 1$ for large τ. Such a crossover behavior can be well described by the mean square displacements of particles governed by the Langevin equation of motion. Thus, θ and D can be considered, respectively, as the temperature-like and diffusivity-like kinetic parameters of the market, and they can be used to characterize the behavior of the market.

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Introduction. – Fluctuations in financial markets are important quantities of practical as well as academic interests. In 1900, Bachelier [1] proposed that fluctuations in financial market follow random walks, which was before Einstein's random walk model for the Brownian particles in the liquid [2]. However, later studies indicate that fluctuations in stocks are not completely random. In 1966, King found that changes in prices of different stocks during time intervals of a day or longer are often highly correlated and the correlation is higher for firms in the same industry [3]. In 1977–1979, Epps studied correlations in log price for four major automakers in the United States AMC (American Motors Corporation, 1954–1987), Chrysler, Ford, and GM during intervals of 10 minutes to three days [4]. Epps found that AMC has less correlation with other companies and the correlations in other three companies increase with the length of the time intervals τ used to calculate changes in log price [4]. This has been called "Epps effect". Such an effect was considered to be related to the information on the degree of transaction synchronicity [5], the lead-lag phenomena between pairs of stocks [6], and other important [7] but less recognized properties of the market.

In 1995, Mantegna and Stanley [8] showed that the scaling of the probability distribution of an economic index (S&P 500) can be described by a non-Gaussian process with dynamics that, for the central part of the distribution, correspond to that for a Lévy stable process. Scaling behavior is observed for time intervals from 1000 min to 1 min. The scaling exponent is remarkably constant over a six-year period (1984–1989). In 2011, Saakian *et al.* obtained exact non-Gaussian distribution of stock returns from the multifractal random walk model [9].

In 1999, Laloux *et al.* [10] calculated the correlation matrix for 406 stocks in S&P 500 in 1991–1996 with a time interval of one day and Plerou *et al.* [11,12] calculated the correlation matrix for 1000 stocks in USA in 1994–1995 with a time interval of 30 minutes. Both groups found that in the eigenvalue distribution of the correlation matrix, there are some discrete distributed larger eigenvalues above the continuous component predicted by the random walk model for stocks. In the eigenvector corresponding to the largest eigenvalue λ_M of the correlation matrix, all stocks in the market move (deviate from the average value) in the same direction. The mode corresponding to λ_M is called the market mode.

In 2004, Ma, Hu and Amritkar [13] proposed a model of coupled random walks for stock-stock correlations (see also [14]); the walks are coupled via a mechanism that the displacement (price change) of each walk (stock) is activated by the price gradients over some underlying network. They assumed that the network has two underlying

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structures: one for the correlations among the stocks of the whole market and another for those within individual groups, and found that such model can reproduce the major spectra features of the US stocks.

To describe fluctuations in the yen-dollar exchange rate, in 2005 Takayasu, Mizuno, and Takayasu [15] introduced a new type of random walk in a moving potential. The properties of resulting random walks from their model are similar to those of ordinary random walks for large time scales; however their short-time properties are approximated by abnormal diffusion with nontrivial exponents [15]. Such short-time behavior deviates from the random walk model. In 2011, Shapira *et al.* [16] showed that the temporal order in the time series of the daily return of financial indices is hidden in the series of the variance of the stock volatility.

In stock market, the price of a stock changes whenever the quotes between the sell and the bid sides agree at a new price and this happens irregularly both in time and in price. This is qualitatively analogous to the motion of a particle at longer observation time interval. Since the trajectory of a particle in a many-particle system can be well characterized as the Brownian motion in spite of the fact that the motions of the particle is governed by some deterministic equations of motion at the shorter time scales, we enquire what kind of scenario we would obtain for the stock prices if the time scales under consideration covered both short and long time scales. To answer this question, we compare the time dependence of the price changes for stocks and displacements of particles in a many-particle system.

The mean squares of log-return for stocks are known to have an asymptotic t^{α}-dependence on the time interval τ for large τ, with the exponent α close to unity. Such a property is shared by the diffusion behavior of particles in their mean square displacement. For particles with continuous trajectories, such "random walk" behaviors are valid only over time scales allowing sufficient exchanges of momenta. In this work, we show that such a limitation on time scales is true also for collections of stocks. We extend our analysis for stocks inwardly to find the properties over the shorter time scales as the analysis of the yen-dollar exchange rate in [15]. We find that the mean square log-returns calculated from the high-frequency one-day moving average of US and Taiwan stocks with the time internal τ show ballistic behavior $\theta\tau^{\alpha_1}$ with the exponent $\alpha_1 \approx 2$ for small τ and show diffusion-like behavior $D\tau^{\alpha_2}$ with the exponent $\alpha_2 \approx 1$ for large τ. Such a crossover behavior can be well described by the mean square displacements of particles governed by the Langevin equation of motion. Thus, θ and D can be considered, respectively, as the temperature-like and diffusivity-like kinetic parameters of the market, and they can be used to characterize the behavior of the market.

Diffusivity and temperature. – Figure 1(a) shows the stock price $P(t)$ of the INTEL Corporation as a function of time t, over 18 trading days in January 1996; each day has trading time 6.5 hours = 23400 seconds. The data are collected in time intervals of 36 seconds. We consider the analogy between particle displacements and the log-return of stocks $r(t,\tau) = \log(P(t+\tau)) - \log(P(t)) \approx (P(t+\tau) - P(t))/P(t)$ for the price $P(t)$ over an interval τ starting at time t. The log-return carries the changes relative to the prices so that it is a quantity that effectively renders all stocks on an equal foot, despite the inherited heterogeneities among the companies and their stocks prices. The dots in fig. 1(b) show the mean square log-return (MSLR) $\langle r^2 \rangle$ of a collection of 345 stocks (the same as those studied in [13]) of the S&P 500, *vs.* τ during the month of January of 1996. The average $\langle\cdot\rangle$ is

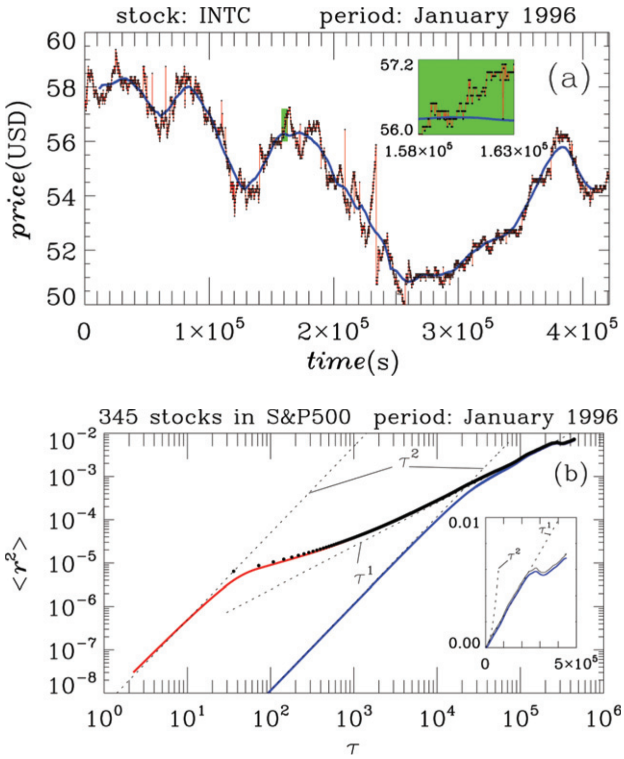

Fig. 1: (Color online) (a) Stock prices of Intel Corporation (INTC), over eighteen trading days in January 1996, each day has trading time 6.5 hours = 23400 seconds. The market data, collected in 36-second intervals, are marked by dots, which are connected by red lines and their the one-day moving averages in steps of 36 seconds are plotted by the blue line. The colored inset shows the enlarged image of the region marked by the same (green) color. (b) Log-log plot for the mean square log-returns (MSLR) *vs.* time interval τ, for a collection of 345 stocks [13], including INTC, picked from S&P500 over January 1996, calculated either based on original data (marked by dots), or based on the moving average (blue curve) shown in (a). The red line shows the result over short time scales, based on the continuous (red) curve of (a); the blue curve shows the results obtained from the blue curve in (a). In the inset, we show the plots of the two former curves in linear scales. For comparison, we plot dashed lines for the guidance of the square or the linear time dependence over the short and the longer time scales, respectively.

taken over all the events for any time interval $(t, t+\tau)$ from the 36-second-step data, for any of the 345 stocks. The result does share the major features with the mean square displacement (MSD) of particles, that facilitate an effective comprehension of these empirical data.

To reveal the analogy in time evolution between the stock prices and the particle trajectories, we analyze the paths of the "motion" of the stocks in the one-dimensional price space. We adopt two different ways to manipulate the data, either by refining the intra-day microscopic temporal features or by eliminating those heterogeneity via a coarse-graining procedures. For the latter, we collect the high-frequency one-day moving averages (HF1MA) $\bar{P}(t)$ of the prices for individual stocks, by taking simple averages over a shifting window which is one-trading-day wide (23400 seconds, or 650 intervals). To retain the high-frequency feature of the data, the window shifts in steps of 36 seconds. The blue line in fig. 1(b) shows the 36-second MSLR, $\langle r^2 \rangle$, of January 1996, calculated from the returns $r(t,\tau) = \log(\bar{P}(t+\tau)) - \log(\bar{P}(t))$ for the collection of 345 stocks.

Alternatively, to keep the discrete features of the price changes, we use the zigzag path (the red line in fig. 1(a)), obtained by linearly interpolating the market data. The MSLRs (fig. 1(b)) for both the 36-second HF1MAs (the blue line) and the linearly interpolated zigzag paths (the red line) contain a τ^2-dependence regime in the shorter times. This is a feature reflecting the validity of a good local linear approximation. The MSLR for the unpolished market data (the dots in fig. 1(b)) is featured by a stretched sub-diffusion regime at the smaller τ before the emergence of diffusion behavior at the larger τ. The feature is retained in the MSLR for the linearly interpolated zigzag paths (the red line), as an intermediate regime in between the τ^2-dependence (exponent-two) regime and the τ-dependence (exponent-one) regime. The kinetics behind the scenario can be realized by referring to the MSD in a dense and cold molecular liquid where an intermediate plateau between the ballistic exponent-two regime and the diffusion-dominant exponent-one regime [17] signals the localized motions of the particles when the system is in a glass-like state [17,18].

In the exponent-two regime of MSD of a fluid system, the velocity v determines the displacement of each particle. The mean square velocity $\langle v^2 \rangle$ depends on the temperature T, space dimension n_d, and mass m of the particle as $\langle v^2 \rangle \sim n_d T/m$. Thus, we have MSD $\approx \langle v^2 \rangle \tau^2 = n_d T \tau^2/m$. Such an asymptotic behavior prevails for a smaller τ over a range for which the trajectories are smooth, facilitating the first-order approximation in τ. With a given T, how far the particles displace over a longer time is affected by the interactions among particles and between particles and the environment. Over a sufficiently large time interval τ, the accumulated momenta exchanges randomize the short-time displacements and, statistically, we see the random walk behavior MSD $\approx n_d D\tau$, where D is the diffusion parameter. Note that, the exponent of the τ-dependence in the diffusive regime can be different from unity, called anomalous transport or diffusion [19], in complex systems.

Guided by the analogy between price changes and the displacements of tracer particles, the temperature-like parameter fitted from the exponent-two regime of MSLR for the HF1MA underscores the net change on coarsening the rapid discrete jumps in prices. The tendency of change is randomized over the longer time scales, where the exponent-one time dependence prevails and the diffusion parameter describes the changes in prices with the inclusion of the randomized effects.

Langevin equation. – A simple model to cover both time regimes, for the motion of a tracer particle of mass m in velocity $v(t) = \frac{\mathrm{d}R}{\mathrm{d}t}$ in $n_d = 1$ dimension, is described by the Langevin equation of motion with white noise $\xi(t)$,

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{\tau_0} v(t) + \xi(t). \tag{1}$$

The balance between the friction $\frac{m}{\tau_0} v(t)$ and the random force $m\xi(t)$ imposes a condition on the amplitude of $\xi(t)$: $\langle \xi(t)\xi(t') \rangle = \frac{2m^2 v(0)^2}{\tau_0} \delta(t-t')$. The MSD is [20]

$$\mathrm{MSD} \equiv \langle (R(t+\tau) - R(t))^2 \rangle = D[\tau - \tau_0(1 - e^{-\tau/\tau_0})], \tag{2}$$

where $D = 2\tau_0 v(0)^2$, which gives asymptotic MSD $\approx \frac{D}{2\tau_0}\tau^2$, for $\tau \approx 0$, and MSD $\approx D\tau$, for $\tau \gg 1$. Note that the ratio μ between the pre-factor D of the latter asymptotic expression to that $\frac{D}{2\tau_0}$ of the former one measures the "mobility" of the tracer particle. The result $\mu = 2\tau_0$ is Einstein's relation.

The averaging procedure adopted in HF1MA effectively eliminates the stretched crossover between the two asymptotic regimes in the MSLR (dots and blue line in fig. 1(b)), rendering the data described by the simple scenario provided by the Langevin equation (eq. (2)). To facilitate such a conjecture, we use the following equation as a master curve (see eq. (2)):

$$\langle r^{*2} \rangle = \tau^* - 1 + e^{-\tau^*} \tag{3}$$

to fit the scaled MSLR $\langle r^{*2} \rangle = \langle r^2 \rangle / (D\tau_0)$ *vs.* $\tau^* = \tau/\tau_0$ for the empirical data. The data are well fitted by $\langle r^2 \rangle \approx \theta \tau^{\alpha_1}$ over the smaller-τ regime to obtain the pre-factor $\theta = \frac{D}{2\tau_0}$. We found that the data are well fitted to give $\alpha_1 = 2.0$ with an error smaller than 1%. For the larger-τ regime, the fitting to the asymptotic form $\langle r^2 \rangle \approx D\tau^{\alpha_2}$ leads to much more scattered values in α_2. To cure this drawback, we impose $\alpha_2 = 1$ and find the best fit for D.

In figs. 2(a) and (b), we show the scaled curves of $\langle r^{*2} \rangle$ *vs.* τ^*, for the 48 months during the years 1996–1999, for 345 stocks from S&P 500 analyzed in fig. 1(b) and for a pool of stocks in the same size from Taiwan stock market (the TSE Intra-Day of TEJ). The trading hours for a trading day in 1996–1999 are six and half hours (9:30AM–4:00PM) for the US market and are three hours

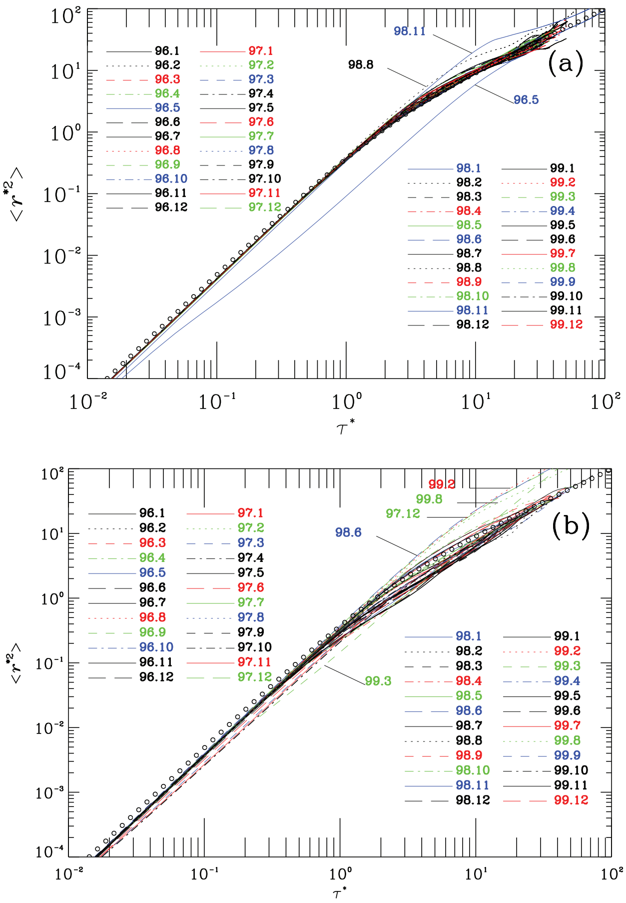

Fig. 2: (Color online) (a) Scaled mean square log-return $\langle r^{*2}\rangle$ *vs.* scaled time internal τ^*, for each month in 1996–1999, for the 345 stocks in S&P500 that have been analyzed in fig. 1(b), and (b) those over the same periods, for a pool of 345 stocks picked from Taiwan markets. They are compared with the master curve (open circles) defined by eq. (3). The trading hours during a trading day in 1996–1999 are six and half hours for the US market and three hours for the Taiwan market. The time sequences for the collections of both markets are in intervals of 36 seconds. The fitting to the asymptotic form θt_1^α and Dt are carried out over the ranges $36 < \tau < 7200$ and $54000 < \tau < 108000$, for the US market (plot (a)) and over $0 < \tau < 7200$ and $54000 < \tau < 108000$ for the Taiwan market (plot (b)). Those months with their scaled MSLRs significantly deviating from the master curve are marked specifically.

(9:00–12:00) for the Taiwan market. The time sequences for the collections of both markets are in intervals of 36 seconds. Treating each market as temporarily stationary over each month, an average $\langle\cdot\rangle$ is taken both time-wise and stock-wise. In spite of a slight degree of bulge at the crossover, we find that the scaled data are in general in a reasonable agreement with eq. (3) (fig. 2(a)), and that the fitted parameters θ and D provide useful information about the condition of the market. The data for Taiwan stocks (fig. 2(b)) are basically also in agreement with the master equation, except that they are apparently more scattered than those for US stocks, probably due to the lesser sampling events over the shorter trading hours. In the data for both markets, there are a few curves deviating significantly from the master curve. These deviations signal temporarily nonstationarity of the markets.

Market mode and ordering. – While the stretching at the crossover between the two asymptotic regimes in the MSLR of the raw data without averaging (dots and red line in fig. 1(b)) can be realized as a localized short-time effect due to the discrete nature of the price changes, we may gain some further insight by pondering their analogy to the properties of dense molecular fluid. In the latter system, it is the interactions with the neighboring molecules that keep the tracer particle from entering the long-time diffusion regime straightforwardly. Such an effect leads to the presence of instantaneously short-lived local structure, which is signalled by a dull peak in the structure factor obtained from a scattering experiment [21].

A similar idea can be applied to the case of stocks. Consider the Karhunun-Loeve expansion of a set of normalized time sequences, each of length T $s_i(t) = \frac{r_i(t)}{\langle r_i^2\rangle^{\frac{1}{2}}}$ $(t = 1,\ldots,T)$ for the log-returns of stocks $i = 1,\ldots,N$,

$$s_i(t) = \sum_k \sqrt{T\lambda_k}\, a_k(i) b_k(t), \tag{4}$$

where the normalized eigenvectors $\{a_k\}$ of the cross-correlation matrix, $[c_{ij}] = [\frac{1}{T}\sum_t s_i(t)s_j(t)]$, and the composite vectors $\{b_k, b_k(t) = \sum_i a_k(i)s_i(t)\}$ form orthonormal spatial and temporal bases, respectively. It is an expansion with the square-root of the eigenvalue λ_k of the k-th mode as the amplitude of that mode. The equalities $c_{ij} = \sum_k \lambda_k a_k(i)a_k(j)$, and $\lambda_k = \sum_i\sum_j c_{ij}a_k(j)a_k(i)$ can be considered as a pair of transformations between λ_k and c_{ij}. While the cross-correlation c_{ij} among stocks corresponds to the spatial correlation function among particles in a many-particle system, we come to the conclusion that the eigenvalue λ_k is the counterpart of the structure factor [22]. The later is the Fourier transform of the spatial correlation function [21], and vice versa. The market mode as well as those with the largest few eigenvalues deviated from the bulk of the rest eigenvalues, therefore, are the counterparts of those of the peaks of a structure factor, with the patterns of their eigenvectors carrying the information of the stock-to-stock ordering. The presence of these modes makes the system more like a system of liquid than that of gas. In the fluid system, the presence of short-range ordering renders the system characterized by retarded relaxations in response to fluctuations [21], and is responsible for the stretched crossover between the two asymptotic regimes in MSD [17].

It is then sensible to inquire the content of the corresponding ordering that had caused the slow sub-diffusion regime at the smaller τ in fig. 1(b). In comparing the eigenvalue spectra of the cross-correlation matrices for the raw data and those for HF1MA, we found that such ordering is indeed local. In fig. 3, we put together the monthly data of the parameters θ, D and their ratio

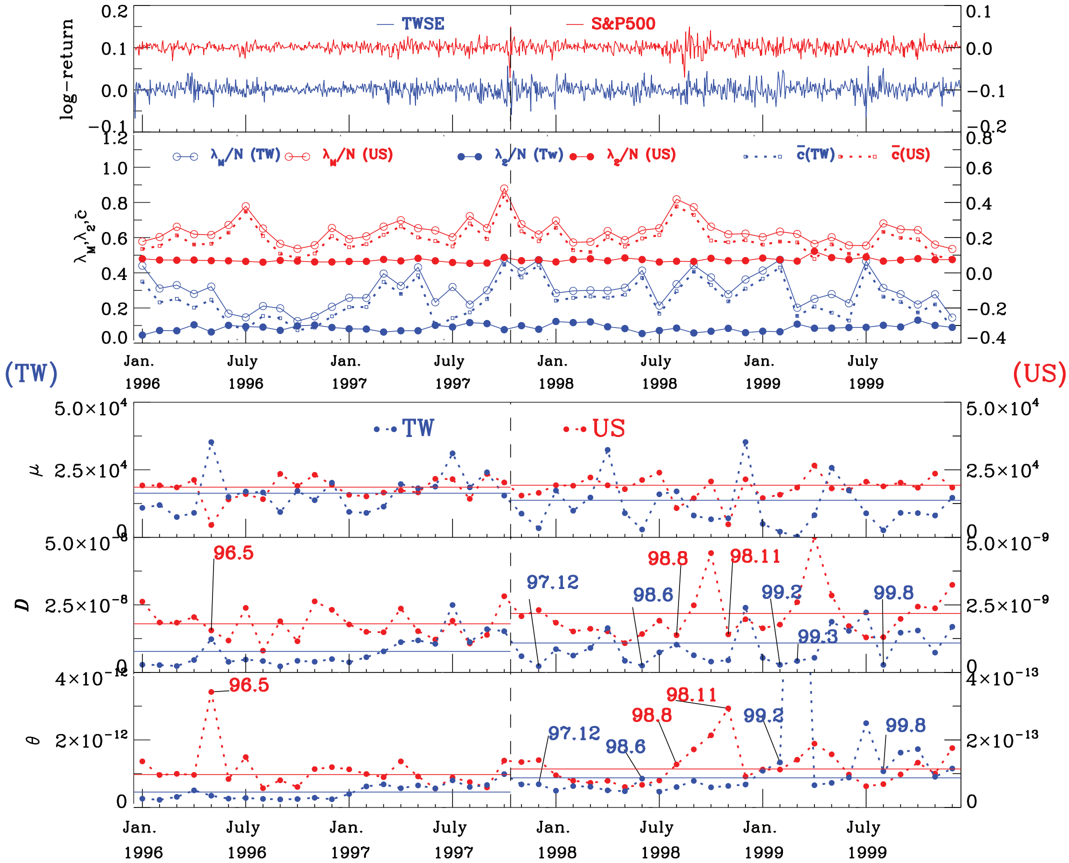

Fig. 3: (Color online) Data for USA S&P500 stocks (red color) and Taiwan TAIEX stocks (blue color) in 1996–1999. (Top down) Daily market indices for S&P500 and TWSE; the largest λ_{M} (open circles) and the second largest eigenvalues and λ_2 (filled circles) for the cross-correlation matrices of monthly data sets; and the parameters θ, D, defined in the main text related to eq. (3), and their ratio $\mu = \frac{D}{\theta}$, for the collection of 345 stocks in S&P500 considered in fig. 2(a) and those in the Taiwan market considered in fig. 2(b). Each correlation matrix is calculated by collecting a time sequence in intervals of 390 minutes (one trading day), for consecutive 25 trading days, beginning the first trading day for each month. The kinetic parameters θ, D are calculated by fitting over ranges (in unit of seconds) (36, 7200) and (54000, 108000), respectively for US stocks and (36, 7200) and (54000, 108000), respectively for Taiwan stocks. They have been used to obtain the scaled curves plotted in fig. 2. The vertical dashed lines in the plots mark the time spot of October 27, 1997, when the markets collapsed during the Asian financial crisis. The horizontal red lines in the three lower panels mark the averaged values over the data of the months, excluding those months with deviated MSLRs in fig. 2(a), before and after, respectively, the market collapsed for US stocks ($\theta = 9.7 \times 10^{-13}$, $D = 1.8 \times 10^{-8}$, $\mu = 1.8 \times 10^4$ before the crisis and $\theta = 1.1 \times 10^{-12}$, $D = 2.2 \times 10^{-8}$, $\mu = 1.9 \times 10^4$ after the crisis.) The horizontal blue lines are for Taiwan stocks, excluding those months with deviated MSLRs in fig. 2(b) ($\theta = 4.6 \times 10^{-12}$, $D = 7.8 \times 10^{-8}$, $\mu = 1.6 \times 10^4$ before the crisis and $\theta = 8.8 \times 10^{-12}$, $D = 1.1 \times 10^{-7}$, $\mu = 1.4 \times 10^4$ after the crisis.)

$\mu = D/\theta$ obtained from the fitting of mean square log-return to the eqs. (2) and (3) for the pools of the stocks in the two markets analyzed in fig. 2. We compare them with the parameters obtained from the analysis of the correlation matrices among the stocks from their daily data. These include the mean correlation coefficients $\bar{c}$, the ratios of the largest and the second largest eigenvalues to the size $N = 345$, λ_M/N and λ_2/N, of each collection of stocks, respectively. We also include the data of daily returns for the market indices, S&P500 and TWSE. The variations in λ_M/N's are dominated by the changes in $\bar{c}$, via $\lambda_M/N \approx \bar{c} + (1-\bar{c})/N$ as the lowest-order approximation [13,23]. The second largest eigenvalue varies differently from the market mode does. There are several major sections of clustered large fluctuations in the returns of market indices, including the one for both markets on October 27, 1997 (the vertical dashed lines in fig. 3) when the markets collapsed due to the Asian financial crisis; and the one between August and October of 1998 in S&P500, and that extended between September 1998 and February 1999 in TWSE. There is a correspondence between the occurrence of such larger-fluctuation sections and the emergent larger values in λ_M/N.

There seems no obvious signatures in the kinetic parameters θ, D or μ corresponding to the former, especially, on the occurrence of the crisis. The results suggest that the kinetic parameters are not fast variables in response to market changes. They, indeed, change when they are viewed over a longer-time span. While their (averaged) values (indicated by colored solid lines in fig. 3) do not change much in crossing the crisis, the number of outliers in fitting to the Langevin master curve, eq. (3) (fig. 2), become larger after the crisis (fig. 3), indicating the on-going adjustments of the markets. The kinetic parameters θ, D and μ are determined by internal properties, which are distinct from the market force that drives the market mode.

In comparing the parameters for the two markets, it is interesting that both D and θ for Taiwan stocks are systematically larger than those for US stocks. Note, the scales of D and θ (on the right vertical axes) for US stocks are one decade smaller than those (on the left vertical axes) for Taiwan stocks. The values for parameter μ, on the other hand, are about the same for the two markets. The results suggest that the price changes (leading to a faster diffusion) are more effective for the stocks in the Taiwan market than those in the US market, due to a higher "temperature". Interestingly, the "mobility" μ turns out to be the same for the two markets.

Conclusion. – In extending the analogy between the particle motion and price change from the diffusion-dominant time regime inwardly to the sub-diffusion time regime, we identify the latter as a regime which features localized price changes. A number of kinetic parameters can be obtained by fitting the mean square log-return for the HF1MA, to the prediction of the Langevin equation. The averaging manipulation facilitates the description of the monthly data based on a reasonably good statistics which would be otherwise feasible only for long period of time [8]. The results obtained from such a coarse-graining procedure turn out to be inspiring. The endogenous kinetic parameters obtained in this study, for example, are passively reacting to the exogenous market turmoils. A generalized "hydrodynamic" scenario [21] could be established to describe the collective behavior in fluctuations of stock returns, so that a refined analysis based on the interplays between spatial and temporal features among stocks [13] would help to comprehend the origin of the hidden correlations [16] behind the financial time series. Indeed, the information of structured collective movements for a many-particle system is often contained in the signatures over various spatial-temporal correlations.

It would be valuable to extend the present work to study the data of a few other markets, not only to specify the market-dependent parameters, such as the temperature and the diffusivity revealed in this study, but also to identify any possible global parameter, of which the mobility-like parameter considered in present work could be a candidate. The analysis may help to quantify the trends of mutual affections among different markets [24]. It is also of interest to study the connection of parameters θ and D defined in the present paper with the similarity measure defined in [25] and to check whether the behaviors of θ and D have correlations with the behavior of the similarity measure under various situations.

* * *

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