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Orthometric Height Improvement in Tainan City using RTK GPS and Local Geoid Corrector Surface Models

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Abstract: Various corrector surface models are proposed to mitigate systematic height errors in order to fit the orthometric heights determined by combining the Tainan City real-time kinematic (RTK) global positioning system (GPS) network and the local geoid model data to the published orthometric heights. Several data sets for Tainan City were tested and analyzed. Two geometric geoid models and one gravimetric-geometric geoid model were generated using the GPS/leveling data. Consequently, three types of orthometric heights were determined (Model I, II, and III). The selection of the optimal corrector surface model for different Models was based on a series of statistical tests. The test results show that: (1) the selection of the optimal corrector surface model is highly related to the geoid model generating method, hence the optimal corrector surface models are a fifth-degree polynomial for Model I and II and a seven-parameter similarity transformation for Model III; and (2) the determined orthometric height is accurate to 2-4 cm after applying an optimal corrector surface model.

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Introduction

The ellipsoidal heights (*h*) from GPS refer to WGS 84; the orthometric heights (*H*) are referenced to the geoid. Thus, geoid heights (*N*) are needed to convert *h* into *H*, using the following equation:

$$H = h - N \tag{1}$$

An accurate geoid model is needed for localized GPS surveys. The geoid model can be generated by the gravimetric method, geometric method, or gravimetric-geometric method (Featherstone and Stewart 2001). Usually, the corrector surface model is applied to absorb datum inconsistencies between *H*, *h*, and *N,* and other systematic errors, such as the polynomial model (Benahmed Daho 2010; Erol et al. 2008; Vella 2003), or the similarity transformation model (Abdalla and Fairhead 2011; Iliffe et al. 2003).

Traditional real-time kinematic (RTK) GPS positioning uses a single reference station, and is often adversely affected by systematic errors, such as ionospheric and tropospheric

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delays. Since fast and stable internet access became available and the mature virtual reference station (VRS) technique was developed, the RTK GPS network has become an important tool of surveying engineering (Edwards et al. 2010; El-Mowafy et al. 2006; Erol et al. 2008; Yeh et al. 2012). There are some problems with height determination using RTK GPS, however, including system-related errors and user-related errors (Featherstone and Stewart 2001). The accuracy of *h* from RTK GPS and a multi-sensor kinematic surveying system are 1.5-4.0 cm (Edwards et al. 2010; El-Mowafy et al. 2006) and 4-7 cm (Gikas et al. 2013), respectively.

The accuracy of the orthometric heights determined by combining the RTK GPS network and the local geoid model data (hereafter referred to as "the determined orthometric heights") is at the level of 2-5 cm (El-Mowafy et al. 2006; Featherstone and Stewart 2001). In order to fit the determined orthometric heights to the published orthometric heights, the four-parameter similarity transformation model was proposed (Andritsanos et al. 2000; Benahmed Daho 2010; El-Mowafy et al. 2006).

Tainan City is located in the southern part of Taiwan Island and has a total area of about 2,192 km^2. In 2007, an RTK GPS reference network was established covering the whole city. Three data sets of Tainan City published or collected at different times (2004, 2009, and 2011) were tested to assess the accuracy of the determined orthometric heights from the RTK GPS reference network. Since three precise local geoid models (two geometric geoid models and one gravimetric-geometric geoid model) were generated from the GPS and leveling data,

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there were three types of determined orthometric height for each tested point. These determined orthometric heights were compared with the published orthometric heights. Any height differences were analyzed and examined statistically. According to the statistical *t*-test results on the mean value of the differences in orthometric height, there are systematic errors of 5-7 cm between the determined orthometric heights and the published orthometric heights.

The systematic errors may be due to: (1) Taiwan is situated at the convergent boundary of the Philippine Sea and the Eurasian plates (Chen et al. 2011); (2) the epochs of the three data sets corresponding to 2004, 2009, and 2011, and variation in the orthometric heights and ellipsoidal heights of the tested points depending on time; (3) the datum inconsistencies inherent among the three height types; (4) systematic errors in the geoid models; and (5) distortions in the orthometric height datum, etc.

In order to properly fit the determined orthometric heights to the published orthometric heights, various types of corrector surface models, such as polynomial models, similarity transformation models, conicoid fitting methods, and artificial neural networks, were proposed and tested using the three data sets. The optimal corrector surface model was selected based on the statistical test results. I will first introduce the concept and methodology of the proposed corrector surface models and the statistical procedures, then present the test results from the three data sets to demonstrate the performance of the optimal corrector surface models.

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Testing Methodology

Corrector Surface Models

Let us assume that the published orthometric height of a benchmark is H, and its determined orthometric height, from the measured ellipsoidal height and estimated geoid height, is $\hat{H}$. The difference between $\hat{H}$ and H is defined as:

$$\Delta H_i = \hat{H} - H_i, i = 1, 2, \cdots, n \tag{2}$$

where n indicates the total number of benchmarks.

In order to mitigate the systematic errors of the determined orthometric heights, various corrector surface models (fitting) were conducted. An appropriate corrector surface model should absorb the inconsistencies of the height sets and allow the determined orthometric heights to fit the published orthometric heights.

In Eq. (3), the function $F(\phi, \lambda)$ or $a_i^T x$ represents the corrector surface model. This function can take various forms and complexity levels (such as a simple bias, a bias and a tilt, higher-order polynomials, etc.) (Erol et al. 2008; Fotopoulos 2003):

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$$\Delta H_i = F(\phi,\lambda) + v_i = a_i^T x + v_i,\ i = 1, 2, \cdots, n \tag{3}$$

where $x(nx1)$ denotes the vector of unknown parameters, $a_i(nx1)$ represents the vector of known coefficients, (ϕ,λ) are the latitude and longitude of a point, and v_i is the residual random noise term.

Polynomial model

The polynomial functions used in this paper are of an order up to degree 8 (M=N=1,⋯,8) (Benahmed Daho 2010; Fotopoulos 2003; Vella 2003):

$$F(\phi,\lambda) = \sum_{m=0}^{M}\sum_{n=0}^{N}\left(\phi_i - \overline{\phi}\right)^n (\lambda_i - \overline{\lambda})^m x_q \tag{4}$$

In Eq. (4), (ϕ_i,λ_i) are the latitude and longitude of a corresponding point; and $\overline{\phi}$, $\overline{\lambda}$ are the mean latitude and longitude of all points in the data set. The q value can be ((N+1) x (M+1)) at most.

Similarity transformation model

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In the following equation of a classic four-parameter similarity transformation, the *x* vector has four elements (Andritsanos et al. 2000; Benahmed Daho et al. 2009; El-Mowafy et al. 2006; Fotopoulos 2003; Iliffe 2003; Kotsakis and Katsambalos 2010; Vella 2003; Ziebart et al. 2004):

$$F(\phi,\lambda) = x_1 + x_2 \cos\phi_i \cos\lambda_i + x_3 \cos\phi_i \sin\lambda_i + x_4 \sin\phi_i \tag{5}$$

The five-parameter similarity transformation is an extended version of Eq. (5) (Fotopoulos 2003; Vella 2003):

$$F(\phi,\lambda) = x_1 + x_2 \cos\phi_i \cos\lambda_i + x_3 \cos\phi_i \sin\lambda_i + x_4 \sin\phi_i + x_5 \sin^2\phi_i \tag{6}$$

The following equation is a more complicated form of a seven-parameter similarity transformation (Abdalla and Fairhead 2011; Benahmed Daho 2010; Fotopoulos 2003; Kiamehr 2011; Vella 2003):

$$\begin{aligned} F(\phi,\lambda) = {} & x_1 \cos\phi_i \cos\lambda_i + x_2 \cos\phi_i \sin\lambda_i + x_3 \sin\phi_i \\ & + x_4 (\frac{\sin\phi_i \cos\phi_i \sin\lambda_i}{W}) + x_5 (\frac{\sin\phi_i \cos\phi_i \cos\lambda_i}{W}) \\ & + x_6 (\frac{1 - f^2 \sin^2\phi_i}{W}) + x_7 (\frac{\sin^2\phi_i}{W}) \end{aligned} \tag{7}$$

where W equals $\sqrt{1-e^2\sin^2\phi_i}$, e^2 denotes the eccentricity, and f is the flattening of the reference ellipsoid.

Conicoid fitting method

The conicoid fitting method is usually used to construct geoid models using the geometric method (Hu et al. 2004; Lin 2007). Three conicoid fitting methods are tested in this paper, four-parameter (see Eq. (8)), six-parameter (see Eq. (9)), and ten-parameter conicoid fitting (see Eq. (10)):

$$F(\phi,\lambda) = x_1 + x_2\phi_i + x_3\lambda_i + x_4\phi_i\lambda_i \tag{8}$$

$$F(\phi,\lambda) = x_1 + x_2\phi_i + x_3\lambda_i + x_4\phi_i\lambda_i + x_5\phi_i^2 + x_6\lambda_i^2 \tag{9}$$

$$\begin{aligned} F(\phi,\lambda) = {} & x_1 + x_2\phi_i + x_3\lambda_i + x_4\phi_i\lambda_i + x_5\phi_i^2 + x_6\lambda_i^2 \\ & + x_7\phi_i^3 + x_8\phi_i^2\lambda_i + x_9\phi_i\lambda_i^2 + x_{10}\lambda_i^3 \end{aligned} \tag{10}$$

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Suppose there are n benchmarks with known latitude, longitude, and ΔH in a certain test area. One of the above-mentioned corrector surface models can be used to fit the values of ΔH, so the matrix system of observation equations can be expressed as follows:

$$Ax = \Delta H + v \tag{11}$$

where A denotes the design matrix composed of one row a_i^T for each observation ΔH_i. The unknown parameters x of the parametric models can be determined through least squares adjustment (Ghilani 2010).

Artificial neural network

Artificial neural networks (ANNs) are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As in nature, the network function is determined largely by the connections between elements. The network is adjusted, based on a comparison of the output and the target, until the network output matches the target. Typically, many such input/target pairs are used in this supervised learning to train a network (Hu et al. 2004; Kavzoglu and Saka 2005; Lin 2007).

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Back-propagation (BP) was created by generalizing the Widrow-Hoff learning rule to multiple-layer artificial neural networks and nonlinear differentiable transfer functions. The architecture of a back-propagation artificial neural network (BP ANN) is the multilayer feed-forward network. Feed-forward networks often have one input layer and one or more hidden layers of sigmoid neurons followed by an output layer of linear neurons (Hu et al. 2004; Kavzoglu and Saka 2005; Lin 2007). From Kavzoglu and Saka (2005), the BP learning algorithm has been used in about 70% of all ANN applications. The BP learning algorithm has also been used in recent studies on geoid approximation using ANN (Hu et al. 2004; Kavzoglu and Saka 2005; Lin 2007). BP ANN was thus selected to model local geoid and corrector surfaces in this paper.

The use of BP ANNs is complicated, essentially due to problems encountered in their design and implementation. From the design perspective, the specification of the number and size of the hidden layer(s) is critical for the network's capability to learn and generalize. A further difficulty in the use of BP ANNs is the choice of appropriate values for network parameters that have a major influence on the performance of the learning algorithm. It is often the case that a number of experiments are required to ascertain the selection of the parameter values that give the highest accuracy. A trial-and-error strategy is frequently used to determine appropriate values for these parameters (Hu et al. 2004; Kavzoglu and Saka 2005; Lin 2007).

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Suppose there are n reference points in a specific region. The reference point set $P=\{P_1, P_2, \cdots, P_n\}$ can be used to train the BP ANN:

$$P_i = (\phi_i, \lambda_i, \Delta H_i),\ i = 1, 2, \cdots, n \tag{12}$$

It should be noted that a three-layer BP ANN with one input layer, one hidden layer, and one output layer was adopted in this paper to generate a corrector surface model. Hence, the input vector consists of (ϕ_i, λ_i) and the output vector consists of ΔH_i. The number of neurons in the hidden layer is determined by trial and error.

After being trained by the reference point set P, the BP ANN establishes the functional relationship between input layer (ϕ_i, λ_i) and output layer ΔH_i:

$$\Delta H_i = F(\phi, \lambda)\ , i = 1, 2, \cdots, n \tag{13}$$

where $F(\phi, \lambda)$ is a function which associates input vectors (ϕ_i, λ_i) with specific output vectors ΔH_i. It should be noted that the main function of $F(\phi, \lambda)$ is similar to that of the coefficients of the parametric models. However, the main function of $F(\phi, \lambda)$ is determined implicitly by the neurons in the hidden layer of the BP ANN.

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Statistical Analysis Procedures

In order to mitigate the systematic errors of the determined orthometric heights, several tests were conducted upon the determined orthometric heights by varying the type of corrector surface model to find an adequate functional representation of the correction necessary for them to fit the published orthometric heights. All the benchmarks of a test data set were divided into two groups, reference points and check points. The reference points were used to estimate the coefficients of the parametric models, or train the artificial neural network. The check points were used to evaluate the performance of the tested corrector surface models.

The selection of the optimal corrector surface model depends on a set of statistical tests (Erol et al. 2008). Firstly, empirical tests identified three optimal corrector surface model candidates – optimal, suboptimal, and the third best corrector surface model – for each determined orthometric height model. After fitting out the systematic errors estimated by the three optimal corrector surface models, the performance statistics of ΔH were analyzed across the check points for the three determined orthometric height models.

Next, a series of statistical tests were performed in order to further evaluate the performance of these three optimal corrector surface model candidates; the tested items included: (1) the improvement in σ ; (2) a two-tailed *t*-test on the mean value of ΔH; and (3) a

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two-tailed χ^2 test on the variance of ΔH.

The improvement in σ is defined as:

$$\text{Improvement in } \sigma = \frac{\sigma_{\text{pre-fit}} - \sigma_{post-fit}}{\sigma_{\text{pre-fit}}} \times 100\% \tag{14}$$

where $\sigma_{pre-fit}$ and $\sigma_{post-fit}$ denote the standard deviation of ΔH at the check points before and after applying a specified corrector surface model, respectively.

A two-tailed *t*-test with a significance level of $\alpha = 5\%$ was performed to check for the presence of any systematic errors in the sample as a whole by testing the deviation of the sample mean from the mean of its population, which is assumed to be zero. This test involves checking the sample mean $(\bar{y})$ of ΔH from the check points against the population mean ($\mu = 0.000$ m). The test is specified by the null hypothesis ($H_0 : \mu = \bar{y}$) versus the alternative hypothesis ($H_a : \mu \neq \bar{y}$) (Ghilani 2010).

A two-tailed χ^2 test with a significance level of $\alpha = 5\%$ was used to check if the variance of ΔH at the check points after applying a suboptimal (or the third best) corrector surface model was the same as after applying an optimal corrector surface model. The test involves checking the variance of ΔH (S^2) after applying a suboptimal corrector surface model against the variance of ΔH (σ^2) after applying an optimal corrector surface model. The test is specified by the null hypothesis ($H_0 : S^2 = \sigma^2$) versus the alternative hypothesis

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($H_a : S^2 \neq \sigma^2$) (Ghilani 2010).

Test Data

Published Orthometric Heights and Ellipsoidal Heights

A new national vertical datum, Taiwan Vertical Datum 2001 (TWVD2001), was established between 2000 and 2003 using the observations of geodetic leveling, GPS, and gravity collected at 2,065 newly established benchmarks within a 4,500 km network of first-order leveling lines by the Ministry of the Interior (MOI), Taiwan. These 2,065 benchmarks are roughly 2 km apart. The vertical datum surface was defined by the mean sea level of the Keelung harbor tide gauge, and the datum point was the benchmark K999 (Chen et al. 2011; You 2006). Strict specifications for fieldwork procedures and comprehensive corrections for systemic errors were applied to the leveling data (Chen et al. 2011). Whole adjustment computation of the networks was carried out in 2004 by fixing the height of the benchmark K999. In 2004, the MOI announced the orthometric heights (H^{04}), geodetic coordinates, and ellipsoidal heights (h^{04}) obtained from static GPS measurements of the 2,065 benchmarks. The accuracy of H^{04} and h^{04} is ± 8.8 mm and ± 36 mm, respectively (Lin 2007).

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Eurasian plates, significant crustal deformation rates have been reported in the region and measured with geodetic techniques (Chen et al. 2011). It is also known that some parts of Taiwan are constantly subject to subsidence due to an excess usage of ground water. The TWVD2001 benchmarks were therefore re-surveyed using geodetic leveling between 2005 and 2008. The MOI announced the new orthometric heights (H^{09}) of the 2,065 benchmarks in 2009.

Tainan City's RTK GPS Network

The Tainan City government started operating its RTK GPS network – which contains six reference stations (SCES, NJES, RFES, WHES, BKBL, and KAWN) and covers the whole city – in September 2007. A seventh reference station, YJLO, was installed in April 2010 to improve the accuracy and efficiency of RTK GPS surveying in the mountainous area (Tainan 2012). The network baseline lengths range from 14 to 38 km. Fig. 1 shows the distribution of the seven reference stations.

The coordinate solutions of Tainan City's RTK GPS network system were referenced to the Taiwan datum 1997 (TWD97), which is a three-dimensional geocentric datum connected to the International Terrestrial Reference Frame (ITRF) at epoch 1994.0 (You 2006). Field testing revealed that Tainan City's RTK GPS network system achieved the following accuracies: ±2 cm in plane coordinates and ±5 cm in ellipsoidal height (Tainan 2012).

Data Set Description

Three data sets of Tainan City region were tested. The first, which included h^{04}, H^{04}, and the geodetic coordinates of the 145 benchmarks, was used to generate the local geoid models.

The second data set included H^{09} of the 145 benchmarks (the same benchmarks as the first data set). The third data set, including the geodetic coordinates and h^{11} of 118 benchmarks (a subset of the 145 benchmarks), was determined in 2011 using an RTK GPS network approach. The second and the third data sets were used to evaluate the performance of the determined orthometric heights and the proposed corrector surface models. The geographical distribution of the first and the third data sets is shown in Fig. 1.

The Determined Orthometric Heights

Geoid Models for Tainan City

Two geometric methods, the back-propagation artificial neural network (BP ANN) and the conicoid fitting method, were used to generate the local geoid model of Tainan City (Hu et al.

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2004; Kavzoglu and Saka 2005; Lin 2007). This paper used a three-layer BP ANN, with one input layer, one hidden layer, and one output layer to generate a local geoid model. $2\times p\times 1$ BP ANN thus signifies that the input layer has two elements, the latitude and longitude (ϕ_i, λ_i) of each benchmark; the hidden layer has p neurons; and the output layer has one element, the geoid height N_i of each benchmark. The neuron number p is determined by trial and error.

The 145 benchmarks of the first data set were divided into two groups; that is, one group of reference points (109 points), and another of check points (36 points). Because the benchmarks' H^{04} and h^{04} are known, their geoid height values (N) can be calculated using Eq. (1). If the geoid height of each benchmark estimated by the trained BP ANN or conicoid fitting method is $\hat{N}$, the difference ΔN of each benchmark is defined as:

$$\Delta N_i = N_i - \hat{N}_i, i = 1, 2, \cdots, n \tag{15}$$

where n is the total number of benchmarks.

Trial and error tests revealed that a $2\times 35\times 1$ BP ANN and six-parameter conicoid fitting method offers superior local geoid model accuracy. Further, the geoid heights of the 36 check points were estimated using a local Taiwanese geoid model (the MOI model) determined by the gravimetric method (Lin 2007; You 2006), thus giving the three local geoid models tested

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in this paper. The geoid heights estimated by these three geoid models are designated as $\hat{N}^{04}$ since they are derived from the values of H^{04} and h^{04} from the first data set.

Table 1 compares BP ANN accuracy with other geoid height estimation methods. The term "$2\times35\times1$ BP ANN" denotes that BP ANN was used to estimate the check points' geoid height in conjunction with the "trainbr" training algorithm and with 35 neurons in the hidden layer. The "trainbr" is a network training function that updates weight and bias values according to Levenberg-Marquardt optimization (Lin 2007). The term "six-parameter conicoid fitting" indicates that the six-parameter conicoid fitting method was used to estimate the check points' geoid height. The term "MOI model (pre-fit)" denotes that the MOI model was used to estimate each check point's geoid height directly. The term "MOI model (post-fit)" denotes that the geoid heights estimated by the MOI model were fitted to the known geoid heights by the fifth-degree polynomial corrector surface model. σ (m), mean (m), max (m), and min (m) indicate the standard deviation, the mean value, the maximum value, and the minimum value of ΔN in meters, respectively.

Some comments can be made from Table 1: (1) the gravimetric geoid model, such as the MOI model, has a systematic error of 7.4 cm; (2) after fitting to the GPS/leveling data with the fifth-degree polynomial surface corrector model, the systematic error of the MOI model (pre-fit) has been mitigated; (3) BP ANN produces a more accurate estimation of geoid height than the other two methods; and (4) the mean values of ΔN produced from the geometric

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and gravimetric-geometric geoid models show that there was no significant systematic error in the estimated geoid heights.

Please note that the accuracy of the predicted geoid height depends on the following factors: the number of reference points, the distribution of reference points, the distance from the check points to the nearest reference point, etc. Taking the results of "six-parameter conicoid fitting" as an example, the distances from the check points to the nearest reference point in the above-mentioned test case (109 reference points and 36 check points), are in the range of 0.9-3.6 km (with a mean distance of 1.6 km). Among the 36 check points, there are 31 points less than 2 km from a reference point (the values of ΔN are -0.073 ~ +0.090 m), and five other points further than 2 km (the values of ΔN are -0.070 ~ +0.092 m). On the other hand, if the number of reference and check points are changed to seven and 138, respectively (the values of σ, mean, max, and min of ΔN are ±0.066, 0.043, 0.251, -0.150 m, respectively), the distances between the check points and the nearest reference point are in the range of 0.9-18.4 km (with a mean distance of 7.1 km). Among the 138 check points, there are 53 less than 5 km from the nearest reference point (the largest value of ΔN is 0.124 m), 52 points between 5 and 10 km away (the largest value of ΔN is 0.173 m), 26 points between 10 and 15 km away (the values of ΔN are -0.150 ~ +0.251 m), and another seven points further than 15 km (the largest value of ΔN is 0.245 m).

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Causes of Systematic Errors in the Determined Orthometric Heights

According to Eq. (1), the determined orthometric height of each benchmark of the third data set was calculated using the formula $\hat{H}^{11} = h^{11} - \hat{N}$. The geoid height $\hat{N}$ (denoted as $\hat{N}^{04}$) was estimated by one of the three generated geoid models, BP ANN, six-parameter conicoid fitting, and MOI (post-fit), using the first data set from 2004. The differences ΔH at the 118 benchmarks could be calculated from $\hat{H}^{11}$ and H^{09} (from the second data set) using Eq. (2).

According to Eq. (1) and (2), the difference between $\hat{H}^{11}$ and H^{09} is derived as:

$$\begin{aligned}
\Delta H &= \hat{H}^{11} - H^{09} = (h^{11} - \hat{N}^{11}) - (h^{09} - N^{09}) = (h^{11} - h^{09}) - (\hat{N}^{11} - N^{09}) \\
&= \left[(H^{11} + N^{11}) - (H^{09} + N^{09}) \right] - (\hat{N}^{11} - N^{09}) \\
&= (H^{11} - H^{09}) + [(N^{11} - N^{09}) - (\hat{N}^{11} - N^{09})] \\
&= (H^{11} - H^{09}) + (N^{11} - \hat{N}^{11}) \\
&= (H^{11} - H^{09}) + (N^{11} - \hat{N}^{04} - \Delta N^{11-04})
\end{aligned} \tag{16}$$

where H^{11} and N^{11} denote the orthometric height and geoid height from 2011, respectively, $\hat{N}^{04}$ represents the estimated geoid height from 2004, and ΔN^{11-04} indicates the geoid height difference between 2004 and 2011.

The difference between h^{11} and h^{04} is derived as:

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$$\begin{aligned} h^{11}-h^{04} &= (H^{11}+N^{11})-(H^{04}+N^{04}) = (H^{11}-H^{04})+(N^{11}-N^{04}) \\ &= (H^{11}-H^{09})+(H^{09}-H^{04})+(N^{11}-N^{04}) \end{aligned} \tag{17}$$

where N^{04} denotes the geoid height from 2004.

From Eq. (17), $\Delta\tilde{\mathrm{H}}$ is defined as:

$$\Delta\tilde{\mathrm{H}} = (\boldsymbol{h}^{11}-h^{04})-(H^{09}-H^{04}) = (H^{11}-H^{09})+(N^{11}-N^{04}) \tag{18}$$

From Eq. (16) and (18), the relationship between $\Delta\mathrm{H}$ and $\Delta\tilde{\mathrm{H}}$ is expressed as:

$$\begin{aligned} \Delta H &= \hat{H}^{11}-H^{09} = (H^{11}-H^{09})+(N^{11}-\hat{N}^{11}) \\ &= (H^{11}-H^{09})+(N^{11}-\hat{N}^{04}-\Delta N^{11-04}) \\ &\approx \Delta\tilde{H} = (\boldsymbol{h}^{11}-h^{04})-(H^{09}-H^{04}) \end{aligned} \tag{19}$$

According to Eq. (18), the $\Delta\tilde{H}$ values of the 118 benchmarks can be calculated from the known or observed values h^{04}, h^{11}, H^{04}, and H^{09} of the three data sets. From Eq. (19), the $\Delta\tilde{H}$ values can provide the estimate of $\Delta\mathrm{H}$. The performance statistics of various height differences at the 118 benchmarks from different epochs are shown in Table 2. “Model I” indicates the *ΔH* between $\hat{H}^{11}$ determined from h^{11} and $\hat{N}$, estimated by the BP ANN geoid model, and H^{09}. σ (m), mean (m), max (m), and min (m) indicate the standard deviation, the mean value, the maximum value, and the minimum value of *ΔH* in meters, respectively.

Further examining the values of $H^{09} - H^{04}$ for the 118 benchmarks, it was found that 57 benchmarks had uplifted (13 points uplifted more than 0.050 m) and 61 others had subsided (34 points subsided more than 0.050 m). On the other hand, examination the values of $h^{11} - h^{04}$ for the 118 benchmarks, found that 39 benchmarks had uplifted (18 points uplifted more than 0.050 m) and 79 others had subsided (64 points subsided more than 0.050 m).

From Eq. (19) and Table 2, it can be seen that the statistical values of $\Delta \mathrm{H}$ and $\Delta \tilde{\mathrm{H}}$ are very similar. Further examining the test results of Table 2 and Eq. (19), it is found that the systematic errors in the determined orthometric heights $\hat{H}^{11}$ may come from: (1) unknown orthometric height variation $(H^{11} - H^{09})$ caused by various geodynamic effects; (2) unknown geoid height variations $(N^{11} - \Delta N^{11-04})$; (3) datum inconsistencies and other possible systematic distortions in the three test data sets; (4) random noise in the values for ellipsoidal heights h^{11} and h^{04}, orthometric heights H^{04} and H^{09}, and geoid height $\hat{N}^{04}$; and (5) theoretic approximations in the computation of either *H* or *N*.

Performance Evaluation of the Determined Orthometric Heights

Table 3 summarizes the performance statistics of *ΔH* at the 118 benchmarks from the three determined orthometric height models. "Model II" denotes the *ΔH* between $\hat{H}^{11}$

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determined from h^{11} and $\hat{N}$, estimated by the six-parameter conicoid fitting geoid model, and H^{09}. "Model III" indicates the *ΔH* between $\hat{H}^{11}$ determined from h^{11} and $\hat{N}$, estimated by the MOI (post-fit) geoid model, and H^{09}.

A two-tailed *t*-test with a significance level of $\alpha = 5\%$ on the mean value of *ΔH* was also conducted to check the presence of any systematic errors in the determined orthometric heights of the 118 benchmarks. Table 3 shows that the standard deviations of *ΔH* from Model I, II, and III are ±0.050, ±0.063, and ±0.051 m, respectively. However, the mean values of Model I, II, and III are -0.051, -0.050, and +0.074 m, respectively. According to the results of the two-tailed *t*-test, all three Models reject the null hypothesis H_0, and there is statistical reason to believe that there are significant systematic errors in the determined orthometric heights $\hat{H}$.

Data Analysis

Strategy of Selecting the Optimal Corrector Surface Models

Three types of *ΔH*, Model I, II, and III, were tested on the 118 benchmarks of the third data set to evaluate the performance of the various corrector surface models. Eighty-nine points of the 118 benchmarks were selected as reference points; the other 29 were defined as check

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points. The selection of the optimal corrector surface models depends on a series of empirical and statistical tests, as described in the section "Statistical Analysis Procedures".

Geoid Model Evaluation

Model I

Table 4 shows the performance improvement statistics of ΔH at the 29 check points from Model I, before and after correcting systematic errors estimated by the three candidates for optimal corrector surface model. The term "pre-fit" denotes that no corrector surface model is applied. The term "2x2x1 BP ANN & six-parameter conicoid fitting" indicates that a 2x2x1 BP ANN is applied first to mitigate the systematic errors of ΔH, followed by a six-parameter conicoid fitting to mitigate the residual systematic errors of ΔH. Note that the output layer of a 2x2x1 BP ANN has one element, the *dH* value of each benchmark. The *dH* value is defined as:

$$dH_i = \Delta H_i - \Delta \bar{H}, i = 1, 2, \cdots, n \tag{20}$$

where $\Delta \bar{H}$ denotes the mean value of ΔH at all benchmarks of the test area.

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The term "2x8x1 BP ANN & 2x5x1 BP ANN" denotes that a 2x8x1 BP ANN is applied first to mitigate the systematic errors of ΔH, then a 2x5x1 BP ANN, to mitigate the residual systematic errors of ΔH. Note that the output layer of a $2 \times p \times 1$ BP ANN has one element, the ΔH value of each benchmark. The term "the fifth-degree polynomial" indicates that a fifth-degree polynomial is applied. It can be seen in Table 4 that the three optimal corrector surface model candidates mitigate the systematic errors of ΔH effectively and improve the accuracy of ΔH. The fifth-degree polynomial performed best, however.

Table 5 summarizes the improvement in σ, *t*-test, and χ^2 test from Model I. One can see that: (1) the improvement in σ after applying 2x2x1 BP ANN & six-parameter conicoid fitting, 2x8x1 BP ANN & 2x5x1 BP ANN, and the fifth-degree polynomial, are 38.9%, 40.7%, and 48.1%, respectively; (2) from the *t*-test results on the mean value of ΔH it can be seen that, after applying any one of the three optimal corrector surface model candidates, all accept the null hypothesis and there is statistical reason to believe that the mean value of ΔH is equal to 0.000 m; and (3) from the χ^2 test results on the variance of ΔH, it can be seen that, after applying 2x2x1 BP ANN & six-parameter conicoid fitting and 2x8x1 BP ANN & 2x5x1 BP ANN, all accept the null hypothesis and there is no statistical reason to believe that the variance of the fifth-degree polynomial is statistically different from that of 2x2x1 BP ANN & six-parameter conicoid fitting or 2x8x1 BP ANN & 2x5x1 BP ANN. In other words, these three candidates are all treated as the optimal corrector surface models of Model I.

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Model II

Table 6 shows the performance improvement statistics of *ΔH* at the 29 check points from Model II, before and after correcting systematic errors estimated by the three optimal corrector surface model candidates. It can be seen from Table 6 that the three optimal corrector surface model candidates mitigate the systematic errors of *ΔH* effectively and improve the accuracy of *ΔH*. Again, the fifth-degree polynomial performed best of the three.

Table 7 summarizes the statistics of performance improvement in σ, *t*-test, and χ^2 test from Model II. It can be seen that: (1) the improvement in σ after applying the 10-parameter conicoid fitting, the 2x2x1 BP ANN & 10-parameter conicoid fitting, and the fifth-degree polynomial, are 41.9%, 43.6%, and 54.8%, respectively; (2) from the *t*-test results on the mean value of *ΔH*, it can be seen that, after applying any one of the three optimal corrector surface model candidates, all accept the null hypothesis; and (3) from the χ^2 test results on the variance of *ΔH*, it can be seen that after applying a suboptimal corrector surface – the 2x2x1 BP ANN & 10-parameter conicoid fitting – it accepts the null hypothesis. However, the χ^2 test of the 10-parameter conicoid fitting rejects the null hypothesis. Thus, only the fifth-degree polynomial and 2x2x1 BP ANN & 10-parameter conicoid fitting are considered optimal corrector surface models of Model II. The 10-parameter conicoid fitting is

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considered a suboptimal corrector surface model.

Model III

Table 8 shows the performance improvement statistics of ΔH at the 29 check points from Model III, before and after correcting systematic errors estimated by the three optimal corrector surface model candidates. We see in Table 8 that the three candidates for optimal corrector surface model all mitigate the systematic errors of ΔH effectively and improve the accuracy of ΔH. The seven-parameter similarity transformation performed best, however.

Table 9 summarizes the statistics of performance improvement in σ, *t*-test, and χ^2 test from Model III. It can be seen that: (1) the improvement in σ after applying the fifth-degree polynomial, 2x11x1 BP ANN & the fifth-degree polynomial, and seven-parameter similarity transformation, are 22.9%, 24.6%, and 34.4%, respectively; (2) from the *t*-test results on the mean value of ΔH, it can be seen that, after applying any one of these three optimal corrector surface model candidates, all accept the null hypothesis; and (3) from the χ^2 test results on the variance of ΔH, it can be seen that, after applying the fifth-degree polynomial and 2x11x1 BP ANN & the fifth-degree polynomial, all accept the null hypothesis. In other words, all three candidates are treated as optimal corrector surface models of Model III.

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Comparative Analysis and Discussion

The performance improvement statistics of ΔH at the 29 check points are summarized in Table 10 in order to compare performance after applying the optimal surface corrector model to the three determined orthometric height models. Statistically, according to the above-mentioned test results, there is more than one optimal corrector surface model for Models I, II, and III. For clarity, only the corrector surface model with the smallest standard deviation of ΔH is treated as the optimal corrector surface model in this section. Thus the fifth-degree polynomial is treated as an optimal corrector surface model of Model I.

"Optimal corrector surface model" denotes that the tested corrector surface model has the smallest standard deviation of ΔH in each determined orthometric height model. $\sigma_{pre-fit}$ and $mean_{pre-fit}$ represent the standard deviation and the mean value of ΔH in meters before applying any corrector surface model, respectively. $\sigma_{post-fit}$ and $mean_{post-fit}$ denote the standard deviation and the mean value of ΔH in meters after applying the optimal corrector surface model, respectively.

It can be seen that: (1) applying an optimal corrector surface model improves the performance of the determined orthometric height; (2) the optimal corrector surface models for Models I, II, and III are the fifth-degree polynomial, the fifth-degree polynomial, and seven-parameter similarity transformation, respectively; (3) after applying an optimal

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corrector surface model, the standard deviation of ΔH from Model I and Model II were identical (±0.028 m) and smaller than that of Model III (±0.040 m); and (4) after applying an optimal corrector surface model, the mean values of ΔH from Models I, II, and III were 0.003 m, 0.003 m, and -0.009 m, respectively.

Table 11 summarizes the statistics of performance improvement in σ, *t*-test, and χ^2 test from Models I, II, and III. It can be seen that: (1) the improvement in σ of Models I, II, and III after applying an optimal corrector surface model are 48.1%, 54.1%, and 22.9%, respectively; (2) from the *t*-test results on the mean value of ΔH, it can be seen that after applying an optimal corrector surface, Models I, II, and III all accept the null hypothesis; and (3) in the case of Model III, the χ^2 test results on variance of ΔH reject the null hypothesis and there is statistical reason to believe that the variance of Model III is statistically different from that of Models I and II.

In applying the corrector surface model to fit the gravimetric geoid model to GPS/leveling derived geoid height, the selection of the optimal corrector surface model varies from case to case. For example, Benahmed Daho (2010) found that a third-degree polynomial model was adequate for application in the north of Algeria. On the other hand, Erol et al. (2008) reported that they applied different degrees of polynomial model in different regions of Turkey.

Regarding the application of the corrector surface model to fit the determined orthometric heights to the published orthometric heights, only the four-parameter similarity transformation was tested. For example, Benahmed Daho (2010) used a third-degree polynomial model to fit a gravimetric geoid model to the GPS/leveling derived geoid height first, then applied a four-parameter similarity transformation to fit the determined orthometric heights to the published orthometric heights, with an accuracy of 2-3 cm.

Based on Eq. (19), the values of ΔH from Model I, II, and III, equal $(H^{11}-H^{09})+(N^{11}-\hat{N}^{04}-\Delta N^{11-04})$. Among them, the common terms are $(H^{11}-H^{09})+N^{11}$, and the different parts are $(-\hat{N}^{04}-\Delta N^{11-04})$ which depend on the geoid models used. Please note that the geoid models of Model I and II are generated by the geometric method from GPS/leveling data. And the geoid models of Model III are generated by the gravimetric-geometric method from GPS/leveling data. Based on the test results, it is found that: (1) a fifth-degree polynomial can establish the functional relationship between the ΔH values and the plane coordinates of 118 benchmarks properly; (2) the ΔH statistics after applying a fifth-degree polynomial pass a series of statistical tests; and (3) therefore, a fifth-degree polynomial is selected as the optimal corrector surface model for Model I and II. On the other hand, after a series of empirical and statistical tests, a seven-parameter similarity transformation is selected as the optimal corrector surface model for Model III.

Assume that $\Delta\tilde{H}^{11}$ and $\Delta\tilde{H}^{13}$ denote the estimate values of ΔH of the 118 benchmarks of different epoch 2011 and 2013 respectively. And, h^{13} denote the h values of

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the 118 benchmarks of 2013. From Eq. (18) and (19), it can be found that the only different part between $\Delta\tilde{H}^{11}$ and $\Delta\tilde{H}^{13}$ is the h value ($h^{11} \neq h^{13}$). From Eq. (19), the $\Delta\tilde{H}^{13}$ values can provide the estimate of $\Delta\mathrm{H}^{13}$. Assume that the accuracies of h^{11} and h^{13} remain the same, then, theoretically speaking, the standard deviation of the $\Delta\mathrm{H}^{13}$ values should equal to that of the $\Delta\mathrm{H}^{11}$ values of the 118 benchmarks. After applying the optimal corrector surface model, the performance of the determined orthometric height should be improved.

In order to further evaluate the performance of the optimal corrector surface model from an epoch different from the current ones (2011), several simulated data set were tested. Let $\Delta h = h^{13} - h^{11}$ denote the ellipsoidal height differences of 118 benchmarks from 2011 to 2013. Different Δh values, such as -0.05m, -0.15m, and -0.50m were added to the values of $\Delta\mathrm{H}^{11}$ of Model I, II, and III, to get the values of $\Delta\mathrm{H}^{13}$. Then, the optimal corrector surface models were applied to correct the systematic errors. It was found that the performance statistics of $\Delta\mathrm{H}^{13}$ at the 29 check points after applying the optimal corrector surface models from Model I, II, and III were almost identical to those of Tables 4 to 11. Hence, it is believe that the selected optimal corrector surface models for Model I, II, and III are still valid for data set of the same area from different epochs. However, it is recommend that the parameters of the optimal surface corrector models must be re-estimated in the future.

Hence, although it is difficult to indicate strict methodologies and specific rules for determination of the optimal corrector surface model for Model I, II, and III based on the

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discussion above, the following comments can be made from the above-mentioned test results: (1) the fifth-degree polynomial, BP ANN & BP ANN, and BP ANN & conicoid fitting models can be applied to fit the determined orthometric height to the published orthometric heights if the geoid models are generated by the geometric method from GPS/leveling data; (2) if the geoid models are generated by the gravimetric-geometric method, similarity transformation and BP ANN & the fifth-degree polynomial models can be applied; and (3) the parameters of the optimal surface corrector models must be re-estimated if the epoch of the data sets used is different from 2011, since the values of $(H^{11}-H^{09})+N^{11}$, $(-\hat{N}^{04}-\Delta N^{11-04})$, etc. are time-dependent variables, especially in a region subject to serious geodynamic effects.

Conclusions

Various corrector surface models were proposed and tested in order to mitigate or eliminate the systematic errors of the determined orthometric heights from the RTK GPS network ellipsoidal heights and local geoid model data. Three data sets were used to test the proposed corrector surface models. An optimal corrector surface model was selected based on a series of empirical and statistical tests.

Based on the test results, the following comments can be made: (1) the selection of the optimal corrector surface model is highly dependent on the geoid model generating method;

(2) the optimal corrector surface models for Model I are the fifth-degree polynomial, 2x8x1 BP ANN & 2x5x1 BP ANN, and 2x2x1 BP ANN & six-parameter conicoid fitting; (3) the optimal corrector surface models for Model II are the fifth-degree polynomial and 2x2x1 BP ANN & 10-parameter conicoid fitting; (4) the optimal corrector surface models for Model III are the seven-parameter similarity transformation, 2x11x1 BP ANN & the fifth-degree polynomial, and the fifth-degree polynomial; (5) the performance of the determined orthometric heights can be improved after applying the optimal corrector surface model; (6) the accuracy of the determined orthometric heights can be improved from 5-6 cm to 2-4 cm after mitigating the systematic errors using an optimal corrector surface model; and (7) the parameters of the optimal surface corrector models must be re-estimated if the epoch of the data sets used is different from 2011, especially in a region subject to serious geodynamic effects, such as Taiwan.

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List of figure captions

Fig 1.pdf

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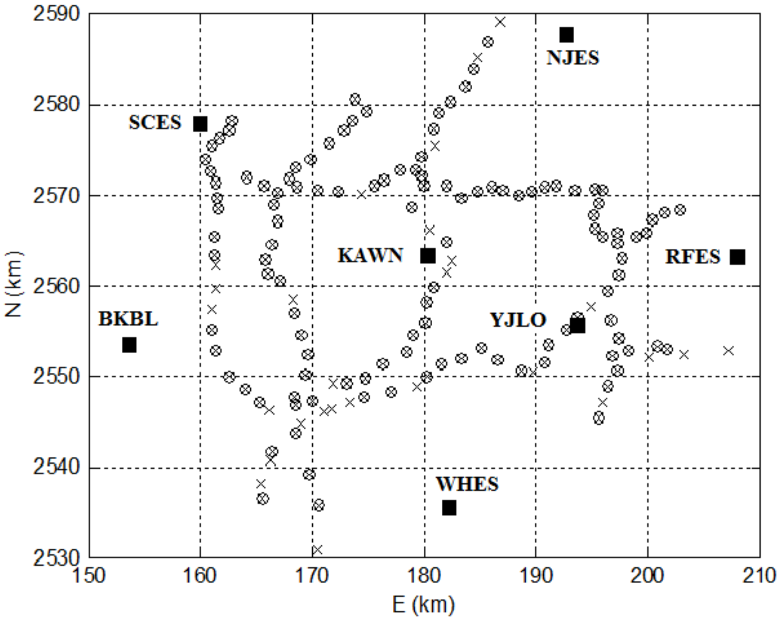

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Table 1. BP ANN accuracy compared with other methods of geoid height estimation

Estimation method	σ (m)	mean (m)	max (m)	min (m)
$2\times35\times1$ BP ANN	±0.024	0.000	0.059	-0.081
six-parameter conicoid fitting	±0.048	0.002	0.125	-0.128
MOI model (pre-fit)	±0.051	0.074	0.260	-0.037
MOI model (post-fit)	±0.043	-0.005	0.096	-0.171

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Table 2. Performance statistics of various height differences at 118 benchmarks from different epochs

Height difference	σ (m)	mean (m)	max (m)	min (m)
$(h^{11} - h^{04})$	±0.130	-0.073	0.322	-0.759
$(H^{09} - H^{04})$	±0.106	-0.021	0.334	-0.771
$\Delta\tilde{H}$	±0.041	-0.052	0.045	-0.119
ΔH (Model I)	±0.050	-0.051	0.061	-0.213

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Table 3. Performance statistics of ΔH at the 118 benchmarks from three determined orthometric height models

Model	σ (m)	mean (m)	max (m)	min (m)	t-test on mean (α=5%)
I	±0.050	-0.051	0.061	-0.213	reject H_0
II	±0.063	-0.050	0.109	-0.241	reject H_0
III	±0.051	+0.074	0.260	-0.037	reject H_0

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Table 4. Performance improvement statistics of ΔH at the 29 check points from Model I, before and after correcting systematic errors estimated by the three optimal corrector surface models

Corrector surface model	σ (m)	mean (m)	max (m)	min (m)
pre-fit	±0.054	-0.049	0.060	-0.213
2x2x1 BP ANN & six-parameter conicoid fitting	±0.033	0.003	0.062	-0.105
2x8x1 BP ANN & 2x5x1 BP ANN	±0.032	0.005	0.075	-0.106
the fifth-degree polynomial	±0.028	0.003	0.055	-0.097

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Table 5. Performance improvement statistics in σ, t-test and χ^2 test from Model I

Corrector surface model	Improvement in σ (%)	t-test (α=5%)	χ^2 test (α=5%)
pre-fit	00.0	reject H_0	reject H_0
2x2x1 BP ANN & six-parameter conicoid fitting	38.9	accept H_0	accept H_0
2x8x1 BP ANN & 2x5x1 BP ANN	40.7	accept H_0	accept H_0
the fifth-degree polynomial	48.1	accept H_0	

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Table 6. Performance improvement statistics of ΔH at the 29 check points from Model II, before and after correcting systematic errors estimated by the three optimal corrector surface model candidates

Corrector surface model	σ (m)	mean (m)	max (m)	min (m)
pre-fit	±0.062	-0.045	0.038	-0.241
10-parameter conicoid fitting	±0.036	0.008	0.095	-0.122
2x2x1 BP ANN & 10-parameter conicoid fitting	±0.035	0.002	0.082	-0.126
the fifth-degree polynomial	±0.028	0.003	0.061	-0.098

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Table 7. Performance improvement statistics in σ, *t*-test and χ^2 test from Model II

Corrector surface model	Improvement in σ (%)	*t*-test (α=5%)	χ^2 test (α=5%)
pre-fit	00.0	reject H_0	reject H_0
10-parameter conicoid fitting	41.9	accept H_0	reject H_0
2x2x1 BP ANN & 10-parameter conicoid fitting	43.6	accept H_0	accept H_0
the fifth-degree polynomial	54.8	accept H_0	

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Table 8. Performance improvement statistics of ΔH at the 29 check points from Model III, before and after correcting systematic errors estimated by the three optimal corrector surface model candidates

Corrector surface model	σ (m)	mean (m)	max (m)	min (m)
pre-fit	±0.061	0.043	0.120	-0.219
the fifth-degree polynomial	±0.047	0.007	0.103	-0.188
2x11x1 BP ANN & the fifth-degree polynomial	±0.046	0.007	0.187	-0.104
seven-parameter similarity transformation	±0.040	-0.009	0.074	-0.114

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Table 9. Performance improvement statistics in σ, *t*-test and χ^2 test from Model III

Corrector surface model	Improvement in σ (%)	*t*-test (α=5%)	χ^2 test (α=5%)
pre-fit	00.0	reject H_0	reject H_0
the fifth-degree polynomial	22.9	accept H_0	accept H_0
2x11x1 BP ANN & the fifth-degree polynomial	24.6	accept H_0	accept H_0
seven-parameter similarity transformation	34.4	accept H_0	

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Table 10. Performance improvement statistics of ΔH at the 29 check points after applying the optimal corrector surface model from Model I, Model II, and Model III

Model	Optimal corrector surface model	$\sigma_{pre-fit}$ (m)	$\sigma_{post-fit}$ (m)	$mean_{pre-fit}$ (m)	$mean_{post-fit}$ (m)
I	the fifth-degree polynomial	±0.054	±0.028	-0.049	0.003
II	the fifth-degree polynomial	±0.062	±0.028	-0.045	0.003
III	seven-parameter similarity transformation	±0.061	±0.040	0.043	-0.009

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Table 11. Performance improvement statistics in σ, t-test and χ^2 test from Model I, II, and III

Model	Optimal corrector surface model	Improvement in σ (%)	t-test (α=5%)	χ^2 test (α=5%)
I	the fifth-degree polynomial	48.1	accept H_0	
II	the fifth-degree polynomial	54.8	accept H_0	accept H_0
III	seven-parameter similarity transformation	22.9	accept H_0	reject H_0