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Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity

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ABSTRACT

Trade credit financing is increasingly recognized as an important strategy to increase profitability in Inventory Management. We revisit an economic order quantity model under conditionally permissible delay in payments, in which the supplier offers the retailer a fully permissible delay of M periods (i.e., there is no interest charge until M) if the retailer orders more than or equal to a predetermined quantity W. However, if the retailer's order quantity is less than W, then the retailer must make a partial payment to the supplier, and enjoy a permissible delay of M periods for the remaining balance. In this paper, we extend the mentioned EOQ under conditionally permissible delay in payments to complement some shortcomings of the model. By contrast to the differential calculus method, we propose a simple arithmetic–geometric method to solve the problem. Furthermore, we establish some discrimination terms to identify the unique optimal solution among three alternatives, and explain those theoretical results by simply economical interpretations. Finally, we solve several numerical examples to illustrate the theoretical results and obtain some managerial implications.

1. Introduction

The effect of the trade credit on the lot sizes has been studied in various researches. In practice, the supplier usually offers to the retailer a permissible delay in payments. During the permissible delay period, there is no interest charge. Hence, the retailer can earn the interest from sales revenue. However, if the payment is not paid in full by the end of the permissible delay period, then the supplier charges to the retailer an interest on the unpaid amount of the purchasing cost. It is important to remark that the permissible delay in payments produces two benefits to the supplier: (1) It should attract new customers who consider it to be a type of price reduction, and (2) It should avoid lasting price competition. On the other hand, the policy of granting a permissible delay adds not only an additional cost (i.e., the opportunity cost for receiving the purchasing amount at the end of the permissible delay, instead of immediately) but also an additional dimension of default risk (i.e., the event in which the buyer will be unable to pay off its debt obligations) to the supplier because the longer the permissible delay, the higher the default risk.

In 1913, the economic order quantity (EOQ) was proposed by Harris (1913). Since then a huge of extensions of the EOQ inventory model have been developed by researchers. A classical paper by Goyal (1985) develops an EOQ inventory model for the buyer when the seller offers a permissible delay in payments. Goyal (1985)'s model can be considered as one of seminal works in trade credit field because a stream of researches have been emerging from it in recent years. Later, Shah (1993) considers a stochastic inventory model when delays in payments are permissible. Subsequently, Jamal et al. (1997) extends Goyal (1985)'s model to allow shortages. At the same time, Hwang and Shinn (1997) add the pricing strategy to the model, and derive jointly the optimal price and lot sizing for a retailer under the condition of permissible delay in payments. Moreover, it should be noted that Teng (2002) provides an alternative conclusion from Goyal (1985), and proves that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. One year later, Chang et al. (2003) develop an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Simultaneously, Huang (2003) extends Goyal (1985)'s model to develop an EOQ model in which the supplier offers the retailer the up-stream trade credit period M, and the retailer in turn provides the down-stream trade credit period N (with $N \le M$) to his/her customers. However, Huang

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(2003)'s model has some flaws. In this context, Teng and Goyal (2007) complement the shortcoming of Huang (2003)'s model and propose a generalized formulation. Further, Teng (2009a) establishes an EOQ model for a retailer who offers distinct trade credits to its good and bad credit customers. After, Chang et al. (2010) present the optimal manufacturer's replenishment policies in a supply chain with up-stream and down-stream trade credits. In a recent study, Teng et al. (2011) obtain the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. In contrast to previous mentioned inventory models, which implicitly assume that whole lot is delivered to the retailer completely perfect, Lin et al. (2012) propose an integrated supplier–retailer inventory model in which both the supplier and the retailer adopt trade credit policies, and the retailer receives some defective items. While Cheng et al. (2012) consider different financial environments when the supplier provides a permissible delay in payments. Recently, Jaggi et al. (2013) develop a new inventory model for imperfect quality items with allowable shortages under permissible delay in payments. Many related articles can be found in Chang et al. (2008), Goyal et al. (2007), Huang and Hsu (2008), Kreng and Tan (2010, 2011), Liao (2008), Ouyang et al. (2005, 2006), Shinn and Hwang (2003), Soni and Shah (2008), Teng and Lou (2012), and their references.

Huang (2007) the first proponent proposes an EOQ model under conditionally permissible delay in payments in which the supplier offers the retailer a fully permissible delay of M periods if the retailer orders more than or equal to a predetermined quantity W. However, if the retailer orders less than W, then the retailer must pay a portion of the total purchasing cost to the supplier immediately meanwhile enjoy a permissible delay of M periods for the remaining balance. The model is realistic and relevant. However, some of his mathematical expressions and graphical figures on both the interest earned and the interest charged are inappropriate. In this paper, we extend Huang (2007)'s model to complement some shortcomings of the model. In contrast to the differential calculus method, we also propose an easy-to-understand and simple-to-apply arithmetic–geometric method to solve the inventory problem. Furthermore, we establish some discrimination terms to identify the unique optimal solution among three alternatives, and explain those theoretical results by simply economical interpretations. Finally, we run computer programs for several numerical examples to illustrate the theoretical results and obtain some managerial implications.

2. Notation and assumption

To develop the EOQ model with conditionally permissible delay in payments, we use the following notation throughout this paper.

D	the annual demand rate in units
A	the ordering cost in dollars per order
W	the pre-determined quantity in units by the supplier at which the fully permissible delay in payments is granted
c	the unit purchase cost in dollars
p	the unit selling price in dollars, which is greater than or equal to the unit purchase cost c
h	the holding cost in dollars per unit per year excluding interest charges.
I_e	the retailer's investment return rate (or interest earned) per dollar per year
I_k	the supplier's interest charged to the retailer per dollar per year
M	the length of the permissible delay in years offered by the supplier
α	the percentage of the purchase amount is granted the permissible delay in payments by the supplier
T	the retailer's replenishment cycle time in years
Q	the retailer's order quantity in units
T_W	the time interval that W units are depleted to zero due to annual demand rate D, hence $T_w = W/D$
$TRC(T)$	the retailer's annual total relevant cost in dollars, which is a function of T
T^*	the retailer's optimal replenishment cycle time of $TRC(T)$
Q^*	the retailer's optimal order quantity, which is DT^*

Likewise, we adopt the following assumptions in developing our proposed model as shown below:

1. If the retailer's order quantity $Q \geq W$ (i.e., $T \geq T_W$), then fully permissible delay in payments is granted by the supplier. Hence, the retailer is allowed to pay the total purchase amount cQ at the end of the permissible delay M. If $Q < W$, then partial permissible delay in payment is granted, in which the retailer must immediately make a partial payment of $(1-\alpha)cQ$, and then must pay off the remaining balance of $\alpha c\, Q$ at the end of the permissible delay M.
2. The supplier charges the retailer the prime rate I_k on unpaid balance after the permissible delay. On the other hand, the retailer may invest sales revenue into stock markets or to develop new products, and get a return rate on investment I_e during the permissible delay period.
3. Demand rate is known and constant.
4. Replenishments are instantaneous.
5. Shortages are not allowed.
6. Only one type of product is considered.
7. The time horizon is infinite.
8. While the account is not settled, the revenue is placed in an interest bank account in order to earn interests.

Now, we are ready to build up the mathematical model for the retailer to obtain its optimal order quantity.

3. Mathematical formulation

The annual total relevant cost $TRC(T)$ based on M and T_W consists of the following two possible cases: (1) $T_W \leq M$, and (2) $T_W > M$. We discuss them separately below.

3.1. Case 1: $T_w \leq M$

(a) The annual ordering cost is A/T.
(b) The annual holding cost excluding interest charges is $h\, DT/2$.
(c) The annual opportunity cost of capital has three possible subcases because the replenishment cycle time T may lie in three possible positions between T_w and M.

Sub-case 1.1: $T_w \leq M < T$. Since $T_w \leq M < T$, the retailer receives the fully permissible delay as shown in Fig. 1. The interest earned per cycle is the return rate I_e multiplied by the area of the triangle OVM (i.e., $pI_eDM^2/2$). Therefore, the annual interest earned is $pI_eDM^2/2T$. Likewise, the interest charged is the interest rate I_k multiplied by the area of the triangle MXT (i.e., $cI_kD(T-M)^2/2$). Hence, the annual interest charged is $cI_kD(T-M)^2/2T$, and the annual opportunity cost of capital is

$$\frac{cI_kD(T-M)^2 - pI_eDM^2}{2T}. \tag{1}$$

Sub-case 1.2: $T_w \leq T \leq M$. Since $T_W \leq T$, the retailer receives the fully permissible delay as shown in Fig. 2. There is no interest

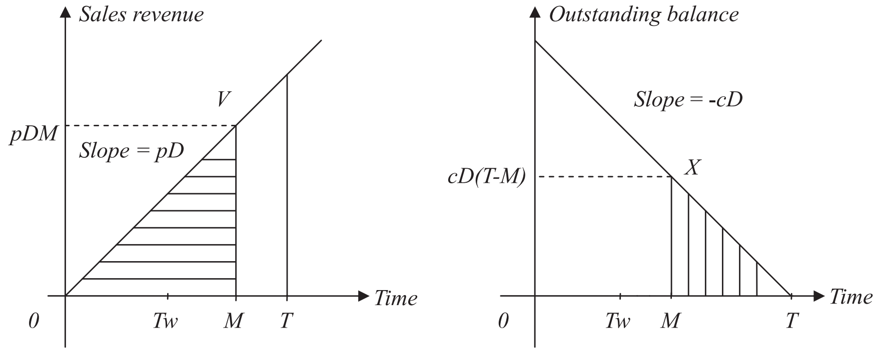

Fig. 1. Delayed payment for $T_w \leqq M < T$.

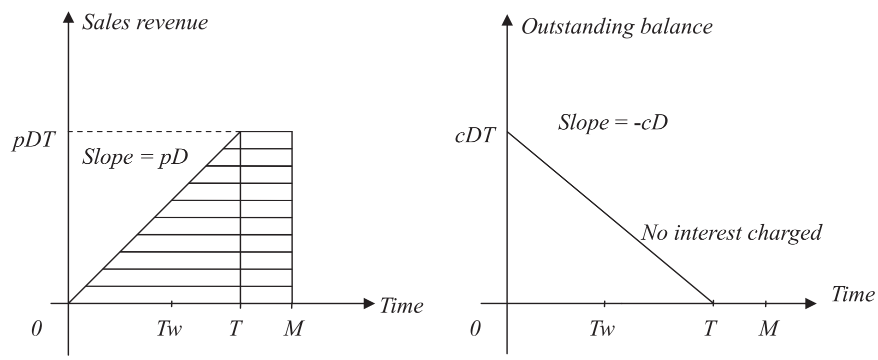

Fig. 2. Delayed payment for $T_w \leqq T \leqq M$.

charge because of $T \leq M$. The interest earned per cycle is the return rate I_e multiplied by the area of the trapezoid on the interval [0, M]. Therefore, the annual interest earned in this case is

$$\frac{pI_eDT(2M-T)}{2T} = pI_eD\left(M - \frac{T}{2}\right). \tag{2}$$

Sub-case 1.3: $T < T_W \leq M$. Since $T < Tw \leq M$, the retailer receives the partially permissible delay as shown in Fig. 3. For the immediate payment, the interest charged per cycle is the interest rate I_k multiplied by the area of the triangle OUT. Hence, the annual interest charged is $(1-\alpha)cI_kDT^2/2T$. As to the delayed payment, there is no interest charged while the interest earned per cycle is the return rate I_e multiplied by the area of the trapezoid on the interval [0, M]. Consequently, the annual interest earned is $\alpha pI_eDT[M+(M-T)]/2T$. As a result, after simplifying terms, the annual opportunity cost of capital is

$$\frac{D}{2}[(1-\alpha)cI_kT - \alpha pI_e(2M-T)]. \tag{3}$$

Notice that Huang (2007) developed inappropriate terms for both interest charged and interest earned as

$$\frac{(1-\alpha)^2cI_kDT^2}{2T} - \frac{cI_eDT(2M-T)}{2T}, \tag{4}$$

which is significantly different from ours in (3).

The annual total relevant cost for the retailer $TRC(T)$ comprises the annual ordering cost, plus annual holding cost, plus annual interest charged and minus annual interest earned. From the above results (1)–(3), we have

$$\begin{aligned} TRC_1(T) &= \frac{A}{T} + \frac{hDT}{2} + \frac{cI_kD(T-M)^2 - pI_eDM^2}{2T} \\ &= \frac{2A - DM^2(pI_e - cI_k)}{2T} + \frac{TD(h+cI_k)}{2} \\ &\quad - cI_kDM, \text{ if } T_W \leq M < T, \end{aligned} \tag{5}$$

$$\begin{aligned} TRC_2(T) &= \frac{A}{T} + \frac{hDT}{2} - \frac{pI_eDT(2M-T)}{2T} \\ &= \frac{A}{T} + \frac{TD(h+pI_e)}{2} - pI_eDM, \text{ if } T_W \leq T \leq M \end{aligned} \tag{6}$$

$$\begin{aligned} TRC_3(T) &= \frac{A}{T} + \frac{hDT}{2} + \frac{(1-\alpha)cI_kDT}{2} - \frac{\alpha pI_eDT(2M-T)}{2T} \\ &= \frac{A}{T} + \frac{TD[h + \alpha pI_e + (1-\alpha)cI_k]}{2} - \alpha pI_eDM, \text{ if } T < T_W \leq M. \end{aligned} \tag{7}$$

It is clear that $TRC_1(M) = TRC_2(M)$. However, $TRC_2(T_W) = TRC_3(T_W)$ only if $\alpha = 1$. Next, we discuss the other case in which $T_W > M$.

3.2. Case 2: $T_w > M$

Similar to Case 1 of $T_W \leq M$, we know that the annual ordering cost is A/T and the annual holding cost excluding interest charges is $h\,DT/2$. Likewise, the annual opportunity cost of capital has three possible sub-cases because the replenishment cycle time T may lie in three possible positions between M and T_W.

Sub-case 2.1: $M < T_w \leq T$. In this case, the retailer receives the fully permissible delay as shown in Fig. 4. The interest earned per cycle is the return rate I_e multiplied by the area of the triangle OVM. Therefore, the annual interest earned is $pI_eDM^2/2T$. Likewise, the interest charged is the interest rate I_k multiplied by the area of the triangle MXT. Hence, the annual interest charged is $cI_kD(T-M)^2/2T$. Consequently, the annual total relevant cost is

$$\begin{aligned} TRC_4(T) = TRC_1(T) &= \frac{2A - DM^2(pI_e - cI_k)}{2T} \\ &\quad + \frac{TD(h+cI_k)}{2} - cI_kDM, \text{ if } M < T_W \leq T. \end{aligned} \tag{8}$$

Sub-case 2.2: $M \leq T < T_w$. Since $M \leq T < T_W$, the retailer receives the partially permissible delay as shown in Fig. 5. For the

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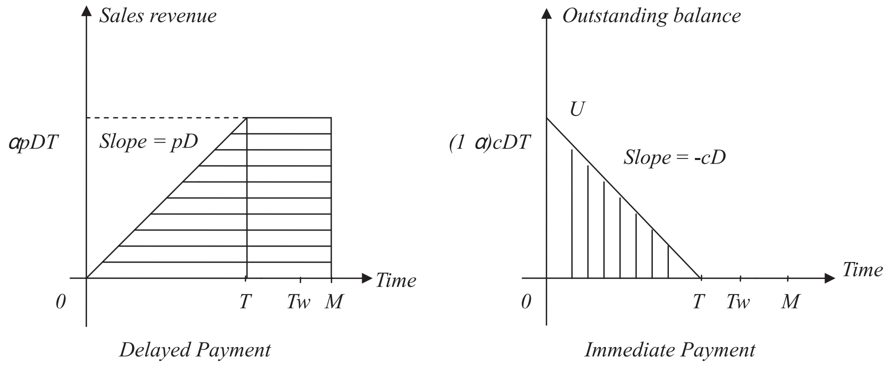

Fig. 3. $T < T_w \leqq M$.

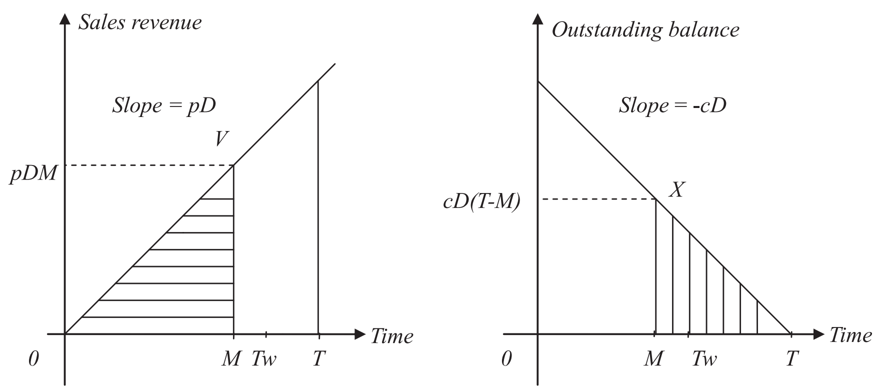

Fig. 4. Delayed payment for $M < T_w \leqq T$.

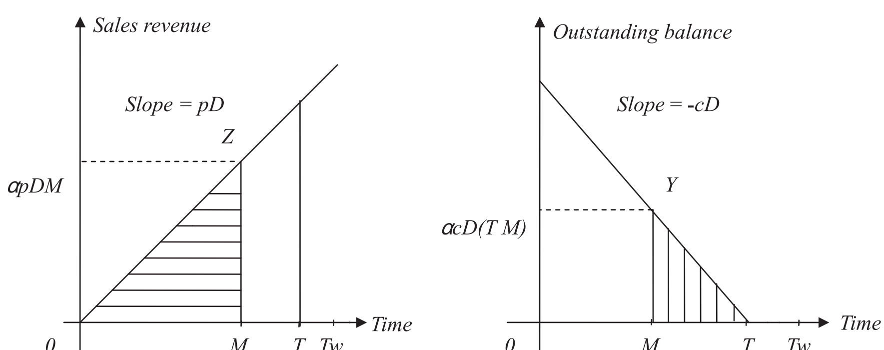

Fig. 5. Delayed payment for $M \leqq T < T_w$.

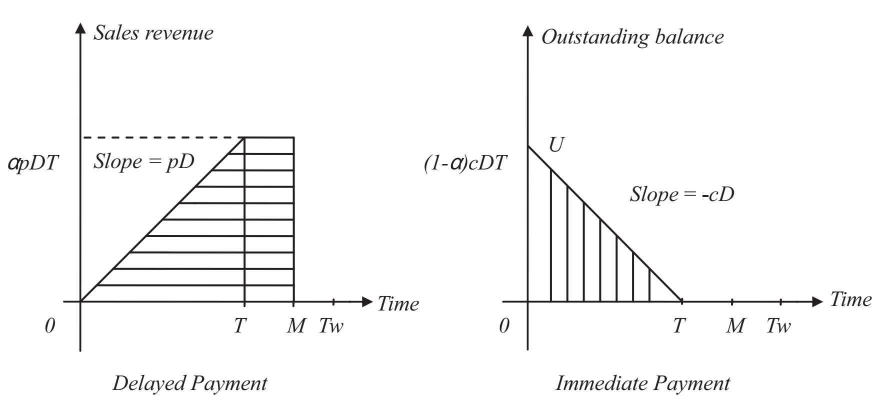

Fig. 6. $T \leqq M < T_w$.

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immediate payment, the annual interest charged is the same as $(1-\alpha)cI_kDT^2/2T$ in Fig. 3. As to the delayed payment, the interest charged per cycle is the interest rate I_k multiplied by the area of the triangle MYT. Thus, the annual interest charged for the delayed payment is $\alpha cI_kD(T-M)^2/2T$. The interest earned per cycle is the return rate I_e multiplied by the area of the triangle OZM. So, the annual interest earned is $\alpha pI_eDM^2/2T$. As a result, the annual total relevant cost is

$$TRC_5(T)=\frac{A}{T}+\frac{hDT}{2}+\frac{(1-\alpha)cI_kDT^2+\alpha cI_kD(T-M)^2-\alpha pI_eDM^2}{2T}$$
$$=\frac{2A-\alpha DM^2(pI_e-cI_k)}{2T}+\frac{TD(h+cI_k)}{2}$$
$$-\alpha cI_kDM,\ \text{if } M\le T<T_W. \tag{9}$$

Sub-case 2.3: $T\le M<T_W$. When $T\le M<T_W$, the retailer receives the partially permissible delay as shown in Fig. 6. In this case, both the interest charged and the interest earned here are the same as those in Sub-case 1.3. Hence, the annual total relevant cost is

$$TRC_6(T)=TRC_3(T)=\frac{A}{T}+\frac{TD[h+\alpha pI_e+(1-\alpha)cI_k]}{2}$$
$$-\alpha pI_eDM \quad \text{if } T\le M<T_W. \tag{10}$$

4. Optimal solution and theoretical results

For simplicity, we propose an easy and simple arithmetic–geometric mean inequality approach (e.g., see Teng (2009b) and Cárdenas-Barrón (2010, 2011)) to obtain the optimal cycle time T that minimizes the annual total relevant cost $TRC(T)$. It is well known that the arithmetic–geometric mean inequality is as follows. For any two real positive numbers, say a and b, the arithmetic mean $(a+b)/2$ is always greater than or equal to the geometric mean $\sqrt{ab}$. Namely, $(a+b)/2\ge\sqrt{ab}$. The equality holds only if $a=b$. Notice that all mathematical methods have their limitation. For example, Calculus cannot deal with discrete mathematics. Likewise, the arithmetic–geometric mean inequality method also has its limitation (e.g., see Chung (2012) and Cárdenas-Barrón (2010)).

From (5)–(10), we know that all TRC_i (T) have two functions minus a constant. One can ignore the constant term because it does not change the shape of the total cost function, but move it down by a constant value. Therefore, it is enough to minimize the two functions and leaving the constant term at the end. Cárdenas-Barrón (2010) establishes that in order to apply the arithmetic–geometric mean inequality as optimization method the following three conditions must be satisfied: (1) the functions must be positive functions, (2) the product of the functions must be a constant, and (3) when these functions are equalized; the system of equations can be solved. These three conditions are completely satisfied in all TRC_i (T) of this paper.

We discuss the case of $T_W\le M$ first, and then the other case of $T_W>M$.

Case 4.1: $T_W\le M$. By using the arithmetic–geometric mean inequality, we can easily obtain that

$$TRC_1(T)=\frac{2A-DM^2(pI_e-cI_k)}{2T}+\frac{TD(h+cI_k)}{2}$$
$$-cI_kDM\ge\sqrt{D(h+cI_k)[2A-DM^2(pI_e-cI_k)]}$$
$$-cI_kDM,\ \text{if } 2A-DM^2(pI_e-cI_k)\ge 0. \tag{11}$$

When the equality $(2A-DM^2(pI_e-cI_k))/T=TD(h+cI_k)$ holds, $TRC_1(T)$ is minimized. Hence the optimal value of T for $TRC_1(T)$ (say T_1^*) is

$$T_1{}^*=\sqrt{\frac{2A-DM^2(pI_e-cI_k)}{D(h+cI_k)}}\quad \text{if } 2A-DM^2(pI_e-cI_k)\ge 0. \tag{12}$$

Therefore, the optimal order quantity Q_1^* is

$$Q_1{}^*=DT_1{}^*=\sqrt{\frac{2AD-(pI_e-cI_k)(DM)^2}{(h+cI_k)}}\quad \text{if } 2A-DM^2(pI_e-cI_k)\ge 0. \tag{13}$$

It is clear that $T_1{}^*$ and Q_1^* are not real numbers if $2A-DM^2(pI_e-cI_k)<0$. To ensure $T_1^*>M$, we substitute (13) into inequality $T_1^*>M$, and obtain that

$$\text{if } \Delta_1=2A-DM^2(h+pI_e)>0 \quad \text{then } T_1^*>M. \tag{14}$$

Likewise, using the arithmetic–geometric mean inequality again, we have

$$TRC_2(T)=\frac{A}{T}+\frac{TD(h+pI_e)}{2}-pI_eDM\ge\sqrt{2AD(h+pI_e)}-pI_eDM.$$

When the equality $A/T=TD(h+pI_e)/2$ holds, $TRC_2(T)$ is minimized. Hence the optimal value of T for $TRC_2(T)$ (say T_2^*) is

$$T_2{}^*=\sqrt{\frac{2A}{D(h+pI_e)}}. \tag{15}$$

Therefore, the optimal order quantity Q_2^* is

$$Q_2{}^*=DT_2{}^*=\sqrt{\frac{2AD}{(h+pI_e)}}. \tag{16}$$

To ensure $T_W\le T_2^*\le M$, we substitute (15) into inequality $T_W\le T_2^*\le M$, and obtain that

$$\text{if } \Delta_1\le 0 \text{ and } \Delta_2=2A-DT_W{}^2(h+pI_e)\ge 0,\ \text{then } T_W\le T_2{}^*\le M. \tag{17}$$

Similarly, we get

$$TRC_3(T)=\frac{A}{T}+\frac{TD[h+\alpha pI_e+(1-\alpha)cI_k]}{2}-\alpha pI_eDM$$
$$\ge\sqrt{2AD[h+\alpha pI_e+(1-\alpha)cI_k]}-\alpha pI_eDM.$$

Hence the optimal value of T for $TRC_3(T)$ (say T_3^*) is

$$T_3{}^*=\sqrt{\frac{2A}{D[h+\alpha pI_e+(1-\alpha)cI_k]}}. \tag{18}$$

Therefore, the optimal order quantity Q_3^* is

$$Q_3{}^*=DT_3{}^*=\sqrt{\frac{2AD}{h+\alpha pI_e+(1-\alpha)cI_k}}. \tag{19}$$

To ensure $T_3^*<T_W$, we substitute (18) into inequality $T_3^*<T_W$, and obtain that

$$\text{if } \Delta_3=2A-DT_W{}^2[h+\alpha pI_e+(1-\alpha)cI_k]<0 \quad \text{then } T_3^*<T_W. \tag{20}$$

In reality, the unit selling price p is usually significantly higher than the unit purchasing cost c. In addition, the return rate of investment is expected to be higher than the interest rate. Otherwise, you would not borrow the money from the supplier. As a result, we may assume without loss of generality that $pI_e\ge cI_k$. For the other irrelevant case, the reader can obtain similar results easily. By comparing Δ_1 through Δ_3, we can easily obtain that if $T_W\le M$ and $pI_e\ge cI_k$, then:

$$\text{if } \alpha=1,\quad \text{then } \Delta_1\le\Delta_2=\Delta_3,\quad \text{and}\quad T_2^*=T_3^*;$$
$$\text{if } 1>\alpha\ge 0,\quad \text{then } \Delta_1\le\Delta_2\le\Delta_3 \tag{21}$$

From the results in (14), (17), (20), and (21), we have the following theoretical results.

Theorem 1. *If $T_W\le M$, and $1>\alpha\ge 0$, then we have*:

(a) *if $\Delta_1>0$ then $T^*=T_1^*$*
(b) *if $\Delta_1=0$ then $T^*=M$*
(c) *if $\Delta_1<0$ and $\Delta_2>0$ then $T^*=T_2^*$*

(d) if $\Delta_2 \le 0$ and $\Delta_3 \ge 0$ then $T^* = T_W$
(e) if $\Delta_3 < 0$ then $T^* = T_3^*$

If $T_W \le M$, and $\alpha = 1$, then we get:

(f) if $\Delta_1 > 0$ then $T^* = T_1^*$
(g) if $\Delta_1 = 0$ then $T^* = M$
(h) if $\Delta_1 < 0$ and $\Delta_2 > 0$ then $T^* = T_2^* = T_3^*$
(i) if $\Delta_2 \le 0$ then $T^* = T_W$.

Proof. The proof follows immediately from (14), (17), (20), and mutually exclusive among Sub-cases 1.1–1.3.

A simple economic interpretation of Theorem 1 is as follows:

(1) It is clear from the classical EOQ model that the optimal order quantity is obtained when the ordering cost is equal to the holding cost.
(2) Whenever a retailer orders DM units, it receives the benefit of $pI_eDM^2/2$ from the supplier's trade credit. Hence, the net ordering cost is reduced to $A–pI_eDM^2/2$.
(3) On the other hand, the inventory holding cost for ordering DM units $h\ DM^2/2$.
(4) Hence, $\Delta_1/2 = A–DM^2(h+pI_e)/2$ represents the net ordering cost minus the holding cost for the order of DM units.
(5) Consequently, if $\Delta_1 > 0$, then the net ordering cost is higher than the holding cost for the order of DM units, and hence we must order more than DM units. Consequently, the optimal order quantity Q^* is higher than DM units (i.e., $T^* = T_1^* > \mathrm{M}$). Similarly, one can easily interpret the rest of the theoretical results by using the analogous argument. Now, we are ready to discuss the other case in which $T_W > M$.

Case 4.2: $T_W > M$. Using arguments similar to those in Case 4.1, one can easily obtain the following results. The optimal value of T for $TRC_4(T)$ (say T_4^*) is

$$T_4{}^* = \sqrt{\frac{2A–DM^2(pI_e–cI_k)}{D(h+cI_k)}} \quad \text{if } 2A–DM^2(pI_e–cI_k) \ge 0 \tag{22}$$

In fact $T_4^* = T_1{}^*$. To avoid confusing, we use T_4^* for the case of $T_W > M$, and T_1^* for the case of $T_W \le M$. Similarly, the optimal order quantity $Q_4^* = Q_1^*$ is

$$Q_4^* = DT_4^* = \sqrt{\frac{2AD–(pI_e–cI_k)(DM)^2}{(h+cI_k)}} \quad \text{if } 2A–DM^2(pI_e–cI_k) \ge 0 \tag{23}$$

It is clear that T_4^* is not a real number if $2A–DM^2(pI_e–cI_k) < 0$. To ensure $T_4^* \ge T_W$, we substitute (22) into inequality $T_4^* \ge T_W$, and obtain that if and only if

$$\Delta_4 = 2A–DM^2(pI_e–cI_k)–DT_W^2(h+cI_k) \ge 0 \quad \text{then } T_4^* \ge T_W \tag{24}$$

The optimal value of T for $TRC_5(T)$ (say T_5^*) is

$$T_5{}^* = \sqrt{\frac{2A–\alpha DM^2(pI_e–cI_k)}{D(h+cI_k)}} \quad \text{if } 2A–\alpha DM^2(pI_e–cI_k) \ge 0 \tag{25}$$

The optimal order quantity Q_5^* is

$$Q_5{}^* = DT_5{}^* = \sqrt{\frac{2AD–\alpha(pI_e–cI_k)(DM)^2}{(h+cI_k)}} \quad \text{if } 2A–\alpha DM^2(pI_e–cI_k) \ge 0 \tag{26}$$

It is clear that T_5^* does not exist if $2A–\alpha DM^2(pI_e–cI_k) < 0$. To ensure $M \le T_5{}^* < T_W$, we substitute (25) into inequality $M \le T_5{}^* < T_W$, and obtain that if and only if $\Delta_5 = 2A–\alpha DM^2(pI_e–cI_k)–DT_W^2(h+cI_k) < 0$, and

$$\Delta_6 = 2A–\alpha DM^2(pI_e–cI_k)–DM^2(h+cI_k) \ge 0 \quad \text{then } M \le T_5^* < T_W \tag{27}$$

The optimal value of T for $TRC_6(T)$ (say T_6^*) is

$$T_6^* = \sqrt{\frac{2A}{D[h+\alpha pI_e+(1–\alpha)cI_k]}} \tag{28}$$

Notice that $T_6^* = T_3^*$. Again, to avoid confusing, we use T_6^* for the case of $T_W > M$. Likewise, the optimal order quantity $Q_6^* = Q_3^*$ is

$$Q_6^* = DT_6^* = \sqrt{\frac{2AD}{h+\alpha pI_e+(1–\alpha)cI_k}} \tag{29}$$

To ensure $T_6^* \le M$, we substitute (28) into inequality $T_6^* \le M$, and obtain that if and only if

$$\Delta_6 \le 0 \quad \text{then } T_6^* \le M. \tag{30}$$

It is obvious that (1) if $\alpha = 1$, then $\Delta_4 = \Delta_5 < \Delta_6$, and $T_4^* = T_5^*$; and (2) if $1 > \alpha \ge 0$, then $\Delta_4 < \Delta_5 < \Delta_6$. Hence, we have the following theoretical results.

Theorem 2. *If $T_W > M$, and $1 > \alpha \ge 0$, then we obtain*:

(a) if $\Delta_4 > 0$ then $T^* = T_4^*$.
(b) if $\Delta_4 \le 0$ and $\Delta_5 \ge 0$ then $T^* = T_W$.
(c) if $\Delta_5 < 0$ and $\Delta_6 > 0$ then $T^* = T_5^*$.
(d) if $\Delta_6 = 0$ then $T^* = M$.
(e) if $\Delta_6 < 0$ then $T^* = T_6^*$.

If $T_W > M$, and $\alpha = 1$, then we yield:

(f) if $\Delta_4 > 0$ then $T^* = T_4^* = T_5^*$
(g) if $\Delta_4 \le 0$, and $\Delta_6 > 0$, then $T^* = T_W$.
(h) if $\Delta_6 = 0$ then $T^* = M$.
(i) if $\Delta_6 < 0$ then $T^* = T_6^*$.

Proof. The proof follows immediately from (24), (27), (30), and mutually exclusive among Sub-cases 2.1–2.3.

A simple economic interpretation of Theorem 2 is similar to that in Theorem 1. Next, we provide some numerical examples to illustrate the results.

5. Numerical examples

In this section, we provide some numerical examples to illustrate several distinct theoretical results as well as to gain an understanding of managerial insights. Let $D = 1000$ units/year, $p =$ \$30/unit, $c = \$10$/unit, $h = \$2.8$/unit/year, $I_e = I_k = 0.04$/\$/year, and $M = 0.20$ years. We then study the sensitivity analyses on $A =$ (\$120, \$80, \$60, \$40, \$20)/order, $W = (125, 250)$ units, and $\alpha = (0.2, 0.5)$. The computational results are shown in Table 1. Table 1 reveals that (1) the higher the fraction of permissible delay payment α, the lower the optimal replenishment cycle time $T^*$and the optimal annual total relevant cost $TRC(T^*)$, and (2) the higher the threshold to fully permissible delay payment W, the higher the optimal replenishment cycle time $T^*$and as well as the optimal annual total relevant cost $TRC(T^*)$.

6. Conclusion

In this paper, we have modified not only Huang (2007)'s mathematical expressions, but also appropriate graphical figures to obtain both the interest earned and the interest charged. We then have extended Huang (2007)'s model to incorporate the facts

Table 1
Optimal solutions under different A, W and α.

W	α	A	Δ_i	Δ_{i+1}	T^*	Q^*	$TRC(T^*)$
150	0.2	120	$\Delta_1=80>0$		$T_1^*=0.2550$	$Q^*=255.0$	735.843
		80	$\Delta_1=0$		$M=0.2000$	$Q^*=200.0$	560.000
		60	$\Delta_1=-40<0$	$\Delta_2=30>0$	$T_2^*=0.1732$	$Q^*=173.2$	452.820
		40	$\Delta_2=-10<0$	$\Delta_3=4.4>0$	$T_W=0.1500$	$Q^*=150.0$	326.667
		20	$\Delta_3=-35.6<0$		$T_3^*=0.1091$	$Q^*=109.1$	318.606
	0.5	120	$\Delta_1=80>0$		$T_1^*=0.2550$	$Q^*=255.0$	735.843
		80	$\Delta_1=0$		$M=0.2000$	$Q^*=200.0$	560.000
		60	$\Delta_1=-40<0$	$\Delta_2=30>0$	$T_2^*=0.1732$	$Q^*=173.2$	452.820
		40	$\Delta_3=-1<0$		$T_3^*=0.1491$	$Q^*=149.1$	416.656
		20	$\Delta_3=-41<0$		$T_3^*=0.1054$	$Q^*=105.4$	259.473
250	0.2	120	$\Delta_4=8>0$		$T_4^*=0.2550$	$Q^*=255.0$	735.843
		80	$\Delta_5=-46.4<0$	$\Delta_6=25.6>0$	$T_5^*=0.2182$	$Q^*=218.2$	685.085
		60	$\Delta_6=-14.4<0$		$T_6^*=0.1884$	$Q^*=188.4$	586.980
		40	$\Delta_6=-54.4<0$		$T_6^*=0.1517$	$Q^*=151.7$	470.459
		20	$\Delta_6=-94.4<0$		$T_6^*=0.1024$	$Q^*=102.4$	318.606
	0.5	120	$\Delta_4=8>0$		$T_4=0.2550$	$Q^*=255.0$	735.843
		80	$\Delta_5=-56<0$	$\Delta_6=16>0$	$T_5^*=0.2108$	$Q^*=210.8$	638.823
		60	$\Delta_6=-24<0$		$T_6^*=0.1803$	$Q^*=180.3$	537.267
		40	$\Delta_6=-64<0$		$T_6^*=0.1414$	$Q^*=141.4$	416.656
		20	$\Delta_6=-104<0$		$T_6^*=0.0866$	$Q^*=86.6$	259.473

that (1) the unit selling price is significantly higher than the unit purchasing cost, and (2) the interest rate charged by the supplier is not necessarily higher than the retailer's investment return rate. In addition, by contrast to the differential calculus method, we have proposed a simple-to-apply arithmetic–geometric-mean-inequality method to obtain the optimal solution to the inventory problem with conditionally permissible delay in payments. Furthermore, we have established the discrimination terms to identify the unique optimal solutions among three alternatives and explained theoretical results by simple economical interpretations. Finally, we have provided several numerical examples to illustrate the theoretical results and obtain some managerial insights.

For future research, the research presented in this paper can be extended in several ways. For instance, we may consider for deteriorating items, increasing demand, ramp-type demand or stock-dependent demand. Also, we could generalize the model to allow for shortages and partial backlogging. Finally, we may consider non-cooperative and cooperative solutions to this supply chain supplier–retailer–buyer model, such as Nash solution, Stackelberg solution, Pareto solution, and integrated solution.

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