Economic Modelling 26 (2009) 234–240

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econbase

Modelling VaR for foreign-asset portfolios in continuous time

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ARTICLE INFO

Article history:
Accepted 3 July 2008

Keywords:
Continuous time
Foreign-asset portfolio
Volatility of exchange rate
Correlation coefficient
Backtesting

ABSTRACT

VaR is widely viewed as a measure of market risk of a portfolio. The purpose of this article is to provide a VaR model for foreign-asset portfolios in continuous time. In the VaR model, the VaRs are not only a function of volatilities of asset returns and exchange rate but also a function of correlation coefficient between foreign assets and exchange rate. Moreover, by backtesting, the empirical results show that the new VaR model can efficiently evaluate the market risk of foreign-asset portfolios.

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1. Introduction

With the rise of the capital market liberalization and globalization, foreign currency denominated assets circulate rapidly in the world. In Taiwan, the official monthly statistic reports illustrate that the fraction of the investment in foreign assets relative to domestic assets is from 20% to 25% in domestic commercial banks during recent years, and the percentage is growing. Thus controlling market risk of foreign portfolios is an increasing concern for financial institutions.

VaR is widely viewed as a measure of market risk of a portfolio. One reason for this is J.P. Morgan's attempt to establish a market standard called RiskMetrics system.

Another is the atmosphere created by uncontrolled risk leading to huge losses taken by Proctor and Gamble, Kidder Peabody, Orange County, and Barings. The third reason is forced upon institutions by national central banks and the Bank of International Settlement (BIS) whose objectives were to use VaR in calculating a bank's required capital in order to minimize the chances of instability and breakdown of the financial system.

Various methods can be used to calculate the VaR amount. Basically, approaches to VaR can be usefully classified into two broad groups, namely, parametric approach and nonparametric approach. In the case of parametric techniques, including the delta-normal method and the RiskMetrics (or variance–covariance method), returns are modeled by normal distribution, and the variations of the market value of financial positions and fluctuations of the risk factors are linked through a linear relation. This approach is very popular, because it is the easiest and fastest method to implement. Literature related to these ideas was introduced by Jorion (1996a,b), Longerstaey and Zangari (1996), Simons (1997), Duffie and Pan (1997), Kupiec (1995, 1999), and Brooks and Persand (2002).

Although the parametric method is commonly adopted, one of the well-known shortcomings of the parametric approach is that it underestimates the frequency of "extreme events", such as outcomes several standard deviations away from the mean.

Thus one can employ a nonparametric method, such as the historical simulation method or Monte Carlo simulation. These simulations are well-known as they do not make any distributional assumptions about asset returns, and take into account the extreme effects of financial crises that have occurred in the past, such as the culmination of the Asian crisis and Russian crisis in August 1998. But, simulation evaluations of VaRs may be costly in terms of the system infrastructure and computation time. Studies in the framework of nonparametric approaches follow the line of Hendricks (1996), Jamshidian and Zhu (1997), Hull and White (1998a,b), Barone-Adesi et al. (1999), Barone-Adesi and Giannopoulos (2001), Linsmeier and Pearson (2000), Lauridsen (2000), Brooks and Persand (2003) and Huang and Lin (2004).

No easy answer exists to the question of which method of calculating VaR is best. The best choice will be determined by which conditions the risk manager considers most important.

Traditionally, previous studies showed that the VaR is mainly a function of the volatility of underlying asset returns. If we want to transform the VaRs in foreign dollars into those valued in domestic currency, it is usually calculated by multiplying the VaRs in foreign dollars with the current exchange rate. Thus there is an explicit drawback that the traditional VaR neglects two important elements:

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doi:10.1016/j.econmod.2008.07.004

the volatility of exchange rate and the correlation coefficient between foreign assets and exchange rate.

Specifically, the volatility of exchange rate, commonly regarded as foreign exchange risk, has been increasingly disclosed by financial institutions as the economies are more globalized. Hence, on the assumption of a normal distribution of asset returns, the aim of this article is to extend the work of Kupiec (1999) and present a new VaR model in continuous time that can efficiently evaluate the market risks of foreign-asset portfolios.

The rest of this article is organized as follows. The next section outlines the model. A closed-form solution for calculating the VaRs of foreign-asset portfolios is shown in this section. Another section provides numerical analyses. In Section 4, from the Taiwanese perspective, we employ ten foreign-issued assets (in U.S. dollars and British pounds) widely circulated in the world to perform a backtesting for evaluation of the accuracy of the new VaR. The time window length is divided into two periods. One is the low oil price years from March 8, 2002 to Aug. 30, 2005. The other period is the years of high oil price from Aug. 31, 2005 to Dec.31, 2007. The respective total of the daily log returns of the ten foreign securities is 908 and 592 in each period. The last section contains a conclusion.

2. VaR model of foreign-asset portfolios

In this section, we first present some assumptions for model formulation and extend the framework of Kupiec (1999) to derive a new VaR model for foreign-asset portfolios. Then, a comparative study of the traditional model and the new model will be discussed.

2.1. Assumptions and model formulation

Following convention, we first assume that (i) the security market is a complete market with no transaction costs or taxes, (ii) there exists a riskless interest rate for lenders and borrowers, and (iii) the dynamic processes of foreign-asset price and exchange rate follow Geometric Brownian motions (GBM) as shown below, respectively:

$$\frac{\mathrm{d}A_t}{A_t} = \mu_f \mathrm{d}t + \sigma_f \mathrm{d}W_{1t}, \tag{1}$$

$$\frac{\mathrm{d}Q_t}{Q_t} = u_Q \mathrm{d}t + \sigma_Q \mathrm{d}W_{2t}, \tag{2}$$

where the μ_f and σ_f are the drift and the volatility of foreign-asset returns, respectively; the μ_Q and σ_Q are the drift and the volatility of exchange rate returns, respectively. Both W_{1t} and W_{2t} are one dimensional Brownian motions defined in a filtered probability space (Ω, F, P) under the original probability measure, P. Moreover, the correlation coefficient of the two Brownian motions is defined as corr ($\mathrm{d}W_{1t}$, $\mathrm{d}W_{2t}$)=$\rho_{f,Q}$. Thus the covariance of foreign assets and exchange rate is denoted by $\mathrm{Cov}\left(\frac{\mathrm{d}A_t}{A_t}, \frac{\mathrm{d}Q_t}{Q_t}\right) = \rho_{f,Q}\sigma_f\sigma_Q \mathrm{d}t$.

From Eqs. (1) and (2), one can obtain $A_t = A_0 e^{\left(\mu_f - \frac{1}{2}\sigma_f^2\right)t + \sigma_f W_{1t}}$ and $Q_t = Q_0 e^{\left(\mu_Q - \frac{1}{2}\sigma_Q^2\right)t + \sigma_Q W_{2t}}$. Obviously, the distributions of the asset price and exchange rate are lognormal, and thus the asset price and exchange rate don't take negative values. This specification is reasonable and has been widely adopted in economics and finance literature.

Now, consider the potential daily loss exposure to long trading positions. Typically, the VaR is a specific left-hand critical value of a potential loss distribution. Following convention, we can define the daily losses valued in domestic dollars transformed from foreign currencies relative to the end-of-period expected asset value, known as relative VaR and denoted by

$$\mathrm{VaR(mean)} \equiv E(Q_T A_T) - V_\alpha, \tag{3}$$

where the E (.) is the expected value, the V_α is the underlying asset value denominated in domestic dollars given a confidence level of α, and the $Q_T A_T$ is the amount valued in domestic dollars of a foreign asset at time T (investment horizon). Alternatively, the VaR (0) represents the VaR in domestic dollars relative to the initial asset value, namely, absolute VaR:

$$\mathrm{VaR}(0) \equiv Q_0 A_0 - V_\alpha. \tag{4}$$

Briefly, we can describe Eqs. (3) and (4) by means of Fig. 1. It shows that when $E(Q_T A_T)>0$, which is generally the case, the value of VaR (mean) is higher than the value of VaR (0).

Before the derivation of the VaR's analytic formula for foreign assets, it is convenient to make use of the following propositions.

Proposition 1. *Given the dynamic processes of foreign currency denominated asset price and exchange rate following the Geometric Brownian motion, the dynamic process of $Q_t A_t$ can be expressed as*

$$\frac{\mathrm{d}X_t}{X_t} = \left(\mu_f + \mu_Q + \rho_{f,Q}\sigma_f\sigma_Q\right)\mathrm{d}t + \sigma_f \mathrm{d}W_{1t} + \sigma_Q \mathrm{d}W_{2t}$$

with $X_t = Q_t A_t$.

Appendix A provides a detailed proof of Proposition 1.

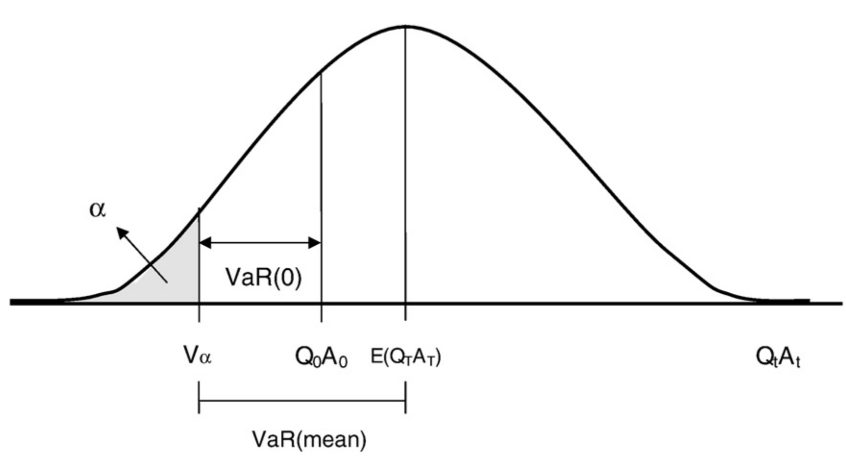

Fig. 1. Plot of VaR (mean) and VaR (0).

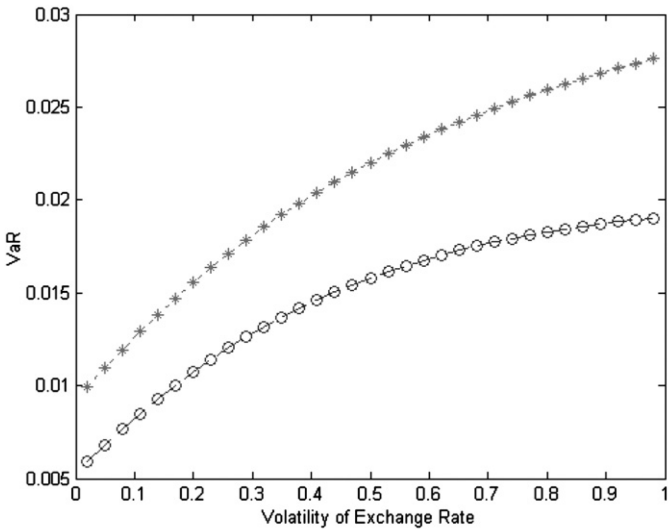

Fig. 2. The impact of volatility of exchange rate on VaR under the new VaR model. *The symbol "–o–" represents VaR relative to the initial asset value; "–*–" denotes VaR relative to the end-of-period expected asset value.

Proposition 2. *Following* Proposition 1*, the analytic formulas of the relative VaR and absolute VaR can be shown as below in the new VaR model, respectively:*

$$\mathrm{VaR(mean)} = \left|\mathrm{Min}\left\{A_0Q_0\left[e^{\left(\mu_f+\mu_Q-\frac{1}{2}\sigma_f^2-\frac{1}{2}\sigma_Q^2\right)T+Z_\alpha\sqrt{\left(\sigma_f^2+\sigma_Q^2+2\rho_{f,Q}\sigma_f\sigma_Q\right)T}}-e^{\left(\mu_f+\mu_Q+\rho_{f,Q}\sigma_f\sigma_Q\right)T}\right],0\right\}\right|,$$

and

$$\mathrm{VaR}(0) = \left|\mathrm{Min}\left\{A_0Q_0\left[e^{\left(\mu_f+\mu_Q-\frac{1}{2}\sigma_f^2-\frac{1}{2}\sigma_Q^2\right)T+Z_\alpha\sqrt{\left(\sigma_f^2+\sigma_Q^2+2\rho_{f,Q}\sigma_f\sigma_Q\right)T}}-1\right],0\right\}\right|,$$

where the Z_α stands for a critical value of the standard normal distribution with a given probability, and T is the investment horizon.

The proof of Proposition 2 is provided in Appendix B. Besides the volatility of foreign assets, the analytic formulas in Proposition 2 include two important elements such as the volatility of exchange rate and the correlation coefficient between foreign assets and exchange rate. This enables us to truly capture market risk of foreign assets.

2.2. Comparative analysis

Kupiec (1999) showed the relative VaR and absolute VaR as follows, respectively:

$$\mathrm{VaR}_k(\mathrm{mean}) = \left|\mathrm{Min}\left\{A_0\left[e^{\left(\mu_f-\frac{1}{2}\sigma_f^2\right)T+Z_\alpha\sqrt{\sigma_f^2T}}-e^{\mu_fT}\right],0\right\}\right|,$$

and

$$\mathrm{VaR}_k(0) = \left|\mathrm{Min}\left\{A_0\left[e^{\left(\mu_f-\frac{1}{2}\sigma_f^2\right)T+Z_\alpha\sqrt{\sigma_f^2T}}-1\right],0\right\}\right|. \tag{5}$$

Traditionally, if one wants to obtain the VaRs in domestic currency, it is usually calculated by multiplying Eq. (5) with the current exchange rate. This is also represented as follows:

$$\mathrm{VaR}_k(\mathrm{mean}) = \left|\mathrm{Min}\left\{A_0Q_0\left[e^{\left(\mu_f-\frac{1}{2}\sigma_f^2\right)T+Z_\alpha\sqrt{\sigma_f^2T}}-e^{\mu_fT}\right],0\right\}\right|,$$

and

$$\mathrm{VaR}_k(0) = \left|\mathrm{Min}\left\{A_0Q_0\left[e^{\left(\mu_f-\frac{1}{2}\sigma_f^2\right)T+Z_\alpha\sqrt{\sigma_f^2T}}-1\right],0\right\}\right|. \tag{6}$$

Comparing Proposition 2 and Eq. (6), the traditional VaR clearly omits the terms of the mean, volatility of exchange rate, and correlation coefficient between foreign assets and exchange rate for the market risk of foreign assets. When $\mu_Q=0$, $\sigma_Q=0$ and $\rho_{f,Q}=0$, the formulas illustrated in Proposition 2 will be the same as Eq. (6). In other words, the new VaR is a general form of the traditional VaR.

3. Numerical analysis of the new VaR model

We perform sensitivity analyses of the impacts of important parameters (volatilities of asset returns and exchange rate and the correlation coefficient between asset returns and exchange rate) on the new VaR model in terms of comparative statics in this section.

Given parameter values of $\mu_f=0.08$, $\mu_Q=0.1$, $A_0=0.1$, $Q_0=0.2$, $Z_{0.01}=-2.33$, and $T=1$, Figs. 2, 3, and 4 demonstrate that both the relative VaR and absolute VaR increase monotonically as volatilities of exchange rate and foreign assets and the correlation coefficient between foreign assets and exchange rate grow. Furthermore, the relative VaR is

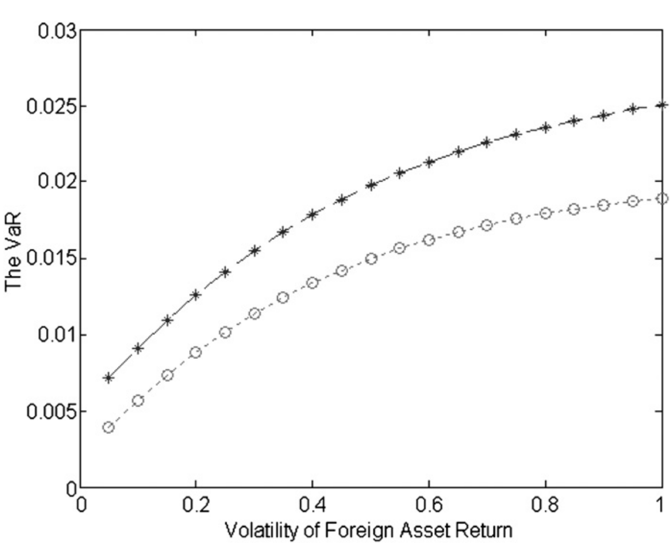

Fig. 3. The impact of volatility of foreign-asset returns on VaR under the new VaR model. *The symbol "–o–" represents VaR relative to the initial asset value; "–*–" denotes VaR relative to the end-of-period expected asset value.

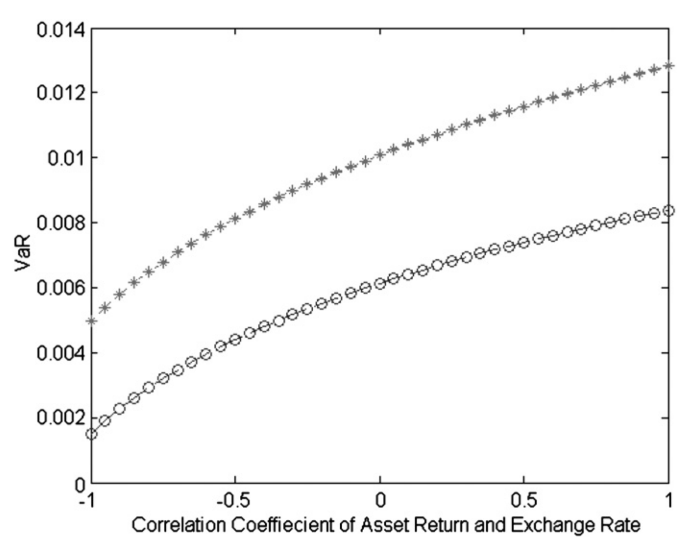

Fig. 4. The impact of correlation coefficient of asset returns and exchange rate on VaR under the new VaR model. The symbol "–o–" represents VaR relative to the initial asset value; "–*–" denotes VaR relative to the end-of-period expected asset value.

greater than absolute VaR. The implication is that the minimum solvency safety margin measured by relative VaR is larger than those valued by absolute VaR in the new VaR model. Again, as shown in Table 1, it is found that the elasticity of volatility of exchange rate on relative VaR or absolute VaR is around 0.3096 or 0.3968; the elasticity of volatility of foreign assets is around 0.4839 or 0.6414; the elasticity of correlation coefficient between foreign assets and exchange rate is around 0.7099 or 1.4746. As a consequence, the VaR is more sensitive to correlation coefficient than to volatility of exchange rate or volatility of foreign assets.

Alternatively, as shown in Table 2, we can see VaR is a nonsymmetrical function of the correlation coefficient between asset return and exchange rate changes. The relative VaRs decline as the correlation coefficient decreases. Generally speaking, the more correlation coefficient rises, the more the risk exposure increases. Further, this is also confirmed in Fig. 5A and B.

Table 2
Value at risk in the new VaR model for alternative correlation coefficients

Asset volatility (%)	Exchange rate volatility (%)	Correlation coefficient	VaR (0)	VaR (mean)
Panel A: negative correlation coefficient				
5	5	−0.8	0.3150	0.1013
10	10	−0.6	0.1407	0.2699
15	15	−0.4	0.0539	0.4602
20	20	−0.2	0.2438	0.6516
25	25	−0.1	0.3898	0.8000
Panel B: positive correlation coefficient				
5	5	0	0.2005	0.2186
10	10	0.2	0.0207	0.4427
15	15	0.4	0.2269	0.6588
20	20	0.6	0.4076	0.8612
25	25	0.8	0.5586	0.9987

Note that the drift term of asset returns is 0.15; the drift term of exchange rate is 0.2; the initial asset return and initial exchange rate are 1; $Z_{0.01}=-2.33$; investment horizon is 1 year.

4. Estimating model parameters and measuring VaR

This section considers long trading positions of several foreign-issued securities issued in U.S.A. and Britain from the Taiwanese perspective. We then want to know the magnitude of the VaR in new Taiwan dollars of the foreign currency securities. Finally, a backtesting is conducted to evaluate the performance of the new VaR.

4.1. Source of the data

We use daily log returns of ten foreign-issued securities issued in U.S.A. and Britain in the sample. For example, IBM, Microsoft (MSFT), PetroQuest Energy Inc (PQ), AFC Enterprise Inc (AFCE) and Mcdonalds Cp (MCD) are issued in U.S.A.; MISYS (MSY.L), ABBOT Group (ABG.L), BP, Expro Intl Group (EXR.L), and Cadbury Schweppes (CSG) are issued in Britain. All of them are well-known to institutional and individual investors in Taiwan. All of the samples in this study span two periods—the low oil price years from Mar. 8, 2002 to Aug. 30, 2005, and the high oil price years from Aug. 31, 2005 to Dec. 31, 2007, so the respective total of the daily log returns of the ten foreign securities of 908 and 592 are obtained in the two periods. The time break occurred on Aug. 30, 2005 because the increment of west Texas Intermediate (WTI) crude oil price broke the record this day. The source of the ten security prices comes from the website of www.finance.yahoo.com. Daily log returns are computed using the following formula:

$$R_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t is the market value of foreign assets at time t. The VaRs are for a one-day-ahead horizon and a 99% confidence level for losses.

Table 1
Sensitivity analysis of VaRs under the new VaR model with respect to model parameters

σ_Q	σ_f	$\rho_{f,Q}$	VaR(mean)	VaR(0)
Panel A: volatility of exchange rate changes				
0.1	0.2	0.9	0.0126	0.0082
0.2	0.2	0.9	0.0155	0.0107
0.3	0.2	0.9	0.0181	0.0128
0.4	0.2	0.9	0.0202	0.0145
0.5	0.2	0.9	0.0220	0.0158
Panel B: volatility of foreign assets changes				
0.1	0.1	0.9	0.0091	0.0057
0.1	0.2	0.9	0.0126	0.0088
0.1	0.3	0.9	0.0155	0.0133
0.1	0.4	0.9	0.0178	0.0134
0.1	0.5	0.9	0.0197	0.0150
Panel C: correlation coefficient between foreign assets and exchange rate changes				
0.1	0.2	1.0	0.0050	0.0015
0.1	0.2	0.5	0.0081	0.0044
0.1	0.2	0.0	0.0101	0.0066
0.1	0.2	−0.5	0.0116	0.0074
0.1	0.2	−1.0	0.0128	0.0084

Note that the drift term of asset return is 0.15; the drift term of exchange rate is 0.2; the initial asset returns and initial exchange rate are 1; $Z_\alpha=-2.33$; investment horizon is 1 year.

4.2. Estimation of model parameters

Before calculating the VaRs, it is necessary to estimate a set of parameter values of μ_f, σ_f, μ_Q, σ_Q and $\rho_{f,Q}$ for the ten securities and exchange rate. First, we can get the respective dynamic processes of log returns of the ten assets and exchange rate from Eqs. (1) and (2) by means of Ito's lemma:

$$\mathrm{d}\ln(A_t) = \left(\mu_f - \frac{1}{2}\sigma_f^2\right)\mathrm{d}t + \sigma_f \mathrm{d}W_{1t}, \tag{7}$$

$$\mathrm{d}\ln(Q_t) = \left(u_Q - \frac{1}{2}\sigma_Q^2\right)\mathrm{d}t + \sigma_Q \mathrm{d}W_{2t}. \tag{8}$$

Through of Eqs. (7) and (8), the estimating results of the expected value and volatilities of log returns of the ten securities in various periods are shown in Table 3. Table 4 reports the expected value and volatilities of log returns of the price of one unit of the U.S. dollars and British pound in new Taiwan dollars in two periods. Consequently, the estimation of μ_f, σ_f, μ_Q and σ_Q is exhibited in Tables 5 and 6. Additionally, the estimation of the correlation coefficient between each foreign asset and exchange rate, and the initial investment valued in new Taiwan dollars of the ten securities in two periods are illustrated in Table 7.

4.3. Measurement of value at risk and implications

Based on the previous estimation results, we can calculate the two VaRs (relative VaR and absolute VaR) under the new VaR and the traditional VaR models. These results are summarized in Table 8 given that $Z_{0.01}=-2.33$, and $T=1$. There exists a common phenomenon exhibited in Panels A and B that the amount of relative VaR is larger than absolute VaR in spite of the traditional VaR and new VaR models.

Additionally, the VaR amount measured in the new VaR model is generally less than the amount in the traditional VaR model in the low oil price period, whereas the VaRs are higher than that in the traditional VaR model in the high oil price years. In the low oil price

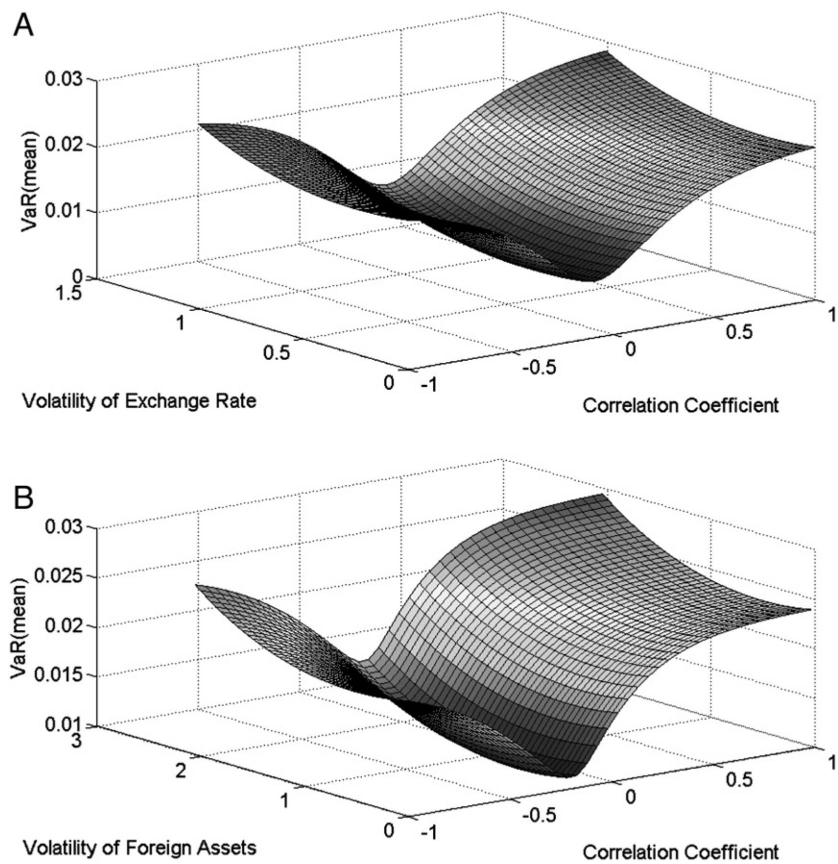

Fig. 5. A) Response surface of relative VaR to volatility of exchange rate and correlation coefficient in the new VaR model. B) Response surface of relative VaR to volatility of foreign assets and correlation coefficient in the new VaR model.

period, the profitability of a firm is high because of its low production cost. This enables default probability to decrease as the equity tends to have a lower capital requirement under the new VaR model. On the contrary, in years of high oil price, it is necessary for a firm to maintain a sufficient capital amount in order to prevent from default risk because of the increase of a firm's production cost. If financial managers adopt the traditional model to evaluate financial risk, the firm's financial ratio such as ROE is better. But the default probability of the firm may increase on the account of a shortage of sufficient capital requirement. Hence the conservative policy of the new VaR model is suitable for financial institutions to control market risk in the high oil price years.

Table 3
The estimation of the expected value and volatility of log returns of ten securities in various periods

Security	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	$E(\mathrm{d}\ \ln A_t)$	σ_f	$E(\mathrm{d}\ \ln A_t)$	σ_f
IBM	−0.00025	0.01700	0.00055	0.01075
MSFT	−0.00002	0.01799	0.00050	0.01309
PQ	0.00024	0.04051	0.00103	0.02544
AFCE	−0.00028	0.02803	−0.00027	0.02822
MCD	0.00022	0.01781	0.00113	0.01259
MSY.L	−0.00034	0.02863	−0.00034	0.02241
ABG.L	0.00056	0.02175	0.00062	0.02141
BP	0.00022	0.01582	0.00003	0.01230
EXR.L	0.00014	0.02885	0.00115	0.02260
CSG	0.00014	0.01396	0.00021	0.01338

Note that $E(\mathrm{d}\ \ln A_t)$ and σ_f stand for the expected value and the volatility of log return of foreign assets, respectively.

Table 4
The estimation of the expected value and volatility of log returns of exchange rate in various periods

Exchange rate	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	$E(\mathrm{d}\ \ln Q_t)$	σ_Q	$E(\mathrm{d}\ \ln Q_t)$	σ_Q
NTD/USD	−0.00008	0.00206	−0.00002	0.00229
NTD/Pound	0.00017	0.00525	0.00015	0.00511

Note that $E(\mathrm{d}\ \ln Q_t)$ and σ_Q stand for the expected value and the volatility of log return of exchange rate, respectively.

5. Evaluation and backtesting

The backtesting is a widely used method of evaluating the VaR accuracy. As was known, the most basic requirement of a VaR model is

Table 5
Parameter estimation of dynamic process of asset return in various periods

Security	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	μ_f	σ_f	μ_f	σ_f
IBM	−0.00010	0.01700	0.00060	0.01075
MSFT	0.00015	0.01799	0.00058	0.01309
PQ	0.00106	0.04051	0.00054	0.02544
AFCE	0.00011	0.02803	0.00013	0.02822
MCD	0.00038	0.01781	0.00121	0.01259
MSY.L	0.00007	0.02863	−0.00009	0.02241
ABG.L	0.00080	0.02175	0.00085	0.02141
BP	0.00035	0.01582	0.00010	0.01230
EXR.L	0.00056	0.02885	0.00141	0.02260
CSG	0.00024	0.01396	0.00030	0.01338

Note that μ_f represents the drift term of foreign-asset returns.

Table 6
Parameter estimation of dynamic process of exchange rate in various periods

Exchange rate	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	μ_Q	σ_Q	μ_Q	σ_Q
NTD/USD	−0.00007	0.00206	−0.00001	0.00229
NTD/Pound	0.00018	0.00525	0.00016	0.00511

Note that μ_Q represents the drift term of exchange rate returns.

Table 7
The estimation of the correlation coefficient between each asset and exchange rate in various periods

Security	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	$A_0\ Q_0$	$\rho_{f,Q}$	$A_0\ Q_0$	$\rho_{f,Q}$
IBM	3411.43	−0.2115	2553.27	0.1024
MSFT	931.06	−0.5124	866.26	0.0362
PQ	205.04	−0.5709	255.46	−0.1529
AFCE	606.36	−0.3971	434.61	0.1289
MCD	862.48	−0.7750	990.39	−0.0147
MSY.L	15,290.21	−0.2938	13,352.31	0.1579
ABG.L	7745.04	0.4658	15,597.38	0.2426
BP	24,313.43	0.2084	35,725.10	−0.6314
EXR.L	22645.50	−0.4687	30,869.82	0.7990
CSG	23,947.16	0.0579	32,317.31	0.5288

Note that $A_0\ Q_0$ and $\rho_{f,Q}$ denote the initial investment of foreign assets measured in new Taiwan dollars (NTD) and the correlation coefficient between each foreign asset and exchange rate, respectively.

that the proportion of times that the losses are larger than VaR should be equal to one minus the VaR confidence level; in other words, the model should provide the correct unconditional coverage. In order to test the null hypothesis that the unconditional coverage equals to the significant level, Kupiec (1995) presents a likelihood ratio statistic. The test procedure is called backtesting.

Given a VaR at the 1% level left-tail over daily horizon for a total of N days, one can count how many times the actual loss exceeds one day's VaR. Define n as the number of exceptions and n / N as the exception rate. The null hypothesis is that a given confidence level for losses, denoted by q for the test, is the true probability. The unconditional coverage is defined by the log-likelihood ratio:

$$\mathrm{LR}_{\mathrm{uc}} = -2\ln\left[(1-q)^{N-n}q^{n}\right] + 2\ln\left\{\left[1-\frac{n}{N}\right]^{N-n}\left(\frac{n}{N}\right)^{n}\right\}.$$

The $\mathrm{LR}_{\mathrm{uc}}$ statistic has a chi-square distribution with one degree of freedom. One would reject the null hypothesis if $\mathrm{LR}_{\mathrm{uc}}>3.84$ at a 95% confidence level.

Table 8
New VaR and traditional VaR in two measures for various periods

Security	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	VaR (0)	VaR (mean)	VaR (0)	VaR (mean)
Panel A: based on the new VaR model				
IBM	113.1735	130.5459	64.5427	66.0586
MSFT	36.2916	36.3419	26.1743	26.6677
PQ	17.9387	18.1320	16.7664	17.1362
AFCE	37.5432	37.5500	28.1403	28.1943
MCD	31.9455	32.1870	27.9445	29.1285
MSY.L	953.1635	956.3211	721.5721	722.7381
ABG.L	426.7432	434.7817	808.9448	825.1581
BP	971.9249	985.2645	808.5752	816.5677
EXR.L	1363.7969	1378.9790	1834.3767	1885.6576
CSG	825.6995	835.8909	1218.6746	1234.8177
Panel B: based on the traditional VaR model				
IBM	113.1735	130.5459	64.5427	66.0586
MSFT	38.2302	38.3669	25.6111	26.1152
PQ	18.4207	18.6389	16.9092	17.2851
AFCE	38.4965	38.5619	27.7679	27.8241
MCD	34.8660	35.1957	27.5398	28.7365
MSY.L	991.6850	992.7000	683.6856	682.4859
ABG.L	378.5896	384.7870	749.8218	763.1369
BP	874.7277	883.1633	1008.2151	1011.9401
EXR.L	1469.1380	1481.7527	1549.1568	1592.6676
CSG	762.8986	768.5733	984.9635	994.7944

Note that the VaRs are valued in new Taiwan dollars of initial investment of each security in various periods.

Table 9
Accuracy of the new VaR model for foreign assets by backtesting in various periods

Security	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	Number of exception	$\mathrm{LR}_{\mathrm{uc}}$	Number of exception	$\mathrm{LR}_{\mathrm{uc}}$
Panel A: based on VaR (0)				
IBM	7	0.5225	11	3.5144
MSFT	7	0.5225	8	0.6651
PQ	4	3.6304	32	57.0117*
AFCE	8	0.1352	11	3.5145
MCD	9	0.0007	10	2.3534
MSY.L	6	1.1988	8	0.6651
ABG.L	14	2.3105	9	1.3962
BP	12	0.8615	25	34.4950*
EXR.L	4	3.6304	26	0.6651
CSG	9	0.0007	8	0.6651
Panel B: based on VaR (mean)				
IBM	7	0.5225	10	2.3534
MSFT	7	0.5225	8	0.6651
PQ	4	3.6304	29	42.9207*
AFCE	8	0.1352	11	3.5145
MCD	9	0.0007	11	3.5145
MSY.L	6	1.1988	6	1.1988
ABG.L	14	2.3105	9	1.3962
BP	12	0.8615	10	2.3534
EXR.L	4	3.6304	25	34.4950*
CSG	8	0.1352	7	0.1880

Note that this table displays Backtests of the new VaR. The critical value is 3.84 at a significant level of 5%. The symbol * denotes the significance at a 5% level.

Table 10
Accuracy of the traditional VaR model for foreign assets by backtesting in various periods

Security	Period I		Period II	
	2002/3/8–2005/8/30		2005/8/31–2007/12/31	
	Number of exception	$\mathrm{LR}_{\mathrm{uc}}$	Number of exception	$\mathrm{LR}_{\mathrm{uc}}$
Panel A: based on VaR (0)				
IBM	7	0.5225	23	28.7719*
MSFT	6	1.1988	9	1.3962
PQ	4	3.6304	32	57.0117*
AFCE	7	0.5225	12	4.8610*
MCD	8	0.1352	21	23.4111*
MSY.L	5	2.2121	8	0.6651
ABG.L	21	11.5336*	10	2.3534
BP	14	2.3105	10	2.3534
EXR.L	2	8.1639*	37	75.1291*
CSG	12	0.8615	13	6.3778*
Panel B: based on VaR (mean)				
IBM	7	0.5225	22	26.0443*
MSFT	6	1.1988	8	0.6651
PQ	4	3.6304	28	43.6991*
AFCE	7	0.5225	11	3.5145
MCD	8	0.1352	21	23.4111*
MSY.L	5	2.2121	8	0.6651
ABG.L	21	11.5336*	10	2.3534
BP	14	2.3105	10	2.3534
EXR.L	2	8.1639*	37	75.1291*
CSG	12	0.8615	13	6.3778*

Note that the critical value is 3.84 at a significant level of 5%. The symbol * denotes the significance at a 5% level.

As shown in Table 9, we fail to reject the null hypothesis for the ten foreign-issued securities at a significance level of 5% in spite of either relative VaR or absolute VaR in the new VaR model, while one cannot accept the null hypothesis for some securities under the traditional model in low oil price years as illustrated in Table 10. Alternatively, the null hypothesis cannot be also accepted in general under traditional VaR model in the period of high oil prices. This discloses that the accuracy of the traditional VaR model is not reliable, and the new VaR model can be used to more accurately calculate VaR for foreign assets.

6. Conclusion

One advantage of VaR is that it is an intuitively appealing measure of risk that can be easily conveyed to a firm's senior manager. Measurement and accuracy of VaRs are two primary concerns for institutional investors in managing financial risk. Especially in the trend of the internationalization of financial products, there is a need for financial managers to control value at risk of foreign assets.

This article provides a new VaR model which is then used to measure the downside risk in the years of low oil price and high oil price for ten foreign-issued securities. We also find that the model is capable of accurately reflecting the loss probability of 5% by means of backtesting. Moreover, we give some improvement on the weakness of the traditional model that VaR is only a function of the volatilities of underlying assets. One can use this method on the other regime, as long as the values are affected by the volatilities of foreign-asset returns and exchange rate and their correlation coefficient. As the economy becomes more globalized, this contribution will give some instructive suggestion to financial corporations and agents.

Appendix A

Given the dynamic processes of foreign-asset price and exchange rate following Geometric Brownian motions, that is

$$\frac{\mathrm{d}A_t}{A_t} = \mu_f \mathrm{d}t + \sigma_f \mathrm{d}W_{1t}, \tag{A.1}$$

and

$$\frac{\mathrm{d}Q_t}{Q_t} = u_Q \mathrm{d}t + \sigma_Q \mathrm{d}W_{2t}, \tag{A.2}$$

where $\mathrm{d}W_{1t}$ is correlated with $\mathrm{d}W_{2t}$, and $\rho_{f,Q}$ is the instantaneous correlation coefficient of the two Brownian motions. The covariance of foreign assets and exchange rate is $\mathrm{Cov}\left(\frac{\mathrm{d}A_t}{A_t}, \frac{\mathrm{d}Q_t}{Q_t}\right) = \rho_{f,Q}\sigma_f\sigma_Q \mathrm{d}t$.

Let $X_t = Q_t A_t$. By Ito's lemma, we can obtain

$$\frac{\mathrm{d}X_t}{X_t} = \frac{\mathrm{d}Q_t}{Q_t} + \frac{\mathrm{d}A_t}{A_t} + \frac{\mathrm{d}Q_t}{Q_t}\cdot\frac{\mathrm{d}A_t}{A_t} \tag{A.3}$$

Substituting Eqs. (A.1) and (A.2) into Eq. (A.3), we get

$$\frac{\mathrm{d}X_t}{X_t} = \left(\mu_f + \mu_Q + \rho_{f,Q}\sigma_f\sigma_Q\right)\mathrm{d}t + \sigma_f \mathrm{d}W_{1t} + \sigma_Q \mathrm{d}W_{2t}.$$

Appendix B

First, we prove the relative VaR under the new VaR model. By the definition of VaR, given a confidence level α, it can be expressed as

$$P_r(A_t \le A_\alpha) = \alpha. \tag{B.1}$$

Based on the relative VaR in domestic dollars of the foreign asset, VaR (mean)$\equiv E\,(Q_T A_T) - V_\alpha$, Eq. (B.1) is transformed to

$$P_r(A_T Q_T - E[A_T Q_T] \le VaR(mean)) = \alpha. \tag{B.2}$$

By the application of Proposition 1 to Eq. (B.2), we can get

$$Z_\alpha = \frac{\ln[\mathrm{VaR(mean)} + E(A_T Q_T)] - \left(\mu_f + \mu_Q - \frac{1}{2}\sigma_f^2 - \frac{1}{2}\sigma_Q^2\right)T - \ln(A_0 Q_0)}{\sqrt{\left(\sigma_f^2 + \sigma_Q^2 + 2\sigma_f\sigma_Q\rho_{f,Q}\right)T}}. \tag{B.3}$$

From Eq. (B.3), the result is

$$\mathrm{VaR(mean)} = \left|\mathrm{Min}\left\{A_0 Q_0\left[e^{\left(\mu_f+\mu_Q-\frac{1}{2}\sigma_f^2-\frac{1}{2}\sigma_Q^2\right)T+Z_\alpha\sqrt{\left(\sigma_f^2+\sigma_Q^2+2\rho_{f,Q}\sigma_f\sigma_Q\right)T}} - e^{\left(\mu_f+\mu_Q+\rho_{f,Q}\sigma_f\sigma_Q\right)T}\right], 0\right\}\right|.$$

Similarly, we can get the absolute VaR as follows:

$$\mathrm{VaR}(0) = \left|\mathrm{Min}\left\{A_0 Q_0\left[e^{\left(\mu_f+\mu_Q-\frac{1}{2}\sigma_f^2-\frac{1}{2}\sigma_Q^2\right)T+Z_\alpha\sqrt{\left(\sigma_f^2+\sigma_Q^2+2\rho_{f,Q}\sigma_f\sigma_Q\right)T}} - 1\right], 0\right\}\right|.$$

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