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On the application of efficient hybrid heuristic algorithms – An insurance industry example

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ABSTRACT

This paper proposes an optimization approach for generating an investment strategy for multi-period asset-liability management of long-term with-profit life insurance policies. Our approach uses models to simulate the processes insurance companies employ when determining multi-period investment strategies over a given planning horizon. The approach utilizes an enhanced heuristic algorithm to determine optimal multi-period investment strategies. Simulation models take into account asset numbers, objective functions, and asset allocation frequency. Strategy performance is evaluated by applying three single-period investment strategies to the simulation models. Computational results not only verify the efficiency and robustness of the algorithm, but also demonstrate the effectiveness of frequent asset reallocation, and dispute the suitability of traditional top-down investment strategies in maximizing investment returns of with-profit insurance policies.

1. Introduction

Over the last few decades, improvements in mortality rates have necessitated a reassessment of the different products offered by the insurance industry. Common products such as pension plans are now facing ever-greater challenges in performing their intended function of protecting retirees from outliving their resources. The recent global financial crisis has drawn this issue into even sharper focus, as overburdened governments and businesses attempt to provide benefits to retirees.

Insurance policy types have many different classifications. The two most common types are referred to as "with-profit" and "unit-linked". A conventional non with-profit and non unit-linked life insurance policy implies that the policyholder earns the assumed (guaranteed) interest rate of the contract, i.e., the interest rate taken into calculations to determine the value of a contract. With conventional policies, the insurance amount is specific, and insurers take on investment risk. This means that policyholders' earnings are guaranteed by that interest rate and are irrelevant to the profit or loss of the insurance company. On the other hand, a with-profit policy (referred to as a "participating policy" in the US) provides an assumed interest rate generally lower than that of a conventional policy, but provides company share dividend payments. This is intended to reduce potential insurer loss when the assumed interest rate is greater than the actual return rate. As the assumed interest rate of a with-profit policy is less than that of a conventional policy, the price of the with-profit policy is greater. This reduces the investment risk faced by insurers and enables them to have the flexibility to pursue a more aggressive investment policy aimed at achieving long-term capital growth. Policyholders are willing to pay a higher premium for the opportunity to share in higher potential profits. Policyholders receive this benefit through dividends, usually an increase in the insurance amount, when the actual return rate is greater than the assumed interest rate. However, the dividend mechanism is not transparent and clear to policyholders, as the dividend amount is determined by the insurer as well as market competition.

"Unit-linked policies" (referred as "variable life policies" in the US, or "segregated funds" in Canada) are a type of insurance that provides both life insurance and investment opportunity. For conventional life insurance policies, including with-profit and unitized with-profit policies, insurers have the authority to manage funds (premiums) collected from policyholders. This type of fund is referred to as a "general fund" when discussing separate funds for

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unit-linked policies. Most of the premiums (minus their deductions to general fund) of a unit-linked policy are located in a separate fund. Any associated deductions are used to buy some low-level life protection and pay expenses. The investment component of unit-linked insurance exists as a separate fund. Policyholders have the authority to allocate his fund, i.e., separate fund, to a basket of mutual funds. However, policyholders take on investment risk and face a high level of uncertainty over the final insurance amount, as these amounts tend to fluctuate in line with stock market movements. On the other hand, policyholders can track the value of their investment as well as the insurance payout and expense charges at any given point in time, and are able to cash in if necessary. While the investment risk for policyholders is higher with unit-linked policies than in conventional policies, unit-linked policies offer policyholders both higher return potential and greater transparency.

Unitized with-profit policies emerged in the UK in the mid-1980s as consumers demanded more transparency into the design of with-profit policies [1]. In reality, unitized with-profit policies are a type of with-profit policy with united feature. The word “unitized” means that the fund is broken into units, just like united-linked policies and general mutual funds. This allows the funds to be an open-ended investment, and investors can pool assets while retaining individual net asset values. This mechanism, combined with a declared rate of interest, helps consumers understand the methods by which with-profit policy returns are determined. Part of the transparency of unitized with-profit policies comes from a specific interest rate declaration process similar to the determination of declared interest rate, r_p, discussed in this paper. Policyholders realize the investment return of their premiums based on this declaration. In contrast, policyholders of with-profit policies cannot appreciate the return rate of their premiums when there is an increase of the insurance amount or when the insurer makes a profit.

Unitized with-profit policies combine the with-profit policy advantages of participating in insurer profits with the transparency of unit-linked policies. Unitized with-profit policyholders are aware of the value of their investment at any given point in time and are able to cash in if necessary. They also have the chance to achieve an increased return without taking on the risk of investment. In addition, most unitized with-profit policies provide policyholders with the flexibility to change premiums. We have outlined the features of each type of life insurance policy in Table 1.

While the risk associated with unitized with-profit contracts, as with virtually all insurance contracts, includes risk from financial markets, surrenders, and mortality, our study focuses on financial risk. Since the interest rate is declared in advance, it is vital that insurers issuing unitized with-profit contracts take the declared interest rate into account. For example, if the insurer pursues an aggressive investment strategy during the life of a contract and an initial bull market is followed by a bear market, the insurer may suffer serious losses due to excess payments during the bull market period. To reduce financial risk posed by unitized with-profit policies, the approach outlined in this paper utilizes a hybridized evolutionary algorithm to explore optimal asset allocation.

2. Models

Typical investment strategies attempt to diversify investment through asset allocation to achieve a high level of return while lowering potential risk. The Markowitz mean-variance (MV) model is widely regarded as the gold standard for asset allocation [2–4]. The MV approach is practical for solving the problem of single-period asset allocation under a restrictive set of assumptions. However, a single-period investment is not suitable for long-term obligations such as insurance liabilities, which extend over a period of five or more years. Applying the MV approach to problems of multi-period asset allocation is also problematic; consequently, mean-variance portfolio optimization is inadequate for insurance liability asset management.

Current popular methods for solving multi-period asset allocation are control theory [5,6] and the Martingale approach [7,8], which are widely applied to financial optimization models. However, the objective of these two approaches is to find a theoretical solution to multi-period asset allocation problems. They fail to deal with realistic objectives owing to their theoretical nature. In order to get a closed-form solution, many real-world issues, such as transaction costs and limitations on the proportion of asset allocation for some specific assets, must be ignored in order to prevent problems from becoming too complex. While these approaches elegantly provide insight into asset allocation problems, they are inapplicable in practice. For example, the proportion of asset allocation for specific assets may be subject to regulatory limits, or there may be a certain probability of insolvency in the objective or constraint function. Neither control theory nor the Martingale approach can obtain a closed-form solution to meet these simple constraints. Linear programming is also incapable of solving complicated asset allocation models, since many are nonlinear [9]. Instead, heuristic algorithms have become the most commonly used techniques for optimization problems, as they provide a general-purpose modeling framework capable of considering a multiplicity of constraints. Practitioners evaluate asset allocation issues through simulations, which enables them to realize potential outcomes over a set of return scenarios with specific asset allocation strategies. Among the heuristic algorithms, the Evolutionary algorithms have been successfully applied to many research fields and became useful simulation tools [9–13].

This paper proposes an optimization approach to generate an optimal investment strategy for multi-period asset liability management of long-term with-profit policies. The objective function considers both investment returns and insolvency risks. The optimal multi-period investment strategy allows insurance companies to achieve the highest investment return while minimizing insolvency risk. Our simulation models replicate the decision processes of insurance companies in determining multi-period investment strategies over a given planning horizon, and a hybrid heuristics algorithm integrated into the simulation models determines optimal multi-period investment strategies. For illustration purposes, a set of equal-probability scenarios of future returns are simulated using Wilkie's investment model [14]. Each scenario represents one potential uncertain return over the planning horizon, and a large scenario set represents highly unlikely market swings. Given a set of plausible scenarios of future returns (e.g., 1000 equal-probability scenarios of returns over 10 years), we developed a simulation model to calculate the terminal profit and insolvency probability for a given multi-period investment strategy. Evolution strategies integrated into the simulation models enable our approach to obtain the optimal investment strategy, and we include different simulation models based on asset numbers, objective functions and asset reallocation frequency. Three single-period investment strategies (conservative, somewhat aggressive and aggressive) were chosen to evaluate the performance of the multi-period investment strategies produced. The information used to generate different scenarios for Wilkie's model is presented in Appendix A. Based on that information, the interested reader may reproduce the approach using the algorithms presented in this study.

Asset liability management for with-profit policies is an important and challenging undertaking for insurance companies, primarily because of the guaranteed returns it provides to policyholders. Since these types of liabilities are long-term in nature, an appropriate investment strategy is required to match them.

Table 1
Comparison of conventional life policy, with-profit, unit-linked and unitized with-profit policies.

	Conventional	With-profit	Unit-linked	Unitized with-profit
Insurance amount	Fixed	Variable, may increase due to dividend	Variable, depends on investment performance	Variable, may increase due to dividend
Participation in insurer profits	No	Yes	No, but can participate in stock market	Yes
Investment decision	Insurer	Insurer	Policyholder	Insurer
Transparency	Low	Low	High	High
Flexibility in premium payment	No	depends on contract	depends on contract	depends on contract

Therefore, insurance companies use standard asset classes including short-term bonds, long bonds, index-linked gilts and equities in conjunction with a multi-period asset allocation approach to achieve optimal results through annual asset reallocation.

Our research uses well-known models familiar to the insurance industry [14,15], and are summarized as follows:

Let the return rate of long bonds be $i_b(t)$ and the return rate of stocks (equity) be $i_s(t)$ during the period $t-1$ to t. Moreover, $w_b(t)$ denotes the weight of long bonds in the portfolio at the start of policy year t and $w_s(t)$ denotes the weight of stocks in the same portfolio. Thus, the terminal value of the portfolio is:

$$A(T) = A(0)\left[\prod_{t=1}^{T}[w_b(t)(1+i_b(t)) + w_s(t)(1+i_s(t))]\right] \tag{1}$$

With-profit life insurance contracts have three key benefits: a certain guaranteed benefit, periodic reversionary bonuses, and a terminal bonus. The reversionary bonus rate is usually determined via a smoothing adjustment to the rate of return of the asset portfolio. The reversionary bonus, once added, becomes part of the guaranteed benefit. The guaranteed payoff and the reversionary bonus constitute the "policy reserve". Upon the claim date of the contract, a terminal bonus is included, based on the final surplus earned by the insurance company. The smoothing and terminal bonus are used to reduce the insurance company's guaranteed risk as illustrated below:

When considering a single premium unitized with-profit contract which will terminate at T, suppose an policyholder pays a single premium, $P(0) = 100$, at the start of the contract. Over the lifetime of the contract, at the beginning of each policy year the policy value, $P(t)$, accumulates at rate r_p so that:

$$P(t) = P(t-1)(1 + r_p(t)) \quad t = 1, 2, \ldots, T \tag{2}$$

where r_p is the declared interest rate of the contract, decided by the equation:

$$r_p(t) = \max\left(r_G, \frac{\beta}{n}\left(\frac{A(t)}{A(t-1)} + \cdots + \frac{A(t-n+1)}{A(t-n)} - n\right)\right), \quad n = \min(t, \varphi) \tag{3}$$

Using the annual guaranteed rate r_G, $\beta \in (0, 1)$ denotes the participating rate, where n is the length of the smoothing periods chosen as $n = \min(t, \varphi)$, and the smoothing factor φ is the length of the averaging period (3 years throughout this study). $A(t)$ denotes the value of the reference portfolio, and the initial portfolio value $A(0)$ is equal to $P(0) = 100$ if there are no other charges. This smoothing adjustment, called the arithmetic smoothing scheme, is the most common method used in the United Kingdom for the accumulation of benefits and reversionary bonuses in unitized with-profit contracts.

At the maturity date of the contract, T, the policyholder will receive the terminal value of the policy plus some percentage of any surpluses generated by the reference portfolio exceeding the value of the benefit. That is, the terminal payoff (henceforth denoted as *gain*) of the policyholder is:

$$gain = P(T) + \gamma R(T) \tag{4}$$

where

$$R(T) = (A(T) - P(T))^+ = \begin{cases} A(T) - P(T), & A(T) \geq P(T) \\ 0 & A(T) < P(T) \end{cases} \tag{5}$$

The $\gamma \in (0, 1)$ is the second participation parameter. The participation parameter β is used to determine the declared interest rate of the contract $r_p(t)$, and the participation parameter γ is used to determine the terminal payoff (*gain*) of the policyholder. Consequently, the terminal profit for the insurer, *Earn*, is:

$$Earn = \begin{cases} (1-\gamma)R(T) & \text{if} \quad A(T) \geq P(T) \\ A(T) - P(T) & \text{if} \quad A(T) < P(T) \end{cases} \tag{6}$$

The contract guarantee is triggered if the reference portfolio is less than the policy value at the maturity date. Thus, the guaranteed cost of the insurer at the maturity date, D, is:

$$D = (P(T) - A(T))^+ = \begin{cases} P(T) - A(T), & P(T) \geq A(T) \\ 0 & P(T) < A(T) \end{cases} \tag{7}$$

Even though insurance investment models can vary, attention has focused on the model that can cover most situations. One criticism of traditional with-profit policies is that their dividend amounts depend partially on the discretion of insurer actuaries. The interest rate declaring rule is not transparent when compared to unitized with-profit policies [1,15]. To improve this deficiency, the interest rate declaring rule of unitized with-profit policies has been adapted to traditional with-profit policies. The model presented in this study can be versatile and applicable for both with-profit and unitized with-profit life insurance contracts.

3. Simulation optimization approaches

A typical investment optimization problem is to obtain the optimal investment strategy for different assets under bearable risks. Optimization theories and applications are generally classified into local and global search algorithms, and many studies have discussed the advantages and drawbacks of each [16,17]. Briefly, local search is faster than global search, but it is sensitive to initial solutions and often becomes trapped in local optimums. Local search also has difficulty obtaining true derivatives for complicated or nonlinear functions. If the functions have constraint conditions, the algorithm cannot guarantee that the solutions generated meet the constraint requirement.

Global search algorithms can identify several critical parameters for the algorithm to converge to the right solution. These include population size, number of generations, solution space, and number of decision variables. However, a global search algorithm can be computationally expensive, as it requires a both large population and a large number of generations for the algorithm to converge. Because the complexity of a problem may vary, no particular set of parameters is capable of generating optimal results for all problems. Poorly chosen parameters may prevent the search algorithm from obtaining a global optimal solution. This problem becomes more severe when the number of decision variables increases, thereby dramatically expanding the solution space [18].

Our study proposes a hybrid approach that applies the advantages of local and global search techniques to the simulation models, combining an evolution strategy algorithm (ES) for global optimization, and the Levenberg–Marquardt algorithm (LM) for the local search method. Solutions obtained by ES then work as the initial solutions for LM, speeding up final optimal solution convergence. Our application of these algorithms, along with our methodology for testing their effectiveness and robustness are discussed in the following sections.

3.1. The local search algorithm

LM is a robust, standard numerical method in nonlinear optimization, representing a combination of steepest descent and the Gauss–Newton method. It iteratively locates a minimum for nonlinear functions (it requires an inverse function for problem finding maximum). The formula is as follows:

$$(J^TJ + \theta diag(J^TJ))\delta = -J^Tf(x_i) \quad (8)$$

where J is the Jacobian matrix and J^TJ is also denoted as a Hessian matrix. δ is the solution vector, defined as $x_{i+1} - x_i$ and θ is a positive scalar and used as the damping term to adjust between the steepest descent and Gauss–Newton methods.

Several issues typically arise in LM optimization. Since Jacobian (and Hessian) matrixes are involved, it is necessary to find the derivatives of the objective function. However, because the objective function is generally high-order, nonlinear, or very complicated, a true derivative is difficult or impossible to obtain. Thus, a numerical approximation is required. To reduce numerical errors, our study adopts a four-point formula approximation [19]. Notice that the error has an order of h^4. In this study, $h = 0.001$ is used.

$$\frac{\partial f}{\partial x_i} = \frac{1}{12h}(f(x_i - 2h) - 8f(x_i - h) + 8f(x_i + h) - f(x_i + 2h)) + \frac{h^4}{30}f^{(5)}(\xi) \quad (9)$$

Another issue relates to constraint conditions. Most optimization problems in real world applications involve constraints to the solutions. When solving mathematical optimization problems, it is generally difficult to find a closed form for the function subject to resolve outside conditions or constraints. LM cannot guarantee that solutions will fall inside the constraints. To overcome this problem, the Lagrange Multiplier method provides a strategy for finding the optimal solution of a constrained function [20]. However, utilizing this method to overcome constraint conditions for a local search increases computational costs, since the solution space increases from $(x_1, x_2, \ldots, x_n)$ to $(x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots, \lambda_M)$. To avoid this issue, LM can be used simply to search for the solution without worrying about any constraints. If a given solution does not meet the constraints, then the solution from LM will be abandoned (the original solution from the global search will remain) and another search will begin. If global search can provide a good initial solution for LM, then LM can reach convergence more efficiently.

3.2. The global search algorithm

Evolutionary algorithms (EAs) are randomized search methods incorporating the principles of evolution [10,21,22] and are common among heuristic algorithms. Instead of single points, EAs use populations to search and solve complex optimization problems. Unlike local searches, the initial populations in EAs are usually generated randomly. Offsprings are produced from the members, or parents, in the population. Favorable offsprings then populate the next generation, based on the theory of "survival of the fittest". This process continues until a termination criterion is satisfied and a superior solution is obtained. As mentioned previously, this approach requires a large population and numerous generations for convergence. While other evolutionary heuristic algorithms exist, some of them were originally designed for integer-base applications, such as the genetic algorithm and Ant Colony algorithm. Even though they can be adjusted and used for real number applications, modifications are usually required to change the natural. On the other hand, evolution strategy (ES) was originated for real number application and has been available for many decades. Many studies have demonstrated the effectiveness of using ES for non-linear, real number problems [12,23,24]. As the financial problem presented in this study is a continuous real number problem, ES was selected for the global searching algorithm.

Evolution strategy (ES) is a type of evolutionary algorithm originally developed for solving nonlinear programming problems [25–27]. The steps of (μ, ζ) ES approach, which μ is the number of parents and ζ is the number of offsprings produced from the parents and ζ is about seven times μ [10,25,28], are summarized as follows:

Step 1. Generate a population for initial generation.

A population of μ solutions is generated. Each solution is a two-part row vector. The elements in the first part are the values of decision variables (x_j), and the elements in the second part are mutation step sizes (σ_j) corresponding to the decision variables in the first part.

Step 2. Apply recombination and mutation to the parents to produce λ offsprings.

Parents (A and B) are randomly chosen from the population, and recombination and mutation are applied to produce child C. Discrete recombination is used to determine the child C decision variable values. The value of each decision variable is random and equally selected from the value of the same variable in A and B. The mutation step sizes for C are determined by intermediate recombination. Determining the mutation step size is usually difficult. The optimal step size depends on the problem, and may vary during the simulation process. Small steps often work well, yet larger mutation steps, if successful, can yield good results much quicker. Our study follows previous research [10,25,28,29] in setting the mutation step size. The recombination approach is implemented by calculating the intermediate mutation step size and then used for the final mutation step size. In other words, the jth mutation step size in C is determined by the average of the jth mutation step size in A and B, as demonstrated by the following equation:

$$\sigma_j(C) = 0.5(\sigma_j(A) + \sigma_j(B)) \quad (10)$$

Then Child C is mutated by first modifying its mutation step sizes, then adding these to mutate the corresponding decision variables. Each mutation step size, σ_j (C), is modified by the following equation:

$$\sigma'_j(C) = \sigma_j(C)\exp(t'N(0, 1) + tN_j(0, 1)) \quad (11)$$

τ and τ' are two exogenous parameters, and are set as the following equations [10,25,28,29].

$$\tau = \left(\sqrt{2\sqrt{n}}\right)^{-1}, \ \tau' = \left(\sqrt{2n}\right)^{-1} \quad (12)$$

n is the size of the decision variables. These two variables were frequently set to 1 according to Nissen and Biethahn [28]. The $N(0, 1)$

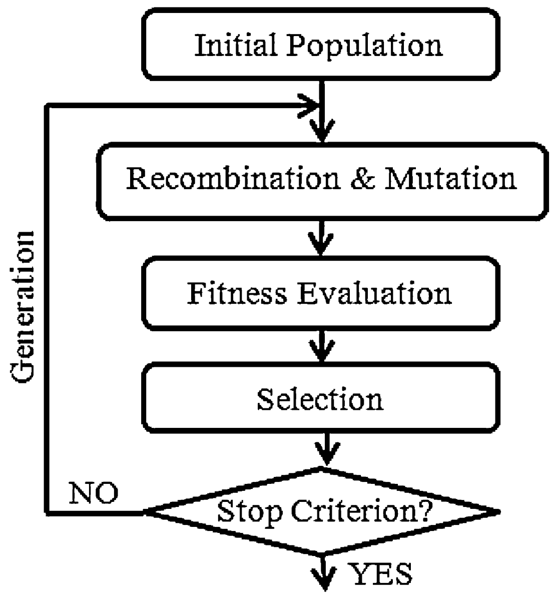

Fig. 1. Evolutionary strategy flowchart.

denotes a normally distributed one-dimensional random number with a mean of zero and standard deviation of one. That random number is generated anew for each value of j, as indicated in $N_j(0, 1)$. The decision variable, x_j, is mutated by the following equation:

$$x'_j(C) = x_j(C) + N_j(0, \sigma'_j(C)) \quad (13)$$

When child C is generated, a normalization method is applied to the four asset variables to ensure that the sum equals one. The reproduction procedure repeats until ζ offsprings are produced. This procedure of determined mutation step size is carried out in each generation. An example of the solution vector (take the size of the decision variable as 2, for example) could be (−82.52, 86.31, 2.87, 1.58). Notice that the first two are the decision variable values, and the last two are the mutation step size corresponding to the decision variables. When the simulation ends, this solution vector will converge and the decision variables will converge to the optimal solution, and the mutation step size values will approach to zero.

Step 3. After all the ζ offsprings are all produced, the fitness function is evaluated with the decision variables with all the offsprings.

Step 4. Select the best μ offsprings to constitute the population for the next generation.

Generated children act as inputs for the simulation model, and we choose the best μ parents from among the ζ offsprings for the next generation.

Step 5. Check the termination criterion.

If the termination criterion is satisfied, stop, otherwise go to Step 2. Our approach sets the number of generations, *maxgen*, as the termination criterion.

The flow chart of this algorithm is presented in Fig. 1.

3.3. Hybrid approach (HA)

Heuristic approaches are population-based algorithms, and require both a large population and a large number of generations to converge globally. The general concern is the performance efficiency if the objective function is computationally costly. The literature shows the performance of current optimization algorithms deteriorates when the size of the determinate variables increase [31,32]. Unfortunately, many real-world problems tend to be large-scale nonlinear problems. This is particularly true for insurance investment models, which contain many complex constraints over a very long simulation horizon. Simulation models that require excessive computational times would be impractical for financial institutions. The motivation behind the hybrid approach (HA) proposed in this study is to utilize the advantages of both local and global algorithms efficiently and provide quality solutions within a reasonable period. In this study, ES is used to find a solution for future diversification, and the best two solutions are used as initial solutions for intensification purposes in the local search algorithm. We attempt to limit the population size and number of generations in the ES algorithm to save computation costs, and use LM to speed up convergence to a proper initial solution. Numerical approaches are used for the derivative functions. The decision variables in our approach are the proportions of the asset allocations at each point of asset reallocation. Note that in the multi-period asset allocation model containing four assets, for a 10-year term policy, we have to determine forty decision variables for a solution. During the (μ, ζ) ES process, when child C is generated, a normalization method is applied to the four asset variables. The proportion of asset allocation is a real value in the range of [0, 1], and the sum of the four asset allocations in a given period is equal to one. The initial mutation step sizes are set to three, following Back's research [10,25]. Since we assume the solutions from ES are valid, we do not require the Lagrange Multiplier for the LM. If the LM solution cannot be improved, the original ES solutions are retained. With this approach, the extra computation load from solving the Lagrange Multiplier equation is alleviated. Experiment results have shown this assumption to be reasonable, and computational costs have been dramatically reduced.

3.4. Effectiveness and robustness test

Research has proven hybrid approaches can improve convergent speed and efficiency, and achieve high precision solutions for complex continuous large-scale applications [31,32]. Ten common verified benchmarks are used to test effectiveness, robustness and efficiency [29,30]. The number of decision variables for the benchmarks in our study was set to 50 to ensure efficacy with a target insurance model of 40 variables. Our research compared the results of our approach with those of basic ES simulation, and also compared them with another evolutionary heuristic algorithm, the genetic algorithm (GA) [33]. While different evolutionary techniques have unique control parameters that can make comparisons difficult, GA is an evolutionary approach similar to ES, and is used here for effectiveness and robustness comparison. Tables 2 and 3 present these benchmark functions and the computational results.

For the robustness test, we independently implemented basic ES (BES), basic GA (BGA) and HA to each benchmark function ten times with a randomly generated initial population. A decision variable for a solution in an initial population was randomly generated from a uniform distribution, with the range constraining the variable in the benchmark functions. The parameters of BES, BGA and HA were set as follows:

For the BES, the number of parents (μ) was twice the decision variables considered in a solution ($\mu = 2n = 100$). The number of offsprings (ζ) was seven times μ. The number of generations for convergence, *maxgen*, was set as 200. In order to provide a fair comparison for BGA, we set the population size to 800 (the sum of parent number and the offsprings of BES) and the same generation number of 200. Note that these numbers are relatively small when compared with those used in previous studies [29,30].

Table 2
Selected Benchmark functions and related information.

Objective function ($n = 50$)	Range	Minimum value
$f_1(\vec{x}) = \sum_{i=1}^{n} x_i^2$	$-5.12 \le x_i \le 5.12$	$f_1(\vec{0}) = 0$
$f_2(\vec{x}) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	$-10 \le x_i \le 10$	$f_2(\vec{0}) = 0$
$f_3(\vec{x}) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_i \right)^2$	$-100 \le x_i \le 100$	$f_3(\vec{0}) = 0$
$f_4(\vec{x}) = \sum_{i=1}^{n-1} (100 \cdot (x_{i+1} - (x_i)^2)^2 + (x_i - 1)^2)$	$-30 \le x_i \le 30$	$f_4(\vec{1}) = 0$
$f_5(\vec{x}) = \sum_{i=1}^{n} i \cdot x_i^4 + rand[0, 1)$	$-1.28 \le x_i \le 1.28$	$f_5(\vec{0}) = 0$
$f_6(\vec{x}) = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	$-5.12 \le x_i \le 5.12$	$f_6(\vec{0}) = 0$
$f_7(\vec{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	$-32 \le x_i \le 32$	$f_7(\vec{0}) = 0$
$f_8(\vec{x}) = \frac{1}{4000}\left(\sum_{i=1}^{n} x_i^2\right) - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i+1}}\right) + 1$	$-600 \le x_i \le 600$	$f_8(\vec{0}) = 0$
$f_9(\vec{x}) = \frac{\pi}{n}\left\{10(\sin^2(\pi y_1)) + \sum_{i=1}^{n-1}((y_i - 1)^2(1 + 10(\sin^2(\pi y_{i+1})))) + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	$-50 \le x_i \le 50$	$f_9(-\vec{1}) = 0$
$f_{10}(\vec{x}) = 0.1\left\{\sin^2(3\pi x_i) + \sum_{i=1}^{n-1}((x_i - 1)^2(1 + \sin^2(3\pi x_{i+1}))) + (x_n - 1) + (1 + \sin^2(2\pi x_n))\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	$-50 \le x_i \le 50$	$f_{10}(1, \ldots 1, -4.76) = -1.142$

As discussed before, heuristic algorithms require significant biological parameters to reach global optimization, and since the population size and *maxgen* in this study were not large enough, computational results show that both BES and BGA did not converge completely (i.e., to good solution quality). In contrast, most results show that HA converged to global optimums. Table 3 also presents mean and standard solution deviations. Most HA results show that they reached optimal solutions, and standard deviations approached zero. This also indicates the solutions obtained from BES are good initial solutions to the local search that LM can converge efficiently to high-precision solutions. Compared to BES and BGA, HA produced superior results, particularly for functions f_1, f_2, f_3, f_7, f_8, f_9, f_{10}. The global minimum, f_4, occurred in a very flat region where LM failed to converge well. Function f_5 was not a continuous function, and LM failed to catch the derivatives. Additionally, function f_6 was a very complicated multimodal function, and the BES results were not close to a global optimum, making LM fail to converge to a good quality solution. However, if the solution domain (n) is reduced from 50 to 40, then both functions converge well under the HA algorithm.

Notice that the goal of this research is not to compare HA with BES and BGA. These results demonstrate that the proposed algorithm is capable of efficiently converging on optimal solutions, regardless of the initial benchmark population. Accuracy and efficiency are very important for many large-scale real-world applications, and this proposed approach meets this requirement. These results also help demonstrate the suitability of applying our approach to insurance investment models.

4. Results and discussion

In this section, we first describe the computational results that illustrate the performance of the application of the proposed HA for the multi-period asset allocation models using four different objective functions. Then, we discuss the sensitivity analysis of β andγ on those models.

4.1. Computational results

The example used for our research is a ten-year unitized with-profit policy. For illustration purposes, we adopted Wilkie's model [14] to generate 1000 simulations of future market conditions. Thus, 1000 simulations of 10-year asset returns would yield 1000 equal-probability scenarios for use by an asset manager to find the best multi-period asset allocation for a participating policy. One strategy is to change asset allocation proportions annually. Insurance companies are generally concerned with one or all three of the following: to increase profit, reduce deficits, or decrease the probability of insolvency. Therefore, we consider four objective functions of an insurer's investment plan. The first objective is to maximize expectation of terminal profit. The second objective is the same, with the constraint of limiting loss. This constraint limits the mean of the top ten percent of deficits to less than fifteen percent of the mean of $P(T)$. The third and fourth objectives attempt to minimize the expectation of guarantee costs and to minimize the probability of deficits. These four objective functions represent four different objectives for the insurer. The insurer selects one of the objectives first, and then finds the optimal asset allocation strategy. They are

Table 3
Computational results for benchmarks of different approaches.

Objective functions ($n = 50$)	Minimum value					
	BES		BGA		HA	
	Mean	STDev	Mean	STDev	Mean	STDev
f_1	1.02E−06	7.12E−07	4.41E+01	1.10E+01	8.49E−22	1.11E−22
f_2	8.94E−06	6.94E−06	3.00E+00	4.61E−01	3.35E−10	3.68E−11
f_3	8.43E+03	6.16E+02	5.44E+02	6.23E+01	2.37E−17	6.25E−18
f_4	1.00E+02	3.63E+01	7.12E+02	8.37E+01	9.49E+01	7.67E+00
f_5	1.87E−02	2.37E−03	3.13E−03	9.61E−04	1.74E−02	4.13E−03
f_6	1.77E+01	2.76E+00	4.41E+01	3.70E+00	1.31E+01	2.76E+00
f_7	2.18E−04	1.11E−04	2.32E+00	1.41E−01	2.16E−07	1.62E−08
f_8	3.22E−02	2.45E−02	1.42E+00	6.90E−02	2.90E−05	4.09E−05
f_9	4.18E−05	3.38E−05	1.77E−01	1.73E−02	1.00E−22	0.00E+00
f_{10}	−0.786	2.17E−01	2.56E+00	8.17E−01	−1.14	4.91E−04

Table 4
Insurer profits and deficits of three single-period investment strategies.

	Long Bonds	Stocks	*E* [*Earn*]	*E* [*D*]	Prob ($D > 0$)
Conservative	0.7	0.3	11.55	2.84	0.176
Less aggressive	0.5	0.5	14.13	4.52	0.212
Aggressive	0.3	0.7	13.59	8.53	0.271

independent and cannot be optimized simultaneously. These four objectives are defined as follows:

Objective 1: $\max\{E[Earn]\}$
Objective 2: $\max\{E(Earn)\}$ subject to $CTE90(D) < (0.15 \cdot E[P(T)])$
Objective 3: $\min\{E(D)\}$
Objective 4: $\min\{\text{Prob}[D > 0]\}$

The parameters used in the simulation models are set as follows: the guaranteed interest rate r_G is 0.07; both the premium $P(0)$ and the initial asset $A(0)$ are 100; the insurance period T is 10; the smoothing factor φ is 3 and both the participating rates β and γ are 0.6. The β is introduced in (3) and γ can be found in (4). HA is applied to the simulation models to find the best multi-period asset allocation. These simulation models were executed with two assets and four assets in the portfolio. To illustrate the changes between risky and non-risky assets in investment strategies more clearly, we first discuss the computational results of the models with two assets (equities and consols) with equities representing risky assets and consols representing non-risky assets. We then compare the performance of the investment strategies generated with two and four assets each. We investigate three single-period investment strategies: an aggressive investment strategy (i.e., holding 70% equities and 30% consols during the 10-year period); a somewhat aggressive investment strategy (50% equities and 50% consols during the 10-year period); and a conservative investment strategy (30% equities and 70% consols during the 10-year period).

Table 4 shows the investment results of insurer profits and guarantee costs for the three single-period investment strategies. The three criteria used to evaluate the performance of different investment strategies are expectation of terminal profit, *E* [*Earn*], expectation of guarantee cost, *E* [*D*], and probability with deficits, Prob ($D > 0$). Studies find that insurers have larger profits and deficits when they switch their investment strategy from conservative to somewhat aggressive [34]. However, when insurers switch strategy from somewhat aggressive to aggressive, profits decrease and deficits rise. This is because insurers declare more to the insured in the earlier years of a policy and increase liability in later years. This is an interesting finding, in that it differs from the general belief that the more aggressive the strategy used, the higher the profits and the corresponding probability of deficits.

Table 5 presents the investment results of the multi-period HA investment strategies for the two-asset models. For convenience, OBJ1 denotes the strategy for the model with the first objective function, OBJ2 for the model with the second objective function, and so on. The first column in Table 5 lists these strategies. The

Table 5
Insurer profits and deficits in the best multi-period investment strategy for the two-asset model.

Objective function	*E* [*Earn*]	*E* [*D*]	Prob ($D > 0$)
OBJ1	20.34	9.27	0.256
OBJ2	18.00	4.10	0.184
OBJ3	12.11	2.49	0.157
OBJ4	14.06	2.68	0.137

Table 6
Mean and standard deviation of the five objective values produced by five different initial populations for the two-asset model.

	Mean (optimal value)	Std dev
OBJ1	20.3399469663	0.0000037557
OBJ2	17.9984818335	0.0000272324
OBJ3	2.4886592916	0.0000125281
OBJ4	0.13700000000	0.0000000000

computational results show that a multi-period investment strategy can simultaneously increase insurer profits and lower risk. For instance, OBJ2 resulted in larger expected profits (18.00) than those using somewhat aggressive (14.13) and aggressive (13.59) strategies. Additionally, OBJ2 had smaller expected deficits and deficit probability (4.1 and 0.184, respectively) than the somewhat aggressive (4.52, 0.211) and aggressive (8.53, 0.271) strategies. Furthermore, both OBJ3 and OBJ4 had larger expected profits, lower expected deficits, and lower deficit probabilities than those of the conservative strategy. We also investigated the robustness of the proposed algorithm by applying it to the two-asset model with each of the four objective functions, using five randomly generated initial populations. For each of the four cases, we observed that the multi-period investment strategies converged from the five initial populations, and all five converged to almost the same investment strategy. Our results demonstrate that the robustness of this hybrid algorithm provides a great advantage for producing effective multi-period investment strategies. For each case, the mean and standard deviations of the four objective values were calculated, and the results are summarized in Table 6.

Traditional top-down investment strategies promote an "aggressive first, conservative later" approach. However, our findings dispute that notion. Fig. 2 shows multi-period investment strategies for the two-asset model using each of the four objective functions. A common property among these strategies is an initially conservative application, which then gradually becomes more aggressive as the term concludes. These findings turn the traditional investment wisdom on its head, advocating instead for a "conservative first, aggressive later" outlook. In other words, insurance companies should hold more riskless assets at the beginning of the liability period in order to limit guarantee costs. They should then gradually switch to risky assets to increase profit.

Our findings also indicated that by holding more assets in the portfolio, when both profits and deficits are considered, a multi-period investment strategy not only increases expected profits but also decreases expected deficits and deficit probability. Suppose that an insurance company holds four assets (equities, consols,

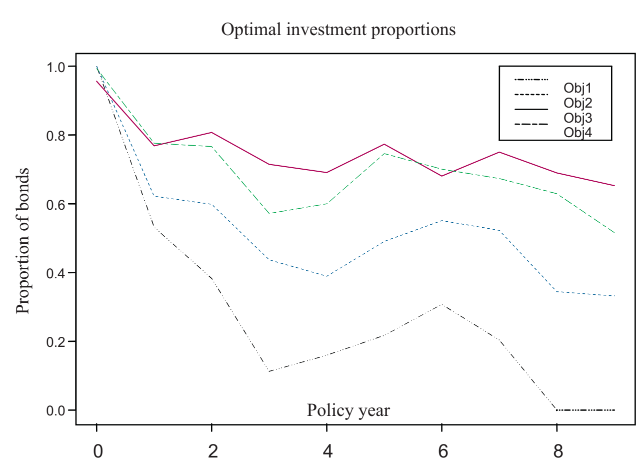

Fig. 2. Optimal asset allocation of four objectives in the two-asset model.

Table 7
Insurer's profits and deficits of the best multi-period investment strategy for the four-asset model.

Strategy (multiple rebalance)	E [Earn]	E [D]	Prob (D > 0)
OBJ1	27.16	8.47	0.232
OBJ2	23.76	3.81	0.164
OBJ3	8.54	0.59	0.092
OBJ4	11.01	0.84	0.057

Table 8
Mean and standard deviation of the five objective values produced by five different initial populations for the four-asset model.

	Mean (optimal value)	Std dev
OBJ1	27.161777736	0.000021845
OBJ2	23.7612186747	0.0000294155
OBJ3	0.5947023490	0.0000090678
OBJ4	0.0574000000	0.0000077226

index-linked gilts and short-bonds) in its portfolio. Table 7 presents the computational results of the multi-period investment strategy for four assets. The computational results of OBJ1 and OBJ2 illustrate this phenomenon. When the number of assets increases from two to four, the expected profit produced by OBJ1 increases from 20.34 to 27.16, the expected deficit decreases from 9.27 to 8.47, and the deficit probability decreases from 0.256 to 0.232. Similarly, the expected profit produced by OBJ2 increases from 18.00 to 23.76, the expected deficit decreases from 4.10 to 3.81, and the deficit probability decreases from 0.184 to 0.164. If only objectives related to deficits are considered, the multi-period investment strategy significantly decreases expected deficits and deficit probability when the number of assets increases from two to four. We applied the same method used to investigate HA robustness of the two-asset models to the four-asset models. The same phenomenon was discovered, and the mean and standard deviations of OBJ1, OBJ2, OBJ3 and OBJ4 are presented in Table 8.

We further studied the effect of the frequency of asset reallocation on the performance of the multi-period investment strategy when considering two assets. Since the results of OBJ1 and OBJ4 were similar to those of OBJ2 and OBJ3, only the results of OBJ2 and OBJ3 are presented in the following tables. Both tables show that, with the exception of the two-year asset reallocation frequency in the E [D] result, the more frequently assets are reallocated, the higher the expected profits, the lower the expected deficits, and the lower the probability of deficits (Tables 9 and 10).

Table 9
Comparison of effectiveness for different frequencies of asset reallocation for OBJ2.

Frequency of changing proportions	E [Earn]	E [D]	Prob (D > 0)
Every ten years	12.27	4.58	0.234
Every five years	14.64	4.12	0.194
Every two years	16.60	4.06	0.19
Every year	18.00	4.10	0.184

Table 10
Comparison of effectiveness for different frequencies of asset reallocation for OBJ3.

Frequency of changing proportions	E [Earn]	E [D]	Prob (D > 0)
Every ten years	9.17	3.48	0.211
Every five years	10.98	2.77	0.17
Every two years	11.72	2.62	0.155
Every year	12.11	2.49	0.157

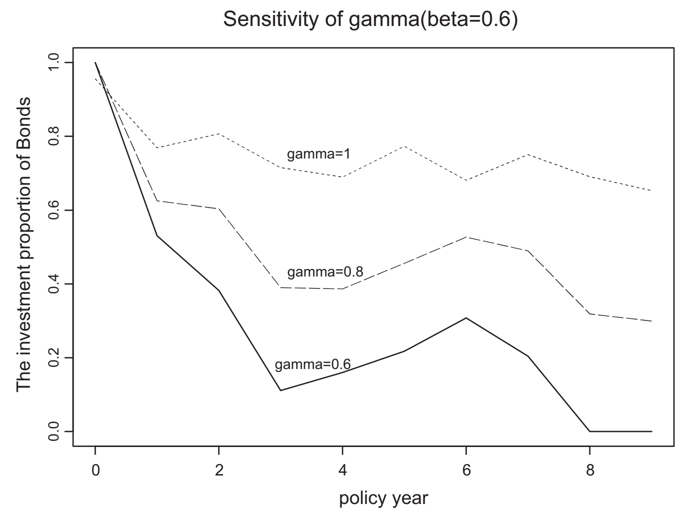

Fig. 3. The asset allocation of different gamma values for OBJ1.

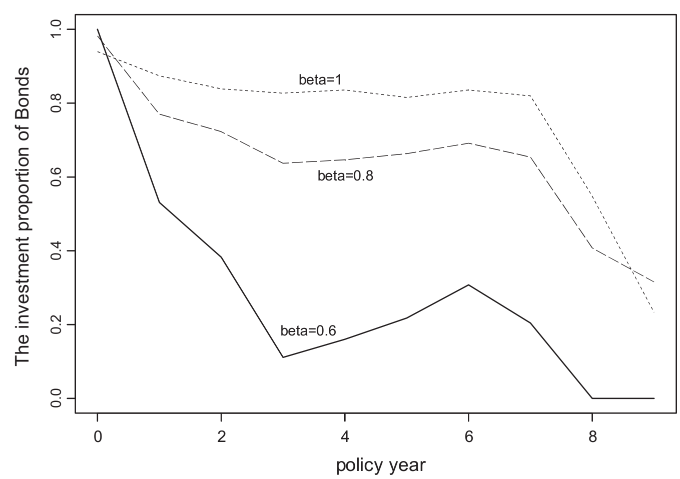

Fig. 4. The asset allocation of different beta values for OBJ1.

4.2. Sensitivity analysis of β and γ

Two parameters, β and γ, are critical to the structure of the contract. Therefore, we investigated the relationship between the multi-period investment strategies (OBJ1, OBJ2, OBJ3, and OBJ4) and β and γ. Since this relationship is not influenced by a related objective function, we only examined the effect of having different β and γ on OBJ1. Fig. 3 shows that OBJ1 becomes more aggressive if γ decreases under a fixed β. From the definition of *Earn*, the decrease in γ leads to an increase in the profit rate. In the case of $\gamma = 1$, profit is zero if $A(T) \geq P(T)$, and will become negative if $A(T) < P(T)$. Thus, an aggressive strategy leads to more deficits, and OBJ1 appears more conservative when compared with other cases with a γ of less than 1. If β decreases, Fig. 4 shows that OBJ1 becomes more aggressive under the same objective function. The reason for this is similar to the preceding case.

5. Discussion and conclusions

This paper proposes an optimization approach for generating an investment strategy for multi-period asset-liability management of long-term with-profit insurance policies. We put forward a hybrid algorithm combining evolution strategies and the Levenberg–Marquardt algorithm. Our proposed algorithm improves on evolutionary approaches by converging on quality solutions more efficiently. When excessive assumptions are imposed on real-world simulation models in an attempt to alleviate computation load, unreasonable results generally occur. Conciseness and efficiency are therefore vital to search techniques, as solutions must be generated within a realistic period. This issue has been crucial in developing large-scale simulation models. However, the fact remains that for complex problems, reaching global optimization is computationally intensive. Practical applications require that an algorithm be both robust and efficient before it can be successfully implemented. The computational results demonstrate that our simulation model is an effective tool for the study of a multi-period asset allocation, and our hybrid approach is an efficient, robust algorithm for generating optimal multi-period investment strategies. Our experiment results show improved returns based on increased asset reallocation frequency.

Given that local search algorithms typically became trapped to a local optimum when poor initial solutions are used, when to apply LM for local search remains an open question. Since the initial solution for LM was obtained from ES, it might not be mature enough to be a good initial solution for LM if ES is far away from global convergence. Solutions obtained from LM will be either outside the constraints or inferior to those obtained from the global search algorithm. In this case, the LM solutions will be abandoned and searching will continue. On the other hand, applying LM with well-converged solutions from EA too late will compromise efficiency. Further research needs to be conducted into how to maximize efficiency and accuracy by using optimal initial solutions to local search. This issue is particularly difficult, given that local search is problem-dependent and requires trial-and-error adjustment for optimal results.

Furthermore, application of our optimization approach disputes the traditional notion of the top-down investment approach to long-term with-profit liability [34]. Rather than investing aggressively in the beginning, and retreating to a more conservative approach as the liability term concludes, our simulation results show that insurance companies need to be conservative during the initial investment stage and gradually apply more aggressive investment strategies near the end of the term to minimize guarantee costs and increase profitability. It is our belief that our research presents a novel approach, with potentially wide-ranging implications for the computing and investment communities.

Appendix A.

The Wilkie Model is a stochastic asset model that represents the behavior of various economic series over time. Wilkie's investment model was first introduced in 1986, with an updated version presented in 1995. In the 1995 version, Wilkie updated the parameter values used in the original model and extended the model to cover short-term interest rates, the yield on long-dated index-linked gilts, property rental yields, the force of property rental growth, and the force of salary growth. We only introduce the series used in this article; detailed information can be found in Wilkie's 1995 publication [14].

- The force of inflation rate:

The force of inflation rate $I(t)$, the driving force for other series, is an $AR(1)$ model, as follows:

$$I(t) = QMU + QA[I(t-1) - QMU] + QE(t),$$

where QMU is the mean force of inflation, QA is the parameter controlling the strength of the autoregression, and $QE(t)$ is an *i.i.d.* (identical independent distribution) random white noise term distributed *Normal* $(0, QSD^2)$. The parameters, including the following, used in this article are consistent with the estimated parameters suggested by Wilkie [14]. The parameters for the force of interest rate are:

$$QMU = 0.047, \quad QA = 0.58, \quad QSD = 0.0425, \quad I(0) = 0.047$$

- Long-term gilt yield:

The model of the long-term gilt yields $C(t)$ at time t is:

$$C(t) = CW \cdot CM(t) + CMU \cdot \exp(CN(t)),$$

with

$$CM(t) = CD \times I(t) + (1 - CD) \cdot CM(t-1), \quad \text{and}$$

$$CN(t) = CA \cdot CN(t-1) + CY \cdot YE(t) + CE(t),$$

where $CE(t)$ is an *i.i.d.* random white noise term distributed *Normal* $(0, CSD^2)$, $YE(t)$ is defined below, and the values of the parameters are:

$$CW = 1.0, \quad CMU = 0.0305, \quad CD = 0.045, \quad CA = 0.8974,$$
$$CY = 0.3371, \quad CSD = 0.1853, \quad CM(0) = 0.047, \quad CN(0) = 0.$$

- Short-term cash rate:

The model of the short-term cash rate $B(t)$ at time t is:

$$B(t) = C(t) \cdot \exp[-BD(t)],$$

with

$$BD(t) = BMU + BA \cdot (BD(t-1) - BMU) + BE(t),$$

where $BE(t)$ is an *i.i.d.* random white noise term distributed *Normal* $(0, BSD^2)$, and the values of the parameters are:

$$BA = 0.74, \quad BMU = 0.23, \quad BSD = 0.18, \quad BD(0) = 0.23.$$

- Share dividend yield:

The share dividend yield $Y(t)$ has two components: a term related to the inflation rate and an $AR(1)$ model for $YN(t)$, as follows:

$$Y(t) = YMU \cdot \exp[YW \cdot I(t) + YN(t)],$$

with

$$YN(t) = YA \cdot YN(t-1) + YE(t),$$

where $YE(t)$ is an *i.i.d.* random white noise term distributed *Normal* $(0, YSD^2)$. The values of the parameters are:

$$YW = 1.8, \quad YMU = 0.0375, \quad YA = 0.55,$$
$$YSD = 0.155, \quad YN(0) = 0.$$

- Yield on long-dated index-linked gilts:

The model of the yield on long-dated index-linked gilts $R(t)$ is

$$\ln R(t) = \ln RMU + RA \cdot [\ln R(t-1) - \ln RMU] + RBC \cdot CE(t) + RE(t)$$

where $RE(T)$ is an *i.i.d.* random white noise term distributed *Normal* $(0, RSD^2)$. The values of the parameters are:

$$RMU = 0.04, \quad RA = 0.55, \quad RBC = 0.22,$$
$$RSD = 0.05, \quad R(0) = 0.04.$$

- Share dividend index:

The share dividend index $D(T)$ at time t is:

$$D(t) = D(t-1) \cdot \exp\left[DQ(t) + DMU + DY \cdot YE(t-1) + DB \cdot DE(t-1) + DE(t)\right],$$

with

$$DQ(t) = DX \cdot I(t-1) + (1 - DX) \cdot DM(t), \quad \text{and}$$

$$DM(t) = DD \cdot I(t) + (1 - DD) \cdot DM(t-1),$$

where $DE(T)$ is an *i.i.d.* random white noise term distributed *Normal* $(0, DSD^2)$. The values of the parameters are:

$$DMU = 0.016, \quad DY = -0.175, \quad DB = 0.57, \quad DX = 0.42,$$
$$DD = 0.13, \quad DSD = 0.07, \quad DM(0) = 0.047, \quad YE(0) = 0,$$
$$DE(0) = 0, \quad \text{and} \quad D(0) \quad \text{can be set arbitrary}.$$

- Share price index:

The share price index $P(t)$ can be derived from the dividend index and the dividend yield, as follows:

$$P(t) = \frac{D(t)}{Y(t)}$$

The return rate of each asset can be derived as follows:

The return rate of short-term bonds from $t-1$ to time t then equals $B(t-1)$.

The return rate of long bonds from $t-1$ to time t, $i_b(t)$, equals $C(t-1) + (C(t-1))/(C(t)) - 1$; or equivalently, $1 + i_b(t) = C(t-1) + (C(t-1))/(C(t))$.

The return rate of index-linked gilts from $t-1$ to time t equals $(R(t-1))/(R(t))[1 + R(t)]\exp[I(t)] - 1$.

The return rate of equities from $t-1$ to time t, $i_s(t)$, equals $(P(t) + D(t))/(P(t-1)) - 1$; or equivalently, $1 + i_s(t) = (P(t) + D(t))/(P(t-1))$.

References

[1] S. Haberman, L. Ballotta, N. Wang, Modelling and Valuation of Guarantees in With-Profit and Unitised With Profit Life Insurance Contracts, Cass Business School Research Paper, 2003, Available at SSRN: http://ssrn.com/abstract=856444.
[2] W.F. Sharpe, I.G. Tint, Liabilities: a new approach, Journal of Portfolio Management 16 (1990) 5–10.
[3] M. Sherris, Portfolio selection and matching: a synthesis, Journal of the Institute of Actuaries 119 (1992) 87–105.
[4] A.J. Wise, Matching and portfolio selection: part 2, Journal of Institute of Actuaries 114 (1987) 551–568.
[5] S. Haberman, J.H. Sung, Optimal pension funding dynamics over infinite control horizon when stochastic rates of return are stationary, Insurance: Mathematics and Economics 36 (2005) 103–116.
[6] K.F.C. Yiu, Optimal portfolios under a value at risk constraint, Journal of Economic Dynamics and Control 28 (2004) 1317–1334.
[7] S. Basak, A. Shapiro, Value at risk based risk management: optimal policies and asset prices, The Review of Financial Studies 14 (2001) 371–405.
[8] A. Lioui, P. Poncet, On optimal portfolio choice under stochastic interest rates, Journal of Economic Dynamics and Control 25 (2001) 1841–1865.
[9] Y. Peng, G. Wang, G. Kou, Y. Shi, An empirical study of classification algorithm evaluation for financial risk prediction, Applied Soft Computing 11 (2) (2011) 2906–2915.
[10] T.H. Back, Evolutionary Algorithms in Theory and Practice, Oxford University Press, New York, 1996.
[11] T. Mantere, J.T. Alander, Evolutionary software engineering, a review, Applied Soft Computing 5 (3) (2005) 315–331.
[12] V. Oduguwa, A. Tiwari, R. Roy, Evolutionary computing in manufacturing industry: an overview of recent applications, Applied Soft Computing 5 (3) (2005) 281–299.
[13] V. Ravi, H. Kurniawan, P. Peter Nwee Kok Thai, Ravi Kumar, Soft computing system for bank performance prediction, Applied Soft Computing 8 (1) (2008) 305–315.
[14] A.D. Wilkie, More on a stochastic asset model for actuarial use, British Actuarial Journal 1 (1995) 777–964.
[15] L. Ballotta, S. Haberman, N. Wang, Guarantees in with-profit and unitized with-profit life insurance contracts: fair valuation problem in presence of the default option, The Journal of Risk and Insurance 73 (1) (2006) 97–121.
[16] S. Andradottir, Simulation optimization, in: J. Banks (Ed.), Handbook of Simulation: Principles, Methodology, Advances, Applications, and Practice, Wiley, New York, 1998, pp. 307–333.
[17] E. Tekin, I. Sabuncuoglu, Simulation optimization: a comprehensive review on theory and applications, IIE Transactions 36 (2004) 1067–1081.
[18] G. Garai, B.B. Chaudhuri, A hierarchical genetic algorithm with search space partitioning scheme, in: International Conference on Integration of Knowledge Intensive Multi-Agent Systems, 2003, pp. 139–144.
[19] R.L. Burden, D.J. Faires, Numerical Analysis, 9th ed., Brooks Cole, 2010, ISBN-10:0538733519.
[20] H. Everett III, Generalized Lagrange multiplier method for solving problems of optimum allocation of resources, Operations Research (1963) 399–417.
[21] Z. Michalewics, Genetic Algorithms + Data Structures = Evolution Programs, 3rd Revised and extended edition, Springer, 1999.
[22] Z. Michalewics, D.B. Fogel, How to Solve It: Modern Heuristics, Springer, 2002.
[23] A. Tiwari, R. Roy, G. Jared, O. Munaux, Evolutionary-based techniques for real-life optimisation: development and testing, Applied Soft Computing 1 (4) (2002) 301–329.
[24] S.B. Andersen, I.F. Santos, Evolution strategies and multi-objective optimization of permanent magnet motor, Applied Soft Computing 12 (2) (2012) 778–792.
[25] T.H. Back, F. Hoffmeister, H.P. Schwefel, A survey of evolution strategies, in: R.K. Belew, L.B. Booker (Eds.), Fourth International Conference on Genetic Algorithms, San Mateo, CA, 1991.
[26] I. Rechenberg, Evolution sstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution, Frommann-Holzboog Verlag, Stuttgart, 1973.
[27] H.P. Schwefel, Numerical Optimization for Computer Models, John Wiley, Chichester, UK, 1981.
[28] V. Nissen, J. Biethahn, An introduction to evolutionary algorithms, in: J. Biethahn, V. Nissen (Eds.), Evolutionary Algorithms in Management Applications, Springer, Berlin, 1995, pp. 3–43.
[29] X. Yao, Y. Liu, Fast evolutionary programming, in: Proceedings of the Fifth Annual Conference on Evolutionary Programming, 1996, pp. 451–460.
[30] J. Vesterstrom, R. Thomsen, A comparative study of differential evolution, particle warm optimization, and evolutionary algorithms on numerical benchmark problems, Congress on Evolutionary Computation (2004) 19–23.
[31] Z. Zhou, Y.S. Ong, M.H. Lim, B.S. Lee, Memetic algorithm using multi-surrogates for computationally expensive optimization problems, Soft Computing 11 (2007) 957–971.
[32] M. Lozano, D. Molina, F. Herrera, Editorial scalability of evolutionary algorithms and other metaheuristics for large-scale continuous optimization problems, Soft Computing 15 (2011) 2085–2087.
[33] J.H. Holland, Adaptation in Natural and Artificial Systems, University of Michigan Press, Ann Arbor, 1975.
[34] H.C. Huang, Y.T. Lee, Optimal asset allocation for a general portfolio of life insurance policies, Insurance: Mathematics and Economics 46 (2) (2010) 271–280.