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A recursive formula for a participating contract embedding a surrender option under regime-switching model with jump risks: Evidence from stock indices☆

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ABSTRACT

This study proposes a recursive formula to value a surrenderable participating contract. To capture the dynamics of stock returns over expansion–recession cycles and the occurrence of catastrophic events, we assume the rate of return of the reference portfolio would follow a regime-switching model with jump risks. Our empirical results show that compared to the Black–Scholes model and the regime-switching model, the regime-switching model with jump risks can better explain the dynamics of the S&P 500 stock index. In addition, we give a recursive formula of a participating contract embedding a surrender option under a regime-switching model with jump risks. Sensitivity analysis shows that the changes of parameters of the regime-switching model with jump risks did influence participating contract premiums. The differences between valuations under the Black–Scholes model, the regime-switching model and the regime-switching model with jump risks suggest that it is critical to apply an appropriate model to value precisely a participating contract.

1. Introduction

In an era of low interest rates and high inflation, it is difficult for investors to accumulate real wealth by only depositing money in the bank. Smart investors put their money in different kinds of capital markets to earn higher return, such as the markets of: stock, bond, insurance, and mutual funds. However, each market has specific risks for investors. It is difficult to select an industry or company to invest in on the stock market as the market risk is high. Regarding the mutual fund market, it is necessary for the investor to discuss with a fund manager about portfolios regularly, for the investment may go back to the original value, depending on the business cycle, after fifteen to twenty years. In relation to the bond market, it is important to consider the credit ranking of the issuer. Compared to the other markets, risk in the insurance market is relatively small. Moreover, insurance products secure benefit when the insured dies, and are therefore good investment instruments.

In the past, insurance policyholders could only buy nonparticipating life insurance, which is cheaper than other insurance products incorporating investment, but only guarantees the benefit on the individual's death. Moreover, under this arrangement surrendering or switching the policy destroys the contract value when the interest rate goes up. Insurance subsequently evolved to incorporate investment, generally categorized as: universal life insurance, variable life insurance, and participating life insurance. Universal life insurance modifies the shortcoming of nonparticipating life insurance, by avoiding returns lower than the technical rate as the interest rate goes up. In fact, the value of universal life insurance is closely linked to the interest rate. More specifically, the value accumulates slowly when the interest rate goes down, and grows quickly when it goes up. The value of variable life insurance is linked to the performance of the portfolio allocated by the policy holder, who has to take full responsibility for profits or losses and therefore it is appropriate for aggressive investors.

Compared with the above mentioned insurance contracts, participating contracts have many advantages. Unlike universal life insurance, which gives only fixed interest income or variable life insurance that has possible investment loss, a participating policy is characterized as allowing policyholders to participate in the upside returns of the reference portfolio. Such a participating mechanism applies when "dividends" are credited to the policy reserve, thus increasing the insured's benefits. A participating contract with a minimum interest rate guarantee forces both the benefit and the periodical premiums to be adjusted annually according to the performance of a special investment portfolio. Moreover, the insured's benefit remains constant if the dividends part that insurance company wants to share with policyholders is lower than the minimum interest rate guarantee. By

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contrast, the benefit increases if the dividend is higher than the minimum interest rate guarantee and thus the payoff mechanism of these contracts is like European call options. Brennan and Schwartz (1976) pioneered the pricing of participating life insurance policy with an asset value guarantee under the Black–Scholes model. Further, Boyle and Schwartz (1977) valued a participating contract with both death and maturity benefit guarantees under this model. In addition, Grosen and Jørgensen (2000), Jensen et al. (2001) and Grosen and Jørgensen (2002) analyzed a participating policy, for which they used the Monte Carlo simulation to derive the percentage of positive performance of firm asset portfolios. Miltersen and Persson (2003) extended this to a multi-period contract, deriving closed-form formulae for pricing under a stochastic interest rate with the Heath–Jarrow–Morton (HJM) model. Bacinello (2001) used the Black–Scholes model to analyze life insurance endowment participating policies with a guaranteed minimum interest rate, and obtained closed-form formulae for those policies in terms of one-year call options.

A surrender mechanism is an American-style put option that entitles the policyholder to sell back the contract to the insurer at the cash surrender value. That is, as a participating contract embedding a surrender option it gives the policyholder the right to terminate the contract early at surrender value. To price a participating contract embedded with a surrender option it is necessary to consider three parts: valuations of the basic contract, participating option and surrender option. Albizzati and Geman (1994) took surrender options into account and derived a single-premium contract under the portfolio consisting of a zero coupon bond and stochastic interest rates. Grosen and Jørgensen (2000) and Jensen et al. (2001) priced surrender options embedded in participating policies with a binomial tree approach and a finite difference one, respectively. Bacinello (2003a) employed Cox et al. (1979) discrete option pricing model to derive a recursive formula to price: the basic contract, the participation option, and the surrender option.

As the pricing of both the participation and surrender options are affected by the value of the reference portfolio, it is important to identify its dynamics. During the past decades, in the high-yielding era, insurance companies were able to put most of their assets into bank deposits or bonds, with only a small portion having to be invested in high-risk assets, such as stocks or mutual funds, as they could still afford the minimum interest rate guarantee embedded in surrenderable participating contracts. However, recent near-zero interest rate policies implemented by governments worldwide have forced insurers to invest most of their assets in high-risk assets, such as stocks or mutual funds. Therefore, when pricing a surrenderable participating contract lasting around twenty years, it is critical to capture the dynamics of stock returns over expansion–recession cycles and the occurrence of catastrophic events.

Fig. 1 shows the dynamics of price and return of the S&P 500 index from 1999 to 2008 and it can be seen that stock prices were trending down from 2000 to 2003, whereas since 2003 the economy has recovered and share prices have been trending up. However, owing to the global financial crisis in 2008, share prices began trending downward again. Generally, the dynamics of price and return of the S&P 500 can be classified as an expansion–recession cycle, in which expansion represents stock price trending upwards, while recession represents it trending downwards. A similar idea was also introduced in Hamilton (1989), who stated that the economy is in expansion if the growth rate of GNP is positive, and the economy is in recession, if the growth rate of GNP is negative. Past research has shown that the regime-switching model can describe features in different market states (Alizadeh and Nomikos, 2004; Bollen et al., 2000; Cai, 1994; Engle, 1994; Haldrup and Nielsen, 2006; Hardy, 2001; Schaller and Norden, 1997; Schwert, 1989; Timmermann, 2000).

During the past two decades, several significant events occurred including the dot-com burst in 2000, the September 11 attacks in 2001, the end of the Iraq war in 2003, the Yen carry trade in 2007 and the global financial crisis in 2008, leading to abrupt jumps in stock prices and returns. Unfortunately, the regime-switching model cannot adequately describe drastic changes in prices and returns and in this paper we propose a regime-switching model with jump risks to address this limitation of the model. More specifically, we show that compared to the Black–Scholes model (BSM) (Black and Scholes, 1973) and the regime-switching model (RSM), the regime-switching model with jump risks (RSMJ) can better explain the dynamics of the S&P 500 stock index, by the estimating parameters of the Expectation–Maximization (EM) algorithm and testing these by computing the likelihood function. Subsequently, we develop a recursive formula to price a participating contract embedding a surrender option under the RSM and the RSMJ.

The remainder of the paper is organized as follows. Section 2 illustrates the framework of the participating contract, the RSM and the RSMJ for the stock index. The empirical estimates and the tests of the three models for the S&P 500 stock index are also reported in this

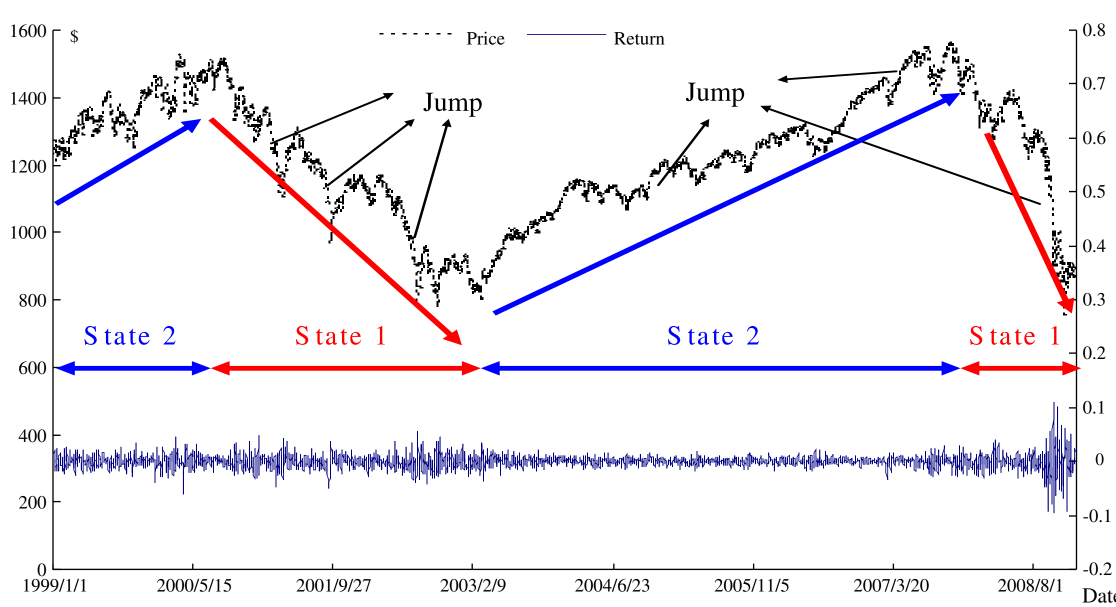

Fig. 1. The dynamics of the price and return, S&P 500 index 1999–2008.

section. In Section 3, a participating contract embedded with a surrender option is priced under the RSM and the RSMJ. Sensitivity analyses are presented in Sections 4 and 5 summarizes the results as well as providing conclusions.

2. The contract and models

This section first describes a life insurance participating policy in detail, including its benefit payments, participation mechanism and surrender option. Since there are many types of participating policies, we follow the definition of Bacinello (2003a). Next, we assume that the performance of a reference portfolio would follow a regime-switching model or a regime-switching model with jump risks.

2.1. Participating contract

Generally, a participating contract contains three parts: a basic contract, a participation mechanism and a surrender mechanism.

2.1.1. Basic contract

Consider a life insurance endowment policy issued at time 0 and maturing T years later, with the insurer paying benefit C_1, if the insured dies before maturity T or if they survive until policy maturity. Under this contract, the insurer is obliged to pay a benefit to the beneficiary, if the insured dies within the term of the contract or survives the maturity date. If the policy is in force at the end of the t-th year, the insurance company must establish a *reserve*[4] so the policyholder can guarantee its ability to meet its obligations. For simplicity, Bacinello (2001, 2003a, 2003b) adopted the guaranteed interest rate i_g as the discount rate for the reserve. x is denoted as the issuing age of the insured. Based on actuarial notation, ${}_{t-1|}q_x$ represents the probability of the insured dying within the t-th year, while ${}_tP_x$ is the probability that the insured remains alive at time t, which can be extracted from the mortality table. The reserve at the end of each policy year is presented as follows:

$$C_1\left[\sum_{h=t}^{T-t}(1+i)^{-h}{}_{h-1|}q_{x+t}+(1+i)^{-(T-t)}{}_{T-t}P_{x+t}\right]=C1A_{x+t:\overline{T-t}|},t=1,2,\dots,T. \tag{1}$$

where $A^{(i)}_{x-t:\overline{T-t}|}$ marks the expected value at time t of the benefit of one dollar of a standard life insurance endowment policy, discounted from the random time of payment to time t with the technical rate i. At time 0, the reserve, $C_1A_{x:\overline{T}|}-U$, equals zero, based on the equivalent principle; hence, the single premium U, can be presented as follows:

$$U=C_1\left[\sum_{t=1}^{T}(1+i)^{-t}{}_{t-1|}q+(1+i)^{-T}{}_Tp_x\right]=C_1A^{(i)}_{x:\overline{T}|}. \tag{2}$$

2.1.2. Participation mechanism

Under the participation mechanism outlined by Bacinello (2003a), each annual dividend (bonus) goes on purchasing an additional paid-up endowment policy with the same maturity date T. Therefore, the total benefit increases by the benefit amount of the additional paid-up endowment policy. For example, the benefit during the t-th year before (after) adjusting the participating mechanism is $C_t(C_{t+1})$ and the dividend during the t-th year is denoted by the dividend function D_t, for $t=1,2,\dots,T-1$. Using actuarial notation, the single premium of the additional paid-up endowment policy can be presented as follows:

$$D_t=(C_{t+1}-C_t)A^{(i)}_{x+t:\overline{T-t}|},\quad t=1,2,\dots,T-1. \tag{3}$$

Since the benefit increases from the before-adjustment C_t to the after-adjustment $C_{t+1}$and the reserve at time t increases accordingly. $F_t(F_t^+)$ is the reserve at time t, before (after) adjustment. The relationship between the reserves F_t and F_t^+ and the dividend D_t is:

$$D_t+F_t=F_t^+,\quad t=1,2,\dots,T-1. \tag{4}$$

The dividend function D_t is derived as follows. G_t is the price of a reference portfolio at time t and during the period t to $t+1$, the performance of the reference portfolio is expressed via:

$$g_t=\frac{G_{t+1}}{G_t}-1,\quad t=1,2,\dots,T-1. \tag{5}$$

where g_t indicates the relative price of the reference portfolio between time t and time $t+1$. Following Bacinello (2003a), the dividend is expressed by δ_tF_t, namely:

$$D_t=\delta_tF_t,\quad t=1,2,\dots,T-1.$$

where the adjustment rate δ_t is defined as:

$$\delta_t=\max\left\{\frac{\eta g_t-i}{1+i},0\right\},\quad t=1,2,\dots,T-1, \tag{6}$$

where the adjustment rate is calculated based on the maximum of the minimum interest rate guaranteed and the dividends of the reference portfolio, called the adjustment bonus rate. We observe that the total return granted to the policyholder during the t-th year of the contract is given by:

$$(1+i)(1+\delta_t)-1=\max\{\eta g_t,i\},$$

where i can be interpreted as a minimum interest rate guaranteed, and η denoted as a participation coefficient and lies between 0 and 1. In the preceding financial year, this participation coefficient is computed and certified by a specific auditor, or decided upon by the board of directors of the company. However, for simplicity, we have used a constant participation coefficient.[5]

Using Eq. (3), the relationship between additional benefit and dividend (bonus) is:

$$(C_{t+1}-C_t)A^{(i)}_{x+t:\overline{T-t}|}=\delta_tF_t,\quad t=1,2,\dots,T-1. \tag{7}$$

From the previously described definition of reserve, the reserve before adjustment $F(t)$ is

$$F_t=C_tA^{(i)}_{x+t:\overline{T-t}|},\quad t=1,2,\dots,T-1. \tag{8}$$

By using Eqs. (7) and (8), the benefit can be adjusted at the rate δ_t, that is:

$$C_{t+1}=C_t(1+\delta_t),\quad t=1,2,\dots,T-1. \tag{9}$$

[4] According to the prospective method of *equivalence principle*, the reserve can be thought of as the present value of the future financial obligations discounted at a constant interest rate, see Bowers et al. (1986).

[5] Insurers and auditors can decide on the participation coefficient based on companies' financial situations or solvency. When facing five previous years with negative returns on the reference portfolio, the insurer can also downgrade the participation coefficient to zero.

The adjusted benefit can be calculated based on the trace of dividend rates, as follows:

$$C_t = C_1 \prod_{j=1}^{t-1} \left(1+\delta_j\right), \quad t = 2, 3, \ldots, T. \tag{10}$$

2.1.3. Surrender mechanism

The presence of a surrender option in a contract means that the policyholders can sell back the contract before maturity. Bacinello (2003a) assumed that this option can be exercised at the start of the policy year immediately following the announcement of the renewal of benefit and the cash value is the payoff to the policyholder upon exercising the surrender option. Generally, the surrender cash value equals the reserve minus a surrender charge, which gradually reduces to zero after the policy has been persistently in place for a specific number of years. For simplicity, in this study we adopt the surrender cash value of Bacinello (2003a):

$$S_t = \rho V_t^{+} = \rho C_{t+1} A^{(i)}_{x+t:\overline{T-t|}}, \quad t = 1, 2, \ldots, T-1 \tag{11}$$

where ρ is a constant percentage of policy and $A^{(i)}_{x+t:\overline{T-t|}}$ is discounted at the guaranteed interest rate i_g.

Policyholders have the right to choose the maximum between the continuation value and the surrender value at the end of each year. Therefore, the contract value would be:

$$F_t = \max(S_t, W_t), \quad \forall t = 1, 2, \ldots, T-1 \tag{12}$$

where F_t, S_t and W_t represents the contract value, surrender value and continuation value at time t, respectively.

2.2. Models

2.2.1. Regime-switching model

The Markov switching model proposed by Hamilton (1989) defines s_t as the market state at time t, where t = 1,2, …, T, and $P = \begin{bmatrix} P_{11} & 1-P_{11} \\ 1-P_{22} & P_{22} \end{bmatrix}$ as the transition matrix. Hence, $P(s_t = j\, s_{t-1} = i) = P_{ij}$ represents the probability from state s_{t-1} to s_t and $\sum_{j=\{1,2\}} P_{ij} = 1$ for all $i \in \{1,2\}$, which can be assumed as $P(s_t\, s_{t-1}, \ldots, s_1) = P(s_t\, s_{t-1})$. We assume that stock markets can be in two states, expansion and recession, thus stock return, R_t, under the regime-switching model, can be shown as:

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z_t & \text{if } s_t = 1 \\ \mu_2 + \sigma_2 Z_t & \text{if } s_t = 2 \end{cases}, \tag{13}$$

where Z_t represents the standard normal distribution at time t, μ_{s_t} and σ_{s_t} are the mean and volatility of stock return at state s_t, which represents the unobservable market state at time t. Because the Markov chain s_t is unobservable, it is called a hidden Markov chain.

After Hamilton (1989) proposed the RSM to examine the persistency of recessions and booms, many papers, including those of Schwert (1989), Schaller and Norden (1997), Hardy (2001), Alizadeh and Nomikos (2004), and Kuswanto and Salamah (2009), applied this model to stock price data. Moreover, the RSM has been applied in different kinds of markets, such as electricity prices research (Haldrup and Nielsen, 2006), and exchange rate data (Bekaert and Hodrick, 1993; Bollen et al., 2000; Dewachter, 2001; Engel and Hamilton, 1990; Engle, 1994). In general, the past research results have shown that the RSM can describe features in different market states, including volatility clustering induced by the business cycle.

2.2.2. Regime-switching model with jump risks

Unfortunately, the RSM cannot adequately describe drastic changes in prices and returns, so Shyu et al. (2011) proposed that this combined with jump diffusion risks could explain asset returns completely. The stock return under the RSMJ can be shown as:

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z_t + \sum_{n=1}^{N(\Delta t)} \log Y_n & \text{if } s_t = 1 \\ \mu_2 + \sigma_2 Z_t + \sum_{n=1}^{N(\Delta t)} \log Y_n & \text{if } s_t = 2 \end{cases} \tag{14}$$

where the definitions of Z_t, μ_i, σ_i and s_t are identical to that in the RSM, $N(\Delta t)$ is a Poisson process describing the number of jumps during period t, and t = 1, 2, …, T. $\{Y_n\}$ is the jump size, and the logarithm of $\{Y_n\}$ follows a normal distribution with mean μ_y and variance σ_y^2. In addition, we assume that $\{Z_t\}$, $\{N_t\}$ and $\{Y_n\}$ are independent from each other.

2.2.3. Empirical results

This study is conducted using the S&P 500 stock index owing to the lack of detailed information regarding reference portfolios collected by insurance companies. The data are taken on a daily basis from Datastream, covering the period from 1 January 1999 to 31 December 2008. Now, we investigate the important features of the S&P 500 stock index, and then estimate the parameters of the BSM, the RSM and the RSMJ by the EM algorithm (Lange, 1995; Meng and Rubin, 1991). Next, we use the likelihood ratio (LR) test to see whether the RSMJ is better than either the BSM or the RSM on its own.

Table 1 shows the descriptive statistics of the returns of the S&P 500 index. We observe the means are negative and standard deviations are

Table 1
The descriptive statistics of the S&P500 index.

S&P 500	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	Total
Number	251	252	248	252	252	252	252	251	251	253	2514
Max	0.0347	0.0465	0.0489	0.0557	0.0348	0.0162	0.0195	0.0213	0.0288	0.1096	0.1096
Min	−0.0285	−0.0600	−0.0505	−0.0424	−0.0359	−0.0165	−0.0169	−0.0185	−0.0353	−0.0947	−0.0947
Mean	0.0007	−0.0004	−0.0006	−0.0011	0.0009	0.0003	0.0001	0.0005	0.0001	−0.0019	−0.0001
Std	0.0114	0.0140	0.0136	0.0164	0.0107	0.0070	0.0065	0.0063	0.0101	0.0258	0.0134
Skewness	0.0598	0.0007	0.0205	0.4251	0.0532	−0.1102	−0.0155	0.1028	−0.4941	−0.0337	−0.1199
Kurtosis	2.8535	4.3882	4.4478	3.6610	3.7589	2.8623	2.8493	4.1553	4.4481	6.6754	11.5406
+2%	14	18	12	22	10	0	0	2	6	31	116
−2%	9	19	14	29	5	0	0	0	12	41	127
±2%	23	37	26	51	15	0	0	2	18	72	243
+3%	1	7	4	10	3	0	0	0	0	19	44
−3%	0	4	4	8	1	0	0	0	2	24	40
±3%	1	11	8	18	4	0	0	0	2	43	84

Note: + k% refers to the number that the returns are up more than k%, − k% indicates the number that the returns are down more than k%, and ± k% represents the number that the returns are up or down more than k%.

Table 2
The empirical analysis of the estimating and testing in the Black–Scholes model, the regime-switching model, the regime-switching model with jump risks.

Index	Model	$\hat{p}_{11}$	$\hat{p}_{22}$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_y$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\sigma}_y$	$\hat{\lambda}$	Λ
S&P 500	BSM			−0.0250			0.2119				
				(0.0003)			(0.0002)				
	RSM	0.9803	0.9893	−0.0250	0.1000		0.3146	0.1249			865.31*
		(0.0064)	(0.0031)	(0.0007)	(0.0002)		(0.0009)	(0.0003)			
	RSMJ	0.9831	0.9929	−0.2750	0.1000	−0.0001	0.3020	0.0964	0.0093	114.8750	33.05*
		(0.0002)	(0.0025)	(0.0457)	(0.0015)	(0.0123)	(0.0042)	(0.0001)	(0.0028)	(0.0558)	

Note: BSM means the Black–Scholes model, RSM means the regime-switching model, and RSMJ means the regime-switching model with jump risks. The parameter estimates and testing statistics are estimated by daily stock returns. However, for comparison purposes, we have the parameter estimates annualized in this table. $\hat{p}_{11}$ and $\hat{p}_{22}$ are the estimates of probabilities staying in the recession state and the expansion state, respectively. $\hat{\mu}_1$ and $\hat{\mu}_2$ are the estimates of mean returns in the recession state and the expansion state, respectively. $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are the estimates of the standard deviation of returns in the recession state and the expansion state, respectively. $\hat{\mu}_y$ and $\hat{\sigma}_y$ are the estimates of the mean and standard deviation of jump sizes, respectively. $\hat{\lambda}$ is the estimate of mean jump times and Λ is the likelihood ratio (LR). Standard errors of the estimated parameters are given in parentheses; * denotes that the null hypothesis can be rejected at the 5% significance level.

large in 2000 to 2002 and 2008, attributable to the dot-com burst and 2008 global financial crisis. Since the economy recovered in 2003, the mean return went back to positive, however, the prices were still fluctuating, causing a greater standard deviation than in other periods. For the other periods, the mean returns are positive, and the standard deviations are small.

No specific pattern is found for skewness, for this was randomly positive or negative. However, it is worth noting that the kurtosis is over 3, which means the returns have heavy tails. If the increase or decrease of returns is over 2%, we identify it as jump and from Table 1, it can be seen that jumps occurred more frequently: in 2000, due to the dot-com bubble; in 2001, because of the September 11 attacks; in 2003, owing to the Iraq war; in 2007, which was down to the large Yen carry trade and in 2008, because of the global financial crisis.

2.2.4. Estimation and testing

The parameter estimates and testing for the three different models, the BSM, the RSM and the RSMJ, are annualized and presented in Table 2. With the BSM, the return is a normal distribution with mean −0.0250 and standard deviation of 0.2119. With the RSM, the transition probabilities P_{11} and P_{22} are 0.9803 and 0.9893, respectively. Both probabilities are close to one, implying that the probabilities of switching from expansion to recession and vice versa are very small. In recession, the mean return is −0.0250, and standard deviation is 0.3146. In contrast, during expansion, the mean return is 0.1000, and the standard deviation of return is 0.1249. Moreover, the volatility of returns in recession is more dramatic than for expansion.

Under the RSMJ, we observe results similar to those under the RSM, with the transition probabilities still being close to one. During recession, the mean return is negative and volatility is high, while for expansion, the mean return is positive and volatility is low. Compared to the estimated results in the RSM, the means are larger and volatilities are smaller as part is explained by the jump term. The mean of the number of jumps is 114.8750, and the logarithm of jump size is a normal distribution with mean −0.0001 and standard deviation 0.0093, thus implying that most of the unanticipated information can be attributed to the catastrophic events, such as: the dot-com burst in 2000, the September 11 attacks in 2001, the end of the Iraq war in 2003, the Yen carry trade in 2007 and the global financial crisis in 2008.

This study uses LR (likelihood ratio) as a testing model, summarized as follows: the null hypothesis is $H_0 : \theta \in \Theta_0$ against the alternative hypothesis $H_1 : \theta \in \Theta_1 \backslash \Theta_0, \Theta_0 \subset \Theta_1$. The testing statistic is:

$$\Lambda = 2(\ln L(R;\Theta_1) - \ln L(R;\Theta_0))$$

where $\ln L(R;\Theta_i)$ is the log maximum likelihood function under H_i. Under the null hypothesis and the sample being large enough, the testing statistic Λ would be distributed as $\chi^2(r)$, where r is the difference between the numbers of parameters in the two models. If $\Lambda > \chi^2_{r,1-\alpha}$, the null hypothesis would be rejected.

In this study, we perform two LR tests as follows: test (a) is based on the BSM against the RSM. That is, when $\Lambda > \chi^2_{4,1-\alpha}$, the BSM is rejected and the RSM is proven to be better than the BSM. Test (b) is based on the RSM against the RSMJ and when $\Lambda > \chi^2_{3,1-\alpha}$, the RSM is rejected, which means that the latter is proven to be better than the former. From the LR test results, it is concluded that the null hypotheses are to be rejected, meaning that, with 95% significance, the RSM is better than the BSM and moreover, the RSMJ is better than the RSM.

Fig. 2 compares the price and return of the S&P 500 index with the probability of recession under the RSMJ and the probability of jumps. In panel A, the economic expansion is in 1999 and from 2003 to 2007, so prices went up. An economic recession occurred in 2000 and from 2002 to 2008, owing to the dot-com burst and the global financial crisis, and then prices went down. Panel B shows the volatilities from 1999 to 2002 are larger than those from 2003 to 2007, implying that the return becomes volatile during recession. In addition, volatility clustering can be observed in panel B.

Panel C indicates that the probability of recession from 1999 to 2002 was high, because of the dotcom bubble, and the probability of recession from 2003 to 2007 was low. This infers that there was a transition of states in 2002 to 2003. In 2008, as the financial crisis occurred, the probability of recession in 2008 also became higher. There was also a switch of states in 2007 to 2008. Panel D shows the probability of jumps was large in 2000, 2001, 2003, 2007 and 2008, pertaining to the of events of the dot-com bubble in 2000, the September 11 attacks in 2001, the end of the Iraq war in 2003, the Yen carry trade in 2007, the global financial crisis in 2008.

3. Valuations of participating contract embedding a surrender option

For an insurance company, there are two kinds of risk in a participating contract to be dealt with. One is mortality risk, meaning that the insurance company must take into account the uncertainty of the insured's death, while the other is financial risk, which means the potential benefit is unknown; whether the insured will surrender the contract is also unknown. In this paper, we assume that the mortality risk and the financial risk are independent of each other (Bacinello, 2001, 2003a, 2003b).[6] For mortality risk, we can estimate survival probabilities through a mortality table. Financial risk, on the other hand, will be reflected in the performance of a reference portfolio.

In this section, we value a participating contract with no-arbitrage condition in the financial markets and derive a very simple closed-

[6] To validate the assumption of independence between mortality risk and financial risk, we check the correlation coefficient between the annual mortality rate in the U.S.A. and the annual rate of return of S&P 500 index from 1961 to 2011. Our results show that the correlation coefficient is −0.0526, which is statistically insignificant (t statistics is −0.365). Consequently, we can validate the assumption that the mortality risk and the financial risk are independent of each other.

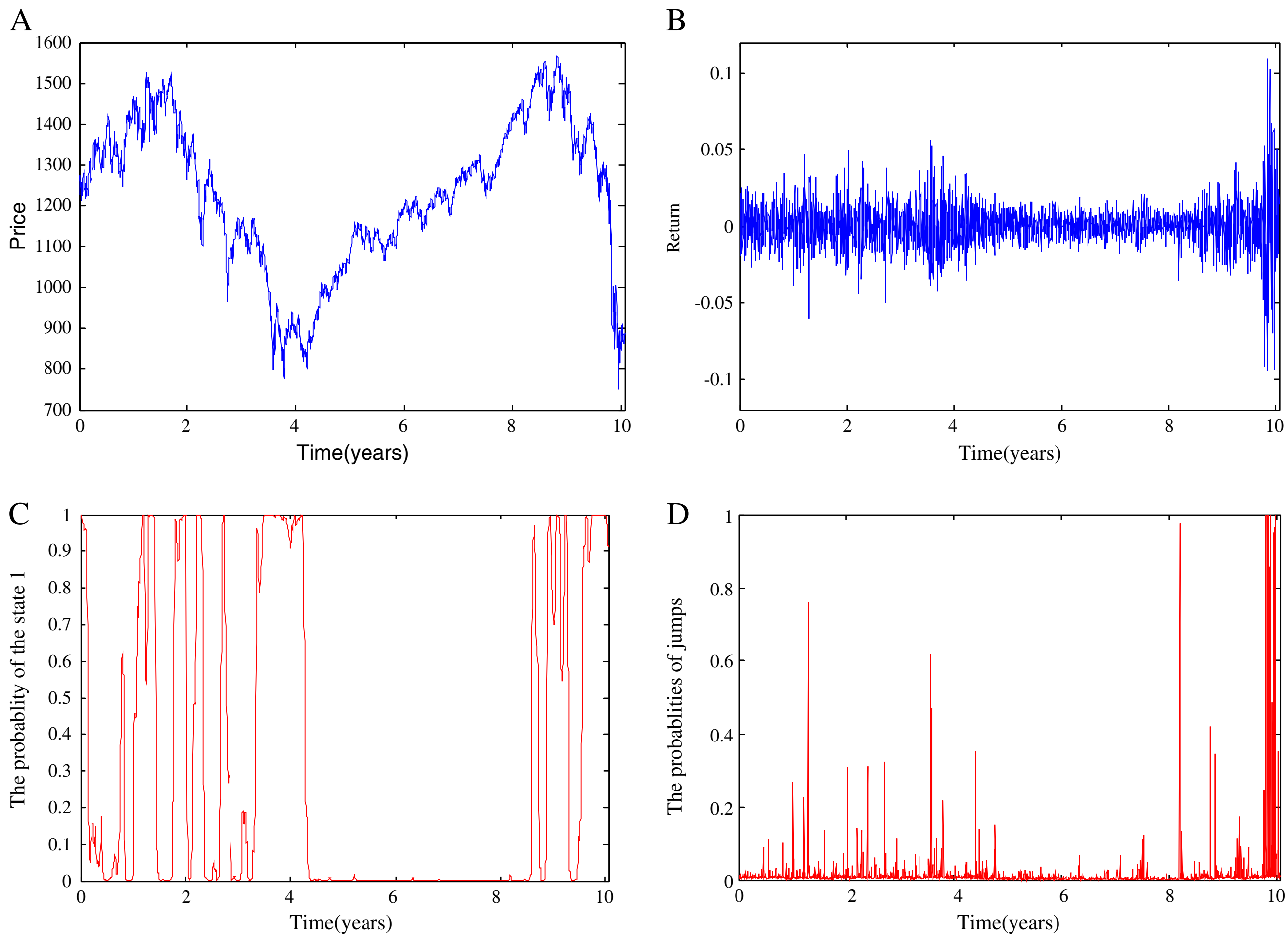

Fig. 2. The price, return, probabilities of the recession and probabilities of the jumps in the S&P 500 index.

form premium. Assuming the dynamics of the reference portfolio follows the RSMJ, given the single premium of a basic contract described in previous section, we value a nonsurrenderable participating contract under the risk-neutral mortality measure, which is only coupled with a bonus option. Next, a surrenderable participating contract is priced under the risk-neutral mortality measure, which combines both the bonus and surrender options with a basic contract.

According to the fundamental theorem of finance, given that the market has no arbitrage, there exists the risk-neutral probability to make that the relative price of the asset is martingale in the incomplete model. Although the risk-neutral probability measure is not unique, the price of option is, based on the law of the one price (Pliska, 1997). In our study, we adopt the Esscher transformation to evaluate the option price in the RSMJ model based on the assumption of a "no-diversified jump risk", but there are some approaches to price the option, such as the extended Girsanov theorem or general equilibrium approaches.

Consider the fixed risk-free interest rate r and the price of the reference portfolio G_t at time t. Under the risk-neutral measure, the dynamics of a reference portfolio can be written as follows (Merton, 1976):

$$\frac{dG_t}{G_t} = rdt + \sigma_{s_t} dw^Q(t) + d\sum_{n=1}^{N(t)}(Y_n - 1), \quad s_t = 1, 2. \tag{15}$$

where $W^Q(t)$ denotes the Brownian motion, the transition matrix of $\{s_t\}$ is $P = \begin{bmatrix} P_{11} & 1-P_{11} \\ 1-P_{22} & P_{22} \end{bmatrix}$, and σ_{s_t} represents the volatility under the market state s_t. The return of the reference portfolio at the t-th year is defined as in Eq. (5). As a basic contract is a non-participating life endowment policy, which is unrelated to the rate of return of the reference portfolio, we do not have to take into account the return process of the reference portfolio in pricing a basic contract. Therefore, the premium of a basic contract is the same as Eq. (1). Next, we value a nonsurrenderable participating contract and a surrenderable one under the RSMJ.

3.0.1. Valuation of a nonsurrenderable participating contract

The cash flow $X_{[T(x)]+1}$ in the effective period is:

$$X_{[T(x)]+1} = \begin{cases} C_{[T(x)]+1}, & T(x) < T \\ C_T, & T(x) \geq T \end{cases} \tag{16}$$

where $T(x)$ denotes the survival time after original time 0, [] is the Gauss symbol, and time of death of the insured is exclusive, $[T(x) = t, t = 1, 2, \ldots T-1]$. Then discounted cash flow is:

$$\pi_{RSMJ}(X_t) = \begin{cases} {}_{t-1|}q\, e^{-rt} C_1 \left(1+\mu_{RSMJ}\right)^{t-1}, & t = 1, 2, \ldots, T-1 \\ {}_{T-1}p_x e^{-rT} C_1 \left(1+\mu_{RSMJ}\right)^{T-1}, & t = T \end{cases}. \tag{17}$$

Subsequently, with the RSMJ, we can find the mean of the adjustment bonus rates with Eq. (6) as in the following equation:

$$\mu_{RSMJ} = \frac{\eta}{1+i} E_{D_1}\left(\sum_{m=0}^{\infty} \frac{e^{-\lambda}\lambda^m}{m!}\left\{\left(e^{r+m(\mu_y+\sigma_y^2/2)}\Phi\left(d_{1,D_1,m}\right)\right) - \left(1+\frac{i}{\eta}\right)\Phi\left(d_{2,D_1,m}\right)\right\}\right), \tag{18}$$

where $d_{1,D_1,m} = \frac{-\ln(1+i/\eta)+\left(r+\frac{1}{2}\left(\sigma_1^2 D_1+\sigma_2^2(1-D_1)\right)+m\left(\mu_y+\sigma_y^2\right)\right)}{\sqrt{\sigma_1^2 D_1+\sigma_2^2(1-D_1)+m\sigma_y^2}}$, and

$$d_{2,D_1,m} = \frac{-\ln(1+i/\eta)+\left(r-\frac{1}{2}\left(\sigma_1^2 D_1+\sigma_2^2(1-D_1)\right)+m\mu_y\right)}{\sqrt{\sigma_1^2 D_1+\sigma_2^2(1-D_1)+m\sigma_y^2}}.$$

D_1 expresses the percentage of a year remaining in recession (state 1), and $\Phi(\cdot)$ is a cumulative distribution function of standard normal distribution, with the detailed derivation being shown in Appendix A.

The valuation of the nonsurrenderable participating contract is the sum of Eq. (17). Therefore, the premium under the RSMJ can be represented as:

$$U^B_{RSMJ} = C_1\left(\sum_{t=1}^{T} {}_{t-1|}q_x e^{-rt}\left(1+\mu_{RSMJ}\right)^{t-1} + {}_T p e^{-rT}\left(1+\mu_{RSMJ}\right)^{T-1}\right). \tag{19}$$

From Eq. (18), we can observe that when the dynamics of the underlying benefit follows the RSMJ, increases of σ_1 or σ_2, the mean and standard deviation of jump sizes μ_y, σ_y and the mean of the number of jumps λ, all cause μ_{RSMJ} to increase, consequently increasing the premium of a nonsurrenderable participating contract, as indicated in Eq. (19).

3.0.2. Valuation of a surrenderable participating contract

Under this arrangement, at the end of each year the policyholder can sell the participating contract back to the insurance company for its surrender value. In general, the policyholder will hold the contract if the continuation value is higher than the surrender value, but will sell it back if the opposite is the case. The mechanism is just like an American put option.

Let S_t, W_t and F_t denote the surrender value, continuation value and contract value, respectively, at $t = 1,2,3\ldots,t-1$. Before $t = T-1$, the surrender value and the continuation value determine the contract value at the end of each year, with the former being decided in Eq. (11). However, for continuation value, we have to consider two cases at time $t = 1,2,\ldots,T-2$: one case is if the insured is dead in t-th year, and the insurer will have to pay the benefit C_{t+1} at the end of t-th year. The other case is whether the insured survives over t years, then the contract value at $t+1$ needs to be considered. Therefore, the continuation value is shown as:

$$W_t = e^{-r}\left[q_{x+t}C_{t+1} + p_{x+t}E^Q\left(F_{t+1}|\mathbb{F}_t\right)\right], \quad t = 1,2,\ldots,T-2. \tag{20}$$

In Eq. (20), we find that the contract value and benefit for the next period decides the current continuation value, which in turn decides current contract value. In other words, there is a forward recursive relationship in determining the initial contract value.

Suppose the contract is still effective at time $T-1$, if the insured dies during time $T-1$ to T, he/she will receive C_T at time T. On the other hand, if the insured survives during time $T-1$ to T, he/she will also receive C_T, at time T. This means whenever the insured dies during time $T-1$ to T, he/she will receive C_T, at time T regardless. Therefore, the continuation value is $e^{-r}C_T$ at time $T-1$, and the contract value is:

$$F_{T-1} = \max\left(\rho C_T A^{(i)}_{x+(T-1):\overline{T-(T-1)}|}, e^{-r}C_T\right) = C_T\max\left(\rho A^{(i)}_{x+(T-1):\overline{T-(T-1)}|}, H_{T-1}\right) \tag{21}$$

where $H_{T-1} = e^{-r}$. The contract value at time $T-1$ is then taken into Eq. (20), and the continuation value at time $T-2$ is shown as follows:

$$W_{T-2} = e^{-r}\left[q_{x+(T-2)}C_{(T-2)+1} + p_{x+(T-2)}E^Q\left(F_{(T-2)+1}|\mathbb{F}_{(T-2)}\right)\right] = C_{T-1}H_{T-2} \tag{22}$$

where $H_{T-2} = e^{-r}\left[q_{x+(T-2)} + p_{x+(T-2)}\max\left(\rho A_{x+(T-1):\overline{T-(T-1)}|}, H_{T-1}\right)\left(1+\mu_{RSMJ}\right)\right]$. Thus, the contract value at time $T-2$ is:

$$F_{T-2} = \max(S_{T-2}, W_{T-2}) = C_{T-1}\max\left(\rho A^{(i)}_{x+(T-2):\overline{T-(T-2)}|}, H_{T-2}\right). \tag{23}$$

From Eq. (22), we observe that when the dynamics of the underlying benefit follows the RSMJ, the increase of μ_{RSMJ} will cause the continuation value of a surrenderable participating contract to increase, consequently affecting customers' surrender decision, as indicated in Eq. (23).

Theorem 1. *P^T_{RSMJ} represents the premium of a surrenderable participating contract using the RSMJ, and H_t is defined as follows:*

$$H_t = e^{-r}\left[q_{x+t} + p_{x+t}(1+\mu)\max\left(\rho A^{(i)}_{x+t+1:\overline{T-t+1}|}, H_{t+1}\right)\right], \quad t = 1,2,\ldots,T-2, \quad H_{T-1} = e^{-r}$$

where μ_{RSMJ} is the mean of adjustment bonus rates under the RSMJ. Then

$$U^S_{RSMJ} = \max\left\{C_1H_0, \rho C_1 A^{(i)}_{x:\overline{T}|}\right\} = C_1\max\left\{H_0, \rho A^{(i)}_{x:\overline{T}|}\right\}.$$

The detailed derivation is in Appendix B. Using the RSM, the mean of adjustment bonus rates μ is μ_{RSM}, whereas using the RSMJ, the mean of adjustment bonus rates μ should be μ_{RSMJ}. With the RSM, if the standard deviations of two states equal each other, that is $\sigma_1 = \sigma_2 = \sigma$, the valuation of a participating contract using the RSM will reduce to the valuation under the BSM.

4. Sensitivity analysis

In this section, we will show the sensitivity analysis for contract value under the estimated parameters of the BSM, RSM and RSMJ for the S&P 500. Bacinello (2003a) estimated the mortality probabilities with Italian statistics for female mortality in 1991, and valued the premium of a surrenderable participating contract with the CRR model (Cox et al., 1979), but the method is time-consuming. To address this shortcoming, we propose a recursive formula to compute the premium of a surrenderable participating contract using the RSM and the RSMJ. Moreover, in this subsection we analyze the sensitivity of parameters on a contract premium.

In this paper, we also adopt the Italian statistics for female mortality in 1991 as in Bacinello (2003a), and define the parameters as follows:

$$r = \log(1.05), \quad i = 0.02, \quad \eta = 0.5, \quad C_1 = 10,000, \quad T = 5, \quad \rho = 0.985.$$

Given these, we follow the recursive formula explicitly described in Section 3 to price a surrenderable participating S&P 500 index contract with: the BSM, the RSM and the RSMJ. Next, we examine the influence

Table 3
The influence of the parameters on the contract premium and option prices under the regime-switching model with jump risks.

RSMJ	Age	r	i	η	ρ	P_{11}	P_{22}	σ_1	σ_2	μ_y	σ_y	λ
U	+	=	−	=	=	=	=	=	=	=	=	=
B_{RSMJ}	−	+	−	+	=	+	−	+	+	+	+	+
U^B_{RSMJ}	+	−	−	+	=	+	−	+	+	+	+	+
S_{RSMJ}	−	+	−	+	+	+	−	+	+	+	+	+
U^S_{RSMJ}	+	−	−	+	+	+	−	+	+	+	+	+

Note: + represents increasing, − represents decreasing, = represents unchanging; U, U^B_{RSMJ} and U^S_{RSMJ} are the premiums of a basic contract, nonsurrenderable participating contract and surrenderable participating contract, respectively. B_{RSMJ} and S_{RSMJ} are the prices of bonus option and surrender option, respectively. r is the risk-free interest rate, i is the minimum interest rate guarantee, η is the participation coefficient, and ρ is the surrender coefficient. P_{11} is the probability of staying in the recession state, and P_{22} is the probability of staying in the expansion state. σ_1 and σ_2 are the standard deviations in the recession state and the expansion state, respectively. μ_y and σ_y are the mean and standard deviation of jump sizes. λ is the mean of jump times.

of parameters on contract value using the RSMJ. The results are reported in Table 3. Under the RSMJ, the results of the influences of age, risk-free interest rate (r), minimum interest rate guarantee (i_g), the participation coefficient (η), and the surrender coefficient (ρ) are consistent with those in Bacinello (2003a). From Table 3, we observe that the contract premium increases along with P_{11}, the probability of staying in a state of recession. As the volatility in the recession state is higher than in an expansion state, total volatility will increase due to the longer time in the former state, thus inducing higher contract premiums and higher option prices. Conversely, when P_{22} increases, the expansion state will persist, causing total volatility decrease, leading to low contract premiums and option prices. Moreover, increases of either σ_1 or σ_2 will cause total volatility increase, thus increasing contract premiums and option prices. Table 3 also shows the influence of the mean and standard deviation of jump sizes μ_y, σ_y and the mean of the number of jumps λ on the contract premiums and option prices. More specifically, an increase in the mean of jump sizes will induce the mean of returns to increase, consequently raising the premiums and the option prices. An increase in the standard deviation of jump sizes will cause a total volatility increase, thus enhancing the premiums and option prices. In addition, an increase in the mean of the number of jumps will cause increases in total volatility, hence also raising premiums and options prices.

Table 4 reports contract premiums and option prices in the different model settings: the BSM, the RSM and the RSMJ. U represents the premium of a basic contract, which is positively associated with the age of the insured. U^B, B, U^S and S represent the premiums of a nonsurrenderable participating contract, the prices of bonus options, the premiums of a surrenderable participating contract and the prices of surrender options using the different models, respectively. The prices of bonus options, B, can be determined from the difference between the premiums of a basic contract (U) and a nonsurrenderable participating contract (U^B). S can be determined from the difference between the premiums of a nonsurrenderable participating contract (U^B) and a surrenderable one (U^S). Our results show that, using the RSMJ, U^B, B, U^S and S are the lowest when compared with the other model settings.

The lower price (U^B, B, U^S and S) obtained with the RSMJ is due to the feature that the jump process belongs to diversifiable risks. When we assume the dynamics of stock returns follow the RSMJ, part of the return volatilities can be explained by the jump process. Since the jump process is assumed to be a kind of diversifiable risk, while the return volatilities estimated by the BSM and the RSM are caused by non-diversified risks, the price estimated by the RSMJ is lower than that using either the BSM or the RSM.

5. Conclusion

In this study we have proposed a recursive formula to value a surrenderable participating contract. Moreover, in order to identify the dynamics of stock returns over expansion–recession cycles and the occurrences of catastrophic events, we have assumed the rate of return of a reference portfolio will follow the regime-switching model (RSM) and this model with jump risks (RSMJ). We have shown that compared to the Black–Scholes model (BSM) and the RSM, the RSMJ can better explain the dynamics of the S&P 500 stock index. Next, we examined the influence of parameters on contract value with the RSMJ, and found that the contract premiums and option prices increase along with the probability of staying in a state of recession, but decrease along with the probability of remaining in an expansion state. Moreover, the increases of standard deviation in either state, the mean of the jump sizes, the standard deviation of the jump sizes, and the mean of the number of jumps, will all increase the mean of the adjustment bonus rates, contract premiums and option prices, consequently affecting the continuation value of contracts and customers' surrender behavior. Our results also show that compared to the BSM and the RSM, the contract premiums and option prices are the lowest under the RSMJ. Moreover, the differences between the valuations under the BSM, the RSM and the RSMJ treatments suggest that it is critical to value a participating contract precisely with an appropriate model.

Surrenderable participating contracts have many advantages for customers. During market expansion periods, such a policy allows policyholders to participate in the upside returns of the reference portfolio, while during recession periods, the contract also provides a minimum interest rate guarantee to protect holder returns. However, during the 2008 global financial crisis period, numerous insurance customers were observed to surrender their policies. We cannot find any explanation for this in our work, but surmise that this was due to financial insolvency rather than the design of any insurance policy. Therefore, in future studies we recommend extension of this research to build a

Table 4
The premiums under the models versus the age from 40 to 60.

Age	U	U^B_{BSM}	U^B_{RSM}	U^B_{RSMJ}	B_{BSM}	B_{RSM}	B_{RSMJ}	U^S_{BSM}	U^S_{RSM}	U^S_{RSMJ}	S_{BSM}	S_{RSM}	S_{RSMJ}
40	7840	9370	9331	9282	1531	1491	1443	9500	9461	9411	130	130	129
41	7840	9370	9331	9282	1530	1491	1442	9500	9460	9411	130	130	129
42	7840	9370	9331	9282	1530	1490	1442	9500	9460	9411	130	130	129
43	7841	9370	9331	9282	1529	1490	1441	9500	9460	9411	130	130	129
44	7842	9370	9331	9282	1529	1489	1441	9500	9460	9411	130	129	129
45	7842	9370	9331	9283	1528	1489	1440	9500	9460	9411	130	129	129
46	7843	9370	9331	9283	1528	1488	1440	9500	9460	9411	130	129	129
47	7844	9370	9331	9283	1527	1488	1439	9500	9460	9411	130	129	129
48	7844	9370	9331	9283	1526	1487	1439	9500	9460	9411	130	129	129
49	7845	9371	9331	9283	1526	1486	1438	9500	9460	9411	130	129	128
50	7846	9371	9331	9283	1525	1486	1437	9500	9460	9411	129	129	128
51	7847	9371	9332	9283	1524	1485	1436	9500	9460	9411	129	129	128
52	7848	9371	9332	9283	1523	1484	1435	9500	9460	9411	129	129	128
53	7849	9371	9332	9284	1522	1483	1434	9500	9460	9411	129	129	128
54	7850	9371	9332	9284	1521	1482	1433	9500	9460	9411	129	128	128
55	7852	9371	9332	9284	1519	1480	1432	9500	9460	9411	129	128	127
56	7853	9371	9332	9284	1518	1479	1431	9500	9460	9411	128	128	127
57	7855	9372	9333	9284	1516	1477	1429	9500	9460	9411	128	128	127
58	7857	9372	9333	9285	1514	1475	1427	9500	9460	9411	128	127	127
59	7859	9372	9333	9285	1513	1474	1426	9500	9460	9412	128	127	126
60	7862	9372	9333	9285	1510	1471	1423	9499	9460	9412	127	127	126

Note: U represents the premium of a basic contract. U^B_{BSM}, U^B_{RSM} and U^B_{RSMJ} are the premium of a non-surrenderable participating contract; B_{BSM}, B_{RSM} and B_{RSMJ} are the price of bonus option; U^S_{BSM}, U^S_{RSM} and U^S_{RSMJ} are the premiums of a surrenderable participating contract; S_{BSM}, S_{RSM} and S_{RSMJ} are the prices of surrender options under the BSM, regime-switching model, and regime-switching model with jump risks, respectively.

model of surrenderable participating contracts that accounts for customers' financial situation so as to elicit any connection between the events in the: financial markets, insurance customers' financial situation and their policy surrender decision.

Some people may be concerned that surrenderable participating policies could cause trouble for insurance companies, in particular, during bear markets, as the minimum interest rate guarantee would be a burden for them. However, the companies could choose to buy options to hedge their possible losses and still be profitable, rather than directly investing in capital markets. Finally, our RSMJ can also be applied in pricing equity index annuity, which is becoming increasingly popular in insurance markets.

Appendix A. Proof of the formula for the mean of adjustment bonus rates in the BSM, the RSM, and the RSMJ

From Eq. (6), the adjustment bonus rate can be rewritten as follows:

$$\begin{aligned}\delta_t &= \max\left\{\frac{\eta g_t - i}{1+i}, 0\right\}\\ &= \frac{\eta}{1+i}\max\left\{g_t - \frac{i}{\eta}, 0\right\}\\ &= \left(\frac{\eta}{1+i}\right)\max\left\{\frac{G_{t+1}}{G_t} - \left(1+\frac{i}{\eta}\right), 0\right\}.\end{aligned}$$

We assume the dynamics of a reference portfolio follows the Black–Scholes model:

$$G_{t+1} = G_t \exp\left\{\left(r - \frac{1}{2}\sigma^2\right) + \sigma W^Q(1)\right\}.$$

Consider the payoff of a European call option is $\max\{G_{t+1} - K, 0\}$, then the European call option formula at time t-th can be derived as follows:

$$\begin{aligned}&E^Q(\max\{G_{t+1} - K, 0\})\\ &\quad = G_t e^r \Phi(d_1) - K\Phi(d_2)\end{aligned}$$

where $d_1 = \frac{\ln\left(\frac{G_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)}{\sigma}$, $d_2 = \frac{\ln\left(\frac{G_t}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)}{\sigma}$, and Φ is the cumulative probability function of a standard normal distribution. Therefore, we apply the Black–Scholes option pricing formula to the mean of the adjustment bonus rates as follows:

$$\begin{aligned}\mu_{BS} &= E^Q(\delta_t)\\ &= \left(\frac{\eta}{1+i}\right)\left[e^r \Phi\left(d_{1,BS}\right) - \left(1+\frac{i}{\eta}\right)\Phi\left(d_{2,BS}\right)\right]\end{aligned}$$

where $d_{1,BS} = \frac{-\ln\left(1+\frac{i}{\eta}\right) + \left(r + \frac{1}{2}\sigma^2\right)}{\sigma}$ and $d_{2,BS} = \frac{\ln\left(\frac{G_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)}{\sigma}$.

Then, we assume the dynamics of the reference portfolio follows the RSM by Eq. (15). Given the percentage of a year staying in recession (state 1), D_1, the dynamics of the reference portfolio can be rewritten as follows:

$$G_{t+1} = G_t \exp\left\{\left(r - \frac{1}{2}\left(D_1\sigma_1^2 + (1-D_1)\sigma_2^2\right)\right) + \sigma_1 W^Q(D_1) + \sigma_2 W^Q(1-D_1)\right\},$$

where the variance of the return of the reference portfolio each year is $D_1\sigma_1^2 + (1 - D_1)\sigma_2^2$. From the Black–Scholes and regime switching option pricing formula (Hardy, 2001), that for the mean of adjustment bonus rates should be related to the volatility of the reference portfolio. Therefore, we can derive the mean of adjustment bonus rates as follows:

$$\begin{aligned}\mu_{RSM} &= E^Q_{\pi_1}(\delta_t)\\ &= \frac{\eta}{1+i}\left(e^r \Phi\left(d_{1,D_1}\right) - \left(1+\frac{i}{\eta}\right)\Phi\left(d_{2,D_1}\right)\right)\end{aligned}$$

where $d_{1,D_1} = \frac{-\ln(1+i/\eta) + \left(r + \frac{1}{2}\left(\sigma_1^2 D_1 + \sigma_2^2(1-D_1)\right)\right)}{\sqrt{\sigma_1^2 D_1 + \sigma_2^2(1-D_1)}}$ and $d_{2,D_1} = \frac{-\ln(1+i/\eta) + \left(r - \frac{1}{2}\left(\sigma_1^2 D_1 + \sigma_2^2(1-D_1)\right)\right)}{\sqrt{\sigma_1^2 D_1 + \sigma_2^2(1-D_1)}}$.

Finally, we assume the dynamics of the reference portfolio follows the RSMJ by Eq. (15). Given the percentage of a year staying in recession (state 1), D_1, the dynamics of the reference portfolio can be rewritten as follows:

$$G_{t+1} = G_t \exp\left\{\left(r - \lambda(k-1)dt - \frac{1}{2}\left(D_1\sigma_1^2 + (1-D_1)\sigma_2^2\right)\right) + \sigma_1 W^Q(D_1) + \sigma_2 W^Q(1-D_1)\right\}\prod_{i=1}^{N(t)} Y_i.$$

From the Black–Scholes and jump diffusion model's option pricing formula (Hardy, 2001), the formula for the mean of adjustment bonus rates should be related to the volatility of the reference portfolio and the volatility of jump events. Therefore, we can derive the mean of the adjustment bonus rates as follows:

$$\mu_{RSMJ} = \frac{\eta}{1+i}\left(\sum_{m=0}^{\infty}\frac{e^{-\lambda}\lambda^m}{m!}\left\{\left(e^{r+m(\mu_y+\sigma_y^2/2)}\Phi\left(d_{1,D_1,m}\right)\right) - \left(1+\frac{i}{\eta}\right)\Phi\left(d_{2,D_1,m}\right)\right\}\right),$$

where $d_{1,D_1,m} = \frac{-\ln(1+i/\eta) + \left(r + \frac{1}{2}\left(\sigma_1^2 D_1 + \sigma_2^2(1-D_1)\right) + m\left(\mu_y + \sigma_y^2\right)\right)}{\sqrt{\sigma_1^2 D_1 + \sigma_2^2(1-D_1) + m\sigma_y^2}}$, and

$$d_{2,D_1,m} = \frac{-\ln(1+i/\eta) + \left(r - \frac{1}{2}\left(\sigma_1^2 D_1 + \sigma_2^2(1-D_1)\right) + m\mu_y\right)}{\sqrt{\sigma_1^2 D_1 + \sigma_2^2(1-D_1) + m\sigma_y^2}}.$$

Appendix B. Proof of Theorems 1

Following the definition in Section 3, S_t, W_t and F_t represent the surrender value, the continuation value and the contract value at time t, respectively, and the relationship can be shown in Eqs. (8), (9), and (22).

It is immaterial whether the insured dies in the T-th year or whether he/she survives the whole insured period, because in both cases the insurer has to pay out the benefit amount C_T. Therefore, the continuation value at $t = T - 1$ is $W_{T-1} = e^{-r}C_T$, and the contract value at $t = T - 1$ is:

$$\begin{aligned}F_{T-1} &= \max(S_t, W_t) = \max\left(\rho C_T A^{(i)}_{x+(T-1):\overline{T-(T-1)}|}, e^{-r}C_T\right)\\ &= C_T \max\left(\rho A^{(i)}_{x+(T-1):\overline{T-(T-1)}|}, e^{-r}\right)\end{aligned}$$

where $H_{T-1} = e^{-r}$. By Eq. (22), we know that the contract value for the next period is a function of the continuation value in the current period. Thus, in $t = T - 2$, the continuation value can be derived as follows:

$$\begin{aligned}W_{T-2} &= e^{-r}\left[q_{x+(T-2)}C_{(T-2)+1} + p_{x+(T-2)}E^Q\left(F_{(T-2)+1}\middle|\mathbb{F}_{(T-2)}\right)\right]\\ &= e^{-r}C_{T-1}\left[q_{x+(T-2)} + p_{x+(T-2)}\max\left(\rho A^{(i)}_{x+(T-1):\overline{T-(T-1)}|}, H_{T-1}\right)(1+\mu)\right]\end{aligned}$$

where $E^Q(\delta_t) = \mu, t = 1, 2, \ldots, T-1$.

As the continuation value is a function of the benefit for the next period, the former equation can be written as:

$$\begin{aligned}W_{T-2} &= C_{T-1}e^{-r}\left[q_{x+(T-2)} + p_{x+(T-2)}\max\left(\rho A_{x+(T-1):\overline{T-(T-1)}|}, H_{T-1}\right)(1+\mu)\right]\\ &= C_{T-1}H_{T-1}\end{aligned}$$

where $H_{T-2} = e^{-r}\left[q_{x+(T-2)} + p_{x+(T-2)} \max\left(\rho A_{x+(T-1):\overline{T-(T-1)}|}, H_{T-1}\right)\right.$ $(1+\mu)]$. Then, the contract value is: $F_{T-2} = \max\left(\rho C_{(T-2)+1} A_{x+}(T-2): \overline{T-(T-2)}|^{(i)}, C_{T-1} H_{T-2}\right) = C_{T-1} \max\left(\rho A^{(i)}_{x+(T-2):\overline{T-(T-2)}|}, H_{T-2}\right)$.

Supposing the relationship that the continuation value is a function of the benefit for next period still holds at $t = T - k$, we get the equation that $W_{T-l} = C_{T-k+1} H_{T-k}$, and the contract value is:

$$F_{T-k} = C_{T-k+1} \max\left(\rho A^{(i)}_{x+(T-k):\overline{T-(T-k)}|}, H_{T-k}\right)$$

where $H_{T-k} = e^{-r}\left(q_{x+(T-k)} + p_{x+(T-k)}(1+\mu) \max \left\{H_{(T-k)+1}, \rho A_{x+(T-k)+1:\overline{k-1}|}\right\}\right)$. Then, if $t = T - (k+1)$, the continuation value is:

$$W_{T-(k+1)} = e^{-r}\left[q_{x+(T-(k+1))} C_{(T-(k+1))+1} + p_{x+(T-(k+1))} E^Q\left(F_{(T-(k+1))+1} \middle| \mathbb{F}_{(T-(k+1))}\right)\right] = C_{T-k} H_{T-(k+1)}$$

where $H_{T-(k+1)} = e^{-r}\left[q_{x+(T-(k+1))} + p_{x+(T-(k+1))} \max\left(\rho A_{x+(T-k):\overline{T-(T-k)}|}, H_{T-k}\right)(1+\mu)\right]$. This means that the continuation value is still a function of the benefit for the next period at $t = T - (k+1)$, and the contract value is:

$$F_{T-(k+1)} = \max\left(\rho C_{T-(k+1)+1} A^{(i)}_{x+T-(k+1):\overline{T-(T-(k+1))}|}, C_{T-k} H_{T-(k+1)}\right),$$

where $H_{T-(k+1)} = e^{-r}\left[q_{x+(T-(k+1))} + p_{x+(T-(k+1))}(1+\mu) \max\left(\rho A_{x+(T-k):\overline{T-(T-k)}|}, H_{T-k}\right)\right]$.

By mathematical induction, we prove that:

(A) the continuation value is a function of the benefit for the next period, $W_t = C_{t+1} H_t$: where $H_t = e^{-r}\left[q_{x+t} + p_{x+t}(1+\mu) \max\left(\rho A^{(i)}_{x+t+1:\overline{T-t+1}|}, H_{t+1}\right)\right]$, $t = 1, 2, \ldots, T-2$,

$H_{T-1} = e^{-r}$.

(B) the contract value is a function of H_t, and can be written as:

$$F_t = \max\left(\rho C_{t+1} A^{(i)}_{x+t:\overline{T-t}|}, W_t\right) = C_{t+1} \max\left(\rho A^{(i)}_{x+t:\overline{T-t}|}, H_t\right), \quad t = 1, 2, \ldots, T-1.$$

The contract value at time t is:

$$F_t = \max\left(\rho C_{t+1} A^{(i)}_{x+t:\overline{T-t}|}, W_t\right) = C_{t+1} \max\left(\rho A^{(i)}_{x+t:\overline{T-t}|}, H_t\right), \quad t = 1, 2, \ldots, T-1$$

where $H_t = e^{-r}\left[q_{x+t} + p_{x+t}(1+\mu) \max\left(\rho A^{(i)}_{x+t+1:\overline{T-t+1}|}, H_{t+1}\right)\right]$, $t = 1, 2, \ldots, T-2$,

$H_{T-1} = e^{-r}$.

Therefore, we can get H_0 by a recursive formula from H_{T-1}, and then compute the surrender value, the continuation value and the contract value at time 0, which can be shown as follows:

$$W_0 = C_1 H_0 = e^{-r} C_1 \left[q_x + p_x \cdot (1+\mu) \cdot \max\left\{H_1, \rho A^{(i)}_{x:\overline{T}|}\right\}\right]$$

$$R_0 = \rho C_1 A^{(i)}_{x:\overline{T}|}$$

$$F_0 = \max\{W_0, R_0\} = \max\left\{C_1 H_0, \rho C_1 A^{(i)}_{x:\overline{T}|}\right\} = C_1 \max\left\{H_0, \rho A^{(i)}_{x:\overline{T}|}\right\}.$$

Thus, the premium of a surrenderable participating contract U^S is derived as:

$$U^S = F_0 = \max\{W_0, R_0\} = \max\left\{C_1 H_0, \rho C_1 A^{(i)}_{x:\overline{T}|}\right\} = C_1 \max\left\{H_0, \rho A^{(i)}_{x:\overline{T}|}\right\}.$$

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