The Geneva Papers, **2011**, **36**, (697–717)

www.genevaassociation.org

Using Reverse Mortgages to Hedge Longevity and Financial Risks for Life Insurers: A Generalised Immunisation Approach

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The launch of new innovative longevity-linked products, such as reverse mortgages, increases the complexity and challenges faced by insurers in implementing an asset-liability management strategy. With the house price dynamic and a large final payment received at the end of the policy year, a reverse mortgage provides a different liability duration pattern from an annuity. In this paper, we propose a generalised immunisation approach to obtain an optimal product portfolio for hedging the longevity and financial risks of life insurance companies. The proposed approach does not rely on specific assumptions regarding mortality models or interest rate models. As long as the scenarios generated by the adopted models are highly correlated, the proposed approach should be effective. By using stochastic mortality and interest rate models and the Monte Carlo simulation approach, we show that the proposed generalised immunisation approach can serve as an effective vehicle to control the aggregate risk of life insurance companies. The numerical results further demonstrate that adding the reverse mortgage to the insurers' product portfolio creates a better hedging effect and effectively reduces the total risk associated with the surplus of the life insurers.
The Geneva Papers (2011) **36,** 697–717. doi:10.1057/gpp.2011.22

Keywords: longevity risk; interest rate risk; reverse mortgage; generalised immunisation approach

Introduction

In the past decade, the longevity phenomenon has increased in human society. Benjamin and Soliman,[1] and McDonald *et al.*[2] confirm that unprecedented improvements in population longevity have occurred worldwide. The decline in unanticipated mortality rates has posed a great risk for insurance company operations. As a testament to the issue's importance, existing literature proposes many solutions to mitigate the threat of longevity risk in life insurance. We can classify these studies into three categories: capital market solutions, industry self-insurance solutions and mortality projection improvements. *Capital market solutions* include mortality

[1] Benjamin and Soliman (1993).
[2] McDonald *et al.* (1998).

securitisation (see, e.g., Dowd;[3] Lin and Cox;[4] Blake *et al.*;[5] Cox *et al.*[6]), survivor bonds (e.g., Blake and Burrows;[7] Denuit *et al.*[8]), and survivor swaps (e.g., Dowd *et al.*[9]). These studies suggest that insurance companies could transfer their exposure to the capital markets that have more funding and participants. Cowley and Cummins[10] provide an overview of the securitisation of life insurance assets and liabilities. *Industry self-insurance solutions* include the natural hedging strategy of Cox and Lin,[11] the duration matching strategy of Wang *et al.*,[12] and the conditional Value-at-Risk approach of Wang *et al.*[13] The advantages of industry self-insurance solutions are that there may exist a lower transaction cost and insurance companies do not need a liquid market. The insurance companies can also hedge longevity risk by themselves or with other counterparties. The third solution, *mortality projection improvements*, provides a more accurate estimation of and more realistic assumptions regarding mortality processes. As Blake *et al.*[14] suggest, these studies fall into two areas: continuous-time frameworks (e.g., Milevsky and Promislow;[15] Dahl;[16] Biffis;[17] Schrager[18]) and discrete-time frameworks (e.g., Brouhns *et al.*;[19] Renshaw and Haberman;[20] Cairns *et al.*[21]). The parameter of uncertainty and model specification in relation to the mortality process have also attracted more attention in recent years (e.g., Melnikov and Romaniuk;[22] Koissi *et al.*;[23] Wang *et al.*[12]).

With the launch of new longevity-linked products, such as reverse mortgages and equity-linked annuities, life insurance companies' operations involve more liability risks and financial risks. These risks in turn increase the complexity and challenges faced by the insurers in implementing their asset-liability management strategy. However, some financial risks may not be positively related to each other. For example, in some markets interest rates and real estate prices were observed to be negatively correlated. In addition, from a liability perspective, with the house price dynamic and

[3] Dowd (2003).
[4] Lin and Cox (2005).
[5] Blake *et al.* (2006a, b).
[6] Cox *et al.* (2006).
[7] Blake and Burrows (2001).
[8] Denuit *et al.* (2007).
[9] Dowd *et al.* (2006).
[10] Cowley and Cummins (2005).
[11] Cox and Lin (2007).
[12] Wang *et al.* (2010b).
[13] Wang *et al.* (2010a).
[14] Blake *et al.* (2006b).
[15] Milevsky *et al.* (2006).
[16] Dahl (2004).
[17] Biffis (2005).
[18] Schrager (2006).
[19] Brouhns *et al.* (2002).
[20] Renshaw and Haberman (2003).
[21] Cairns *et al.* (2006).
[22] Melnikov and Romaniuk (2006).
[23] Koissi *et al.* (2006).

a huge final payment received at the end of the policy year, reverse mortgages provide different cash flows and liability durations from those of annuities.

In this paper, we propose a generalised immunisation approach to obtain an optimal product portfolio for life insurance companies aimed at hedging longevity and financial risks. We use a delta hedge strategy to mitigate the risks associated with the movements of these financial risk factors. We also consider the simultaneous shocks to the mortality curve, interest rate curve and other financial risk factors existing in the product portfolio. We first adopt the two-factor stochastic mortality model the Cairns-Blake-Dowd (CBD model)[21] to construct future mortality processes and corresponding liability distributions. We then simulate interest rate curves by using the Cox *et al.*[24] stochastic interest model (the Cox-Ingersoll-Ross or CIR model). By means of the Monte Carlo simulation of the movements in the mortality curve, the interest rate curve and related risk factors, we show that the proposed generalised immunisation approach can serve as an effective vehicle to reduce the aggregate risk significantly for life insurance companies. Moreover, adding reverse mortgages to the product portfolio also serves as a risk diversification strategy. The numerical results further demonstrate that adding the reverse mortgage to the insurers' product portfolio gives rise to a better hedging effect and effectively reduces the total risk associated with the surplus of the life insurers.

The remainder of this paper is organised as follows. We briefly introduce the reverse mortgage and discuss some important issues in the next section. In the subsequent section, we introduce the stochastic interest rate and mortality models along with our proposed immunisation model as a natural hedging strategy for life insurers. In the section following that section, we demonstrate how our proposed generalised immunisation model can be implemented by using different numerical examples for various product mixes. Finally, we analyse the simulation results in the penultimate section, and conclude in the last section.

Reverse mortgage

Since the 1970s, the United Kingdom, the United States and many other countries have been developing a house mortgage mechanism, known as "Reverse Mortgage" or "Housing Endowment", which enables elderly homeowners to consume some of the home equity but still maintain the ownership and residence of the home. In a typical reverse mortgage arrangement (see Chen *et al.*[25]), the lender advances a lump sum or periodic payments to elderly homeowners. The loan accrues with interest and is settled using the sale proceeds of the property when borrowers die, sell or vacate their homes to live elsewhere. There are various styles of loan services provided by financial institutions, including:

(a) lump-sum payment: the borrower receives a fixed amount of the entire available principal limit at closing of the loan;
(b) tenure payments: equal monthly payments are made as long as the borrower lives;

[24] Cox *et al.* (1985)
[25] Chen *et al.* (2010).

(c) term payments: equal monthly payments are made for a fixed period of months selected by the borrower;
(d) line of credit: instalments are paid to the borrower at times and in amounts of the borrower's choosing until the line of credit is exhausted.

Previous studies suggest that reverse mortgage can help to increase retirement income for homeowners. Mayer and Simons[26] estimate that homeowners can increase their monthly incomes by at least 20 per cent through a reverse mortgage. The empirical results of recent research[27] support that higher housing prices have a positive impact on reverse mortgage originations, and reverse mortgage indeed improve the retirement income for the elderly. Benedict[28] and Davidoff[29] suggest that annuity insurance products, long-term care insurance and reverse mortgage are primary tools for retirement income management. Although reverse mortgages appear to be a useful way for homeowners to access their equity, the market is current extremely small. Certain factors could explain the small size of the reverse mortgage market. First, there are several factors that could discourage homeowners from taking out reverse mortgages. These include high up-front costs, low borrowing limits, concerns about future medical expenses and fear of debt. Venti and Wise[30] and Caplin[31] suggest that bequest motives and the expectation of moving out may also be major reasons for stagnation in the U.S. reverse mortgage market.

On the other hand, despite substantial government subsidies and protection, many lenders have been unable to generate enough profit to justify maintaining costs or risks for this specialised product, and have exited the market. In addition to low origination fees and the uncertainties from regulatory and legal problems, the lenders are also exposed to the following risks: (a) interest rate risk (see Boehm and Ehrhardt[32]); (b) longevity risk (see Chen *et al.*[25]); (c) housing price risk (see Mitchell and Piggott[33]); and (d) borrower maintenance risk (see Ong).[34] The overall risk measure can be referred to as the crossover risk (see Chinloy and Megbolugbe;[35] Wang *et al.*[36]) that the outstanding loan balance will not be repaid in full when the loan is terminated because the loan balance is larger than the property value and the lender will recover only up to the sale price of the property. The crossover risk is usually insured via mortgage insurance premiums, and the fair premium is determined by the present value of the non-recourse provision (see Chen *et al.*[25]).

[26] Mayer and Simons (1994).
[27] Benjamin and Brian (2009), Joan *et al.* (2010) and Shan (2011).
[28] Benedict (2009).
[29] Davidoff (2009).
[30] Venti and Wise (2000).
[31] Caplin (2002).
[32] Boehm and Ehrhardt (1994).
[33] Mitchell and Piggott (2004).
[34] Ong (2008).
[35] Chinloy and Megbolugbe (1994).
[36] Wang *et al.* (2007).

The research models

In this section, we first provide brief descriptions of the two-factor stochastic CBD mortality model of Cairns *et al.*[21] and the stochastic CIR model of Cox *et al.*[37] We then describe the house price index dynamic model and introduce the proposed generalised immunisation approach in this paper.

The two-factor stochastic mortality model

In traditional insurance pricing and reserve calculations, an actuary treats mortality rates as being constant over time, which means that unanticipated mortality improvements can cause serious financial burdens or even bankruptcy for life insurers. In actuarial literature, the question of how to model mortality rates dynamically continues to be an important issue. Earlier developments of stochastic mortality modelling rely on the one-factor model and the pioneering work of Lee and Carter[38] (the LC model). The LC model is easy to apply and provides fairly accurate mortality estimations and population projections. More recent studies have proposed the use of two-factor mortality models. The most distinguishing feature of these models is the consideration of a cohort effect in mortality modelling.[20,39] In particular, Cairns *et al.*[21] allow not only for a cohort effect but also for a quadratic age effect in their CBD model. Cairns *et al.*[40] extend their earlier work by comparing an analysis of eight stochastic models based on the mortality experiences of England, Wales and the United States. Other developments in two-factor models (e.g., Milevsky and Promislow;[41] Dahl;[16] Dahl and Møller;[42] Miltersen and Persson;[43] Luciano and Vigna;[44] Biffis;[17] Schrager[18]) employ a continuous-time framework and thus offer an important means of pricing mortality-linked securities.

Following the recent literature on mortality modelling, we employ a stochastic mortality model to test the proposed hedging strategy. In particular, we adopt the CBD model as our mortality model to test the effectiveness of the hedging strategies. We choose the CBD model as the underlying mortality process for two reasons. First, the CBD model characterises not only a cohort effect but also a quadratic age effect. The two factors $A_1(t)$ and $A_2(t)$ in the CBD model represent all age-general improvements in mortality over time and different improvements for different age groups. These two factors reflect the "trend effect" and "age effect". Thus, the mortality improvement forecasted by the CBD model is more significant than that of other mortality models, such as the LC model, because the cohort effect of the CBD model captures the greater mortality rate dynamics for older consumers, compared to

[37] Cox *et al.* (1985).
[38] Lee and Carter (1992).
[39] Currie (June 2006).
[40] Cairns *et al* (2007).
[41] Milevsky and Promislow (2001).
[42] Dahl and Moller (2006).
[43] Miltersen and Persson (2005).
[44] Luciano and Vigna (2005).

younger consumers. Second, the CBD model is a discrete time model and is more convenient to implement in practice. We offer a brief description of the CBD model; for a more detailed discussion, see Cairns *et al.*,[21] who propose that the mortality curve has the following logistic functional form:

$$q(t,x) = \frac{e^{A_1(t+1)+A_2(t+1)\cdot(x+t)}}{1+e^{A_1(t+1)+A_2(t+1)\cdot(x+t)}}, \tag{1}$$

where $q(t,x)$ is the realised mortality rate for age x insured from time t to $t+1$.

By setting $A(t)=(A_1(t),\ A_2(t))^T$, the two stochastic trends $A_1(t)$ and $A_2(t)$ follow a discretised diffusion process with a drift parameter μ and a diffusion parameter C:

$$\mathbf{A(t+1)} - \mathbf{A(t)} = \boldsymbol{\mu} + \mathbf{CZ(t+1)}, \tag{2}$$

where $\boldsymbol{\mu}=(\mu_1,\mu_2)^T$ is a 2×1 constant parameter vector, the $Z(t)$'s $(t=0,1,2,\ldots)$ are independent two-dimensional standard normal random vectors, and $\mathbf{C}$ is a 2×2 constant lower-triangular Cholesky square root matrix of the covariance matrix $\mathbf{V}$ of Z(t).

There are two steps to implementing a stochastic mortality model from real mortality data. The first involves calibrating the model to fit a set of historical data for mortality experiences in the past. The second is to take the calibrated model and generate simulations to forecast future mortality rates. We assume that the male population of the United States includes clients of the insurance product. Thus, we use the deaths and populations database, in which the ages are distributed from 0 to 114 and the time period from 1969 to 2007, as the input data for calibrating the CBD model. We briefly describe the deaths and population database as follows.

Annual population data

In this paper, we adopt population data through the Surveillance, Epidemiology and End Results[45] (SEER) programme, which builds a complete and available population database for each single age person up to the age of 84 years. Those aged 85 and older make up the last group. The United States Population information is available on the website for the U.S. Census Bureau.[46] Data for sex and age are published in five-year age groups up to the age of 85 and older.

Annual deaths data

In the United States, the vital statistics are published annually by the Centers for Disease Control and Prevention and the National Center for Health Statistics.[47] The provisional tables are published in five-year age groups. LifeMetrics[48] provides single

[45] Surveillance *et al.* (2010).
[46] U.S. Census Bureau (2010).
[47] Centers for Disease Control and Prevention (2010).
[48] JP Morgan (2007).

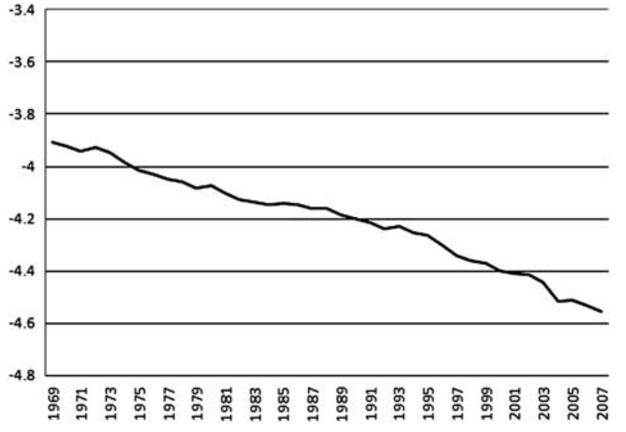

Figure 1. $A_1(t)$.

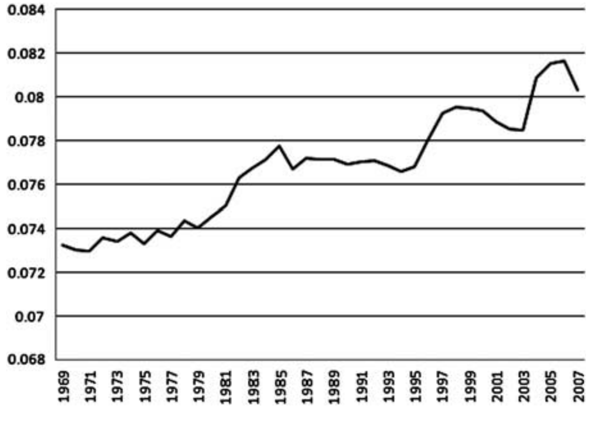

Figure 2. $A_2(t)$.

age deaths data from ages 40 to 84 years. For the deaths data of those above the age of 85 years, we refer to the most updated mortality report on the Center for Disease Control and Prevention (CDC) website.[25]

After collecting data, we estimate $A_1(t)$ and $A_2(t)$ using the maximum likelihood method. The estimated results are shown in Figures 1 and 2. Figure 1 shows that $A_1(t)$ is generally declining over time, which corresponds to the characteristic that mortality rates exhibit improvement effects for all ages.[49] In addition, Figure 2 shows that $A_2(t)$ is generally increasing over time. This suggests that the mortality improvements are more significant at lower ages than at higher ages. These results are consistent with the previous research. Based on the estimation of $A_1(t)$ and $A_2(t)$, we use LifeMetrics to generate 1,000 scenarios of $q(t, x)$ for x=50, 60 and 70.

[49] Pitacco *et al.* (2009).

The stochastic interest rate model

There is a very rich literature on stochastic interest rate models. Some of them are no-arbitrage models such as the Hull–White model or LIBOR market models. No-arbitrage models are suitable for pricing fixed income derivatives and are specified in risk-neutral measures. The others are equilibrium models such as the Vasicek model[50] and CIR model.[37] These models require only a small number of parameters. The parameters of these models can be estimated from historical data and are specified in real-world measures. The equilibrium models are more suitable for the purpose of testing hedging strategies. Thus, we select the CIR model to test the effectiveness of our hedging approach.

The CIR model describes the evolution of the instantaneous short-term interest rates (short rates) r_t as

$$dr_t = a(b - r_t)dt + \sigma\sqrt{r_t}dz,$$

where a, b and σ are constants and z follows a standard Brownian motion. The drift term of dr_t is $a(b-r_t)$, which ensures mean reversion of the short rate towards the long-run value b. The diffusion term of dr_t is $\sigma\sqrt{r_t}$, which guarantees that the short rate remains positive. Given the short rate at time u, r_u $(u<t)$, it can be shown that the short rate at time t, r_t, is, up to a scale factor, a noncentral chi-square distribution. Based on this result, we are able to simulate 1,000 scenarios of the short rates. The CIR model can construct the whole interest rates curve based on the levels of the short-rate and model parameters. Therefore, the simulated 1,000 scenarios of the short rates are sufficient for our purposes. In our numerical examples in the next section, the parameters in the CIR model are as shown in Table 1.

The house price index dynamic

In this paper, we use a geometric Brownian motion (also known as a lognormal process) to model the house price dynamic. Following the assumptions of Kau *et al.*[51] and Szymanoski,[52] we use a lognormal process to model the house price dynamic.

The lognormal process of house price H can be described via a stochastic differential equation of two parameters μ and σ:

$$dH = \mu H\,dt + \sigma H\,dz.$$

The parameter μ is the expected rate of return and the parameter σ is the volatility of the house price.

We use the quarterly data for the House Price Index (HPI) from the first quarter of 1975 to the fourth quarter of 2009 to estimate the parameters μ and σ. HPI is reported by the Office of Federal Housing Enterprise Oversight (OFHEO). OFHEO and the Federal Housing Finance Board (FHFB) were combined to form the new Federal

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[50] Vasicek (1977).
[51] Kau *et al.* (1993).
[52] Szymanoski (1994).

Table 1 Parameters of the CIR model used in numerical examples

a	Speed of mean reversion	0.15
b	Long-run mean of short rate	0.05
σ	Volatility	0.06
r_0	Initial short rate	0.01

Housing Finance Agency (FHFA) in 2008. The estimated values for μ and σ are 5.52 per cent and 3.86 per cent, respectively.

The proposed generalised immunisation approach

To help life insurers achieve a better hedging effect, we propose a general immunisation model that incorporates a stochastic mortality dynamic to calculate the optimal level of a product mix that includes insurance, annuities and reverse mortgages.

Let q, r and S be the current mortality curve, the interest rates curve and other financial risk factors, respectively. The current value of the product portfolio with hedging assets is $V(q, r, S)$. Let us assume that $\hat{q}_s$, $\hat{r}_s$ and $\hat{s}_s$ are typical shocks of the mortality curve, the interest rates curve and other financial risk factors, respectively. These typical shocks can be estimated from the simulated scenarios of the adopted stochastic models for interest rates, mortality rates and other financial risk factors, respectively.

It is common to assume that $\hat{r}_s$ follows a parallel shift.[53] However, other types of shocks have been proposed in the literature.[53,54] In our proposed approach, we use the Principal Components Analysis (PCA) technique to determine these typical shocks. In particular, we first simulate n scenarios from each adopted stochastic model and compute the sample covariance matrix of the simulated scenarios. We then compute the first principal component of the sample covariance matrix. The first principal component can then be used to determine the typical shocks of interest. For a fixed income portfolio, we use the PCA technique to select the most plausible interest rates shock. This method is considered as an important extension of non-stochastic risk methodologies based on durations and convexities (Chapter 3, Golub and Tilman[55]). More precisely, Δq, Δr and ΔS are the lengths of $\hat{q}_s$, $\hat{r}_s$ and $\hat{s}_s$, respectively. Vectors $\mathbf{q}_s$, $\mathbf{r}_s$ and $\mathbf{S}_s$ are normalised versions of $\hat{q}_s$, $\hat{r}_s$ and $\hat{s}_s$. That is, their lengths are equal to one. In Golub and Tilman,[55] the square root of the eigenvalue and the eigenvector of the first principal component of the covariance matrix of $\hat{r}_s$ are considered to be the most plausible magnitude and the most plausible shape of $\hat{r}_s$. We extend this idea in our generalised immunisation approach. In particular, the most

[53] Willner (1996).
[54] Golub and Tilman (1997).
[55] Golub and Tilman (2000).

plausible magnitude (Δq, Δr and ΔS) of the typical shocks are the square roots of the eigenvalues of the corresponding first principal component, and the most plausible shape ($\mathbf{q}_s$, $\mathbf{r}_s$ and $\mathbf{S}_s$) of the typical shocks are the eigenvectors of the corresponding first principal components.

We denote V as the value of the product portfolio and hedging asset. We assume that V contains n products and hedging assets and can be regarded as a hedging programme. Let V_k denote the unit price of the k^{th} product or hedging asset of V and w_k denote the number of units sold or the number of shares purchased. The value of V_k is negative if it is a liability and is positive if it is a hedging asset. The mathematical expression of V is

$$V = \sum_{k=1}^{n} W_k V_k.$$

The value of V is not required to be zero. However, if we include cash in the hedging programme V, then the value of V can be set to zero. For example, in the section of numerical examples, V represents a surplus and its initial value is set to zero.

Let $V(q, r, S)$ be the initial value of V. From the discussion above, the value of V after the shocks can be approximated by $V(q + \Delta q \times \mathbf{q}_s,\ r + \Delta r \times \mathbf{r}_s,\ S + \Delta S \times \mathbf{S}_s)$. Through the multivariate Taylor's formula, the change in V can be approximated as follows:

$$\begin{aligned} &V(q + \Delta q q_{S^r} \quad r + \Delta r r_{S^r} \quad S + \Delta S S_r) - V(q, r, S) \\ &\quad \approx \left(\frac{\partial V}{\partial \Delta q} \Delta q + \frac{\varepsilon V}{\partial \Delta r} \Delta r + \frac{\partial V}{\partial \Delta S} \Delta S \right) \\ &\quad + \frac{1}{2} \left(\frac{\partial^2 V}{\partial \Delta q^2} (\Delta q)^2 + \frac{\partial^2}{\partial \Delta r^2} (\Delta r)^2 + \frac{\partial^2 V}{\partial \Delta S^2} (\Delta S)^2 \right) \\ &\quad + \left(\frac{\partial^2 V}{\partial \Delta q \partial \Delta r} \Delta q \Delta r + \frac{\partial^2 V}{\partial \Delta r \partial \Delta S} \Delta r \Delta S + \frac{\partial^2 V}{\partial \Delta q \partial \Delta S} \Delta q \Delta S \right). \end{aligned} \tag{3}$$

It is easy to see that the first-order and second-order partial derivatives of V are linear functions of the w_k's:

$$\begin{aligned} &\frac{\partial V}{\partial \Delta q} = \sum_{k=1}^{n} \omega_k \frac{\partial V_k}{\partial \Delta q}, \frac{\partial V}{\partial \Delta r} = \sum_{k=1}^{n} \omega_k \frac{\partial V_k}{\partial \Delta r}, \frac{\partial V}{\partial \Delta S} = \sum_{k=1}^{n} \omega_k \frac{\partial V_k}{\partial \Delta S}; \\ &\frac{\partial^2 V}{\partial \Delta q^2} = \sum_{k=1}^{n} \omega_k \frac{\partial^2 V_k}{\partial \Delta r^2}, \frac{\partial^2 V}{\partial \Delta r^2} = \sum_{k=1}^{n} \omega_k \frac{\partial^2 V_k}{\partial \Delta r^2}, \frac{\partial^2 V}{\partial \Delta S^2} = \sum_{k=1}^{n} \omega_k \frac{\partial^2 V_k}{\partial \Delta S^2}; \\ &\frac{\partial^2 V}{\partial \Delta q \partial \Delta r} = \sum_{k=1}^{n} \omega_k \frac{\partial^2 V_k}{\partial \Delta q \partial \Delta r}, \frac{\partial^2 V}{\partial \Delta r \partial \Delta S} = \sum_{k=1}^{n} \omega_k \frac{\partial^2 V_k}{\partial \Delta r \partial \Delta S}; \\ &\frac{\partial^2 V}{\partial \Delta q \partial \Delta S} = \sum_{k=1}^{n} \omega_k \frac{\partial^2 V_k}{\partial \Delta q \partial \Delta S}. \end{aligned} \tag{4}$$

From Eq. (4), by choosing a suitable w_k, we can make

$$\frac{\partial V}{\partial \Delta q} = 0, \; \frac{\partial V}{\partial \Delta r} = 0, \; \frac{\partial V}{\partial \Delta S} = 0. \tag{5}$$

In addition, it is also possible to make

$$\frac{\partial^2}{\partial \Delta q^2} = 0, \frac{\partial^2}{\partial \Delta r^2} = 0, \frac{\partial^2}{\partial \Delta S^2} = 0, \frac{\partial^2 V}{\partial \Delta q \partial \Delta r} = 0, \frac{\partial^2 V}{\partial \Delta r \partial \Delta S} = 0, \frac{\partial^2 V}{\partial \Delta q \partial \Delta S} = 0. \tag{6}$$

When Eq. (5) holds, we have immunised V against small movements in the mortality curve, the interest rates curve and other financial risk factors simultaneously. In this case, V is similar to a zero-duration and zero-convexity portfolios or a delta-gamma-neutral portfolio. When both Eqs. (5) and (6) hold, we have immunised V against larger movements in the mortality curve, the interest rates curve and other financial risk factors simultaneously. In this case, V is similar to zero-duration and zero-convexity portfolios or delta-gamma-neutral portfolio.

In actual application, it is not required that all of the partial derivatives in Eqs. (5) and (6) be equal to zero. If a specific partial derivative is zero or close to zero for all products and assets in V, then we do not need to consider it when choosing the w_k's. For example, in our numerical examples, $\partial^2/\partial \Delta q^2$, $\partial^2/\partial \Delta S^2$, $\partial^2 V/\partial \Delta q \partial \Delta r$, $\partial^2 V/\partial \Delta r \partial \Delta S$ and $\partial^2 V/\partial \Delta q \partial \Delta S$ are not considered. In addition, if V contains a large number of insurance products and invested assets, then Eqs. (5) and (6) are underdetermined linear equations. This implies that the number of solutions of the w_k's is infinite. Therefore, it is reasonable to arrive at a unique solution by fixing a percentage of "key" hedging products or assets. In our numerical examples, a "key" hedging product is a whole life product. Supposing that there are two hedging strategies that satisfy Eqs. (5) and (6), then any convex combination of the solutions of these two hedging strategies also satisfies Eqs. (5) and (6). That is, to include both a term life and whole life in the hedging programme, we just need to obtain a convex combination of the solutions for "Hedging Strategy 1" and "Hedging Strategy 2".

The various partial derivatives in Eqs. (5) and (6) are usually very complex or even have no closed-form formulas, so we use the finite difference method to compute these partial derivatives. The most challenging step in applying the finite difference method is to choose suitable difference sizes in the function parameters. In our approach, we can simply use the most plausible magnitude (Δq, Δr and ΔS) computed by the first principal components mentioned above.

It is important to note that the proposed approach in this paper does not rely on specific assumptions regarding the mortality models or interest rate models. As long as the scenarios generated by the adopted models are highly correlated (this is a basic property for any sensible mortality or interest rate model), the proposed approach should be effective based on the results of the principal components analysis. Thus, compared with other asset-liability management models, our proposed approach is more flexible and much easier to apply. Thus, it can be a more effective tool in calculating the product mix or asset allocation strategies according to the needs of the insurance companies in actual practice.

Numerical examples

To demonstrate our proposed hedging strategy, in this section we construct a numerical analysis for insurance companies. Assume that an insurance company sells three different types of products: life insurance, annuities and reverse mortgages. The hedging strategy depends on the policy condition, such as the issuance age, gender, coverage period, payment method and so on. The whole-life annuity is issued for a 60-year-old man. The life insurers pay US$10,000 at the end of each year if the annuitant lives. The whole-life annuity has no deferred period and the premiums are collected in a single premium. The life insurer has sold 1,000 whole-life annuity contracts and wishes to hedge its longevity and interest rate risk by using different insurance products (term life insurance, whole life insurance and reverse mortgages) and long-term bonds (10-year coupon bonds and 30-year coupon bonds) to hedge the mortality and interest rate risks. The term-life insurance is issued for a 50-year-old man with a 20-year insurance period. The whole-life insurance is also issued for a 50-year-old man. For both life insurance products, the payout benefit is US$1,000,000. We also assume that the life insurer uses the premiums received to buy two types of long-term bonds. The 10-year coupon bond is issued at a 3 per cent coupon rate with a face value of US$1,000,000. The 30-year coupon bond is issued at a 5 per cent coupon rate with a face value of US$1,000,000. The reverse mortgage is provided to a 70-year-old man who has home equity valued at US$1,000,000.

As discussed in the section "Reverse mortgage", there are four variations of reverse mortgages. In the numerical examples of this paper, we assume that the insurer provides a reverse mortgage of the first types. That is, the insurer (lender) makes the payment to the insured (borrower) in the form of a lump sum payment and not as a fixed annuity. At the end of the contract, the insurer will receive the market value of the house when the insured dies or the contract is terminated. Thus, the insurer bears longevity risk because the insurer will get back the lower part of: (a) the initial value plus accrued interest, (b) the value of the house; if the house price is higher than the initial value plus accrued interest, the house owner's estate will get the residual. On the other hand, the increase in the value of the house price will also increase the value of the loan. Under this setting, a reverse mortgage is more like an asset to the insurer. Based on the house price data for the United States, the house price inflation will be high enough to overcome the potential longevity risk presented in the U.S. mortality data, especially due to the reverse mortgage we considered having a low loan-to-value ratio (=65 per cent). In addition, the second type of reverse mortgage can be constructed from the first type by converting the initial loan amount into a fixed annuity. Therefore, the second type of reverse mortgage can be regarded as a combination of the first type of reverse mortgage and a fixed annuity. This implies that our approach can also apply to the second type of reverse mortgage. The basic assumptions are summarised in Table 2.

The total value of the hedged product portfolio (V) depends on mortality rates (q), market interest rates (r) and/or the market house price index (S). We first simulated 1,000 scenarios for the mortality rate shock, interest rate shock and house price shock

Table 2 Basic assumptions for the numerical analysis

Product or hedging asset	*Age/gender*	*Coverage*	*Sum insured*	*Coupon rate*	*Maturity*	*Face value*
Whole-life annuity	60	Whole life	10,000			
Term-life insurance	50	20 years	1,000,000	—	—	—
Whole-life insurance	50	Whole life	1,000,000	—	—	—
Bond 1	—	—	—	3%	10 years	1,000,000
Bond 2	—	—	—	5%	30 years	1,000,000
Reverse mortgages	70	Whole life	1,000,000	—	—	—

using the CBD, CIR and lognormal models. It is important to recognise the age effect under the CBD model. We have three mortality rate shock vectors associated with age x=50, 60 and 70. To apply the PCA technique mentioned previously, we concatenate these three vectors to form a long mortality rate shock vector. Based on these scenarios, we can compute the most plausible magnitude (Δq, Δr) and the most plausible shape ($\mathbf{q}_s$, $\mathbf{r}_s$) via the first principal component of the sample covariance matrix of the simulated interest rate and mortality curve scenarios.

In addition, we compare the results of the surplus distribution for five different hedging strategies as follows:

	Product portfolio
Hedging Strategy 1	Hedged annuity by term-life and long-term bonds
Hedging Strategy 2	Hedged annuity by whole-life and long-term bonds
Hedging Strategy 3	Hedged annuity by reverse mortgage and long-term bonds
Hedging Strategy 4	Hedged annuity by reverse mortgage and long-term bonds with the house price being hedged by put option
Hedging Strategy 5	Hedged annuity by reverse mortgage and long-term bonds with the house price being perfectly hedged

In our numerical examples, we consider different parameter settings in the CIR model as shown in Table 1. The parameter values of the CIR model are: a=0.15, b=0.05, σ=0.06 and r_0=0.01. The partial derivatives required to compute the holding amounts for each of the products/investments are reported in Table 3 and the original portfolio mix without hedging is shown in Table 4. In addition, we conduct the analyses by not/partially/fully hedging the house price risk in reverse mortgages. In Hedging Strategy 3, the house price risk in reverse mortgage is not hedged. Hedging Strategy 4 is a partially hedged strategy that includes a put option to mitigate the house price risk. The put option is an at-the-money option where maturity equals one year. The sensitivities to house price $\Delta V/\Delta H$ of reverse mortgage and put option are 0.76 and −0.37, respectively. In Hedging Strategy 5, the house price risk in reverse mortgage is fully hedged.

The results of the portfolio mix for different hedging strategies are summarised in Tables 5–8. The results show that the proposed generalised immunisation approach can serve as an effective vehicle to calculate the required portfolio mix to control the

Table 3 Partial derivatives computed by finite difference method

	Annuity	*Term life*	*Whole life*	*Bond 1*	*Bond 2*	*Reverse mortgages*
V	−21.8	−15.1	−33.4	98.8	120.2	75.8
$\Delta V/\Delta r$	38	25.5	66.4	−163.9	−218	−0.1
$\Delta V/\Delta q$	−0.6	22.1	54.6	0	0	19.7
$\Delta^2 V/\Delta r^2$	−76.1	−47.1	−138.3	285.3	431.7	−1.3

Table 4 Portfolio mix without any Hedging Strategy ($\times 10^4$)

	Annuity	*Cash*
Unit price	−21.8	
Holding amount	1,000	
Total value of each product/investment	−21,800	21,800

Table 5 Portfolio mix for Hedging Strategy 1 ($\times 10^4$)

	Annuity	*Term life*	*Bond 1*	*Bond 2*	*Cash*
Unit price	−21.8	−15.1	98.8	120.2	
Holding amount	1,000	26.1	25.1	162.5	
Total value of each product/investment	−21,800	−394.1	2,480	19,532.5	181.7

Table 6 Portfolio mix for Hedging Strategy 2 ($\times 10^4$)

	Annuity	*Whole life*	*Bond 1*	*Bond 2*	*Cash*
Unit price	−21.8	−33.4	98.8	120.2	
Holding amount	1,000	10.6	21	165.8	
Total value of each product/investment	−21,800	−354	2,075	19,929.2	150

Table 7 Portfolio mix for Hedging Strategy 3 ($\times 10^4$)

	Annuity	*Bond 1*	*Bond 2*	*Reverse mortgages*	*Cash*
Unit price	−21.8	98.8	120	75.8	
Holding amount	1,000	20	163	30	
Total value of each product/investment	−21,800	1,976	19,560	2,274	−2,010

Table 8 Portfolio mix for Hedging Strategy 4 ($\times 10^4$)

	Annuity	*Bond 1*	*Bond 2*	*Reverse mortgages*	*Put option*	*Cash*
Unit price	−21.8	98.8	120	75.8	0.9	
Holding amount	1,000	19.8	163.7	29.5	61	
Total value of each product/investment	−21,800	1,958	19,649	2,236	55	−2,098

aggregate risk for life insurance companies. The results show that, to hedge the same amount of longevity risk, the required amounts of the term-life products are greater than the required amounts of the whole-life products. We can find that the insurer needs 26.1 units of term life to hedge the annuity in Table 5. However, to hedge the same amount of the annuity, the insurer only needs 10.6 units of term life in Table 6. This implies that the whole-life products do have a better hedging effect on the longevity risk than the term-life products.

The detailed surplus distributions for different hedging strategies are summarised in Figures 3–8. We find that the standard deviation of the product portfolio (V) without any hedging strategy is 6,774,633 (Figure 3). The standard deviation of the product

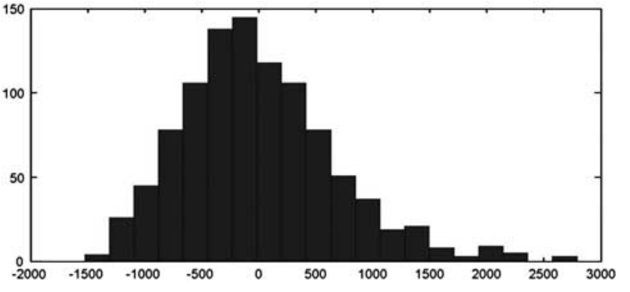

Figure 3. Surplus distribution without any Hedging Strategy ($\times 10^4$).

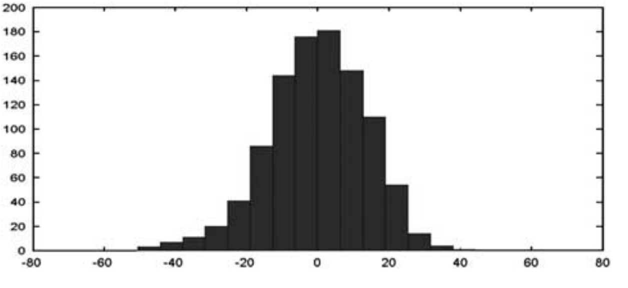

Figure 4. Surplus distribution of Hedging Strategy 1 ($\times 10^4$).

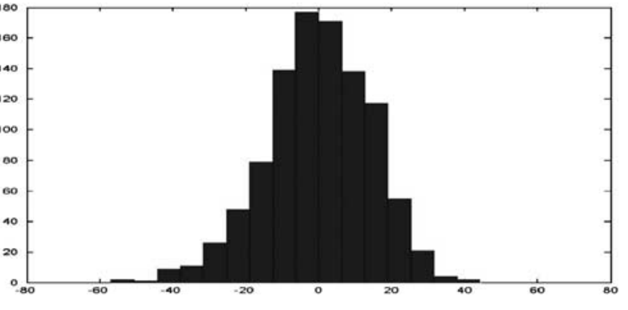

Figure 5. Surplus distribution of Hedging Strategy 2 ($\times 10^4$).

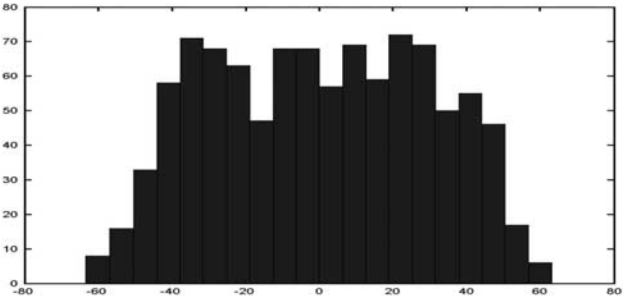

Figure 6. Surplus distribution of Hedging Strategy 3 ($\times 10^4$).
Note: House price risk in reverse mortgage is not hedged.

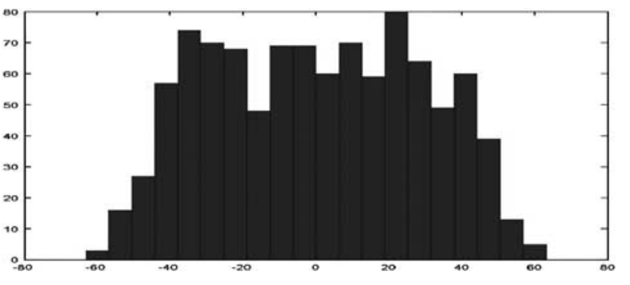

Figure 7. Surplus distribution of Hedging Strategy 4 ($\times 10^4$).
Note: House price risk in reverse mortgage is partially hedged with a put option.

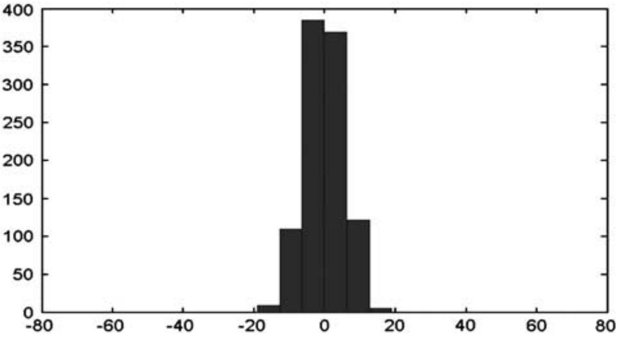

Figure 8. Surplus distribution of Hedging Strategy 5 ($\times 10^4$).
Note: House price risk in reverse mortgage is fully hedged.

portfolio (V) for Hedging Strategy 1 is 137,463 (Figure 4). The standard deviation of the product portfolio (V) for Hedging Strategy 2 is 144,655 (Figure 5). The standard deviation of the product portfolio (V) for Hedging Strategy 3 is 298,076 (Figure 6). The standard deviation of the product portfolio (V) for Hedging Strategy 4 is

286,926 (Figure 7). The standard deviation of the product portfolio (V) for Hedging Strategy 5 is 52,786 (Figure 8). From Figures 4 and 5, we can find that there is not much difference between the results for Hedging Strategy 1 and those for Hedging Strategy 2. This implies that the hedging strategy using whole-life products did not exert a more significant risk-reduction effect than did the strategy using term-life products.

Figures 6, 7 and 8 depict the results with Hedging Strategies 3, 4 and 5, respectively. From Figure 8 we find that the hedging strategy using a reverse mortgage exerts a better risk-reduction effect than the other two strategies when the house price risk is perfectly hedged.[56] However, when the house price risk is considered and is unhedged, the total hedging effect is reduced (Figure 6). It is mainly because the house price dynamic would introduce the potential return and risk into the portfolio. In addition, when the house price risk is partially hedged by a short-term put option on the house price, the risk can be slightly reduced in terms of standard deviation. We find that the left tail of the surplus distribution in Hedging Strategy 4 is thinner than that of Hedging Strategy 3 and the right tail is thicker (Figure 7). However, when the house price risk is perfectly hedged, the hedging effect is much better than for the other hedging strategies (Figure 8). These results demonstrate that adding the reverse mortgage to the product portfolio effectively reduces the total risk associated with the surplus because the increase in the value of the house price can help to offset the longevity risk of the annuity. Thus, the numerical results further demonstrate that adding a reverse mortgage to the product portfolio (Hedging Strategy 5) creates a much better hedging effect and effectively reduces the total risk of the surplus in both cases.

To sum up, the numerical results suggest that the proposed generalised immunisation approach can effectively calculate the required product mix portfolio to eliminate most of the interest rate and mortality risks simultaneously. We also show that a reverse mortgage can serve as an effective hedging vehicle for hedging longevity risk. It is important to note that the insurers need to separate the aggregate longevity risk from the idiosyncratic longevity risk when considering a hedging strategy. As the numbers of policy-holders increase, the idiosyncratic longevity risk should be effectively diversified. The hedging effectiveness will also improve as the number of policy-holders increases.

Conclusions

To hedge the interest rate risk in the insurer's liability, asset-liability managers commonly adopt a classical immunisation strategy. A similar hedging approach for

[56] A perfectly hedged house price risk can be achieved by using a delta hedging strategy. This idea is similar to the hedging concept in the Black–Scholes model for stock options. We also assume that there exists a security whose return is the same as the return on the house price, that the short selling of this security is permitted, there are no transaction costs and taxes, and that the security is perfectly divisible. However, in the real world, only REIT-type securities are available for hedging, and their returns may be highly correlated with the return on the house price, but not identical to the return on the house price. Therefore, the net hedging effect lies in between the effects of no hedging and perfect hedging.

longevity risk, which involves matching the mortality duration of life insurance and annuities, has been proposed by Wang *et al.*[57] The launch of new innovative longevity-linked financial products, such as reverse mortgages, increases the complexity and challenges faced by the insurers as they seek to implement their asset-liability management strategy. With the house price dynamics and a large final payment received at the end of the policy year, a reverse mortgage provides a different liability duration pattern from the annuity. In this paper, we propose a generalised immunisation approach to obtain an optimal product portfolio so that life insurance companies can hedge longevity and financial risks.

A key contribution of this paper is that it uses the PCA technique to determine the most plausible shocks brought about by interest rates and mortality rates. We consider this approach to be an important extension of the non-stochastic risk methodologies that are based on durations and convexities. The proposed approach in this paper does not rely on specific assumptions with regard to mortality models or interest rate models. As long as the generated scenarios from the adopted models are highly correlated, the proposed approach should be effective based on the results of the principal components analysis. Thus, it can be a more effective tool used in calculating the product mix or asset allocation strategies according to the needs of the insurance companies in actual practice.

From our numerical examples, we demonstrate that our proposed approach can effectively reduce the longevity risk and interest rate risk existing in a product portfolio simultaneously. By using stochastic mortality and interest rate models and the Monte Carlo simulation approach, we show that the proposed generalised immunisation approach can serve as an effective vehicle in controlling the aggregate risk of life insurance companies. The numerical results further demonstrate that adding reverse mortgages to the product portfolio creates a better hedging effect and effectively reduces the total risk associated with the surplus of the life insurers.

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