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ORIGINAL RESEARCH

Foreign exchange option pricing in the currency cycle with jump risks

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Abstract This paper examines regime switching behavior and the nature of jumps in foreign exchange rates, as well as their implications in currency option pricing. Considering the characteristics of long swing as well as the short term jumps in exchange rates, we adopt the regime-switching model with jump risks to capture the movement of exchange rates in the developed and emerging countries. Our results show that 'high-variance' and 'low-variance' describes most of our sample currencies' trajectories. The regime-switching model with jump risks is proven to capture better exchange rate changes than the regime-switching model (RSM) and the Black–Scholes model (BSM). In addition, our results show that the currency option pricing model when considering regimes of high-variance or low-variance states as well as the jump nature of exchange rates, is better than the traditional BSM and RSM.

Keywords Exchange rate · Currency option · Regime-switching · Jump risks

JEL Classification C58 · F31 · G13

1 Introduction

The foreign exchange market is dramatically affected by monetary policy and at the most basic level, the most important driver of money supply growth is the business cycle. Since

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money supply is closely related to currency values (the more there is of a currency, the less its value will be), forex trends also display cyclical behaviors. For example, when a nation is going through the boom phase of the cycle, international capital will flow there in search of better returns on investment, through channels like foreign direct investment, or international loans. These will create inflows of capital, and cause the nation's currency to appreciate. Conversely, when a nation is going through the bust phase of the cycle, international capital will shut down and forex flows will dry up, causing the currency to depreciate. As with Newton's First Law, these developments will keep going on until they are checked by market developments or turned around by government action.

Past research has shown that the cyclical behavior of exchange rates can be well captured by the regime switching models. Froot and Obstfeld (1991), Engel and Hakkio (1996), Dahlquist and Gray (2000) have elicited that the changes of regimes have linkage with the underlying currency policy alternation, such as a switch from a free float regime to a target zone, target bands, or an exchange rate peg, and vice versa. More recent papers, such as Ichiue and Koyama (2011), confirm the regime switching model can also explain the most popular theme in the currency markets during the past decade, carry-trade. This refers to a strategy where investors borrow low-yielding currencies and lend high-yielding ones, which is mostly executed in times of global financial and exchange rate stability and pulled back during liquidity shortages. Ichiue and Koyama (2011) indicate that in a low exchange rate volatility regime, low-interest-rate currencies tend to depreciate, because lower volatility results in lower margins and this enables speculators to take more carry-trade positions. By contrast, in a high exchange rate volatility regime, such as the recent global financial meltdown, many investors turn away from commonly practiced carry-trade strategies and seek a "safe-haven" in these uncertain times, causing low-interest-rate currencies to appreciate rapidly.

Over the past two decades, some financial or catastrophic events, such as the devaluation of a currency, an announcement of default on sovereign debt obligations, earthquakes, hurricanes, or terrorism and political unrest, have triggered an immediate and startling adverse chain reaction on currency values among countries within a region and in some cases across regions. For instance, the floatation of the Thai baht on Jul. 2, 1997 triggered financial turmoil across East Asia, with the currencies of Indonesia, Korea, Malaysia, and the Philippines depreciating by about 75 %. Similarly, the financial crisis of 2007 affected many countries in terms of falls in equity prices, spikes in the cost of borrowing, scarcity in the availability of international capital, and decline in the value of their currencies and in output. Recently, the severe earthquake that occurred in Japan on Mar. 11, 2011 and the European sovereign debt crisis, led to large capital flows among countries, further inducing exchange rate fluctuations.

Cyclical movements in foreign exchange rates have been proven to be well captured by the regime switching model (Engel and Hamilton 1990; Engel 1994). However, events such as: the currency crises that occurred in Mexico in 1994, Thailand in 1997, Brazil in 1999, and Turkey in 2001, which were related to the abandonment of an exchange rate peg and subsequent devaluation; Russian's defaults on its domestic bond debt in 1998; the subprime mortgage crisis in 2007 and the European sovereign debt crisis in late 2009, which were associated with the suspension of convertibility and change in political nature; the terrorist attacks on New York on Sep. 11, 2001 as well as the huge aforementioned earthquake in Japan, might induce collapse in exchange rate, leaving economic fundamentals being unable to forecast foreign exchange rate movements. To summarize, as Mussa (1979), Frenkel (1981) and Flood and Hodrick (1986) have argued, jumps in exchange rates generated by discontinuities in the arrival of "news" or by changes in monetary policies directed at affecting the external value of a currency, should be considered as one of the predominant causes of exchange rate movements.

In this study, we choose our research target as follows: the dollar exchange rate changes of three currencies from the developed world, the euro (EUR), British pound (GBP) and the Japanese yen (JPY) as well as those of three emerging countries, the Brazilian real (BRL), the Indonesian Rupiah (IDR) and the Mexican Peso (MXN). Table 1 gives summary statistics of the daily logarithmic returns for the six currencies from 1999 to 2010. Here we use an identifying assumption to distinguish crisis episodes from tranquil periods based on a threshold approach, such as that found in Eichengreen et al. (1995, 1996), Lowell et al. (1998), Favero and Giavazzi (2002), and Bae et al. (2003), whereby basically we assume that all movements of less than a certain size (2 or 3 %) are noise. From Panel D of Table 1, it is noticeable that shocks occurred more frequently in 1999, 2001–2002 and 2008 in BRL. As is well known, in 1999, Brazil devalued the real and eventually adopted a floating exchange rate system, causing the real to depreciate by over 70 %.[1] From 2001 to 2002, it suffered from an energy crisis and a series of external shocks, such as the economic slowdown in the United States, which was worsened by the dotcom bubble bursting as well as the terrorist attacks in New York on Sep. 11, 2001, and Argentina's mounting difficulties in obtaining external financing. Growing fears over the type of economic policy the next administration might pursue were reflected in the fall in the prices of financial assets, which, in turn, brought a large depreciation in the exchange rate. In 2008, the subprime mortgage crisis also resulted in a large devaluation of the real, and thus affected the value of currency options.

Traditionally, the development of valuations for European-style currency options can be found in Garman and Kohlhagen (1983), Grabbe (1983), and Biger and Hull (1983). However, recently there has been a growing interest in the use of regime-switching models for option valuation, solely for the purpose of better understanding the impact of structural changes in economic conditions on this valuation. The history of the regime-switching models can be traced back to the early works of Quandt (1958) and Goldfeld and Quandt (1973), where a class of regime-switching models was applied to model nonlinear economic data. The idea of regime-switching was also designed to capture changes in the underlying economic mechanism that generated the data and examples of this approach include Hamilton (1989) and Gray (1996), Bekaert and Hodrick (1993) and Durland and McCurdy (1994). In the area of option valuation, the regime-switching model fills the gap of models between the Black–Scholes model (BSM) (Black and Scholes 1973), which in the regime-switching framework can be viewed as a special case of a single volatility regime. For example, Bollen (1998) presents a lattice-based algorithm that permits American options to be priced under the regime-switching model (RSM). In his paper, the BSM is shown to generate significant pricing errors when a regime-switching process governs the underlying asset returns. In addition, option values under the RSM are validated to capture an implied volatility smile commonly found in empirical studies. In the subsequent research, Costabile et al. (2013) present a binomial approach for pricing contingent claims when the parameters governing the underlying asset process follow a regime-switching model, while Lin et al. (2013) propose a regime-switching model with jump risk for pricing the European call option of the stock index. In their papers, numerical

[1] The Brazilian currency crisis occurred in 1999. On Jan. 13, 1999, the Brazilian Central Bank devalued the real by 8 % and on Jan. 15, 1999 the Cardoso government announced that the real would no longer be pegged to the US dollar. Immediately, it lost more than 30 % of its value, and the subsequent devaluation resulted in a further loss of 45 % of the original value.

Table 1 The summary statistics of daily logarithmic returns in exchange rates from 1999 to 2010

Period Observation	Year Num.	1999 260	2000 260	2001 261	2002 261	2003 261	2004 262	2005 260	2006 260	2007 261	2008 262	2009 261	2010 261	All 3,130
Panel A : EUR	Mean	0.0006	0.0003	0.0002	−0.0006	−0.0007	−0.0003	0.0005	−0.0004	−0.0004	0.0002	−0.0001	0.0003	−4.3E−05
	SD	0.0056	0.0073	0.0069	0.0055	0.0059	0.0064	0.0055	0.0048	0.0037	0.0088	0.0079	0.0070	0.0064
	Skewness	−0.8236	−0.6399	−0.1179	0.0309	0.2215	0.1634	−0.1104	−0.5207	0.2655	0.0067	−0.6882	−0.2012	−0.2252
	Kurtosis	4.8036	4.7146	3.2202	4.6164	2.7982	3.3357	2.9103	4.2668	3.2344	6.5544	7.0449	2.9149	5.7224
	Change over 2 %	2	3	1	2	0	0	0	0	0	12	4	1	25
	Change over 3 %	0	1	0	0	0	0	0	0	0	2	1	0	4
Panel B: GBP	Mean	0.0001	0.0003	0.0001	−0.0004	−0.0004	−0.0003	0.0004	−0.0005	−6E−05	0.0012	−0.0004	0.0001	1.9E−05
	SD	0.0041	0.0053	0.0048	0.0042	0.0047	0.0062	0.0053	0.0051	0.0042	0.0091	0.0095	0.0062	0.0060
	Skewness	−0.4195	−0.5362	−0.0778	−0.0055	0.2052	0.1872	0.0563	−0.3895	0.5897	0.0090	0.0226	−0.0601	0.0353
	Kurtosis	3.7719	4.5670	3.1925	5.4252	3.2295	3.1093	3.0414	3.3509	4.3011	6.7273	5.6555	3.4334	7.6313
	Change over 2 %	0	1	0	0	0	1	0	1	0	11	10	1	25
	Change over 3 %	0	0	0	0	0	0	0	0	0	3	3	0	6
Panel C: JPY	Mean	−0.0004	0.0004	0.0005	−0.0004	−0.0004	−0.0002	0.0005	0.0000	−0.0002	−0.0008	0.0001	−0.0005	−0.0001
	SD	0.0077	0.0062	0.0060	0.0063	0.0053	0.0060	0.0057	0.0052	0.0056	0.0095	0.0085	0.0068	0.0067
	Skewness	−0.0362	−0.4697	−0.1905	−0.5460	−0.0391	0.1257	−0.8443	−0.2040	−0.6322	−0.8450	−0.3865	0.4885	−0.4200
	Kurtosis	3.9893	5.0859	3.2521	5.3806	3.5492	3.9343	5.9013	3.7161	4.9490	6.3343	5.6596	5.6689	6.2354
	Change over 2 %	8	3	0	4	0	1	2	0	1	9	5	4	37
	Change over 3 %	0	0	0	0	0	0	0	0	0	3	1	1	5
Panel D: BRL	Mean	0.0015	0.0003	0.0006	0.0016	−0.0008	−0.0003	−0.0005	−0.0003	−0.0007	0.0010	−0.0011	−0.0002	0.0001
	SD	0.0177	0.0046	0.0108	0.0176	0.0090	0.0060	0.0089	0.0083	0.0083	0.0172	0.0101	0.0075	0.0114
	Skewness	0.9593	0.0867	−0.4319	−0.8065	0.1626	0.8594	0.7177	1.3764	1.1695	0.4384	0.1417	0.1025	0.4371
	Kurtosis	16.6807	3.7998	3.9613	14.3753	5.3023	5.5910	4.0323	10.9587	11.5094	8.5273	3.4022	5.5055	20.0586
	Change over 2 %	24	0	19	36	12	4	5	9	5	38	18	7	177
	Change over 3 %	14	0	4	18	2	0	2	4	2	22	1	1	70
Panel E: IDR	Mean	−0.0005	0.0012	0.0003	−0.0006	−0.0002	0.0004	0.0002	−0.0003	0.0002	0.0006	−0.0006	−0.0002	4E−05
	SD	0.0179	0.0104	0.0132	0.0068	0.0042	0.0049	0.0053	0.0053	0.0041	0.0085	0.0057	0.0029	0.0085

Table 1 continued

Period Observation	Year Num.	1999 260	2000 260	2001 261	2002 261	2003 261	2004 262	2005 260	2006 260	2007 261	2008 262	2009 261	2010 261	All 3,130
	Skewness	0.0019	0.0531	−0.9205	1.1481	0.6725	0.2580	0.2115	1.2274	0.5570	0.5347	0.1191	0.0130	−0.1342
	Kurtosis	6.1576	4.6905	11.5141	8.9828	8.1808	8.4593	16.0128	13.0607	9.7539	51.5682	9.8762	6.2595	21.1885
	Change over 2 %	47	19	26	4	1	1	2	1	1	7	2	0	111
	Change over 3 %	28	4	11	2	0	0	2	1	0	3	1	0	52
Panel F: MXN	Mean	−0.0002	0.0000	−0.0002	0.0005	0.0003	−3.1E−05	−0.0002	6.8E−05	3.1E−05	0.0009	−0.0002	−0.0002	7.0E−05
	SD	0.0064	0.0051	0.0052	0.0050	0.0063	0.0043	0.0040	0.0048	0.0034	0.0114	0.0091	0.0065	0.0063
	Skewness	0.4178	−0.3372	−0.1077	0.6727	0.5669	0.7396	0.3206	0.4437	0.2065	1.1839	−0.1073	0.0217	0.7464
	Kurtosis	21.1821	7.4476	4.1040	5.5445	4.1108	5.7950	3.3009	3.8443	3.1519	15.1159	5.5748	8.5754	19.1693
	Change over 2 %	4	2	1	2	1	1	0	0	0	19	7	4	41
	Change over 3 %	2	0	0	0	0	0	0	0	0	12	3	1	18

analysis shows that the proposed algorithm is efficient and computes accurate values in comparison to the existing pricing models.

In this paper, we apply a regime-switching model with jump risks (RSMJ) to capture cyclical movements as well as large abrupt changes in exchange rates, and subsequently validate the valuing of currency options. Our results show that 'high-variance' and 'low-variance' regimes describe most of our sample currencies' trajectories. Moreover, the RSMJ is proven to be better than either the BSM or the RSM at capturing the movement of exchange rates. In addition, our results show that the currency option pricing model when considering regimes of high-variance or low-variance states as well as the jump nature in exchange rates is superior to these two earlier models.

The remainder of the paper is structured as follows. The following section outlines the economic framework of the RSM as well as the RSMJ, focusing on the economic causes and effects of the latter. Section 3 describes the data and develops the RSMJ for exchange rates as well as discussing the estimation and testing issues. The RSM is also discussed for comparison purposes and the impact factors of jumps in currencies are summarized in this section as well. Section 4 compares the currency option values generated from the BSM, the RSM and RSMJ with the market values. Section 5 contains the conclusion.

2 The economic model

2.1 Regime-switching model

Since 1989, when Hamilton (1989) adopted the RSM to describe business cycles in the US, there has been a surge of empirical research employing this approach as well as substantial extension of the model. That is, because the RSMs can match the propensity of financial markets to change often their behavior abruptly and the phenomenon that the new behavior of financial variables commonly persists for several periods after such a change, they are an important class of financial time series models. A key feature of an RSM is that the model parameters are functions of a hidden Markov chain, whose states represent hidden states of an economy or different stages of business cycles. Engel and Hamilton (1990) and Engel (1994) investigated quarterly changes in exchange rates and found the RSMs to be a good approximation to the underlying processes. Other studies that have employed an RSM in exchange rate analysis include: Kirikos (2000), Caporale and Spagnolo (2004), Bergman and Hansson (2005) and Ismail and Isa (2007). For example, Bergman and Hansson (2005) have suggested that the real exchange rate between the major currencies in the Post-Bretton Woods period can be described by a stationary, two state Markov switching AR(1) model. Ismail and Isa (2007) employed the RSM to capture regime shifts behavior in Malaysia ringgit exchange rates against four other currencies, namely: the British pound sterling, the Australian dollar, the Singapore dollar and the Japanese yen, from 1990 to 2005. They concluded that the RSM is found successfully to capture the timing of regime shifts in the four series.

The basic idea of an RSM is that it assigns probabilities to the occurrence of different regimes, which have to be inferred from the data. The nonlinearity feature of the financial time series that can be in two or more regimes has motivated the used of RSMs. Suppose there are two states, S_t is defined as the exchange rate of one US dollar being converted into other currencies, here the focal ones being the: EUR, GBP, JPY, BRL, MXN, and the IDR. The log return of exchange rate, R_t, denotes the difference of the natural logarithm of the general price levels ($R_t = \ln(S_t) - \ln(S_{t-1})$), and the RSM can be expressed as follows:

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z & \text{if } q_t = 1 \\ \mu_2 + \sigma_2 Z & \text{if } q_t = 2 \end{cases}, \quad P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}, \tag{1}$$

where q_t is the state at time t, μ_{q_t} is the mean under the state q_t, $\{q_t = 1 \text{ or } 2\}$; σ_{q_t} is the volatility under the state q_t; Z represents a standard normal distribution; P is the transition matrix, $p_{q_{t-1}q_t}(\Delta t)$ denotes the probability from state q_{t-1} to q_t at the time segment Δt, $\{q_t = 1 \text{ or } 2 \text{ and } q_{t-1} = 1 \text{ or } 2\}$. This model assumes that state 1 represents currency depreciation, and state 2 appreciation. When in the depreciation state, the mean value is μ_1, and the volatility is σ_1. On the other hand, in state 2, the appreciation state, the mean value is μ_2, and the volatility is σ_2. The transition probability of appreciation-depreciation cycles can be defined by P. According to Engel and Hamilton (1990), the model could describe a variety of processes depending on the values taken by the six parameters: μ_1, μ_2, σ_1, σ_2, p_{11} and p_{22}. Most importantly from their perspective, is the ability of this model to capture so-called long swings in the exchange rate, which would exhibit opposite signs on μ_1 and μ_2 and large values of p_{11} and p_{22}. Supposing that the exchange rate is in state 1 and that μ_1 is positive, under the long swings hypothesis it is expected to remain in this state for $1/(1 - p_{11})$ periods and increase by μ_1 in each period. Once the state switches to state 2, the exchange rate is expected to remain there for $1/(1 - p_{22})$ periods and to fall by μ_2 on average in each period. Clearly this process has parallels with the desires of chartists to identify long-lived periods of currency appreciation or depreciation.

2.2 Regime-switching model with jump risks

Overall, in fact, the literature on regime-switching modeling of exchange rates has produced models that fit satisfactorily and forecast well in sample, but fail to beat simple random walk models or linear specifications in out of sample forecasting (Kaminsky 1993; Marsh 2000). Since past research has shown that many macroeconomic fundamentals contain valuable information for forecasting future spot exchange rates and that exchange rate dynamics display nonlinearities, Kaminsky (1993) incorporated monetary announcements to help predict the future path of the exchange rate. Apart from macroeconomic fundamentals, as pointed out above, events such as a currency crisis, related to the abandonment of an exchange rate peg and devaluation; the suspension of convertibility and changes in the political situation; terrorist attacks as well as catastrophes, might also induce a collapse in the exchange rate. In order to capture the temporary shock in exchange rate movements, we include jumps in the RSMs.

The economic literature dealing with jump processes in exchange rates and their pricing implications has been growing ever since the seminal work of Merton (1976). Jorion (1988) and Bates (1996a) were among the first to assert that the outliers in exchange rate series can be accounted for by a jump diffusion process. Many studies have since documented the statistical significance of jumps in exchange rates (Bates 1996b; Jiang 1998; Doffou and Hilliard 2001; Andersen et al. 2001; Chaboud et al. 2008; Barndorff-Nielsen and Shephard 2006; Neely 2011). Bates (1996a, b), Jiang (1998), and Doffou and Hilliard (2001) found that jumps are important components of the currency exchange rate dynamics. Chaboud et al. (2008), Barndorff-Nielsen and Shephard (2006) and Neely (2011), on the other hand, indicated that the jumps in exchange rates come from monetary policy announcements. Lin et al. (2013) address a regime-switching model with jump risk can successfully capture the cycle and jump features for the stock index, and find the empirical validity in option pricing and hedging.

Suppose a shock such as a policy change or a financial crisis occurs, causing a large depreciation or appreciation of exchange rate, the RSMJ can be shown as follows:

$$R_t = \begin{cases} \mu_1 + \sigma_1 Z + \sum_{n=1}^{Nt} Z_n & \text{if } q_t = 1 \\ \mu_2 + \sigma_2 Z + \sum_{n=1}^{Nt} Z_n & \text{if } q_t = 2 \end{cases}, \quad P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}, \tag{2}$$

where N_t represents the news arrival during every time segment, including good news and bad news, which would cause large deprecation or appreciation of the exchange rate. We assume the news arrival follows a Poisson distribution with the expected number of occurrences in this interval being λ, and Z_n the level of depreciation or appreciation upon news arrival, which follows a normal distribution, with mean μ_y and variance σ_y.

3 Estimation and test

In this section, we first carry out data analysis for the six sample exchange rates, and then perform parameter estimation with both the RSM and the RSMJ. The estimated results and testing are presented in this section as well. Finally, we summarize the impact factors of jumps in currencies.

3.1 Data

As described above, this study is conducted using the dollar exchange rates of three countries from the developed world (EUR, GBP and JPY) as well as those of three emerging countries (BRL, IDR and MXN). The data are daily rates, covering the period from January 1, 1999 to December 31, 2010 and were obtained from Datastream. The descriptive statistics of the daily logarithmic differences for the six sample exchange rates are presented in Table 1. From the table, we observe that mean returns are, on average positive (currency depreciation), for GBP, BRL, IDR and MXN and negative (currency appreciation) for EUR and JPY. The average standard deviation analysis shows that except for BRL and IDR, with large mean volatility, equal to 0.0114 and 0.0085, respectively, the other currencies have a value of about 0.0065. It is noteworthy that generally for all countries, the mean returns are positive (currency depreciation, with the exception being JPY, with currency appreciation) and the standard deviations are large in 2008, attributable to the global financial crisis at that time. However, the mean returns switch to negative and the standard deviations decline after 2009. Moreover, the aggregate foreign exchange rate market returns display a pattern with the kurtosis being over three, which means the returns have heavy tails.

Table 1 also shows the number of the periods that shocks occurred over 2 and 3 %, which we identify as jumps. From Panel A to Panel C in table 1, we observe that shocks occurred more frequently in 2008–2009 in the developed countries than in their counterparts. This reflects the occurrence of the subprime mortgage crisis, resulting in a large depreciation in EUR and GBP, but there was a large appreciation in JPY, which can be put down to the liquidation of the yen's carry trade. Moreover, as JPY appreciated there was pressure to cover any debts in yen by converting foreign assets into that currency, which had an accelerating effect on its valuation changes and when a large swing occurs this can cause a carry reversal. The large depreciation of EUR and GBP can be attributed to capital outflow from Europe and the United Kingdom.

Panel D to Panel F in Table 1 present the summary statistics of the three emerging countries. Panel D shows the summary statistics for BRL, which as we mentioned earlier, had its major shocks in 1999, 2001–2002 and 2008, consistent with the events of: the introduction of a floating exchange rate system; the energy crisis and the dotcom bubble bursting, as well as the 911 terrorist attacks and the subprime mortgage crisis, respectively. Panel E shows the summary statistics of IDR, with the most frequent shocks being in 1999–2001 and 2008, due to the catastrophic damage to the rupiah caused by the 1997 Asian financial crisis as well as political instability in 2001, in the first case and the 2008 global financial crisis in the second. Panel F shows the summary statistics of MXN, where it is noticeable that shocks occurred more frequently in 1999–2002 and 2008–2009, due to the Brazilian crisis in 1999, Argentina crisis in 2001–2002 regarding the first period, and the 2008 global financial crisis in relation to the second.

3.2 Parameter estimating under the regime-switching model

We estimate parameters under the RSM.[2] Suppose $\tilde{R} = \{R_1, R_2, \ldots, R_T\}$ and $\tilde{q} = \{q_1, q_2, \ldots, q_T\}$ are the observations and state variables of exchange rate changes from time 1 to time T, then we can write down the space for the model's parameters as:

$$\Theta_{RSM} = \{(p_{11}, p_{22}, \mu_1, \mu_2, \sigma_1, \sigma_2) | 0 < p_{11} < 1, 0 < p_{22} < 1, \mu_1 \text{ and } \mu_2 \in \mathbf{R}, \sigma_1 \text{ and } \sigma_2 \in \mathbf{R}^+\}$$

Define $L^c_{RSM}\left(\Theta_{RSM} \middle| \tilde{R}, \tilde{q}\right)$ as a complete-data likelihood function under the RSM:

$$L^c_{RSM}\left(\Theta_{RSM} \middle| \tilde{R}, \tilde{q}\right) = P\left(\tilde{R} | \tilde{q}, \Theta_{RSM}\right) P(\tilde{q} | \Theta_{RSM}) = \pi_{q_1} \prod_{t=2}^{T} p_{q_{t-1} q_t} \left(\prod_{t=1}^{T} P(R_t | q_t, \Theta_{RSM}) \right) \quad (3)$$

In this study, we use the Expectation–maximization (EM) algorithm to find the maximum likelihood estimates of parameters. Under the RSM setting, the log complete-data likelihood function is:

$$\log L^c_{RSM}\left(\Theta_{RSM} \middle| \tilde{R}, \tilde{q}\right) = \log \pi_{q_1} + \sum_{t=2}^{T} \log p_{q_{t-1} q_t} + \sum_{t=2}^{T} \left[-\frac{1}{2} \log\left(2\pi\sigma^2_{q_t}\right) - \frac{\left(R_t - \mu_{q_t}\right)^2}{2\sigma^2_{q_t}} \right] \quad (4)$$

If we already have the estimates of the $(k-1)$th parameter, $\Theta^{(k-1)}_{RSM}$, the estimates of the *kth* parameter can be obtained by the step E given the observable data and the $(k-1)$th parameter estimates. The conditional expectation of the complete-data likelihood function, $Q_{RSM}\left(\Theta_{RSM} \middle| \Theta^{(k-1)}_{RSM}\right)$, can be shown as:

[2] If regime switching is ignored, that is $\mu_1 = \mu_2 = \mu$ and $\sigma_1 = \sigma_2 = \sigma$, the regime-switching model degenerates into the Black–Scholes model, and the parameters, μ and σ, are also estimated by the MLE method.

$$
\begin{aligned}
Q_{RSM}\left(\Theta_{RSM}\middle|\Theta_{RSM}^{(k-1)}\right) = & \sum_{i=1}^{2} \log \pi_i P\left(q_1 = i\middle|\tilde{R}, \Theta_{RSM}^{(k-1)}\right) \\
& + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{t=2}^{T} \log p_{q_{t-1}q_t} P\left(q_{t-1} = i, q_t = j\middle|\tilde{R}, \Theta_{RSM}^{(k-1)}\right) \\
& + \sum_{i=1}^{2}\sum_{t=1}^{T}\left(\log P\left(R_t|q_t = i, \Theta_{RSM}^{(k-1)}\right)\right) P\left(q_t = i\middle|\tilde{R}, \Theta_{RSM}^{(k-1)}\right).
\end{aligned} \tag{5}
$$

Next, we can use the step M to find the space of parameters that can maximize $Q_{RSM}\left(\Theta_{RSM}\middle|\Theta_{RSM}^{(k-1)}\right)$, and through the Lagrange multiplier, we can finally get the estimates of $\hat{p}_{11}, \hat{p}_{22}, \mu_1, \mu_2, \sigma_1$ and σ_2 from the EM gradient algorithm, which can be shown as follows:

$$
\Theta_{RSM}^{(k)} = \Theta_{RSM}^{(k-1)} - a\left(d^{20} Q\left(\Theta_{RSM}\middle|\Theta_{RSM}^{(k-1)}\right)\right)^{-1} d^{10} Q\left(\Theta_{RSM}\middle|\Theta_{RSM}^{(k-1)}\right), \tag{6}
$$

Here, $\Theta_{RSM}^{(k)} = \arg\max_{\Theta} Q_{RSM}(\Theta|\Theta^{(k-1)})$, where $a \in (0, 1)$, d^{10} and d^{20} are the first and second order conditions of $Q_{RSM}\left(\Theta_{RSM}\middle|\Theta_{RSM}^{(k-1)}\right)$ with respect to Θ_{RSM}. Under the condition that $Q_{RSM}\left(\Theta_{RSM}\middle|\Theta_{RSM}^{(k-1)}\right)$ is monotonically increasing, we repeat the step E and the step M until the parameter estimates converge. Then we can estimate the parameters' standard deviation by Supplemented Expectation–Maximization (SEM) as proposed by Meng and Rubin (1991).

3.3 Parameter estimating under the regime-switching model with jump risks

By the same logic, we estimate parameters under the RSMJ. $\tilde{N} = (N_1, N_2, \ldots, N_T)$ is defined as the number of news arrivals causing a large depreciation or appreciation of the exchange rate between every time segment.

The space for the model's parameters can be written as follows:

$$
\begin{aligned}
\Theta_{RSMJ} = \{ & (p_{11}, p_{22}, \mu_1, \mu_2, \sigma_1, \sigma_2, \lambda, \mu_y, \sigma_y) | 0 < p_{11} < 1, 0 < p_{22} < 1, \mu_1, \mu_2 \\
& \text{and } \mu_y \in \mathbf{R}, \lambda, \sigma_1, \sigma_2 \text{ and } \sigma_y \in \mathbf{R}^+ \}
\end{aligned}
$$

We define $L_{RSMJ}^{C}\left(\Theta_{RSMJ}\middle|\tilde{R}, \tilde{q}, \tilde{N}\right)$ as a complete-data likelihood function under the RSMJ:

$$
L_{RSMJ}^{c}\left(\Theta_{RSMJ}\middle|\tilde{R}, \tilde{q}, \tilde{N}\right) = \pi_{q_1} \prod_{t=2}^{T} p_{q_{t-1}q_t} \left(\prod_{t=1}^{T} P(R_t|q_t, N_t, \Theta_{RSMJ})\right)\left(\prod_{t=1}^{T} P\left(N_t|\Theta_{RSMJ}\right)\right), \tag{7}
$$

The EM algorithm is used to find the maximum likelihood estimates of the parameters. Under the RSMJ setting, the log complete-data likelihood function is:

$$
\begin{aligned}
\log L_{RSMJ}^{c}\left(\Theta_{RSMJ}\middle|\tilde{R}, \tilde{q}, \tilde{N}\right) = & \log \pi_{q_1} + \sum_{t=1}^{T} \log p_{q_{t-1}, q_t} \\
+ \sum_{t=1}^{T} & \left[-\lambda + n_t \log \lambda - \log(n_t!) - \frac{1}{2}\log\left(2\pi\left(\sigma_{q_t}^2 + n_t\sigma_y^2\right)\right) - \frac{\left(R_t - \mu_{q_t}\right)^2}{2\left(\sigma_{q_t}^2 + n_t\sigma_y^2\right)}\right],
\end{aligned} \tag{8}
$$

If we already have the estimates of the $(k-1)$th parameter, $\Theta_{RSMJ}^{(k-1)}$, the estimates of the kth parameter can be obtained by the step E given the observable data and the $(k-1)$th parameter estimates. The conditional expectation of the complete-data likelihood function, $Q_{RSMJ}\left(\Theta_{RSMJ}\middle|\Theta_{RSMJ}^{(k-1)}\right)$ can be shown as follows:

$$
\begin{aligned}
Q_{RSMJ}\left(\Theta_{RSMJ}\middle|\Theta_{RSMJ}^{(k-1)}\right) = & \sum_{i=1}^{2}(\log\pi_i)P\left(q_1 = i\middle|\tilde{R},\Theta_{RSMJ}^{(k-1)}\right) \\
& + \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{t=2}^{T}(\log p_{ij})P\left(q_{t-1} = i, q_t = j\middle|\tilde{R},\Theta_{RSMJ}^{(k-1)}\right) \\
& + \sum_{i=1}^{2}\sum_{t=1}^{T}\sum_{n=0}^{\infty}(\log P(R_t|q_t = i, N_t = n)h(n,\lambda)) \\
& P\left(N_t = n\middle|\tilde{R},\Theta_{RSMJ}^{(k-1)}\right)P\left(q_t = i\middle|\tilde{R},\Theta_{RSMJ}^{(k-1)}\right).
\end{aligned} \tag{9}
$$

Next, we can use the step M to find the space of the parameters that can maximize $Q_{RSMJ}\left(\Theta_{RSMJ}|\Theta_{RSMJ}^{(k-1)}\right)$, and through the Lagrange multiplier, we can finally obtain the estimates of $\hat{p}_{11}$, $\hat{p}_{22}$, μ_1, μ_2, μ_y, σ_1, σ_2, σ_y and $\lambda$from the EM gradient algorithm, which can be shown as follows:

$$
\Theta_{RSMJ}^{(k)} = \Theta_{RSMJ}^{(k-1)} - a\left(d^{20}Q\left(\Theta_{RSMJ}\middle|\Theta_{RSMJ}^{(k-1)}\right)\right)^{-1}d^{10}Q\left(\Theta_{RSMJ}\middle|\Theta_{RSMJ}^{(k-1)}\right), \tag{10}
$$

Here $\Theta_{RSMJ}^{(k)} = \arg\max_{\Theta} Q_{RSMJ}(\Theta|\Theta^{(k-1)})$, where $a \in (0,1)$, d^{10} and d^{20} are the first and second order conditions of $Q_{RSMJ}\left(\Theta_{RSMJ}\middle|\Theta_{RSMJ}^{(k-1)}\right)$ with respect to Θ_{RSMJ}. Under the condition that $Q_{RSMJ}\left(\Theta_{RSMJ}\middle|\Theta_{RSMJ}^{(k-1)}\right)$ is monotonically increasing, we repeat steps E and M until the parameter estimates converge. Then we can estimate the parameters' standard deviations by SEM as proposed by Meng and Rubin (1991).

3.4 Likelihood ratio tests

This study uses Likelihood ratio (LR) as a testing model, summarized as follows: the null hypothesis is $H_0 : \theta \in \Theta_0$ against the alternative hypothesis $H_1 : \theta \in \Theta_1/\Theta_0$, $\Theta_0 \subset \Theta_1$. Testing statistic is:

$$
\Lambda = 2(\ln L(R;\Theta_1) - \ln L(R;\Theta_0)), \tag{11}
$$

where $\ln L(R;\Theta_i)$ is the log maximum likelihood function under H_i. Under the null hypothesis and if the sample is large enough, the testing statistics Λ would be distributed as $\chi^2(r)$, where r is the difference between the numbers of parameters in the two models. If $\Lambda > \chi^2_{r,1-\alpha}$, the null hypothesis would be rejected.

In this study, we perform two LR tests as follows: test (a) is based on the BSM against the RSM. When $\Lambda > \chi^2_{4,1-\alpha}$, the BSM is rejected and the RSM is proven to be better than the BSM. Test (b) is based on the RSM against the RSMJ. When $\Lambda > \chi^2_{3,1-\alpha}$, the RSM is rejected and the RSMJ is proven to be better than the RSM.

3.5 Empirical results of estimation and testing

In Table 2, we report the parameter estimates for the three models: the BSM, the RSM and the RSMJ. Due to the statistically insignificant results of μ_1 and μ_2, we eliminate the mean parameters in our model. Specifically, we set $\mu = 0$ in the BSM; $\mu_1 = \mu_2 = 0$ in the RSM; $\mu_1 = \mu_2 = \mu_y = 0$ in the RSMJ. Therefore, we only estimate the parameters σ_1, σ_2, p_{11} and p_{22} in the RSM, while estimating σ_1, σ_2, λ, p_{11} and p_{22} in the RSMJ.[3] In Table 2 of Sect. 3.5, under the RSM, two regimes of 'high variance' and of 'low variance' classify most of our sample countries' exchange rates, except JPY, which exhibits regimes of 'low variance' in state 1 and 'high variance' regime in state 2 due to the phenomenon of carry reversal during large swing periods of exchange rates. For example, in EUR, the estimated standard deviations, σ_1 and σ_2, are 0.0091 and 0.0054. Moreover, the standard deviations in the emerging countries are larger than those in the developed countries, consistent with the estimation in the BSM. For example, in BRL, the estimated standard deviations, σ_1 andσ_2, are 0.0225 and 0.0065. The parameters of the transition probabilities, p_{11} and p_{22}, are close to one for almost all the sample countries, indicating that switching from a high variance regime to a low one or vice versa did not frequently happen. For example, the transition probabilities p_{11} and p_{22} in EUR are 0.9818 and 0.9945, respectively, which means that when the euro becomes more volatile for the current time, the probability for it to remain in a high variance state for the next time is 0.9818. In addition, the EUR's stability for the current time will also lead to its stabilization persisting for a long time.

Under the RSMJ, the estimated results of the standard deviations and transition probabilities are also similar to those under the RSM, with the latter still being close to one. Moreover, compared to the estimated results under the RSM, the volatilities are generally smaller, which is in part explained by the jump term. The parameter, λ, defined as the number of jumps causing a large movement of the exchange rate between every time segment, is estimated to be 0.12 in the developed countries, whilst it is between 0.12 and 0.42 for the emerging countries, meaning that the exchange rates in the latter have suffered jumps more often than those in the former. Additionally, the AIC/SIC test statistics are included in Table 2 and we can see that the RSMJ has lower such statistics than the BSM and the RSM. Moreover, the results show that the RSMJ does not have an overkill problem. Finally, from the LR test results, it is concluded that the null hypotheses are to be rejected, meaning that with 95 % significance, the RSM is better than the BSM. Moreover, the RSMJ is better than the RSM.

3.6 The impact factors of jumps in currencies

Figures 1, 2, 3, 4, 5 and 6 compare the price level and log return of exchange rates with the probability of a high variance state under the RSMJ and the probability of jumps for EUR, GBP, JPY, BRL, IDR and MXN, respectively. From panel A and B, we observe most currencies display large volatilities during the 2008 global financial crisis periods. Moreover, the emerging countries have more volatile periods than the developed ones. For example, in addition to the 2008 global financial crisis period, BRL shows large volatility during two additional Brazilian crisis periods, 1999 and 2002; IDR shows large volatility in 1999 and 2001, due to the catastrophic damage to the rupiah caused by the 1997 Asian financial crisis and political instability, respectively; MXN shows large volatility during the Brazilian real devaluation period in 1999. We can also observe volatility clustering in Panel B.

[3] To save space, we did not report the full model estimation (with mean parameters) here. However, the results would be available upon request.

Table 2 Parameter estimation and testing for the Black–Scholes model, the regime-switching model and the regime-switching model with jump risks

Currency Pair	Model	p_{11}	p_{22}	σ_1	σ_2	σ_y	λ	LLH	AIC	SIC	Λ
Panel A: EUR	BSM			0.0064 (8.1E−05)				11,371	−22,739	−22,733	
	RSM	0.9818 (0.0068)	0.9945 (0.0018)	0.0091 (0.0004)	0.0054 (0.0001)			11,498	−22,987	−22,963	254.0***[a]
	RSMJ	0.9926 (0.0038)	0.9972 (0.0013)	0.0079 (0.0005)	0.0048 (0.0002)	0.0077 (0.0016)	0.1218 (0.0204)	11,516	−23,020	−22,983	36.4***[b]
Panel B: GBP	BSM			0.0059 (7.4E−05)				11,632	−23,262	−23,256	
	RSM	0.9867 (0.0030)	0.9973 (0.0004)	0.0094 (0.0013)	0.0049 (0.0004)			11,831	−23,653	−23,629	397.3***[a]
	RSMJ	0.9882 (0.0056)	0.9982 (0.0008)	0.0096 (0.0004)	0.0047 (0.0001)	0.0052 (0.0012)	0.1228 (0.0256)	11,834	−23,655	−23,619	6.1**[b]
Panel C: JPY	BSM			0.0067 (8.5E−05)				11,223	−22,444	−22,438	
	RSM	0.9656 (0.0186)	0.8369 (0.0048)	0.0054 (0.0071)	0.0110 (0.0007)			11,364	−22,720	−22,696	281.7***[a]
	RSMJ	0.9969 (0.0083)	0.9840 (0.0024)	0.0050 (0.0007)	0.0085 (0.0002)	0.0097 (0.0028)	0.1184 (0.0362)	11,389	−22,767	−22,731	50.7***[b]
Panel D: BRL	BSM			0.0114 (1.4E−04)				9,568	−19,133	−19,127	
	RSM	0.9385 (0.0138)	0.9856 (0.0046)	0.0225 (0.0011)	0.0065 (0.0003)			10,428	−20,848	−20,824	1,720.4***[a]
	RSMJ	0.9537 (0.0083)	0.9901 (0.0044)	0.0223 (0.0005)	0.0055 (0.0002)	0.0087 (0.0065)	0.1926 (0.0034)	10,453	−20,894	−20,858	50.2***[b]
Panel E: IDR	BSM			0.0085 (1.1E−04)				10,470	−20,938	−20,932	
	RSM	0.8645 (0.0163)	0.9435 (0.0070)	0.0150 (0.0004)	0.0029 (0.0001)			11,825	−23,643	−23,619	2,710.9***[a]
	RSMJ	0.9202 (0.0181)	0.9775 (0.0104)	0.0160 (0.0005)	0.0020 (0.0001)	0.0052 (0.0038)	0.4261 (0.0013)	11,923	−23,834	−23,798	195.1***[b]
Panel F: MXN	BSM			0.0063 (8.0E−05)				11,405	−22,807	−22,801	
	RSM	0.9225 (0.0147)	0.9873 (0.0027)	0.0130 (0.0005)	0.0042 (0.0001)			12,029	−24,050	−24,026	1,248.6***[a]
	RSMJ	0.9369 (0.0090)	0.9902 (0.0031)	0.0130 (0.0004)	0.0039 (0.0001)	0.0051 (0.0035)	0.1281 (0.0029)	12,037	−24,061	−24,025	15.6***[b]

BSM means the Black–Scholes model, RSM means the regime-switching model, and RSMJ means the regime-switching model with jump risks. p_{11} and p_{22} are the estimates of probabilities staying in the high variance and low variance states, respectively. σ_1 and σ_2 are the estimates of the standard deviation of returns in states 1 and 2, respectively. σ_y is the estimate of the standard deviation of jump sizes. λ is the estimate of the mean number of jumps. LLH is the value of the maximized log Likelihood. AIC and SIC calculates the Akaike and Schwarz information criteria. Λ is the likelihood ratio (LR). Standard errors of the estimated parameters are given in parentheses

*** and ** the null hypothesis can be rejected at the 1 and 5 % significance levels, respectively

[a] Likelihood ratio test based on the BSM against the RSM

[b] Likelihood ratio test based on the RSM against the RSMJ

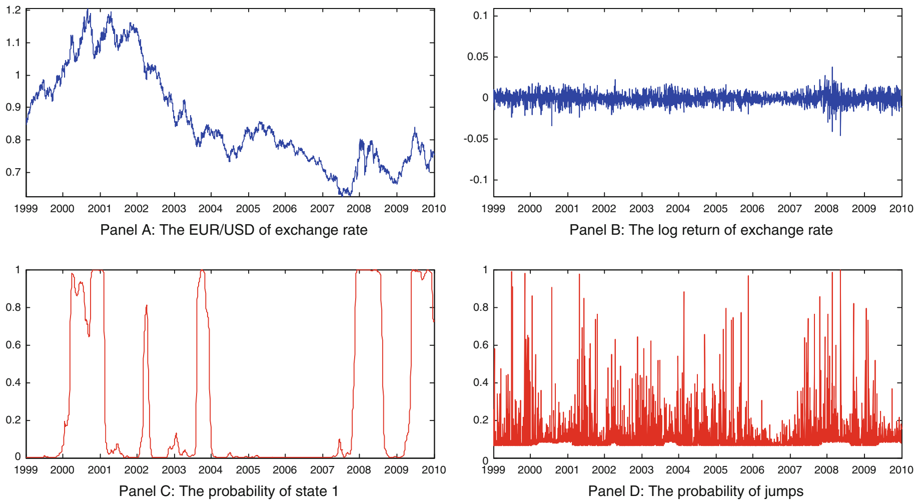

Fig. 1 The dynamics of exchange rate, log return, the probability of state 1 and the probability of jumps in EUR/USD

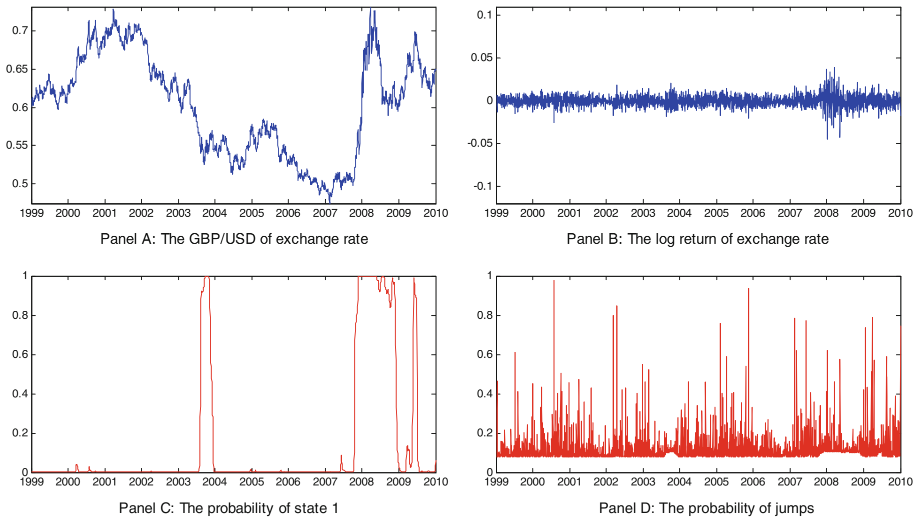

Fig. 2 The dynamics of exchange rate, log return, the probability of state 1 and the probability of jumps in GBP/USD

Panel C plots the probability of a high volatility state of currency. For example, EUR shows the probability of high volatility was high from 2000 to 2001 due to the global economic slowdown, from 2003 to 2004 owing to the Iraq War as well as the weak macroeconomic index announcement, in 2008 due to the global financial crisis, and in 2010 due to the occurrence of the European sovereign debt crisis. Otherwise, the probability was low from 1999 to 2000, because of the initiation of the Euro, from 2002 to 2003, owing to the economic recovery, from 2004 to 2008, which was associated with a rising interest rate policy implemented by the ECB, and in 2009 due to the economic recovery

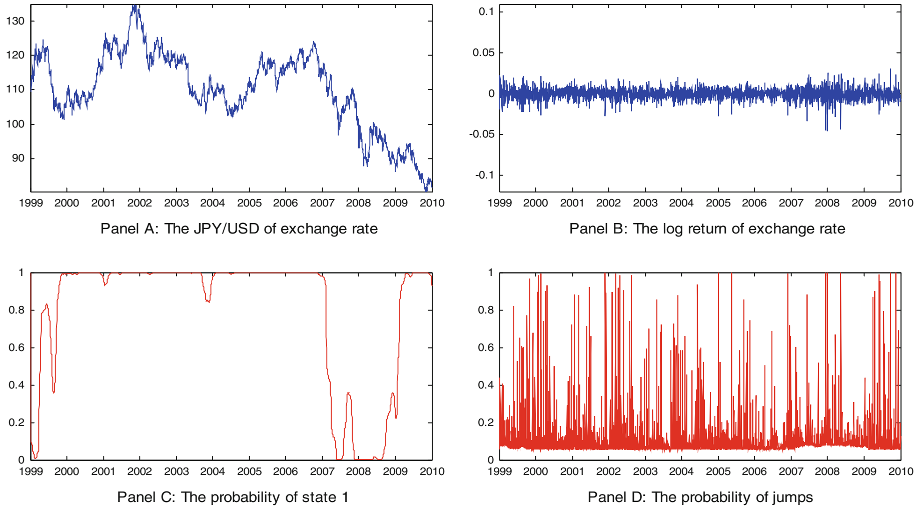

Fig. 3 The dynamics of exchange rate, log return, the probability of state 1 and the probability of jumps in JPY/USD

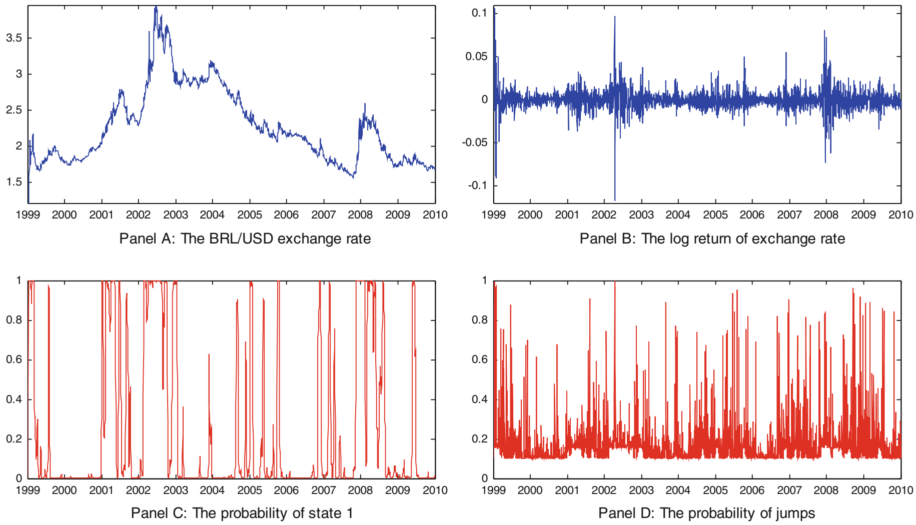

Fig. 4 The dynamics of exchange rate, log return, the probability of state 1 and the probability of jumps in BRL/USD

after the 2008 global crash. Hence, it can be seen that there were switches of state in 2000, 2002, 2003, 2004, 2008 and 2009.

Panel D shows the probability of jumps. Most currencies show the probability of jumps large in 1999–2002, and 2008, consistent with the events of the Brazilian crisis in 1999, the Argentine currency crisis and debt default from 2001 to 2002, and the global financial crisis in 2008. Moreover, the emerging countries show higher probabilities of jumps than

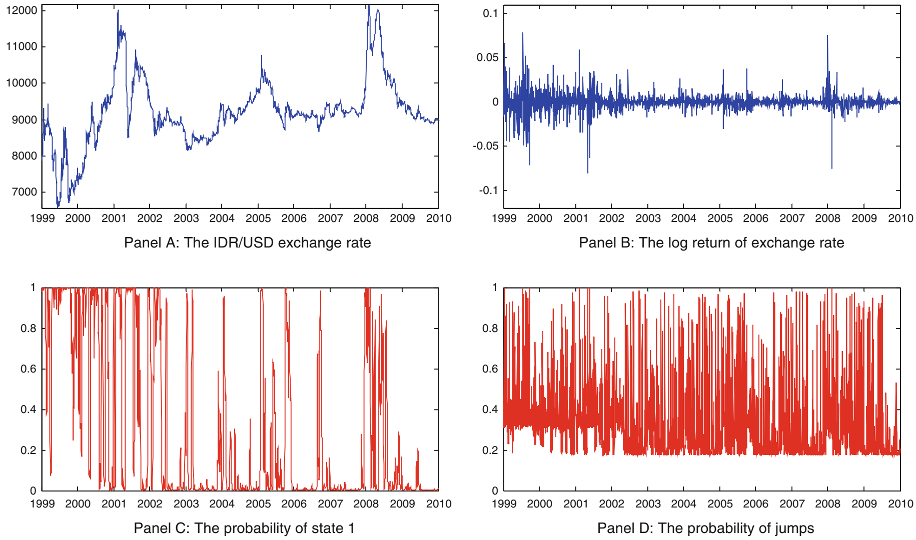

Fig. 5 The dynamics of exchange rate, log return, the probability of state 1 and the probability of jumps in IDR/USD

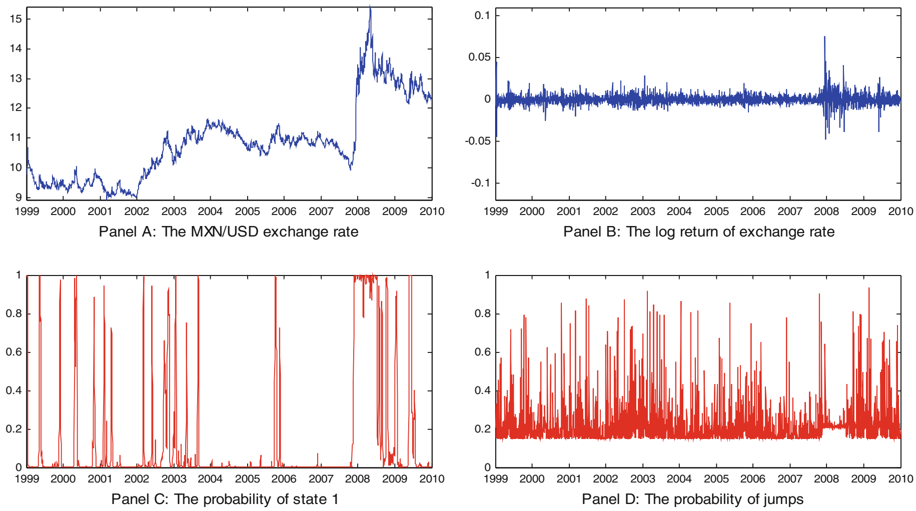

Fig. 6 The dynamics of exchange rate, log return, the probability of state 1 and the probability of jumps in MXN/USD

the developed ones, indicating that they have occurred more frequently in the former than in the latter.

Table 3 summarizes the jumps in exchange rates and provides detailed information about their impact factors. Generally, the jumps can be attributed to the following three factors: announcement of monetary policies associated with the alternation of exchange rate regimes, open market operation, change of interest rates as well as quantitative easing

Table 3 The impact factor of jumps in currencies

Date	Impact factor	The exchange rate changes
1999/1/13	*Monetary policy in the change of exchange rate regime: Brazil's currency crisis* Brazil adopts a floating exchange rate regime, thereby abandoning a managed float system	BRL (0.0660) IDR (0.0854) MXN (0.0454)
2000/7/3	*Political factor* The presidential election in 2000, ending the Mexico one-party regime	MXN (−0.0258)
2000/9/22	*Monetary policy in open market operations* The US Federal Reserve joins counterparts to Europe and Japan to intervene in currency markets in support of the Euro	EUR (−0.0332) GBP (−0.0251) JPY (0.0116)
2001/7/23	*Political factor* Impeachment and removal of Indonesian president	IDR (−0.0808)
2002/08/02	*Monetary policy in open market operations: Brazil's economic crisis* The International Monetary Fund (IMF) agrees to lend Brazil an extra $30 billion and lower the limit of the Brazilian foreign exchange reserves from $15 billion to $5 billion so as to help to pull the country and the region out of crisis	BRL (−0.1178)
2008/10/06	*Financial crisis* The crisis begins in the United States and rapidly spreads to Europe and Latin America.	GBP (0.0232) JPY (−0.0453) IDR (0.0168) BRZ (0.0721) MXN (0.0513)
2008/10/08	*Monetary policy in open market operations: financial crisis* The growing impact of the global financial crisis on Mexico and Brazil, forces the central banks in Mexico and Brazil to deploy billions of dollars of reserves	MXN (0.0755) BRL (0.0812) JPY (−0.0251)
2008/10/24	*Financial crisis* Traders are expecting that the Bank of England and European Central Bank will have interest rates cut to prevent contagion of the financial crisis	GBP (0.0233) JPY (−0.0460)
2008/10/29	*Monetary policy in open market operations* The IMF, the European Union (EU) and the World Bank, agree to offer a $25.1 billion rescue package to prevent the global financial crisis deteriorating	EUR (−0.0297) GBP (−0.0447) MXN(−0.0334) BRZ (−0.0270)

Table 3 continued

Date	Impact factor	The exchange rate changes
2008/12/15	*Monetary policy in open market operations* The FED announces it will buy $500 billion mortgage backed securities from Freddie and Fannie	EUR (−0.0252) MXN (−0.0409) BRL (−0.0462)
2008/12/16	*Monetary policy in low interest rate policy* The FED implements a near-zero interest rate policy	EUR (−0.0402) JPY (−0.0240)
2009/1/07	*Monetary policy in low interest rate policy* The Bank of England implements an "asset purchase program", purchasing high quality assets as part of a quantitative easing (QE) program	GBP (−0.0447)
2009/3/19	*Monetary policy in open market operations* The Fed announces that it will initiate a program to purchase $300 billion in treasury securities before Sep. 2009	EUR (−0.0462) GBP (−0.0433) JPY (−0.0443)
2010/5/10	*Monetary policy in open market operations: European sovereign debt crisis* European finance ministers approve a rescue package worth €750 billion aimed at ensuring financial stability across Europe by creating the European Financial Stability Facility (EFSF)	EUR (−0.0148) GBP (−0.0205) JPY (0.0244) IDR (−0.0126) BRZ (−0.0343) MXN (−0.0383)
2010/9/15	*Monetary policy in open market operations* Japanese government buys dollars to weaken the surging yen which is already harming Japanese manufactures	JPY(0.0308)
2011/3/18	*Monetary policy in open market operations: the 311 earthquake in Japan* The G7 countries agree to have a joint intervention in the yen to stabilize foreign exchange markets	JPY (0.0298)

The events are selected by the criteria of the top 10 appreciations or depreciations in the six sample exchange rates policy; political risk; and financial crises. In addition, the jumps in the emerging countries' currencies are more likely to be induced by political risk factors, while those in the developed ones are prone to be triggered by the announcement of monetary policies.

4 The valuation of currency options

In this section, we price currency options for the following six exchange rates: EUR, GBP, JPY, BRZ, IDR and MXN, assuming that the dynamics of these follow the BSM, the RSM and the RSMJ.

4.1 Data description

We collect currency option quotes for the following six exchange rates: EUR/USD, GBP/USD, JPY/USD, USD/BRZ, USD/IDR and USD/MXN from the Bloomberg database, with our samples spanning from January 2, 2008 to December 31, 2012. We collect options on each pair with fixed time to maturities at 1 month, 3 months and 12 months. The option quotes we obtained are the same as those from Carr and Wu (2007). At each maturity, quotes are available at five deltas in the form of delta-neutral straddle implied volatilities, 10- and 25-delta risk reversals, and 10- and 25-delta butterfly spreads. For the straddle to be delta-neutral under the Garman–Kohlhagen model, the strike price K needs to satisfy:

$$e^{-r_f\tau}N(d_1) - e^{-r_f\tau}N(-d_1) = 0 \tag{12}$$

where r_f refers the foreign interest rate, $N(.)$ denotes the cumulative normal distribution, and

$$d_1 = \frac{\ln(F_t/K)}{IV\sqrt{\tau}} + \frac{1}{2}IV\sqrt{\tau},$$

with F_t being the forward currency price, τ the time to maturity in years, and IV the implied volatility quote. Eq. (12) implies that $d_1 = 0$. Therefore, the strike price is very close to the spot or forward price and this quote can be referred to as the at-the-money implied volatility quote (ATMV).

The ten-delta risk reversal (RR10) measures the difference in implied volatilities between a ten-delta call option and a ten-delta put one, while the ten-delta butterfly spread (BF10) measures the difference between the average implied volatility of the two ten-delta options and the delta-neutral straddle implied volatility. Additionally, the 25-delta risk reversals (RR25) and butterfly spreads (BF25) are defined analogously. From the above five quotes, we can derive the implied volatilities at the five levels of delta. To convert the implied volatilities into option prices and the deltas into strike prices, we need the currency price and the domestic and foreign interest rates, which are obtained from Datastream.

4.2 Stylized features of currency option implied volatilities

Summarizing the currency option implied volatility quotes, we observe several important features. First, we find a U shape for each currency and at each maturity when we plot the time-series average of implied volatilities against delta. Fig. 7 shows the average implied volatility smile across moneyness at selected maturities of 1 month, 3 months and 12 months. This has long been regarded as evidence for return non-normality under a risk-

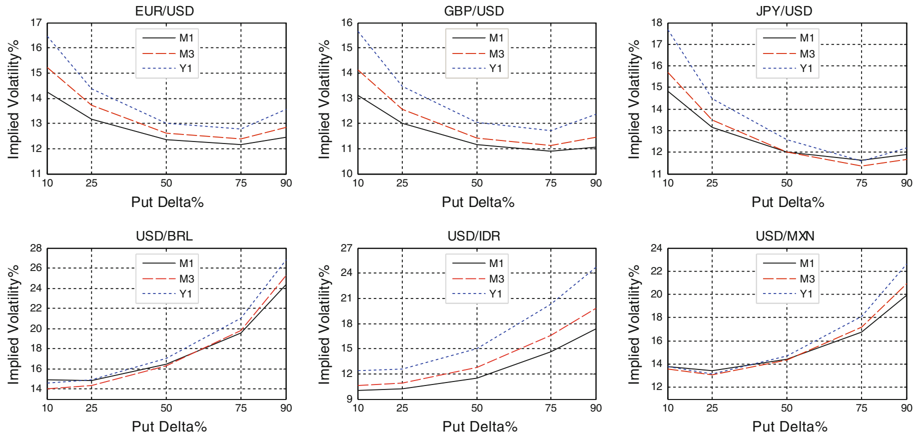

Fig. 7 Average implied volatility smiles on currency options. The *lines plot* the time-series averages of the implied volatility quotes in percentage points against the put delta of the currency options at three selected maturities: 1 month, 3 months, and 1 year. The averages are on daily data from January 2, 2008, to December 31, 2012, with 1,303 observations for each series

neutral measure. That is, the curvature of the smile reflects fat tails in the return distribution, while asymmetry reflects skewness in the return distribution. Here, the relatively asymmetric mean implied volatility smiles on all exchange rates show that, on average, the risk-neutral return distributions of all currency pairs are not only fat tailed but also highly asymmetric. Moreover, Fig. 7 reveals that the average smile remains highly curved as the option maturity increases from 1 month to 1 year, which indicates that the return distribution remains highly non-normal as the horizon increases. To account for the slow convergence of the return distribution to normality, we have proposed the RSMJ to incorporate the features of currency cycles as well as sudden jumps.

Figure 8 plots the time series of the delta-neutral straddle implied volatility for the six currency pairs of 1 month, 3 months and 12 months. The plots show that the implied volatilities of 1 month experience larger variations than those of 3 and 12 months, particularly during the crisis periods. Moreover, if we use the implied volatility as a proxy for the currency return volatility level, the plots in Fig. 8 suggest that a reasonable model should accommodate the feature of time-varying volatility as Baharumshah and Wooi (2007) indicated in their paper. The RSM as well as the RSMJ proposed in this paper can incorporate this feature of the data.

Table 4 reports the mean, standard deviation, and the daily autocorrelation of risk reversals, butterfly spread, and delta-neutral straddle implied volatilities. The risk reversals and butterfly spreads are normalized as percentages of the delta-neutral straddle implied volatility. For the emerging countries' currencies, USD/BRL, USD/IDR and USD/MXN, the sample averages of the risk reversals are positive, implying that the out-of-money call options are, on average, more expensive than the corresponding out-of-money put options during the sample period. The average butterfly spreads range from 0.7 to 4.05 % at ten delta and span less than 1 % at 25 delta. For the developed countries' currencies, EUR/USD, GBP/USD and JPY/USD, the average implied volatility smile of the risk reversals is negative, implying that the out-of-money call options are, on average, less expensive than the corresponding out-of-money put options during the sample period. In addition, the average butterfly spreads range from 0.96 to 2.3 % at ten delta and span less than 1 % at 25 delta.

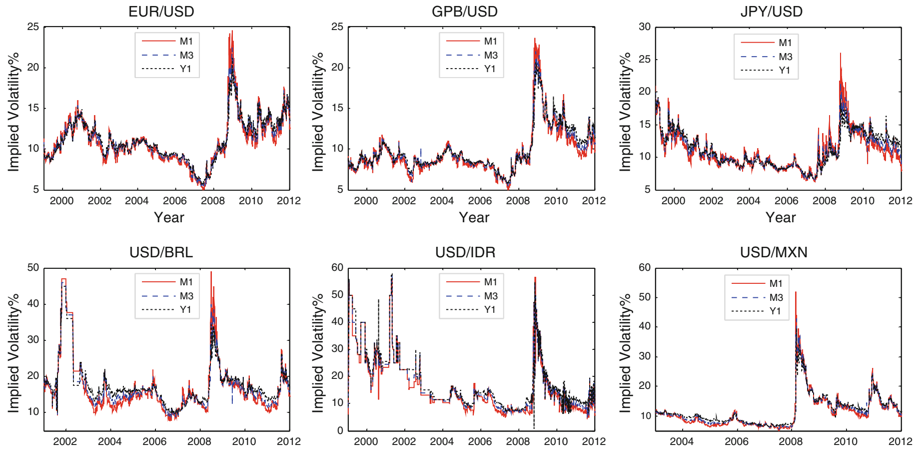

Fig. 8 The time variation of currency option implied volatilities. The *lines plot* the time series of the 1-month, 3-months, and 1-year delta-neutral straddle implied volatility quotes in percentage points on the dollar price of EUR, GBP, JPY, BRL, IDR and MXN. The data is from January 4, 1999 to December 31, 2012, with 3,657 observations for EUR, GBP, JPY, IDR series; from January 2, 2001 to December 31, 2012, with 2,974 observations for the BRL series; and from October 2, 2003 to December 31, 2012, with 2,609 observations for the MXN series

For all currencies, the standard deviations of the risk reversals are much larger than the standard deviations of the butterfly spreads. For the developed countries' currencies, EUR/USD, GBP/USD and JPY/USD, the standard deviations are around 2 % for ten-delta risk reversals and are less than 1 % for ten-delta butterfly spreads. The standard deviations of 25-delta risk reversals are 0.7–2 %, but those for the 25-delta butterfly spreads are less than 0.15 %. The same pattern holds for the emerging countries' currencies, USD/BRL, USD/IDR and USD/MXN in that the standard deviations for the risk reversals are about three times larger than those for the corresponding butterfly spreads. Further, the delta-neutral straddle implied volatilities have standard deviations about 3–4 % for the developed countries' currencies and about 6–9 % for the emerging countries' currencies. Finally, all currencies show strong serial correlation that increases with the option maturity.

4.3 Currency option pricing under the Black–Scholes model

Garman and Kohlhagen (1983) adopted Merton's (1973) results to derive the pricing formula for currency option. The dynamics of exchange rates can be written as:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t, \tag{13}$$

where S_t is the exchange rate for converting from one US dollar to the other currencies, μ is the instantaneous mean value of exchange rate changes, σ is the instantaneous standard deviation, and W_t denotes the Brownian motion. The currency call option can be written down as follows:

$$C_{BSM,T} = Max(S_T - K, 0), \tag{14}$$

where K denotes the strike exchange rate, and S_T is the currency spot rate at maturity T. Assuming the local risk free rate and the US dollar interest rate are r and r_f, respectively, by

Table 4 Summary statistics of currency option implied volatilities

Maturity	ATMV			10RR			10BF			25RR			25BF		
m	Mean	SD	Auto	Mean	SD	Auto	Mean	SD	Auto	Mean	SD	Auto	Mean	SD	Auto
EUR															
1 M	12.3967	3.5876	0.9850	−1.8603	1.8708	0.9845	0.9619	0.5620	0.7275	−1.0199	1.0139	0.9847	0.2940	0.1514	0.9447
3 M	12.6152	3.0940	0.9889	−2.4104	2.1339	0.9921	1.4615	0.5801	0.9512	−1.3407	1.1754	0.9911	0.4066	0.1514	0.9615
12 M	12.9871	2.3152	0.9931	−2.9650	2.3579	0.9942	2.0566	0.5950	0.9588	−1.6526	1.3101	0.9925	0.5816	0.1768	0.8593
GBP															
1 M	11.1991	4.3164	0.9915	−2.1051	1.3055	0.9760	0.9838	0.5512	0.9533	−1.1504	0.7243	0.9782	0.3169	0.1633	0.9546
3 M	11.4410	3.7358	0.9944	−2.6556	1.2965	0.9832	1.3772	0.5922	0.9599	−1.4785	0.7114	0.9832	0.4165	0.1684	0.9723
12 M	12.0467	2.9327	0.9963	−3.3229	1.3641	0.9874	1.9787	0.6011	0.9674	−1.8297	0.7381	0.9484	0.5589	0.1774	0.9598
JPY															
1 M	12.0551	4.1487	0.9703	−1.5920	2.0921	0.9914	1.3166	0.6569	0.9555	−1.5920	2.0921	0.9914	0.3166	0.1076	0.7595
3 M	12.0293	3.1121	0.9804	−2.1150	2.2169	0.9935	1.6177	0.7273	0.9820	−2.1150	2.2169	0.9935	0.3427	0.1131	0.7557
12 M	12.5879	1.9395	0.9882	−5.7682	4.7553	0.9975	2.3090	0.8352	0.9863	−2.9159	2.4023	0.9968	0.3947	0.2158	0.6550
BRL															
1 M	16.2901	8.3492	0.9771	8.0886	4.7300	0.9713	2.1429	2.8491	0.6811	4.4064	2.6270	0.9877	0.5756	0.2780	0.5371
3 M	16.1790	6.3315	0.9796	9.6827	4.2509	0.9597	2.8201	2.3343	0.7521	5.2109	2.3596	0.9841	0.6876	0.3007	0.4108
12 M	16.9972	4.2865	0.9831	11.2460	4.1812	0.9695	4.0563	2.1797	0.7954	5.9823	2.3288	0.9839	0.8759	0.2590	0.6228
IDR															
1 M	11.6131	8.5494	0.9912	5.9408	6.0298	0.9743	0.7392	2.7010	0.8648	3.2452	3.1723	0.9600	0.4813	0.9209	0.8332
3 M	12.8240	8.0027	0.9931	8.2914	6.2194	0.9786	1.5886	2.8462	0.8809	4.6188	3.3053	0.9663	0.6717	0.9229	0.8232
12 M	15.1098	7.1922	0.9935	13.9255	8.3931	0.9076	3.2571	3.9058	0.9718	7.0111	3.8421	0.9833	1.0314	1.2840	0.9110
MXN															
1 M	14.4532	8.9245	0.9767	6.0370	4.5774	0.9707	1.8466	2.1193	0.5071	3.2256	2.5783	0.9802	0.5509	0.3515	0.7957
3 M	14.2326	6.8499	0.9906	7.4488	4.3602	0.9723	2.3760	1.8755	0.7232	3.9980	2.4798	0.9822	0.6423	0.3419	0.7728

Table 4 continued

Maturity	ATMV			10RR			10BF			25RR			25BF		
m	Mean	SD	Auto	Mean	SD	Auto	Mean	SD	Auto	Mean	SD	Auto	Mean	SD	Auto
12 M	14.6740	5.3530	0.9922	9.0025	4.3409	0.9741	3.2326	1.7571	0.7303	4.8735	2.4328	0.9818	0.8031	0.3650	0.7782

The three columns under each contract report the mean (Mean), standard deviation (SD), and daily autocorrelation (Auto) of the contract on risk reversal (RR), butterfly spread (BF), and delta-neutral straddle implied volatilities (ATMV). Risk reversals and butterfly spreads are in percentages of the delta-neutral straddle implied volatility. The numbers following RR and BF denote the delta of the contract. The data are daily covering from January 02, 2008 to December 31, 2012, with 1,303 observations for each series. The first column denotes the option maturities, with ''M'' denoting months

Merton's (1973) model, we can replace continuous cash dividend payment with r_f in the stock option pricing formula. Under these assumptions, the value of a European-style call option with exercise price K and time to maturity T is given by:

$$C_{BSM,T} = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2), \tag{15}$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - r_f + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

$$\sigma = \sqrt{Var(d\ln S_t)/_{dt}}$$

is the standard deviation of exchange rate return, and $N(x)$ is the cumulative distribution function for a standard normal random variable with upper integral limit x.

4.4 Currency option pricing under the regime-switching model

The RSM is constructed to capture the salient features of exchange rate dynamics and incorporates the majority of popular models used in the literature as its special cases. Under the physical probability measure, we assume that the instantaneous currency spot rate S_t follows the RSM:

$$S(t+s) = S(t)\exp\left\{\left(\mu_{q_t} - \frac{1}{2}\sigma_{q_t}^2\right)s + \sigma_{q_t} W(s)\right\}, \tag{16}$$

where q_t is the state at time t, representing the return of the exchange rate either in the deprecation (state 1) or the appreciation state (state 2); the transition probability of $\{q_t\}$ is $P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$; μ_{q_t} and σ_{q_t} represent the mean and standard deviation under the market state q_t, respectively; and $W(s)$ is the Brownian motion with duration s days in the state q_t, following a normal distribution $N(0, s)$. Suppose that the Markov process q_t is not varied with the probability measure transformation, and no risk premium occurs during the state transition, we can use the Esscher transform to transfer the dynamics of exchange rates under the physical probability measure to that under the risk neutral measure. Under the risk neutral probability measure, the dynamics of the exchange rates follows the RSM:

$$S(t+s) = S(t)\exp\left\{\left(r - \frac{1}{2}\sigma_{q_t}^2\right)s + \sigma_{q_t} W^Q(s)\right\}. \tag{17}$$

Under the risk neural measure Q at time $t+s$ and under the market state q_t, the Brownian motion, $W^Q(s)$ can be represented as:

$$W^Q(s) = W(s) - h_{Bq_t}\sigma_{q_t} s \sim N(0, s),$$

where h_{Bq_t} denotes the Esscher transform parameter of the Brownian motion under the market state q_t. If the local risk free rate and the foreign interest rate are r and r_f, respectively, under the risk neutral measure, the dynamics of the exchange rates at time $t+s$ can be presented as follows:

$$S(t+s) = S(t)\exp\left\{\left(r - r_f - \frac{1}{2}\sigma_{q_t}^2\right)s + \sigma_{q_t} W^Q(s)\right\}. \tag{18}$$

Given the initial market state $q_0 = i$ and the duration days staying in state 1, k_1, the dynamic process of exchange rate with the maturity date T under the risk neutral measure can be represented in the following form:

$$S(T) \stackrel{dist}{=} S(0)\exp\left\{(r - r_f)T - \frac{1}{2}\theta_{k_1}^2 + \theta_{k_1} Z\right\}, \tag{19}$$

where Z is an identically independent standard normal distribution and the weighted variance under the state q_i is $\theta_{k_1}^2 = k_1\sigma_1^2 + (T - k_1)\sigma_2^2$. Then we can price a European-style currency call option at the maturity date T under the RSM, $C_{RSM,T}(0)$:

$$C_{RSM,T}(0) = \sum_{k_1=0}^{T}\sum_{i=1}^{2}\pi_i \cdot \gamma_{T,k_1|q_0=i}\left[S(0)e^{-r_f T}N(d_{1,k_1}) - Ke^{-rT}N(d_{2,k_1})\right], \tag{20}$$

where

$$d_{1,k_1} = \frac{\ln\frac{S(0)}{K} + (r - r_f)T + \frac{1}{2}\theta_{k_1}^2}{\theta_{k_1}}$$

and

$$d_{2,k_1} = d_{1,k_1} - \theta_{k_1};$$

π_i is the steady-state probability of the initial market state i; $\gamma_{T,k_1|q_0=i}$ represents the probability that over the T periods, k_1 periods are assigned to the state 1 conditional on the initial market state i. According to Duan et al. (2002), we can solve $\gamma_{T,k_1|q_0=i}$ from $\gamma_{t,k_1|q_0=i}$ recursively.

$$\gamma_{t,k_1|q_0=1} = \begin{cases} p_{12}p_{22}^{t-1}, & \text{for } k_1=0 \text{ and } t=1,2,\ldots,T \\ p_{11}, & \text{for } k_1=1 \text{ and } t=1 \\ p_{11}p_{12}p_{22}^{t-2} + (t-2)p_{12}^2 p_{21}p_{22}^{t-3} + p_{12}p_{21}p_{22}^{t-2}, & \text{for } k_1=1 \text{ and } t=2,3,\ldots,T \\ \sum\limits_{m=1}^{t-k_1+1} F(m|q_{t-m-1}=1)\cdot\gamma_{t-m,k_1-1|q_{t-m-1}=1}, & \text{for } k_1=2,3,\ldots,t \text{ and } t=2,3,\ldots,T \end{cases}$$

where

$$F(m|q_{t-m-1}=1) = \begin{cases} p_{11}, & \text{for } m=1 \\ p_{12}p_{22}^{m-2}p_{21}, & \text{for } m=2,3,\ldots,t \end{cases}$$

and

$$\gamma_{t,k_1|q_0=2} = \begin{cases} p_{22}^t, & \text{for } k_1=0 \text{ and } t=1,2,\ldots,T \\ p_{21}, & \text{for } k_1=1 \text{ and } t=1 \\ (t-1)p_{21}p_{12}p_{22}^{t-2} + p_{22}^{t-1}p_{21}, & \text{for } k_1=1 \text{ and } t=2,3,\ldots,T \\ \sum\limits_{m=1}^{t-k_1+1} F(m|q_{t-m-1}=2)\cdot\gamma_{t-m,k_1-1|q_{t-m-1}=1}, & \text{for } k_1=2,3,\ldots,t \text{ and } t=2,3,\ldots,T \end{cases}$$

where

$$F(m|q_{t-m-1}=2)=\begin{cases} p_{21}, & \text{for } m=1 \\ p_{22}^{m-1}p_{21}, & \text{for } m=2,3,\ldots,t \end{cases}.$$

k_1 denotes the duration days staying in state 1; m is the start point of k_1; $q_0=1$ means the initial state is in state 1, otherwise, the initial state is state 2; and $F(m|q_{t-m-1}=1)$ is the probability at time m conditional on the initial state being state 1.

4.5 Currency option pricing under the regime-switching model with jump risks

As the RSM cannot capture unanticipated information in markets, we incorporate jump terms as introduced by Merton (1976) to complete our pricing model. Merton (1976) first introduced the jump diffusion model and the pricing framework with the jump term $\sum_{n=1}^{N_t}\log Y_n$, where N_t represents the news arrival during every time segment, including good news and bad news, which would cause large deprecation or appreciation in exchange rates, and Y_n is the jump size.

Suppose the exchange rate follows the RSMJ, then the dynamic process can be written as follows:

$$S(t+s)=S(t)\exp\left\{\left(\mu_{q_t}-\frac{1}{2}\sigma_{q_t}^2-\lambda(e^{\mu_y+\frac{\sigma_y^2}{2}}-1)\right)s+\sigma_{q_t}W(s)+\sum_{m=0}^{N(s)}\ln Y_m\right\}, \tag{21}$$

where the definitions of q_t, $W(s)$ are identical to those in the RSM, and $N(s)$ is a Poisson process with mean value λ describing the number of jumps in the exchange rate. $\{Y_m\}$ is the jump size, and the logarithm of $\{Y_m\}$ follows a normal distribution with mean μ_y and variance σ_y^2. In addition, we assume that $\{W(s)\}$, $\{N(s)\}$ and $\{Y_m\}$ are independent from each other.

Suppose that the Markov process q_t is not varied with the probability measure transformation and no risk premium occurs during state transition, then the dynamic process of the exchange rate under the physical probability measure can be transferred to that under the risk neutral probability measure according to the Esscher transform.[4] If the local risk free rate and the US dollar interest rate are r and r_f, respectively, then the dynamic process of exchange rates under the RSMJ with the risk neutral measure can be denoted as follows:

$$S(t+s)=S(t)\exp\left\{\left(r-\frac{1}{2}\sigma_{q_t}^2-\lambda\left(e^{\mu_y+\frac{\sigma_y^2}{2}}-1\right)\right)s+\sigma_{q_t}W^Q(s)+\sum_{m=0}^{N^Q(s)}\ln Y_m^Q\right\}, \tag{22}$$

where $W^Q(s)=W(s)-h_{Bq_t}\sigma_{q_t}s$, $N^Q(s)$ denotes the Poisson process under the risk neutral measure with time interval s, $\ln(Y_m^Q)$ represents the jump size which follows a normal distribution with mean $-\frac{1}{2}\sigma_y^2$ and variance σ_y^2 under the risk neutral measure. Given the initial market state $q_0=i$, the duration days staying in state 1, k_1, and the number of jumps, $N^Q(T)=n$, the dynamic process of the exchange rate with the maturity date Tunder the risk neutral measure can be represented as follows:

$$\begin{aligned} S(T)&=\left(S(0)\exp\left\{\int_0^T rdt-\frac{1}{2}\int_0^T\sigma_{q_t}^2dt+\int_0^T\sigma_{q_t}dW^Q(t)+\sum_{m=1}^{n}\ln Y_m^Q\right\}\right)\\ &\overset{dist}{=}S(0)\exp\left\{rT-\frac{1}{2}\theta_{k_1}^2+\theta_{k_1}Z+\sum_{m=1}^{n}\ln Y_m^Q\right\}, \end{aligned} \tag{23}$$

[4] The detailed derivation is shown in "Appendix".

where Z is an identically independent standard normal distribution and the weighted variance under the state q_t is$\theta_{k_1}^2 = k_1\sigma_1^2 + (T - k_1)\sigma_2^2$. Therefore, $\sum_{m=1}^{n} \ln Y_m^Q$ and $\theta_{k_1} Z_n + \sum_{m=1}^{n} \ln Y_m^Q$ also follow normal distributions:

$$\sum_{m=1}^{n} \ln Y_m^Q \sim N\left(-\frac{1}{2}n\sigma_y^2, n\sigma_y^2\right)$$

and

$$\theta_{k_1} Z_n + \sum_{m=1}^{n} \ln Y_m^Q \sim N\left(-\frac{1}{2}n\sigma_y^2, \theta_{k_1}^2 + n\sigma_y^2\right)$$

Then we can price a European-style currency call option at maturity date T under the RSMJ, $C_{RSMJ,T}(0)$:

$$C_{RSMJ,T}(0) = \sum_{n=0}^{\infty} \sum_{k_1=0}^{T} \sum_{i=1}^{2} \pi_i \cdot \gamma_{T,k_1|q_0=i} \frac{e^{-\lambda *}\left(\lambda^*\right)^n}{n!} \left[S(0)e^{-r_f T + n(\mu_y + \frac{\sigma_y^2}{2}) - \lambda(\xi^{(1)} - 1)T} N(d_{1,k_1}) - Ke^{-rT} N(d_{2,k_1})\right], \tag{24}$$

where $\lambda^* = \lambda T$,

$$d_{1,k_1} = \frac{\ln\frac{S(0)}{K} + \left(r - r_f\right)T + \frac{1}{2}\theta_{k_1}^2 + n\left(\mu_y + \sigma_y^2\right)}{\sqrt{\theta_{k_1}^2 + n\sigma_y^2}},$$

$$d_{2,k_1} = d_{1,k_1} - \sqrt{\theta_{k_1}^2 + n\sigma_y^2}.$$

When no jump occurs, $n = 0$ and $\lambda T = 0$, a European-style call option under the RSMJ will degenerate that under the RSM. The detail derivation is shown in "Appendix". The pricing formula of the European put option under the regime-switching model with jumps risks is:

$$P_{RSMJ,T}(0) = \sum_{n=0}^{\infty} \sum_{k_1=0}^{T} \sum_{i=1}^{2} \pi_i \cdot \gamma_{T,k_1|q_0=i} \frac{e^{-\lambda *}(\lambda^*)^n}{n!} \left[Ke^{-rT} N(-d_{2,k_1}) - S(0)e^{-r_f T + n(\mu_y + \frac{\sigma_y^2}{2}) - \lambda(\xi^{(1)} - 1)T} N(-d_{1,k_1})\right], \tag{25}$$

where $\lambda^* = \lambda T$,

$$d_{1,k_1} = \frac{\ln\frac{S(0)}{K} + \left(r - r_f\right)T + \frac{1}{2}\theta_{k_1}^2 + n\left(\mu_y + \sigma_y^2\right)}{\sqrt{\theta_{k_1}^2 + n\sigma_y^2}},$$

$$d_{2,k_1} = d_{1,k_1} - \sqrt{\theta_{k_1}^2 + n\sigma_y^2}.$$

4.6 The validity of the currency options

In this section, we compare the values of currency options generated by the following three models: the BSM, the RSM and the RSMJ with the observed market value. We compare the pricing errors generated by the different models, which are computed using the mean square error (MSE):

$$MSE_t = \min_{\Theta} \frac{1}{N} \left(\sum_{i=1}^{N} \left(O_{i,t}^{MODEL} - O_{i,t}^{DATA} \right)^2 \right), \tag{26}$$

where N denotes the number of days in the sample, $O_{i,t}^{DATA}$ represents the market values of currency option and $O_{i,t}^{MODEL}$ are the corresponding values of currency options computed from the three different models: the BSM, the RSM and the RSMJ. The model parameters are chosen to minimize the MSE. Hence, the MSE formula evaluates the models based on comparison of the pricing errors between the market value and the estimated value for the models. As we want to check for the existence of any structural patterns in the pricing errors of three different models, we compute the MSEs at five levels of delta: 10, 25, 50, 75, and 90 put delta; and at three maturities: 1 month, 3 months, and 12 months. The model with the MSE close to zero and showing no obvious structures along both the moneyness and maturity dimensions is considered the best choice.

4.6.1 In-sample and out-of-sample performance comparison

Our sample is taken from January 2, 2008 to December 31, 2010 and from January 4, 2011 to December 31, 2012 for in-sample and out-of-sample testing, respectively. We investigate how the RSMJ performs against the BSM and the RSM by comparing the summation of MSEs over the five levels of deltas among the three different models. Table 5 reports in-sample and out-of-sample model performance comparisons at three fixed maturities: 1 month, 3 months, and 12 months as well as the average performance over the three maturities. From the in-sample test in table 5, the RSMJ markedly outperforms the BSM and RSM at both 1 month and 3 months maturities. For the 12 months maturity, the RSMJ performs the best among models for JPY, BRL and MXN, while the RSM dominates for EUR, GBP and IDR. Moreover, for average performance, the RSMJ also outperforms the other two models for all sample currencies.

The out-of-sample test in the right panel of Table 5 shows a similar pattern to the in-sample test. That is, the RSMJ performs better than the BSM and the RSM for 1 month and 3 months maturities, but there is mixed evidence at 12 months maturity. The results of average performance for the out-of-sample test indicate that the superior in-sample performance of the RSMJ over both the BSM and the RSM extends to such a comparison.

4.6.2 Structural patterns in the pricing errors

Figures 9 and 10 plot the summation of MSEs over the three different maturities (1 month, 3 months and 12 months) along the moneyness dimension for the in-sample and out-of-sample periods, respectively. As mentioned above, the model with the MSE close to zero and showing no obvious structures along the moneyness would be the best choice. Under the BSM, the pricing errors display obvious structural patterns for both the in-sample and out-of-sample performances along the moneyness dimension. Specifically, the pricing

Table 5 In-sample and out-of-sample performance comparison

Currency pair	Model	In-sample test				Out-of-sample test			
		1 month	3 month	1 year	Average	1 month	3 month	1 year	Average
Panel A: EUR	BSM	5.64E−06	6.14E−06	9.12E−06	6.97E−06	1.38E−04	3.65E−04	1.07E−03	5.24E−04
	RSM	5.28E−06	5.35E−06	7.52E−06	6.05E−06	5.79E−06	6.33E−06	8.97E−06	7.03E−06
	RSMJ	4.21E−06	4.30E−06	8.22E−06	5.58E−06	4.79E−06	5.34E−06	9.73E−06	6.62E−06
Panel B: GBP	BSM	6.73E−06	7.90E−06	1.95E−05	1.14E−05	1.10E−04	3.34E−04	1.40E−03	6.15E−04
	RSM	6.98E−06	7.14E−06	7.27E−06	7.13E−06	7.36E−06	7.93E−06	8.33E−06	7.87E−06
	RSMJ	6.42E−06	6.11E−06	8.76E−06	7.10E−06	6.82E−06	6.94E−06	9.83E−06	7.86E−06
Panel C: JPY	BSM	8.30E−02	1.30E−01	4.24E−01	2.12E−01	7.19E−01	1.89E+00	5.92E+00	2.84E+00
	RSM	6.43E−02	7.14E−02	2.14E−01	1.17E−01	6.74E−02	7.88E−02	2.33E−01	1.26E−01
	RSMJ	5.75E−02	6.37E−02	2.12E−01	1.11E−01	6.22E−02	7.25E−02	2.33E−01	1.23E−01
Panel D: BRL	BSM	6.48E−05	9.39E−05	2.87E−04	1.49E−04	4.01E−04	1.11E−03	4.00E−03	1.84E−03
	RSM	6.64E−05	7.24E−05	1.94E−04	1.11E−04	7.26E−05	8.76E−05	2.16E−04	1.25E−04
	RSMJ	4.07E−05	6.35E−05	1.79E−04	9.42E−05	4.83E−05	8.19E−05	2.04E−04	1.11E−04
Panel E: IDR	BSM	3.24E+01	4.86E+01	8.56E+01	5.55E+01	3.43E+01	5.17E+01	9.51E+01	6.04E+01
	RSM	2.60E+01	3.28E+01	6.31E+01	4.07E+01	2.98E+01	3.96E+01	7.73E+01	4.89E+01
	RSMJ	2.61E+01	3.24E+01	6.35E+01	4.07E+01	3.00E+01	3.98E+01	7.83E+01	4.94E+01
Panel F: MXN	BSM	1.72E−03	2.26E−03	6.46E−03	3.48E−03	1.50E−02	3.27E−02	1.10E−01	5.27E−02
	RSM	1.70E−03	1.86E−03	4.99E−03	2.85E−03	1.83E−03	2.16E−03	5.75E−03	3.25E−03
	RSMJ	9.78E−04	1.44E−03	4.70E−03	2.37E−03	1.19E−03	1.79E−03	5.57E−03	2.85E−03

Entries report the summation of mean square errors (MSEs) in option price percentage points over the five levels of deltas among the three different models. We use daily European-style dollar pair call option prices of three fixed maturities: 1 month, 3 months, and 12 months. The models are estimated using data from January 2, 2008 to December 31, 2010. The in-sample statistics are from the same period. The out-of-sample statistics are computed from the remaining 2 years of data from January 4, 2011 to December 31, 2012, based on model parameters estimated from the first subsample

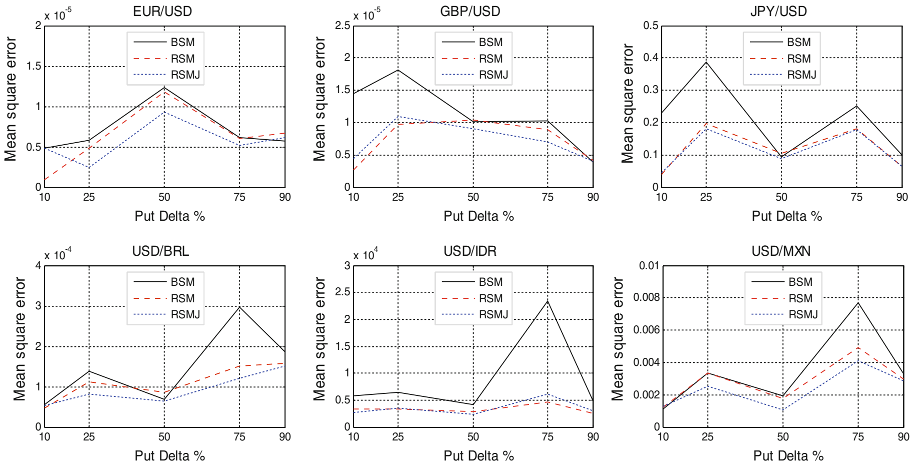

Fig. 9 In-sample mean pricing errors. The *three lines* in each panel denote the summation of mean square errors (MSEs) in option price percentage points over the three different maturities (1 month, 3 months and 12 months) along the moneyness dimension for the in-sample period for the three different models: the BSM, the RSM, and the RSMJ

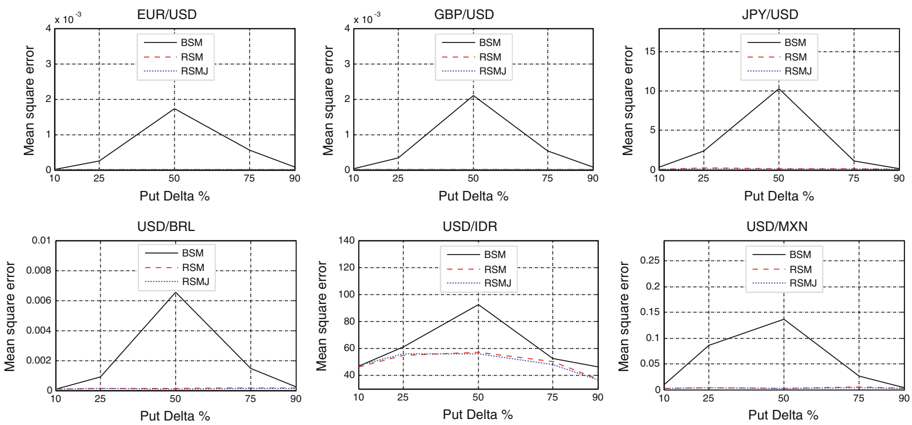

Fig. 10 Out-of-sample mean pricing errors. The *three lines* in each panel denote the summation of mean square errors (MSEs) in option price percentage points over the three different maturities (1 month, 3 months and 12 months) along the moneyness dimension for the out-of-sample period for the three different models: the BSM, the RSM, and the RSMJ

errors are larger on at-the-money options than on out-of-money options for out-of-sample performance, indicating that the BSM cannot fully account for the observed pricing bias. The pricing errors under the RSMJ are generally smaller than those under the BSM or the RSM across moneyness for all currency pairs and for both in-sample and out-of-sample performance. However, for deep out-of-money options (10 and 90 put delta), the RSM might generate smaller pricing errors than the RSMJ. Furthermore, under the latter, the pricing error is relatively invariant to moneyness comparing to the other two models for each underlying currency pair, indicating that it captures the pricing bias better at all terms and for all currencies.

5 Conclusion

This study has examined the dynamic behavior of exchange rates for both developed and emerging countries over the past decades. Our empirical results show that 'high-variance' and 'low-variance' describes most of our sample currencies. The LR tests also show that the RSMJ is better than both the BSM and the RSM for capturing the movement of exchange rates. Moreover, the jumps in exchange rates can be attributed to the following three factors: announcement of monetary policies associated with the alternation of exchange rate regimes, open market operation, change of interest rates as well as quantitative easing policy; political risks; and financial crises.

We then compared the values of the currency options generated by the following three models: the BSM, the RSM and the RSMJ with the market value. Our results show that the pricing model regarding the regimes of high-variance or low-variance states as well as the jump nature in exchange rates is better than the traditional BSM and the RSM.

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Appendix: Change of measure in Esscher transform and currency option pricing under the RSMJ model

Suppose the exchange rate follows the RSMJ with Eq. (19), and there is no risk premium occurring during the transition of the states, h_B and h_J are the parameters of the Brownian motion and the systematic jump risks in the Esscher transform, respectively. $h_B = \begin{bmatrix} h_{B1} & 0 \\ 0 & h_{B2} \end{bmatrix}$; where h_{B1} and h_{B2} refer to the h_B under the states 1 and 2, respectively. Because the jump risk is non-diversifiable, the exchange rate dynamics under the physical probability measure can be transferred to that under the risk neutral probability measure by the Esscher transform in this paper.

Let

$$A = \left\{ \left(\mu_{q_t} - \frac{1}{2}\sigma_{q_t}^2 \right) s + \sigma_{q_t} W(s) \right\} \quad \text{and } B = \sum_{m=1}^{N(s)} \ln Y_m. \tag{27}$$

The Randon–Nikodym derivatives of the Brownian motion and the systematic jumps risk term, η_B and η_J are derived as follows:

$$\eta_B = \begin{bmatrix} \exp\left\{ -\frac{h_{B1}^2 \sigma_1^2 s}{2} + h_{B1}\sigma_1 W(s) \right\} & 0 \\ 0 & \exp\left\{ -\frac{h_{B2}^2 \sigma_2^2 s}{2} + h_{B2}\sigma_2 W(s) \right\} \end{bmatrix},$$

where

$$\begin{aligned} \eta_{Bq_t} &= \frac{e^{h_{Bq_t}A}}{E(e^{h_{Bq_t}A})} = \frac{\exp\left\{ h_{Bq_t}(\mu_{q_t} - \frac{1}{2}\sigma_{q_t}^2)s + h_{Bq_t}\sigma_{q_t} W(s) \right\}}{\exp\left\{ h_{Bq_t}(\mu_{q_t} - \frac{1}{2}\sigma_{q_t}^2)s + \frac{h_{Bq_t}^2 \sigma_{q_t}^2 s}{2} \right\}} \\ &= \exp\left\{ -\frac{h_{Bq_t}^2 \sigma_{q_t}^2 s}{2} + h_{Bq_t}\sigma_{q_t} W(s) \right\} \end{aligned}$$

and

$$\eta_J = \prod_{m=0}^{N(s)} Y_m^{h_J} \cdot e^{-\lambda s \xi^{(h_J)}}.$$

Because the Brownian motion term and the systematic jumps risk term are independent from each other, the Brownian motion under the risk neutral measure can be transferred from that under the physical measure by change of measure:

$$dQ(W^Q(s)) = dP(W(s)) \cdot \eta_{Bq_t} = \frac{1}{\sqrt{2\pi s}} \exp\left\{ -\frac{\left[W(s) - h_{Bq_t}\sigma_{q_t} s\right]^2}{2s} \right\}. \tag{28}$$

Next, we deal with the jump terms in the Esscher transform, which can be separated into two parts: jump sizes and jump frequencies. Given the number of jumps, n, the jump terms under the risk neutral measure can be rewritten from those under the physical measure by change of measure:

$$\begin{aligned} dQ(N^Q(s) &= n, \ln Y_1^Q, \ldots, \ln Y_n^Q) \\ &= dP(N^Q(s) = n) dP(\ln Y_1^Q, \ldots, \ln Y_n^Q) \cdot \prod_{m=1}^{N(s)} Y_m^{h_J} \cdot e^{-\lambda s \xi^{(h_J)}} \\ &= dP(N^Q(s) = n) \left\{ \prod_{m=1}^{n} [f(\ln Y_m) \exp(h_J \ln Y_m)] \right\} \\ &\{E[\exp(h_J \ln Y_m)]\}^{-n} \{E[\exp(h_J \ln Y_m)]\}^n e^{-\lambda s \xi^{(h_J)}} \end{aligned} \tag{29}$$

According to the assumption of independent jump sizes, the individual jump size under the risk neutral measure is distributed as follows:

$$f^Q(\ln Y_m) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left\{ -\frac{\left[\ln Y_m - (\mu_y + h_J \sigma_y^2)\right]^2}{2\sigma_y^2} \right\}. \tag{30}$$

Next, the probability of the number of jumps n under the risk neutral measure is evaluated as follows:

$$dQ(N^Q(s) = n) = \frac{e^{-\lambda s\left(\xi^{(h_J)}+1\right)} \left[\lambda s \left(\xi^{(h_J)} + 1\right)\right]^n}{n!}. \tag{31}$$

Therefore, the jump sizes under the risk neutral measure follow a normal distribution with $\ln Y_m^Q \sim N(\mu_y + h_J\sigma_y^2, \sigma_y^2)$, and the number of jumps under the risk neutral measure follows a Poisson process with an arrival rate $\lambda s\left(\xi^{(h_J)} + 1\right)$.

The Esscher transform parameters have to satisfy the martingale condition, which is derived under the RSMJ dynamics of the exchange rate with the risk neutral measure:

$$\mu_{q_t} - r + h_{Bq_t}\sigma_{q_t}^2 + \lambda\left(\xi^{(h_J+1)} - \xi^{(h_J)} - \xi^{(1)} + \xi^{(0)}\right) = 0. \tag{32}$$

From the infinite solutions of the Esscher parameters, we can find one condition to satisfy the martingale condition:

$h_B = \begin{bmatrix} \frac{r-\mu_1}{\sigma_1^2} & 0 \\ 0 & \frac{r-\mu_2}{\sigma_2^2} \end{bmatrix}$ and $h_J = 0$.

Given the jump size and the jump frequency under the risk neutral measure is: $\ln Y_m^Q \sim N(\mu_y, \sigma_y^2)$ and $N^Q(s) \sim Poi(\lambda s)$.

Under the risk neutral measure (also called the Merton measure, 1976), the dynamic process of exchange rates under the RSMJ can be solved in the following form:

$$\begin{aligned} S(T) &= \left(S(0)\exp\left\{ \int_0^T (r - r_f - \xi^{(1)} + \xi^{(0)})dt - \frac{1}{2}\int_0^T \sigma_{q_t}^2 dt + \int_0^T \sigma_{q_t} dW^Q(t) + \sum_{m=1}^n \ln Y_m^Q \right\} \right) \\ &\overset{dist}{=} S(0)\exp\left\{ \left(r - r_f - \xi^{(1)} + \xi^{(0)}\right)T - \frac{1}{2}\theta_{k_1}^2 + \theta_{k_1} Z + \sum_{m=1}^n \ln Y_m^Q \right\}, \end{aligned} \tag{33}$$

where Z is an identically independent standard normal distribution and the weighted variance under the state q_i is$\theta_{k_1}^2 = k_1\sigma_1^2 + (T - k_1)\sigma_2^2.\sum_{m=1}^n \ln Y_m^Q$ and $\theta_{k_1} Z_n + \sum_{m=1}^n \ln Y_m^Q$ also follow normal distributions:$\sum_{m=1}^n \ln Y_m^Q \sim N\left(n\mu_y, n\sigma_y^2\right)$ and $\theta_{k_1} Z + \sum_{m=1}^n \ln Y_m^Q \sim N\left(n\mu_y, \theta_{k_1}^2 + n\sigma_y^2\right)$.Therefore, we can price a European-style call option, $C_{RSMJ,T}(0)$ with strike price K, the local and foreign risk-free interest rates, r and r_f, and maturity date T as follows:

$$\begin{aligned} C_{RSMIJ}(0) &= e^{-rT} E_{N(T)}^Q E_D^Q E_{q_0}^Q E^Q\left[\max\{S(T) - K, 0\} \middle| N^Q(T) = n, q_0 = i, D = k_1\right] \\ &= E_{N(T)}^Q E_D^Q E_{q_0}^Q \left\{ S(0)e^{-rT} E^Q\left[e^{\ln\frac{S(T)}{S(0)}} I_{\left\{\ln\frac{S(T)}{S(0)} > \ln\frac{K}{S(0)}\right\}} \middle| N^Q(T) = n, q_0 = i, D = k_1 \right] \right\} \\ &\quad - Ke^{-rT} E_{N(T)}^Q E_D^Q E_{q_0}^Q \left\{ E^Q\left[I_{\left\{\ln\frac{S(T)}{S(0)} > \ln\frac{K}{S(0)}\right\}} \middle| N^Q(T) = n, q_0 = i, D = k_1 \right] \right\} \\ &= E_{N(T)}^Q E_D^Q E_{q_0}^Q \{A - B\}. \end{aligned} \tag{34}$$

The term A of Eq. (34) can be represented as:

$$\begin{aligned} A &= S(0)e^{-rT} E^Q\left[e^{\ln\frac{S(T)}{S(0)}} I_{\left\{\ln\frac{S(T)}{S(0)} > \ln\frac{K}{S(0)}\right\}} \middle| N^Q(T) = n, q_0 = i, D = k_1 \right] \\ &= S(0)e^{-r_f T + n\left(\mu_y + \frac{\sigma_y^2}{2}\right) - \lambda(\zeta^{(1)} - 1)T} N(d_{1k_1}) \end{aligned}$$

where $N(\cdot)$ is the cumulative distribution function for a standard normal random variable,

$$d_{1k_1} = \frac{\ln\frac{S(0)}{K} + (r - r_f - \lambda(\zeta^{(1)} - 1))T + \frac{1}{2}\theta_{k_1}^2 + n(\mu_y + \sigma_y^2)}{\sqrt{\theta_{k_1}^2 + n\sigma_y^2}}.$$

The term B can be shown as:

$$Ke^{-rT}E^{Q}\left[I_{\left\{\ln\frac{S(T)}{S(0)} > \ln\frac{K}{S(0)}\right\}}\middle|N^{Q}(T)=n,q_0=i,D=k_1\right] = Ke^{-rT}N(d_{2k_1}),$$

where

$$d_{2k_1} = \frac{\ln\frac{S(0)}{K} + (r-r_f-\lambda(\zeta^{(1)}-1))T - \frac{1}{2}\theta^2_{k_1} + n\mu_y}{\sqrt{\theta^2_{k_1}+n\sigma^2_y}} = d_{1k_1} - \sqrt{\theta^2_{k_1}+n\sigma^2_y}.$$

Given the steady-state probability of the initial market state, π_i; the probability that over the T periods, k_1 periods are assigned to the state 1 conditional on the initial market state i, $\gamma_{T,k_1|q_0=i}$, we can further rewrite the pricing equation as:

$$\begin{aligned} C_{RSMJ,T}(0) = & \sum_{n=0}^{\infty}\sum_{k_1=0}^{T}\sum_{i=1}^{2}\pi_i\cdot\gamma_{T,k_1|q_0=i}\frac{e^{-\lambda*}\left(\lambda^*\right)^n}{n!} \\ & \left[S(0)e^{-r_fT+n\left(\mu_y+\frac{\sigma_y^2}{2}\right)-\lambda(\zeta^{(1)}-1)T}N(d_{1,k_1}) - Ke^{-rT}N(d_{2,k_1})\right], \end{aligned} \tag{35}$$

where $\lambda^* = \lambda T$,

$$d_{1,k_1} = \frac{\ln\frac{S(0)}{K} + (r-r_f-\lambda(\zeta^{(1)}-1))T + \frac{1}{2}\theta^2_{k_1} + n(\mu_y+\sigma^2_y)}{\sqrt{\theta^2_{k_1}+n\sigma^2_y}},$$

$d_{2,k_1} = d_{1,k_1} - \sqrt{\theta^2_{k_1}+n\sigma^2_y}$, and $N(x)$ is the cumulative distribution function for a standard normal random variable with upper integral limit x.

References

Andersen TG, Bollerslev T, Diebold FX, Labys P (2001) The distribution of exchange rate volatility. J Am Stat Assoc 96:42–55

Bae K, Karolyi A, Stulz R (2003) A new approach to measuring financial contagion. Rev Financ Stud 16:717–763

Baharumshah AZ, Wooi HC (2007) Exchange rate volatility and the Asian financial crisis: evidence from South Korea and ASEAN-5. Rev Pac Basin Financ Mark Polic 10:237–264

Barndorff-Nielsen OE, Shephard N (2006) Econometrics of testing for jumps in financial economics using bipower variation. J Financ Econom 4:1–30

Bates DS (1996a) Dollar jump fears, 1984–1992: distributional abnormalities implicit in currency futures options. J Int Money Financ 15:65–93

Bates DS (1996b) Jumps and stochastic volatility: exchange rate processes implicit in deutsche mark options. Rev Financ Stud 9:69–107

Bekaert G, Hodrick R (1993) On biases in the management of foreign exchange risk premiums. J Int Money Financ 12:115–138

Bergman UM, Hansson J (2005) Real exchange rates and switching regimes. J Int Money Financ 24:121–138

Biger N, Hull J (1983) The valuation of currency options. Financ Manag 12:24–28

Black F, Scholes M (1973) The pricing of options and corporate liabilities. J Polit Econ 81:637–654

Bollen N (1998) Valuing options in regime-switching models. J Deriv 6:38–49

Caporale GM, Spagnolo N (2004) Modeling East Asian exchange rates: a Markov-switching approach. Appl Financ Econ 14:233–242

Carr P, Wu L (2007) Stochastic skew in currency options. J Financ Econ 86:213–247

Chaboud AP, Chernenko SV, Wright JH (2008) Trading activity and macroeconomic announcements in high-frequency exchange rate data. J Eur Econ Assoc 6:589–596

Costabile M, Leccadito A, Massabo' I, Russo E (2013) A reduced lattice model for option pricing under regime-switching. Rev Quant Financ Acc. doi:10.1007/s11156-013-0357-9

Dahlquist M, Gray SF (2000) Regime-switching and interest rates in the European monetary system. J Int Econ 50:399–419

Doffou A, Hilliard J (2001) Pricing currency options under stochastic interest rates and jump-diffusion processes. J Financ Res 24:565–585

Duan JC, Popova I, Ritchken P (2002) Option pricing under regime switching. Quant Financ 2:116–132

Durland J, McCurdy T (1994) Duration-dependent transitions in a markov model of US GNP growth. J Bus Econ Stat 12:279–288

Eichengreen B, Rose AK, Wyplosz C (1995) Exchange market mayhem: the antecedents and aftermath of speculative attacks. Econ Policy 10:249–296

Eichengreen B, Rose AK, Wyplosz C (1996) Contagious currency crises: first tests. Scand J Econ 98:463–484

Engel C (1994) Can the Markov switching model forecast exchange rates? J Int Econ 36:151–165

Engel C, Hakkio CS (1996) The distribution of the exchange rate in the EMS. Int J Financ Econ 1:55–67

Engel C, Hamilton J (1990) Long swings in the dollar: are they in the data and do markets know it? Am Econ Rev 80:689–713

Favero C, Giavazzi F (2002) Is the international propagation of financial shocks non-linear? Evidence from the ERM. J Int Econ 57:231–246

Flood R, Hodrick R (1986) Real aspects of exchange rate regime choice with collapsing fixed rates. J Int Econ 21:215–232

Frenkel J (1981) Flexible exchange rates, prices, and the role of news: lessons from the 1970s. J Polit Econ 89:665–705

Froot KA, Obstfeld M (1991) Exchange-rate dynamics under stochastic regime shifts. J Int Econ 31:203–229

Garman M, Kohlhagen S (1983) Foreign currency option values. J Int Money Financ 2:231–237

Goldfeld S, Quandt R (1973) A Markov model for switching regressions. J Econom 1:3–15

Grabbe J (1983) The pricing of call and put options on foreign exchange. J Int Money Financ 2:239–253

Gray S (1996) Modeling the conditional distribution of interest rates as a regime-switching process. J Financ Econ 42:27–62

Hamilton J (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica 57:357–384

Ichiue H, Koyama K (2011) Regime switches in exchange rate volatility and uncovered interest rate parity. J Int Money Financ 30:1436–1450

Ismail MT, Isa Z (2007) Detecting regime shifts in Malaysian exchange rates. J Fundam Sci 3:211–224

Jiang GJ (1998) Jump diffusion model of exchange rate dynamics: estimation via indirect inference. Working paper. University of Groningen, The Netherlands

Jorion P (1988) On jump processes in the foreign exchange and stock market. Rev Financ Stud 1:427–445

Kaminsky G (1993) Is there a peso problem? Evidence from the Dollar/Peso exchange rate. Am Econ Rev 83:450–472

Kirikos DG (2000) Forecasting exchange rates out of sample: random walk v.s. Markov switching regimes. Appl Econ Lett 7:133–136

Lin SK, Shyu SD, Wang SY (2013) Option pricing under stock market cycle with jump risks: evidence from Dow Jones industrial average and S&P 500 index. Working paper

Lowell J, Neu CR, Tong D (1998) Financial crises and contagion in emerging market countries. RAND Working paper MR-962

Marsh IW (2000) High-frequency Markov switching models in the foreign exchange market. J Forecast 19:123–134

Meng XL, Rubin DB (1991) Using EM to obtain asymptotic variance-covariance matrices: the SEM algorithm. J Am Stat Assoc 86:899–909

Merton RC (1973) An intertemporal capital asset pricing model. Econometrica 41:867–887

Merton R (1976) The impact on option pricing of specification error in the underlying stock price returns. J Financ 31:333–350

Mussa M (1979) The two-sector model in terms of its dual: a geometric exposition. J Int Econ 9:513–526

Neely CJ (2011) A foreign exchange intervention in an era of restraint. Fed Reserve Bank St. Louis Rev 93:303–324

Quandt R (1958) The estimation of the parameters of a linear regression system obeying two separate regimes. J Am Stat Assoc 53:873–880